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Study on Flow of Rivlin-Ericksen Fluid past a Porous Vertical Wall with Constant Suction

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Abstract- The flow of Rivlin-Ericksen fluid past a porous vertical wall with the constant suction under the influence of uniform transverse magnetic field is studied. The velocity distribution, temperature distribution, rate of heat transfer, and the coefficient of skin friction are analyzed under the initial and boundary conditions. The effects of magnetic field, visco-elastic parameter, Prandtl number and suction parameter have been investigated on the flow of Rivlin-Ericksen fluid and the results are computed graphically for different values of the parameters.

Index Terms- Rivlin-Ericsen fluid, magnetic field, suction, skin friction

I. INTRODUCTION

Rheology is the science that studies the deformation and flow of matter. It is a relatively young and multidisciplinary science that encompasses such different industrial areas of activity as plastics, ceramics, cosmetics, pharmaceutics, food and biotechnology, but also paints and inks, adhesives, lubricants and surfactants. In fact, it is quite straightforward to list situations where the deformation or the flow of matter (which depend on the rheological characteristics of the materials involved) determines the performance of a product, the effectiveness of a service, the rate of a manufacturing process. The truly polyvalent character of Rheology can also be ascertained from the fact that, increasingly, techniques and procedures developed for specific types of materials and industries are later found to be of usefulness in completely different areas. A good example is the increasing use of computational, theoretical and experimental tools developed for polymers and the plastics industry in the food industry. As a multidisciplinary science Rheology requires the integration of computational, theoretical and experimental tools from different sciences. Rheology is the part of fluid mechanics that deals with non-Newtonian fluids. Merrill explains clearly the difference between a Newtonian fluid with a high viscosity and a Non-Newtonian fluid. With the growing importance of non-Newtonian fluids in modern technology and industries, investigations on such fluids are desirable. Oldroyd [6] proposed a theoretical model for a class of viscoelastic fluids. Tom's and Strawbridge [7] showed that a dilute solution of methyl methacrylate in n-butylacetate agrees well with the theoretical model of Oldroyd fluid. Sharma and Sharma [8] have studied the stability of the plane interface separating two Oldroydian viscoelastic superposed fluids of uniform densities. There are manyelastico-viscous fluids that cannot be characterized by Oldroyd's constitutiverelations. One such class of elastico-viscous fluids is Rivlin-Ericksen fluid.

Johri (1978a) [1] has considered the flow of visco-elastic fluid induced by elliptic harmonic oscillations of a disc. Johri (1978b) [2] also studied the unsteady channel flow of an elastico-viscos liquid. Bhasker and Bathaiah (1980) [3] have studied the hydromagnetic transient flow of visco-elastic fluid down an inclined plane. Bathaiah and Usha Rani (1981) [4] have investigated the magneto hydrodynamics (MHD) unsteady Hele-Shaw flow of visco-elastic fluid. Beard and Wallters (1964) [5] have studied elastic-viscous boundary-layer flows. In this paper Hydromagnetic free convection laminar flow of an incompressible Visco-elastic fluid past a porous vertical wall with constant suction is studied using Matlab

1.1 Nomenclature

T - Temperature
T_∞ - Temperature at infinity
t - Time
P - Pressure
C_p - Specific heat at constant pressure
K - Thermal conductivity
u - Horizontal velocity
v - Vertical velocity
g - Acceleration due to gravity

Greek symbols

β - Visco-elasticity
β_1 - Coefficient of thermal expansion
ν - Kinetic viscosity
σ - Electrical conductivity
μ_e - Magnetic permeability
ρ - Density

II. MATHEMATICAL FORMULATION

The flow of an incompressible Rivlin-Ericksen fluid past a porous vertical plate with constant suction under the influence of uniform transverse magnetic field is considered. The X-axis is taken along the vertical wall and a straight line perpendicular to that as the Y-axis. The magnetic field of small intensity H_0 is introduced in the Y-direction. Since the fluid is slightly conducting, the magnetic Reynolds number is much less than unity, and hence, the induced magnetic field is neglected in comparison with the applied magnetic field (Sparrow and Cess, 1962). In the absence of an input electric field, the equations governing the flow are:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=g\beta_1\left[T-T_\infty\right]+\nu\frac{\partial^2 u}{\partial y^2}+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right)-\frac{\sigma\mu_e^2H_0^2}{\rho}u \qquad \ldots(1)$$

