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Probabilistic Load Flow Analysis of the 9 Bus WSCC System

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Abstract- Load flow analysis is one of the most important tools being used in analysing power systems. The significant uncertain parameters of the system planning and reliability are neglected in the deterministic method. In this study, the stochastic technique is proposed to examine the power flow issues in power systems. The main aim of this study is to provide a solid solution by considering the system uncertainties while keeping the system topology constant. The prospective values and the standard deviation of the power flow of each power line are estimated. The sensitivity coefficients which are quite significant for each power line are calculated as well. These coefficients tell us that how the changes occurring in the node data influence the power flow of each power line. This technique investigates the possibilities of all power flows taking place in the system. Additionally, by applying the stochastic approach to 9 bus WSCC system it is verified that the probabilistic analysis method gives more detailed information in power systems than that the deterministic analysis does.

Index Terms- Probabilistic load flow, Load distribution, Power systems, Probability

I. Introduction

Load flow analysis is the most widely used tool for steady-state studies in power systems. Its application is based on the assumption that the system loading is precisely known. That is, the electric load and generation are deterministically known quantities. However, in many cases the load needs to be assumed stochastic in nature and therefore the power system operation has to be studied based on estimates of this demand and taking into consideration the probabilistic nature of the load. This is performed using the probabilistic power flow analysis. Probabilistic power flow is a term that refers to power flow analysis methods that directly treat the uncertainty of electric load and generation [1].

There are very well developed deterministic methods that are used in power flow studies that permit them to be made very quickly, accurately and efficiently [2]. In these deterministic methods the nodal loads, generation and topology of the network is kept constant. In practice however the data of the nodal loads and generation can only be known with limited accuracy. In this reason the PLF (Probabilistic Load Flow) was proposed by Borkowska for evaluation of the power flow considering uncertainties.

The uncertainties encountered may be due to; measurement error, forecast inaccuracy or outages of system elements [2].

The main purpose and reason of using probabilistic analysis instead of deterministic is to consider the uncertainties and model these statistical variations in the input data of the power system. A number of papers have been published that have modeled the load flow problem probabilistically [2]-[9].
In this paper the probabilistic method has been explained.

II. Load Flow Equations

Although well-known it is useful to explain the load-flow equations [10-14] :

$$P_i = V_i \sum_{k=1}^{n} V_k (G_{ij}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad (1)$$

$$P_{ik} = -t_{ik} G_{ik} V_i^2 + V_i V_k (G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad (2)$$

$$Q_i = V_i \sum_{j=1}^{n} V_j (G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}) \quad (3)$$

$$Q_{ik} = t_{ik} B_{ik} V_i^2 - B_{ik}' V_i^2 + V_i V_k (G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad (4)$$

$$Q_{i(sh)} = V_i^2 B_{i(sh)} \quad (5)$$

where B_{ik} is the imaginary part of element ik of admittance matrix, G_{ik} is the real part of element ik of the admittance matrix, n is the number of nodes, P_i is the injected active power at node i, P_{ik} is the active power flow in line i-k, t_{ik} is the transformer tap ratio, V_i is the voltage magnitude at node i, δ_i is the angle at node i referred to slack node and δ_{ik} is the difference in angles between nodes i and k.

III. The Probabilistic Approach

The loads in an electrical power system vary continuously, so the system operation must adapt to this kind of variation at any time. Therefore, to build an appropriate load model is very important for system security assessment. According to the requirement of security assessment, the load model can be divided into time instance load model and time-period load model [15].

The nodal data for the system are considered as random variables. The nodal loads and generation are defined as random variables because of factors like the change in the load demand and the generator outages. The traditional deterministic load flow only finds line flows under a specified operating condition. On the other hand, the probabilistic load flow takes the uncertainties into considerations, such as the probability of a line flow being greater than its thermal rating under load uncertainties and random contingencies.

The nodal data are specified in terms of probability density functions. In this study of the 9 bus WSCC system, these are normal distributions for representing nodal-load estimate, binomial distributions for representing a set of identical generator units, discrete variables when neither of the previous distributions are suitable and one point values when a power has a unity probability of occurrence [3].

