On the Techniques for Constructing Even-order Magic Squares using Basic Latin Squares

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Abstract: Tomba (May, 2012) introduced a technique for construction (n x n) magic squares (when n is odd) using basic Latin Squares by fixing the pivot element and arranging other elements in an orderly manner [10]. However, even-order magic squares can't be constructed using the same procedure because of duplication in diagonal elements. Again, Tomba (July, 2012) developed a technique for constructing (n x n) magic squares (when n is doubly-even) using basic Latin square by fixing the column associated with the elements, adjacent to the pivot element and arranging in an orderly manner, making symmetric transformations that generates a magic parametric constant (T) and finally derived by minor adjustment on the pair-numbers of satisfying T [11]. The technique can provide weak magic squares for singly-even cases. In this paper, a technique for constructing singly-even magic squares is developed and illustrated with suitable examples.

Key-words: Latin square (basic), singly even and doubly even magic square (normal), weak magic squares, magic parametric constant (Tomba's constant),

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1. INTRODUCTION

In a Latin square, Latin letters are seen once in each row and column whereas the sums of rows and columns are equal but not the sums of diagonals. The idea of Latin Squares and basic Latin squares, normal magic squares have studied in [10] and [11]. A (n x n) basic Latin square is symmetric and non-duplicated (if n is odd) and symmetric but duplicated (if n is even).

Normal magic square has the following properties;

(a) Elements or numbers (n ≥ 0) are consecutive and not repeated

(b) Sums of the rows, columns and diagonals (using diagonal notation as d_{ij}) are equal to the magic sum, S

$$\Rightarrow \sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij} \quad i, j = 1, 2, \ldots.., n$$

(c) Equality property of the sum of rows, columns and diagonals remain unaltered for rotations and reflections.

A (n x n) array $\{b_{ij}\}$(with diagonal notation d_{ij}) satisfying the following properties are weak magic squares;

(a) Elements or numbers (n ≥ 0) are consecutive and not repeated

(b) Sums of diagonals are equal to the magic sum, S

$$\Rightarrow \sum_i d_{ij} = \sum_j d_{ij} = S \quad i, j = 1, 2, \ldots.., n$$

(c) Sums of the rows, columns are equal to the magic sum (S), except for some i and j

$$\Rightarrow \sum_i b_{ij} = \sum_j b_{ij} = S \quad i, j = 1, 2, \ldots.., n \qquad [1]$$

(d) Equality property of the rows, columns and diagonals remain unaltered for rotations and reflections.

Then the matrix $\{b_{ij}\}$ is a weak magic square.

Let $\{a_{ij}\}$ be a magic square satisfying the properties (a) to (c). The alternate structures of a magic square can be expressed (clockwise or anticlockwise rotation) as $\{a_{ij}(k)\}$ for $(k\frac{\pi}{2}); k = \pm 1, \pm 2, \ldots \pm m)$ where, $\{a_{ij}\} = \{a_{ij}(k)\}$ for all $i = 0, 4, 8, \ldots\ldots$

2. TECHNIQUE

(For constructing doubly-even magic squares)

The technique of constructing doubly-even magic square using basic Latin square principle can be expressed as follows:

Let the n^2 matrix $\{a_{ij}\}$; $i, j = 1, 2, \ldots n$ with the consecutive numbers be arranged in basic Latin square format as;

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n-1} & a_{1n} \\ a_{22} & a_{23} & \ldots. & a_{2n} & a_{21} \\ \ldots & & & & \\ a_{nn} & a_{n1} & \ldots.. & a_{nn-2} & a_{nn-1} \end{bmatrix} \text{ where, } \sum_i a_{ij} = S \text{ for all i,}$$

$$S = \tfrac{1}{2}\{n(n^2+1)\} \qquad [2]$$

This condition is true for all n (odd or even) due to basic Latin square property.

The pivot element is unique (for n is odd) but it lies between two numbers (for n is even).

Since the pivot element is not fixed, we select the column, associated with the numbers adjacent to the pivot element, assign it as the diagonal elements and arrange the other elements in an

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orderly manner to get a new matrix satisfying the property of $\sum_i d_{ij} = \sum_j d_{ij} = S$ for all i and j [3]

Again, make symmetric transformations of other elements, retaining the diagonal elements un-changed to find out the extreme corner blocks and central block of (2 x 2) each.