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+2\left[2\beta+\nu\right]\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \qquad \ldots(2)$$

$$\frac{\partial v}{\partial y}=0 \qquad \ldots(3)$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2} \qquad \ldots(4)$$

Where u,v are the velocity components along the axis of coordinates respectively, g the acceleration due to gravity, β_1 the coefficient of thermal expansion, T the temperature of the plate, T_∞ temperature at infinity, ν the kinetic viscosity, β the visco-elasticity, t the time, σ the electrical conductivity of the fluid, μ_e the magnetic permeability, ρ the fluid density, p the fluid pressure, ν the kinematic cross viscosity, c_p the specific heat at constant pressure and k the thermal conductivity of the fluid. The Continuity equation (3) shows that ν is the function of time only. In order to obtain a steady solution of the boundary layer type, it is known that ν must be a negative non-zero constant ν_0. In the unsteady case also, we shall make this restriction (Stuart, 1955).

The boundary conditions are:

At $y=0, u=0, v=-v_0, T=\theta(t)$ $\qquad \ldots(5a)$

As $y\to\infty, u=0, \dfrac{\partial u}{\partial y}=0, T=0$ $\qquad \ldots(5b)$

We introduce the following non-dimensional quantities:

$$y^*=\frac{|V_0|y}{\nu}=\frac{y}{L};\quad t^*=\frac{\nu t}{L^2};\quad u^*=\frac{uL}{\nu};\qquad R=\frac{V_0L}{\nu}$$

$$sT^*=\frac{L^3g\beta_1\left(T-T_\infty\right)}{\nu^2};\qquad S=\frac{\beta}{L^2};\qquad P_r=\frac{PL^2}{\rho\nu^2} \qquad \ldots(6)$$

In view of Equation (6), the Equations (1) to (4) reduce to (dropping the superscripts star).

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$$\frac{\partial\left(\frac{u^*\nu}{L}\right)}{\partial\left(\frac{t^*L^2}{\nu}\right)}+v\frac{\partial\left(\frac{u^*\nu}{L}\right)}{\partial\left(y^*L\right)}=\frac{ST^*\nu^2}{L^3}+\nu\frac{\partial^2\left(\frac{u^*\nu}{L}\right)}{\partial\left(y^*L\right)^2}+SL^2\frac{\partial^2}{\partial\left(y^*L\right)^2}\left[\frac{\partial\left(\frac{u^*\nu}{L}\right)}{\partial\left(\frac{t^*L^2}{\nu}\right)}+v\frac{\partial\left(\frac{u^*\nu}{L}\right)}{\partial\left(y^*L\right)}\right]-\frac{\sigma\mu_e^2H_0^2}{\rho}\left(\frac{u^*\nu}{L}\right)$$

$$\frac{\frac{\nu}{L}}{L^2/\nu}\frac{\partial u^*}{\partial t^*}+v\frac{\nu}{L\times L}\frac{\partial u^*}{\partial y^*}=\frac{\nu^2}{L^3}ST^*+\frac{\nu^2}{L^3}\frac{\partial^2u^*}{\partial y^{*2}}+\frac{SL^2}{L^2}\frac{\partial^2}{\partial y^{*2}}\left[\frac{\nu/L}{L^2/\nu}\frac{\partial u^*}{\partial t^*}+v\frac{\nu/L}{L}\frac{\partial u^*}{\partial y^*}\right]-\frac{\sigma\mu_e^2H_0^2}{\rho}u^*\frac{\nu}{L}$$

$$\frac{\nu^2}{L^3}\frac{\partial u^*}{\partial t^*}+v\frac{\nu}{L^2}\frac{\partial u^*}{\partial y^*}=\frac{\nu^2}{L^3}ST^*+\frac{\nu^2}{L^3}\frac{\partial^2u^*}{\partial y^{*2}}+S\frac{\partial^2}{\partial y^{*2}}\left[\frac{\nu^2}{L^3}\frac{\partial u^*}{\partial t^*}+v\frac{\nu/L^2}{1}\frac{\partial u^*}{\partial y^*}\right]-\frac{\sigma\mu_e^2H_0^2}{\rho}u^*\frac{\nu}{L}$$