All the input data are first converted to the expected values (μ) and their variance (σ^2) by their distributions. Then the active load flows and bus angles are calculated by using the DC load flow assumptions. Consider Eq. 1 assuming $V_i=V_k=1p.u.$, $G_{ik}=0$ (zero line resistance) and $\sin\delta_{ik} \approx \delta_{ik}$, we obtain [3],

$$P_i = \frac{1}{X_{ik}}\delta_{ik} \tag{6}$$

where X_{ik} is the reactance of the line joining buses i and k. The above equation in matrix form and inverting gives;

$$\delta = Y^{-1}P \tag{7}$$

where Y_{ik} =-1/X_{ik} and $Y_{ii} = \sum_{i \neq k} 1/X_{ik}$, in which the slack bus row and column are deleted. This equation is known as a dc form of the load flow problem. Under these circumstances Eq. 2 becomes,

$$P_{ik} = \frac{\delta_i - \delta_k}{X_{ik}} = \sum_j H_{(ik)j} P_j \tag{8}$$

The H matrix contains network distribution factors. It is called the sensitivity coefficients and the notation "(ik)j" represents the amount of real power flowing in line l (line between buses i and k) as a result of injection of 1MW at bus j. If node j is slack, $H_{(ik)j}=0$ [16].

To obtain the voltages and reactive load flows, equation 3 was linearized by assuming the voltage $V_i=1p.u.$ Equation 3 becomes,

$$Q_i = \sum_{k=1}^{n} A_{ik} V_k \tag{9}$$

where; $A_{ik} = G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}$ and $A_{ii}=- B_{ii}$.

Writing in matrix form $Q=AV$ and partitioning into the load and generation quantities gives [6],

$$\begin{vmatrix} Q_l \\ Q_g \end{vmatrix} = \begin{vmatrix} M & L \\ N & J \end{vmatrix} \begin{vmatrix} V_l \\ V_g \end{vmatrix} \tag{10}$$

where Q_l has n_1 elements and Q_g has n_2 elements. From eqn. 10;

$$Q_l = MV_l + LV_g \tag{11}$$

$$V_l = \hat{M}Q_l + \hat{M}H \tag{12}$$

where; $H = - LV_g$. From eqn. 10 and 12 the reactive power of the generation bus,

$$Q_g = NV_l + JV_g \tag{13}$$

To obtain the reactive load flows in the lines, equation 4 was linearized by assuming $V_i^2 = V_i$ and, $V_i V_k = V_k$ after the assumptions the reactive load flows,

$$Q_{ik} = \alpha_{ik} V_i + A_{ik} V_k \tag{14}$$

$$Q_{i(sh)} = V_i B_{i(sh)} \tag{15}$$

where $\alpha_{ik} = t_{ik} B_{ik} - B_{ik}'$.

IV. TEST SYSTEM

To exemplify the probabilistic approach, the 9 bus WSCC test system is used and shown in Figure 1. This system has 8 nodes that are independent, one reference node (node 1) and 9 lines. The nodal data used are shown in Table I and the network data are shown in Table II. The nodal data contains binomial and discrete distributions.