Reverting $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns systematically, a magic parametric constant (T) and a set of sub magic parametric constants are generated.

For doubly-even cases, by adjusting the values in the parametric constants and sub parametric constants, the required magic squares (doubly-even) can be constructed as

$$\begin{bmatrix} b_{11} & b_{12} & b_{1j} & b_{1n} \\ b_{21} & b_{22} & b_{2j} & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & b_{nj} & b_{nn} \end{bmatrix}$$

Satisfying $\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$ [4]

$\Rightarrow \{b_{ij}\}; i, j = 1,2,...n$ is a n^2 (doubly-even) magic square.

3. THEOREMS

3.1: Odd order Magic Squares derived using basic Latin Squares

For any n (n is odd), a magic square constructed using basic Latin square, the following theorem holds;

Theorem 1: A (n x n) matrix $\{b_{ij}\}$, developed by using basic Latin square format when the pivot element is fixed, assigned the row associated with the pivot element (keeping the pivot element in the middle cell) as diagonal element and arranging in an orderly manner represents a magic square for any odd n. [10]

3.2: Doubly-even Magic Squares derived using basic Latin Squares

For any n (n is doubly-even), a magic square constructed using basic Latin square, the following theorem holds;

Theorem 2: A (n x n) matrix $\{b_{ij}\}$, developed by using basic Latin square format when the column, associated with the numbers adjacent to the pivot element is selected, assign it as the diagonal elements and arrange the other elements in an orderly manner, making symmetric transformations gives a new matrix satisfying the equality property of diagonal sums. Selecting a suitable central block (retaining the diagonal elements unchanged) and revering the $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns systematically, the system generates a magic parametric constant (T) and sub-magic parametric constants. With some adjustment on the pair-numbers satisfying T, it generates magic squares of any doubly-even n. [11]

3.3: Singly-even Weak Magic Squares derived using basic Latin Squares

For any n (n is singly-even), the following theorem holds

Theorem 3: A (n x n) matrix $\{b_{ij}\}$, developed by using basic Latin square format when the column, associated with the numbers adjacent to the pivot element is selected, assign it as the diagonal elements and arrange the other elements in an orderly manner, making symmetric transformations of other elements gives a new matrix satisfying the equality property of diagonal sums. Selecting a suitable central block (retaining the diagonal elements unchanged) and reverting the rows and columns systematically, the system generates a magic parametric constant (T) and sub-magic parametric constants. With some adjustments on the pair-numbers satisfying T, it generates weak magic squares only for any singly-even n. [11]

4. METHODOLOGY

(For constructing singly-even magic squares)

The technique of constructing singly-even magic square using basic Latin square principle can be expressed as follows:

Let the n^2 matrix $\{a_{ij}\}; \ i, j = 1,2,...n$ with the consecutive numbers be arranged in basic Latin square format as;

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n-1} & a_{1n} \\ a_{22} & a_{23} & \dots & a_{2n} & a_{21} \\ \dots & \dots & \dots & \dots & \dots \\ a_{nn} & a_{n1} & \dots & a_{nn-2} & a_{nn-1} \end{bmatrix} \text{ where, } \sum_i a_{ij} = S \text{ for all i}$$

$$S = \tfrac{1}{2}\left\{n(n^2+1)\right\} \quad [5]$$

This condition is true for all n (odd or even) due to basic Latin square property. Find T= $[n^2+1]$.

Select the column, associated with the minimum and maximum elements, assign it as the diagonal elements and arrange the other elements in an orderly manner to get a new matrix satisfying $\sum_i d_{ij} = \sum_j d_{ij} = S$ for all i and j Again, retaining the diagonal elements un-changed, make symmetric transformations, to find the extreme corner blocks and central block of (2 x 2) each.