Multiply both sides by $\frac{L^3}{\nu^2}$

$$\frac{\partial u^*}{\partial t^*}+v\frac{L}{\nu}\frac{\partial u^*}{\partial y^*}=ST^*+\frac{\partial^2u^*}{\partial y^{*2}}+S\frac{\partial^2}{\partial y^{*2}}\left[\frac{\partial u^*}{\partial t^*}+v\frac{L}{\nu}\frac{\partial u^*}{\partial y^*}\right]-\frac{\sigma\mu_e^2H_0^2}{\rho}u^*\frac{L^2}{\nu}$$

$$\frac{\partial u^*}{\partial t^*}-R\frac{\partial u^*}{\partial y^*}=ST^*+\frac{\partial^2u^*}{\partial y^{*2}}+S\frac{\partial^2}{\partial y^{*2}}\left[\frac{\partial u^*}{\partial t^*}-R\frac{\partial u^*}{\partial y^*}\right]-\frac{\sigma\mu_e^2H_0^2}{\rho}u^*\frac{L^2}{\mu/\rho}$$

Since $R=\frac{V_0L}{\nu},\ v=-v_0,\ R=-\frac{vL}{\nu}$

$$\frac{\partial u^*}{\partial t^*}-R\frac{\partial u^*}{\partial y^*}=ST^*+\frac{\partial^2u^*}{\partial y^{*2}}+S\frac{\partial^2}{\partial y^{*2}}\left[\frac{\partial u^*}{\partial t^*}-R\frac{\partial u^*}{\partial y^*}\right]-Mu^*$$

Dropping the superscript star, Equation (1) reduce to,

$$\frac{\partial u}{\partial t}-R\frac{\partial u}{\partial y}=T+\frac{\partial^2u}{\partial y^2}-S_0\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}-R\frac{\partial u}{\partial y}\right)-Mu \qquad \ldots(7)$$

Where $M=\frac{\sigma\mu_e^2H_0^2L^2}{\mu}$ (Magnetic parameter)

And $S=S_0$ is the visco-elastic parameter. For Rivlin-Ericksen 2^{nd} order fluid the visco-elastic parameter S is necessarily negative $\left(0<S_0\le\frac{1}{4}\right)$.

From Equation (4)

$$\rho c_p\left[\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right]=K\frac{\partial^2 T}{\partial y^2}$$

$$\rho c_p\left[\frac{\partial\left(\frac{ST^*\nu^2}{L^3 g\beta_1}+T_\infty\right)}{\partial\left(\frac{t^*L^2}{\nu}\right)}+v\frac{\partial\left(\frac{ST^*\nu^2}{L^3 g\beta_1}+T_\infty\right)}{\partial\left(y^*L\right)}\right]=K\frac{\partial^2\left(\frac{ST^*\nu^2}{L^3 g\beta_1}+T_\infty\right)}{\partial\left(y^*L\right)^2}$$

$$\rho c_p\left[\frac{\frac{S\nu^2}{L^3 g\beta_1}}{L^2/\nu}\frac{\partial T^*}{\partial t^*}+\frac{\nu}{L^2}\frac{\partial T_\infty}{\partial t^*}+v\frac{\frac{S\nu^2}{L^3 g\beta_1}}{L}\frac{\partial T^*}{\partial y^*}\right]=K\frac{\frac{S\nu^2}{L^3 g\beta_1}}{L^2}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{K}{L^2}\frac{\partial^2 T_\infty}{\partial y^{*2}}$$

$$\rho c_p\frac{S\nu^2}{L^4 g\beta_1}\left[\frac{\nu}{L}\frac{\partial T^*}{\partial t^*}+v\frac{\partial T^*}{\partial y^*}\right]=K\frac{S\nu^2}{L^5 g\beta_1}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\rho c_p\left[\frac{\nu}{L}\frac{\partial T^*}{\partial t^*}+v\frac{\partial T^*}{\partial y^*}\right]=\frac{K}{L}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\frac{\nu}{L}\frac{\partial T^*}{\partial t^*}+v\frac{\partial T^*}{\partial y^*}=\frac{K}{L\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\frac{\partial T^*}{\partial t^*}+v\frac{L}{\nu}\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho c_p\nu}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\frac{\partial T^*}{\partial t^*}-R\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho c_p\,\mu/\rho}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\frac{\partial T^*}{\partial t^*}-R\frac{\partial T^*}{\partial y^*}=\frac{K}{\mu c_p}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\frac{\partial T^*}{\partial t^*}-R\frac{\partial T^*}{\partial y^*}=\frac{1}{\Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$$

$\Pr=\dfrac{\mu c_p}{K}$ (Prandtl number)

Dropping the superscript star,

$$\frac{\partial T}{\partial t}-R\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} \qquad \ldots(8)$$