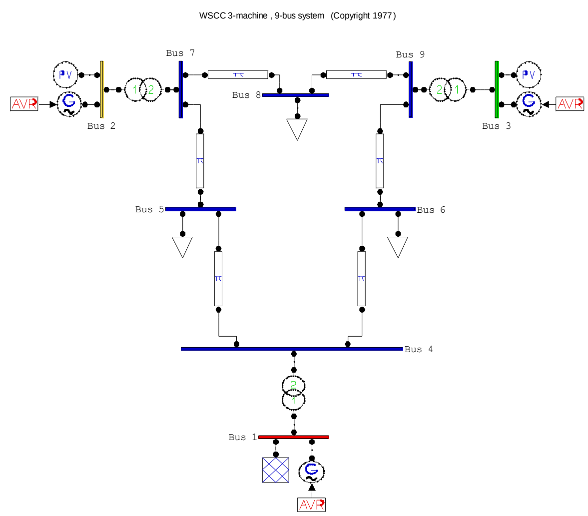

Fig. 1 9 bus WSCC test system

Table I. Nodal data used for the system

Node	Probability Function	Number of Units	Active Power (MW)	Reactive Power (MVAR)	
					Outage Coefficient
2	Binomial	10	17,1589		0,05
3	Binomial	10	10		0,15
					Probability of Occurrence
4	-	-	0	0	1
5	Discrete		-123	-45	0,2
			-124	-47	0,2
			-125	-50	0,2
			-126	-53	0,2
			-127	-55	0,2
6	Discrete		-88	-22	0,2
			-89	-26	0,2
			-90	-30	0,2
			-91	-34	0,2
			-92	-38	0,2
7	-	-	0	0	1
8			-100	-35	1
9	-	-	0	0	1

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Table II. Network data for system shown in Fig. I

Node	To Node	R (p.u.)	X (p.u.)	Y (p.u.)	Transformer Tap
2	7	0.0000	0.0625	0.0000	1.00
7	8	0.0085	0.0720	0.0745*2	1.00
7	5	0.0320	0.1610	0.1530*2	1.00
5	4	0.0100	0.0850	0.0880*2	1.00
4	1	0.0000	0.0576	0.0000	1.00
4	6	0.0170	0.0920	0.0790*2	1.00
6	9	0.0390	0.1700	0.1790*2	1.00
9	3	0.0000	0.0586	0.0000	1.00
9	8	0.0119	0.1008	0.1045*2	1.00

Table III. Active power flow results

Line	Expected Value μ (MW)	Standard Deviations σ (%)	Deterministic Solution (MW)
2-7	163	7.2552	163
7-8	76.036	8.0234	76.4
7-5	86.9656	10.0237	86.6
4-5	38.0318	23.1482	40.9
1-4	67.7053	24.5863	71.6
4-6	28.9706	28.4699	30.7
9-6	61.0344	13.3688	60.8
3-9	85	13.2842	85
8-9	-23.9721	25.4448	-24.1

V. RESULTS

The expected values and standard deviation of the power flow results in each line is shown in Table III. The deterministic values were assumed to be expected values. In the deterministic solution only the expected flows were obtained and the solution includes no information about the standard deviation.

The density functions of the line between nodes 4-6 is shown in Fig. 2. The density functions show that the change in the input data affects the power flows in each line. It is seen from the figure that the probability of expected value is not always the greatest. This fact should be known when a power system is being planned and operated. This kind of information is in great importance in situations where security and reliability is very important.

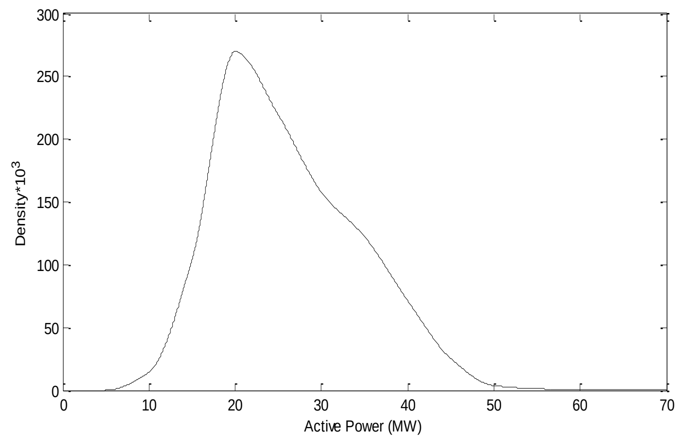

Fig. 2 Active power flow between nodes 4 and 6

Table IV. Voltages of the load buses and their standard deviation

Load Bus	Expected Value μ (p.u.)	Standard Deviations σ (p.u.)	Deterministic Solution (p.u.)
4	1.0255	2.5136×10^{-4}	1.0258
5	0.9982	2.9155×10^{-4}	0.99563
6	1.0092	3.784×10^{-4}	1.0127
7	1.0337	1.595×10^{-4}	1.0258
8	0.9905	1.745×10^{-4}	1.0159
9	1.0238	1.8464×10^{-4}	1.0324

Table V. Reactive power injected at the generation bus

Generation Bus	Expected Value μ (MVAR)	Standard Devitions σ (MVAR)	Deterministic Solution (MVAR)
1	26.48	4.9	27.046
2	6.14	2.9	6.654
3	4.03	3.5	-10.8