Reverting $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns systematically, a magic parametric constant (T) and a set of sub magic parametric constants are generated. Reverting the diagonal elements (keeping the central block unchanged) and selecting the pair-numbers satisfying magic parametric constant T, assigning the pair-numbers properly and making minor adjustments in the values in the parametric constant and other elements, the required magic squares (singly-even) can be constructed as;

$$\begin{bmatrix} b_{nn} & b_{12} & \dots & b_{1j} & b_{1n} \\ b_{21} & b_{n-1,n-1} & & b_{n-j,n-j} & b_{2n} \\ \dots & \dots & \dots & \dots & \\ b_{n1} & b_{n2} & \dots & b_{nj} & b_{11} \end{bmatrix}$$

Satisfying $\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$ [6]

$\Rightarrow \{b_{ij}\}; i, j = 1,2,...n$ is a n^2 (singly-even) magic square.

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Hence, for any singly-even n, the following theorem is developed;

Theorem 4: A (n x n) matrix $\{b_{ij}\}$, developed by using basic Latin square format when the column associated with the maximum and minimum numbers is selected, assign it as diagonal elements and arrange the other elements in an orderly manner get a new matrix satisfying the equality property of diagonal sums. Again, make symmetric transformations, retaining the diagonal elements un-changed to find the extreme corner blocks and central block of (2 x 2) each. Revering $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns systematically (keeping the central block unchanged), the system generates a magic parametric constant (T) and sub-magic parametric constants.
Reverting one of the diagonal element (keeping the central block unchanged), assigning the pair-numbers satisfying the properties of T in selective positions, followed with some adjustments gives magic squares of any singly-even n.

5. STEPS FOR CONSTRUCTION

5.1: Doubly-even magic squares

Construction of magic square using basic Latin square is expressed in the following steps:

Step-1: First arrange the consecutive numbers 1 to n^2 or (a_{11} *to* a_{nn}) in basic Latin square format. Find T= $[n^2+1]$.

Step-2: Find the range of $\frac{1}{2}(a_{11}+a_{nn})$ and select the column associated with this range assign it as diagonal elements and arrange other elements in an orderly manner, giving diagonals sums equal

Step-3: Retaining the diagonal elements unchanged, make transformations of other elements to construct the extreme corner and central blocks of (2 x 2) each .

Step-4: Reverting $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns in a systematic manner, a magic parametric constant (T) and a set of sub-magic parametric constants are generated...
The reversion process will satisfy the following condition.
(i) $2 \le \left\{\frac{n-2}{2} \pm 2k\right\} < (n-2)$ for k = 0, 1, 2 ... .n

Step-5: Main adjustments should be made on the elements corresponding to the magic parametric constant (T), whereas minor adjustments should be made on other elements of sub-magic parametric constants to get the magic square satisfying the properties of

$$\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$$

Note: For singly-even n, it generate weak magic squares only.

5.2: Singly-even magic squares

Construction of magic square using basic Latin square is expressed in the following steps:

Step-1: First arrange the consecutive numbers 1 to n^2 or (a_{11} *to* a_{nn}) in basic Latin square format. Find T= $[n^2+1]$.

Step-2: Select the column associated with maximum and minimum numbers, assign it as diagonal elements and arrange other elements in an orderly manner, giving diagonals sums equal

Step-3: Retaining the diagonal elements unchanged, make transformations of other elements to construct the extreme corner and central blocks of (2 x 2) each.

Step-4: Reverting $\left\{\frac{1}{2}(n-4)\right\}$ rows and columns in a systematic manner, a magic parametric constant (T) and a set of sub-magic parametric constants ($T_1\, T_2 \ldots$) can be determined. The reversion process will satisfy the following condition.
(i) $2 \le \left\{\frac{n}{2} \pm 2k\right\} < (n-2)$ for k = 0, 1, 2 ....n

Step-5: Revert one of the diagonal elements keeping the central block unchanged. Select the pair-numbers satisfying T and arranged on selective basis which is normally complicated.

Step-6: Adjustments should be made on the elements corresponding to the magic parametric constant (T), whereas minor adjustments should be made on other elements to get the magic square satisfying the properties of

$$\sum_i b_{ij} = \sum_j b_{ij} = \sum_i d_{ij} = \sum_j d_{ij}$$

6. NUMERICAL EXAMPLES

(Doubly-even and singly-even cases)

6.1: To construct a (4 x 4) magic square (doubly-even)

Step-1: The numbers (1 to 16) in 4 rows and 4 columns, arranged in basic Latin square format be;

1	2	3	4
6	7	8	5
11	12	9	10
16	13	14	15

The arrangement gives the column totals equal, $\sum_i a_{ij} = 34$ for all i. Here, S = 34 and T = 17.

Step-2: Select the column associated with the numbers 8 and 9 (say 3, 8, 9, 14) and assign it as diagonal elements.

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Rearranging the other elements in an orderly manner gives a new matrix satisfying $\sum_i d_{ij} = \sum_j d_{ij} = 34$

15	11	7	3
16	12	8	4
3	9	5	1
4	10	6	2

Step-3: Retaining the diagonal elements un-changed, make transformations of other elements to generate the extreme corner and central blocks of (2 x 2) each.

15	6	10	3
1	12	8	13
4	9	5	16
14	7	11	2

Extreme corner blocks: $\begin{bmatrix} 15 & 6 \\ 1 & 12 \end{bmatrix}, \begin{bmatrix} 10 & 3 \\ 8 & 12 \end{bmatrix}, \begin{bmatrix} 4 & 9 \\ 14 & 7 \end{bmatrix}, \begin{bmatrix} 5 & 16 \\ 11 & 2 \end{bmatrix}$

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\} = 0$ for n = 4. No reversion process needed.

Step-5: Hence, no adjustment needed and therefore the construction of (4 x 4) magic squares is completed in Step-3.

6.2 **To construct (8 x 8) magic square** (doubly-even)

Step-1: Let the consecutive numbers be (1, 2, 3,... 64) in 8 rows and 8 columns be arranged in basic Latin Square format as:

1	2	3	4	5	6	7	8
10	11	12	13	14	15	16	9
19	20	21	22	23	24	17	18
28	29	30	**31**	**32**	25	26	27
37	38	39	**40**	**33**	34	35	36
46	47	48	41	32	43	44	45
55	56	49	50	51	52	53	54
64	57	58	59	60	61	62	63

Step-2: Select the row associated with the element (32 and 33) and assign this as diagonal elements. Rearranging the other elements in an orderly manner gives a new matrix giving the diagonal sums equal satisfying the diagonal sums equal.

61	53	45	37	29	21	13	5
62	54	46	38	30	22	14	6
63	55	47	39	31	23	15	7
64	56	48	**40**	**32**	24	16	8
57	49	41	**33**	**25**	17	9	1
58	50	42	34	26	18	10	2
59	51	43	35	27	19	11	3
60	52	44	36	28	20	12	4

Step-3: Retaining the diagonal elements un-changed, make transformations of other elements to construct the extreme corner and central block of (2 x 2) each.

61	12	20	28	36	44	52	5
3	54	19	27	35	43	14	59
2	10	47	26	34	23	50	58
1	9	17	40	32	41	49	57
8	16	48	33	25	24	56	64
7	15	42	31	39	18	55	63
6	51	22	30	38	46	11	62
60	13	21	29	37	45	53	4

Extreme corner blocks: $\begin{bmatrix} 61 & 12 \\ 3 & 54 \end{bmatrix} \begin{bmatrix} 52 & 5 \\ 14 & 59 \end{bmatrix} \begin{bmatrix} 6 & 51 \\ 60 & 13 \end{bmatrix} \begin{bmatrix} 11 & 62 \\ 53 & 4 \end{bmatrix}$

Let the Central block be assumed as: $\begin{bmatrix} 40 & 32 \\ 33 & 25 \end{bmatrix}$

Note: Taking the central block in different forms, different magic squares can be constructed.

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\} = 2$ for n = 8.

Reverting 3th column, 5th column and 3th row , 5th row (keeping the diagonal elements, extreme corner blocks and central block unchanged), a magic parametric constants (T) can be determined

61	12	21	28	37	44	52	5	**260**
3	54	22	27	38	43	14	59	**260**
58	50	47	26	39	23	10	2	**255**
1	8	48	**40**	**32**	24	49	56	**260**
64	57	17	**33**	**25**	41	16	9	**260**
7	15	42	31	34	18	55	63	**265**
6	51	19	30	35	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**236**	**244**	**276**	**284**	**260**	**260**	**260**

Here, T = 65 and sub-magic parametric constants: $T_1 = 49$, $T_2 = 81$, $T_3 = 59$ $T_4 = 71$