Table VI. Reactive power flow results

Line	Expected Value μ (MVAR)	Standard Devitions σ (MVAR)	Deterministic Solution (MVAR)
2-7	6.434	2.98623	6.7
7-5	-5.165	3.825	-8.3
5-4	-38.868	6.1318	-38.6
4-1	-23.31	2.9124	-23.7
4-6	0.9663	6.9105	0.9
7-8	-24.94	2.7075	-0.8
6-9	-13.421	4.595	-13.5
9-3	14.565	3.5775	14.9
9-8	3.232	3.7193	3.1

The voltages of the load buses and their standard deviations is shown in Table IV, the reactive power and the reactive load flows are shown in Table V and Table VI.

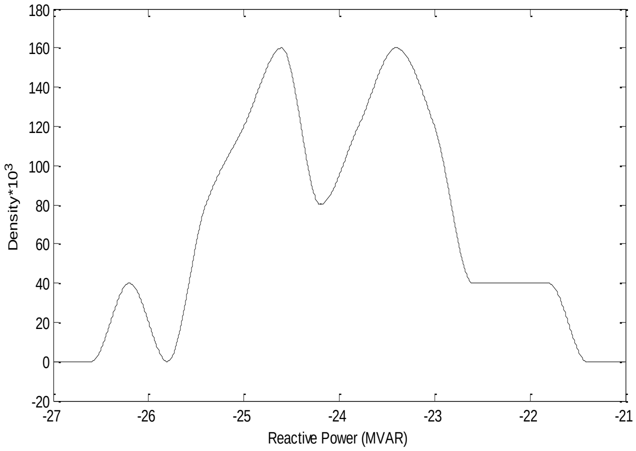

Fig. 1 Reactive power flow between nodes 4 and 1

VI. CONCLUSIONS

Although the expected value of the power flows calculated in probabilistic method is not exact as the deterministic method, several advantages appear. The error is solved by shifting the density curve until its expected coincides with the value obtained from the deterministic analysis. The previous sections have shown the wide information that can be gained by the PLF. The probable power flows were calculated with their probability. Even though many of the input data have normal distribution, the binomial and discrete variables that exists in the system takes the results away from being normal distributed.

The active power cumulative distribution function of the line between nodes 4 and 6 is shown in Fig. 4. Applying this cumulative distribution function to the active power flow in line 4-6 can give the probabilities for exceeding various limits shown in Table VII.

This method objectively solves the uncertainties problem in the load flow problem and considers all the possibilities like generator outages and load changes. It helps to think about the worst conditions that can happen with the wider information the density functions give. Applying probabilistic studies in power systems allow gaining information for the future conditions.

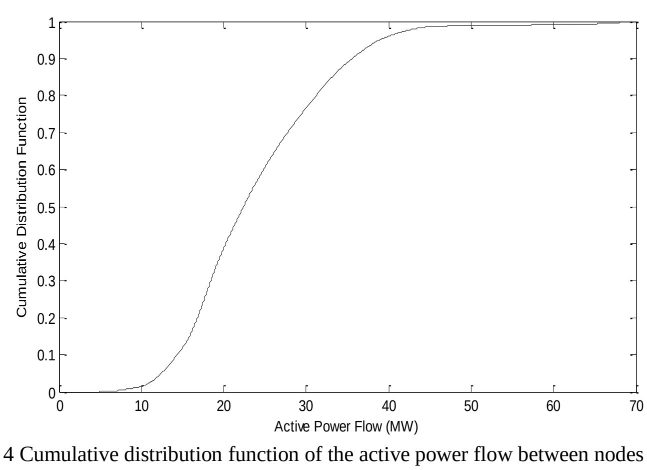

Fig. 4 Cumulative distribution function of the active power flow between nodes 4 and 6.

Table VII. Probability of power flow exceeding stated limits in line 4-6

Limit (MW)	Probability of Exceeding Limit
20	0.5713
25	0.3520
30	0.1941
35	0.1022
40	0.0307
45	0.0052
50	0.0015
55	0.0003
60	0

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