Pair-numbers satisfying T : [28, 37], [27, 38], [26, 39], [56, 9], [49, 16], [41, 24], [48, 17] , [8, 57], [1, 64], [31, 34], [30, 35] and [29, 36]

Other pair-numbers satisfying T:

(i) In extreme corner blocks: [61, 4], [3, 62], [12, 53], [5, 60] [6, 59], [13, 52]

(ii) In central block: [40, 25] and [32, 33]

Step-5: Adjustments on the values of magic parametric constants, T are made to get the magic square as follows;

61	12	21	28	37	44	52	5	**260**
3	54	22	27_{+11}	38_{-11}	43	14	59	**260**
58	50	47	26_{+5}	39	23	10	2	255_{+5}
1	9	48	**40**	**32**	24	49	57	**260**
64	56	17_{+24}	**33**	**25**	41_{-24}	16	8	**260**
7	15	42	31_{-5}	34	18	55	63	265_{-5}
6	51	19	30_{+5}	35_{-5}	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**236** **+24**	244 +16	276 -16	**284** **-24**	**260**	**260**	**260**

Finally the necessary magic square of 8x8 is constructed as;

61	12	21	28	37	44	52	5	**260**
3	54	22	27	38	43	14	59	**260**
58	50	47	26	39	23	10	2	**260**
1	9	48	**40**	**32**	24	49	57	**260**
64	56	41	**33**	**25**	17	16	8	**260**
7	15	42	31	34	18	55	63	**260**
6	51	19	30	35	46	11	62	**260**
60	13	20	29	36	45	53	4	**260**
260	**260**	**260**	**260**	**260**	**260**	**260**	**260**	**260**

As stated earlier, for singly-even cases, the technique generates weak magic squares only

6.3 To construct a (6 x 6) weak magic square (Singly-even)

Step-1: Let the consecutive numbers be (1, 2, ... 36) in 6 rows and 6 columns be arranged in basic Latin square format;

1	2	3	4	5	6
8	9	10	11	12	7
15	16	17	18	13	14
22	23	24	**19**	**20**	21
29	30	25	**26**	**27**	28
36	31	32	33	34	35

Satisfying $\sum_i a_{ij}$ = S for all i, where, S =111.

The pivot element lies between 18 and 19.

Find T = 37

Step-2: Select the column associated with the element (18, 19) as (4, 11, 18, 19, 26, 33) and assign this column as diagonal elements, Rearrange the other elements in an orderly manner to get a new matrix having the diagonal sums equal.

34	28	22	16	10	4
35	29	23	17	11	5
36	30	24	18	12	6
31	25	19	13	7	1
32	26	20	14	8	2
33	27	21	15	9	3

Step-3: Retaining the diagonal elements un-changed, make symmetric transformations of other elements to get the extreme corner and central blocks of (2 x 2) each.

34	9	15	21	27	4
2	29	14	20	11	32
1	7	**24**	**18**	25	31
6	12	**19**	**13**	30	36
5	26	17	23	8	35
33	10	16	22	28	3

Extreme corner blocks: $\begin{bmatrix} 34 & 9 \\ 2 & 29 \end{bmatrix} \begin{bmatrix} 27 & 4 \\ 11 & 32 \end{bmatrix} \begin{bmatrix} 5 & 26 \\ 33 & 10 \end{bmatrix} \begin{bmatrix} 8 & 35 \\ 28 & 3 \end{bmatrix}$

Central block assumed: $\begin{bmatrix} 24 & 18 \\ 19 & 13 \end{bmatrix}$

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\}$ = 1 for n = 6.

Reverting the 3rd row and 3rd column (retaining the diagonal elements and central block unchanged), a magic parametric constant T is generated.

34	9	16	21	27	4	**111**
2	29	17	20_{-6}	11	32	**111**
31	25_{+5}	**24**	**18**	7	1	**106**
6	12	**19**	**13**	30_{-5}	36	**116**
5	26	14_{+6}	23	8	35	**111**
33	10	15	22	28	3	**111**
111	**111**	**105**	**117**	**111**	**111**	**111**

Pair-numbers satisfying T: [16, 21], [17, 20], [1, 36], [7, 30] [25, 12], [31, 6], [14, 23] and [15, 22]

Other pair-numbers satisfying T:
(i) In extreme corner blocks: [34, 3], [9, 28], [2, 35], [4, 33] [27, 10], [32, 5]
(ii) In central block: [24, 13] and [18, 19]

34	9	16	21	27	4	**111**
2	29	17	14	11	32	**105**
31	30	**24**	**18**	7	1	**111**
6	12	**19**	**13**	25	36	**111**
5	26	20	23	8	35	**116**
3	10	15	22	28	3	**111**
111	**116**	**111**	**111**	**106**	**111**	**111**

Step-5: After simple adjustments in the pair-numbers, satisfying T, we get weak magic square, generated from basic Latin Square

Note: Adjustment to make row and column sums equal will affect the sum of the diagonals and therefore generates weak magic squares only.

6.4: To construct a (6 x 6) magic square (Singly-even)

Step-1: Let the consecutive numbers be (1, 2, ...36) in 6 rows and 6 columns be arranged in Latin square format as;

1	2	3	4	5	6
8	9	10	11	12	7
15	16	17	18	13	14
22	23	24	19	20	21
29	30	25	26	27	28
36	31	32	33	34	35

It satisfies $\sum_i a_{ij}$ = S for all i, where, S =111. Here, T = 37

Step-2: Select the column associated with the minimum and maximum numbers as (1, 8, 15, 22, 29, 36) and assign this column as diagonal elements, Rearrange the other elements in an orderly manner to get a new matrix giving diagonal sums equal,

31	25	19	13	7	1
32	26	20	14	8	2
33	27	21	15	9	3
34	28	22	16	10	4
35	29	23	17	11	5
36	30	24	18	12	6

Step-3: Retaining the diagonal elements un-changed, make symmetric transformations of other elements to get the extreme corner and central blocks of (2 x 2) each.

Extreme corner blocks: $\begin{bmatrix} 31 & 9 \\ 5 & 261 \end{bmatrix} \begin{bmatrix} 30 & 1 \\ 11 & 35 \end{bmatrix} \begin{bmatrix} 2 & 29 \\ 36 & 10 \end{bmatrix} \begin{bmatrix} 8 & 32 \\ 25 & 6 \end{bmatrix}$

Consider the central block as $\begin{bmatrix} 16 & 15 \\ 22 & 21 \end{bmatrix}$

31	12	18	24	30	1
5	26	17	23	8	35
4	10	**16**	**15**	28	34
3	9	**22**	**21**	27	33
2	29	14	20	11	32
36	7	13	19	25	6

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\}$= 1 for n = 6.
Reverting 3rd row and 3rd column (retaining the diagonal elements and central block unchanged), the magic parametric constant T is generated.

31	12	13	24	30	1
5	26	14	23	8	35
34	28	**16**	**15**	10	4
3	9	**22**	**21**	27	33
2	29	17	20	11	32
36	7	18	19	25	6

Pair-numbers satisfying T: [13, 24] [14, 23] [4, 33] [10, 27] [28, 9] [34, 3] [17, 20] [18, 19]

Step-5: Revert one of the diagonal elements (keeping the central block unchanged). Select the pair-numbers satisfying T in a very selective manner (which is normally very complicated, not satisfied for many cases) as;

6	12_{+20}	3	34	30_{+5}	1	86_{+25}
5_{+2}	11	9_{+18}	28	8	35_{-5}	96_{+15}
19	17_{-3}	16	15	24	24	114_{-3}
18	20	22	21	14_{+3}	13	108_{+3}
2_{+23}	29	10	27_{-18}	26	32_{-20}	126_{-15}
36	7_{-2}	33	4	25_{-23}	31	136_{-25}
86_{+25}	96_{+15}	93_{+18}	129_{-18}	126_{-15}	136_{-25}	111

Step-6: Make simple adjustments in the value of the magic parametric constants and other elements to get the magic square, generated from basic Latin Squares

6	32	3	34	35	1	**111**
7	11	27	28	8	30	**111**
19	14	16	15	23	24	**111**
18	20	22	**21**	17	13	**111**
25	29	10	9	26	12	**111**
36	5	33	4	2	31	**111**
111	**111**	**111**	**111**	**111**	**111**	**111**

6.5: To construct a (10 x10) magic square

Step-1: Let the consecutive numbers be (1, 2, ...100) in 10 rows and 10 columns be arranged in Latin square format as;

1	2	3	4	5	6	7	8	9	10
12	13	14	15	16	17	18	19	20	11
23	24	25	26	27	28	29	30	21	22
34	35	36	37	38	39	40	31	32	33
45	46	47	48	49	50	41	42	43	44
56	57	58	59	60	51	52	53	54	55
67	68	69	70	61	62	63	64	65	66
78	79	80	71	72	73	74	75	76	77
89	90	81	82	83	84	85	86	87	88
100	91	92	93	94	95	96	97	98	99

It satisfies $\sum_i a_{ij}$ = S for all i, where, S = 505. Here, T = 101

Step-2: Select the column associated with the minimum and maximum numbers and assign this column as diagonal elements, Rearrange the other elements in an orderly manner to get a new matrix giving diagonal sums equal,

91	81	71	61	51	41	31	21	11	1
92	82	72	62	52	42	32	22	12	2
93	83	73	63	53	43	33	23	13	3
94	84	74	64	54	44	34	24	14	4
95	85	75	65	**55**	**45**	35	25	15	5
96	86	76	66	**56**	**46**	36	26	16	6
97	87	77	67	57	47	37	27	17	7
98	88	78	68	58	48	38	28	18	8
99	89	79	69	59	49	39	29	19	9
100	90	80	70	60	50	40	30	20	10

Step-3: Retaining the diagonal elements un-changed, make symmetric transformations of other elements to get the extreme corner and central blocks of (2 x 2) each.

91	20	30	40	50	60	70	80	90	1
9	82	29	39	49	59	69	79	12	99
8	18	73	38	48	58	68	23	88	98
7	17	27	64	47	57	34	77	87	97
6	16	26	36	**55**	**45**	66	76	86	96
5	15	25	35	**56**	**46**	65	75	85	95
4	14	24	67	44	54	37	74	84	94
3	13	78	33	43	53	63	28	83	93
2	89	22	32	42	52	62	72	19	92
100	11	21	31	41	51	61	71	81	10

Step-4: Here, $\left\{\frac{1}{2}(n-4)\right\}$= 3 for n = 10.

Reverting 3[rd] 5[th] 7[th] rows and 3[rd] 5[th] 7[th] columns (retaining the diagonal elements and central block unchanged), the magic parametric constant T is generated.

91	20	21	40	41	60	61	80	90	1
9	82	22	39	42	59	62	79	12	99
98	88	73	63	58	43	38	23	18	8
7	17	24	64	44	57	34	77	87	97
96	86	76	66	**55**	**45**	36	26	16	6
5	15	25	35	**56**	**46**	65	75	85	95
94	84	74	67	54	47	37	27	14	4
3	13	78	33	48	53	68	28	83	93
2	89	29	32	49	52	69	72	19	92
100	11	30	31	50	51	70	71	81	10

The pair-numbers satisfying T: [41, 60] [42, 59] [58, 43] [44, 57] [6, 95] [16, 85], [26, 75], [36, 65], [50, 51] [49, 52] [48, 53], [54, 47], [5, 96] [44, 57] [96,5] [86, 15], [76, 25], [66, 35]

Other pair-numbers satisfying T:

(i) In extreme corner blocks: [34, 3], [9, 28], [2, 35], [4, 33], [27, 10], [32, 5]

(ii) In central block: [24, 13] and [18, 19]

(iii) In diagonals: [100,1], [89,12], [67,34], [91,10], [83, 18], [73, 28], [64, 37]

Step-5: Consider the central block as $\begin{bmatrix} 45 & 46 \\ 55 & 56 \end{bmatrix}$

Revert one of the diagonal elements (keeping the central block unchanged). Select the pair-numbers satisfying T in a very selective manner (shifting its positions in 90^0)

10	20_{+61}	21	40_{+31}	5	96	61	80	90	1
9_{+83}	19	22	39_{-6}	15	86	62_{-30}	79	12	99
98	88	28	63	25	76	38	23	18	8
7	17	24_{+44}	37	35	66	34	77	87	97
50	49	48	54	**55**	**45**	44	58	42	41
51	52	53	47	**56**	**46**	57	43	59	60
94	84	74	67	65	36_{-20}	64	27	14	4
3	13	78	33_{+6}	75	26	68_{-44}	73	83	93
2	89	29_{+43}	32_{+30}	85	16_{+20}	69	72_{-43}	82	92_{-83}
100	11	30	31	95	6	70	71_{-31}	81_{-61}	91

Step-6: Make simple adjustments in the value of the magic parametric constants and other elements to get the magic square, generated from basic Latin Squares

10	81	21	80	5	86	70	61	90	1
92	19	22	40	16	96	30	79	12	99
98	88	28	69	36	75	62	23	18	8
3	11	77	37	26	65	34	68	87	97
51	54	53	52	**45**	**46**	58	44	42	60
48	49	50	41	**55**	**56**	47	59	57	43
94	84	74	67	66	6	64	27	14	9
7	17	78	24	76	25	29	73	83	93
2	89	71	32	85	35	72	33	82	4
100	13	31	63	95	15	39	38	20	91

Precautions

In the construction of singly-even magic squares using basic Latin squares, selection of proper central block, assigning the pair-numbers satisfying the magic parametric constant T, is normally complicated. In many cases, it will generate weak magic squares.

Example:6.6: Using step-4 of example 6.4.

Let $\begin{bmatrix} 21 & 15 \\ 22 & 16 \end{bmatrix}$ be the central block.

If we proceed without assigning the pair-numbers satisfying T properly, after making adjustments, it generates Weak magic square as

6	32	13	24	35	1	**111**
25	11	23	14	8	30	**111**
34	28	**21**	**15**	10	4	**111**
3	9	**22**	**16**	27	33	**111**
5	29	17	20	26	12	**111**
36	5	18	19	5	31	**111**
111	**111**	**116**	**106**	**111**	**111**	**111**

7. CONCLUSION

The technique can be used for finding magic squares from basic Latin Squares of any order ($n \geq 1$, for n is singly-even). The construction is done by fixing the column, associated with the maximum and minimum elements, assigning it as diagonal element and arranging other elements in an orderly manner, making symmetric transformations, reverting one the diagonal elements (keeping the central block un-changed), selecting and arranging the pair-numbers satisfying the magic parametric constant, T in a very selective manner and finally adjusting the elements on the magic parametric constant and other elements.

Weak magic squares of different structures can be generated depending upon the choice of central block, assigning the pair-numbers satisfying T. and therefore expected to provide more applications in cryptology than that of the actual magic squares for any n (odd or even).

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References

[1]. Abe, G.: *Unsolved Problems on Magic Squares*; *Disc. Math.* **127**, 3-13, 1994

[2] Barnard, F. A. P: *Theory of Magic Squares and Cubes*; *Memoirs Natl. Acad. Sci.* **4**, 209-270, 1888.

[3] Carl, B Boyer (Revised by Uta, C. Merzbach): *A History of Mathematics*, Revised Edition, 1998

[4] Ganapathy G and Mani K: Add-on Security Model of Public-key Cryptosystem Based on Magic Square Implementation, Proceedings of the World Congress on Engineering and Computer Science, (Vol-1), Oct 22-27, San Francisco, USA, 2009

[5] Flannery, S. and Flannery, D.: In code: *A Mathematical Journey*, London's Profile Books, p16-24, 2000

[6] Heinz, H and Hendricks J. R.: *Magic Squares Lexicor*, Illustrated Self Published, 2001

[7] Hirayama, A. and Abe, G: *Researches in Magic Squares;* Osaka, Japan: Osaka Kyoikutosho, 1983.

[8]. McCranie, Judson: *Magic Squares of All Orders*, Mathematics Teacher, 674-678, 1988

[9] Pickover, C. A.: *The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures Across Dimensions.* NJ: Princeton University Press, 2002

[10] Tomba I. : *A Technique for Constructing Odd-order Magic Squares using Basic Latin Squares*, International Journal of scientific and research Publications, Volume-2, Issue-5, May 2012 pp 550-554

[11] Tomba I. : *A Technique for Constructing Even-order Magic Squares using Basic Latin Squares*, International Journal of scientific and research Publications, Volume-2, Issue-7, July 2012 pp 403-411