The non-dimensional boundary conditions are:

At $y=0, u=0, v=-v_0, T=\theta(t)$ where $v_0>0$ $\ldots(9a)$

As $y\to\infty, u=0, \dfrac{\partial u}{\partial y}=0$ and $T=0$ $\ldots(9b)$

In the neighborhood of the plate, the solution of the form

$$u(y,t)=u_0(y)+\in u_1(y)e^{iwt} \qquad \ldots(10)$$

$$T(y,t) = T_0(y) + \in T_1(y)e^{iwt} \qquad \ldots(11)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u_0}{\partial t} + \varepsilon u_1(y) i\omega e^{i\omega t}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u_0}{\partial y^2} + \varepsilon e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2}$$

$$\frac{\partial^3 u}{\partial y^3} = \frac{\partial^3 u_0}{\partial y^3} + \varepsilon e^{i\omega t}\frac{\partial^3 u_1}{\partial y^3}$$

Substitute the above equations in (7),

$$\left[\frac{\partial u_0}{\partial t} + \varepsilon u_1 e^{i\omega t} i\omega\right] - R\left[\frac{\partial u_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y}\right] = T + \left[\frac{\partial^2 u_0}{\partial y^2} + \varepsilon e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2}\right]$$
$$- S_0\left[\frac{\partial^3 u}{\partial y^2 \partial t} - R\frac{\partial^3 u}{\partial y^3}\right] - M\left[u_0 + \varepsilon u_1 e^{i\omega t}\right]$$

$$\left[\frac{\partial u_0}{\partial t} + \varepsilon u_1 e^{i\omega t} i\omega\right] - R\left[\frac{\partial u_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y}\right] = T + \left[\frac{\partial^2 u_0}{\partial y^2} + \varepsilon e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2}\right]$$
$$- S_0\frac{\partial^2}{\partial y^2}\left(\frac{\partial u_0}{\partial t} + \varepsilon u_1 i\omega e^{i\omega t}\right) + S_0 R\left[\frac{\partial^3 u_0}{\partial y^3} + \varepsilon e^{i\omega t}\frac{\partial^3 u_1}{\partial y^3}\right] - M\left[u_0 + \varepsilon u_1 e^{i\omega t}\right]$$

$$\frac{\partial u_0}{\partial t} + \varepsilon u_1 e^{i\omega t} i\omega - R\frac{\partial u_0}{\partial y} - R\varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y} = T_0 + \varepsilon e^{i\omega t}T_1 + \frac{\partial^2 u_0}{\partial y^2} + \varepsilon e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2}$$
$$- S_0 \varepsilon i\omega e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2} + S_0 R\frac{\partial^3 u_0}{\partial y^3} + S_0 R\varepsilon e^{i\omega t}\frac{\partial^3 u_1}{\partial y^3} - Mu_0 - M\varepsilon u_1 e^{i\omega t}$$

Comparing the harmonic and non harmonic terms,

$$- R\frac{\partial u_0}{\partial y} = T_0 + \frac{\partial^2 u_0}{\partial y^2} + S_0 R\frac{\partial^3 u_0}{\partial y^3} - Mu_0$$

$$- S_0 R\frac{\partial^3 u_0}{\partial y^3} - R\frac{\partial u_0}{\partial y} - \frac{\partial^2 u_0}{\partial y^2} + Mu_0 = T_0$$

$$- S_0 R u_0^{111} - u_0^{11} - R u_0^{1} + Mu_0 = T_0 \qquad \ldots(12)$$

Equating the coefficient of $\varepsilon e^{i\omega t}$,

$$\varepsilon u_1 i\omega e^{i\omega t} - R\varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y} = \varepsilon e^{i\omega t}T_1 + \varepsilon e^{i\omega t}\frac{\partial^2 u_1}{\partial y^2} - S_0 i\omega\frac{\partial^2 u_1}{\partial y^2} + S_0 R\varepsilon e^{i\omega t}\frac{\partial^3 u_1}{\partial y^3}$$
$$- M\varepsilon e^{i\omega t}u_1$$

$$u_1 i\omega - R\frac{\partial u_1}{\partial y} = T_1 + \frac{\partial^2 u_1}{\partial y^2} + S_0 R\frac{\partial^3 u_1}{\partial y^3} - S_0 i\omega t u_1^{11} - Mu_1$$

$$S_0 i\omega u_1^{11} - S_0 R\frac{\partial^3 u_1}{\partial y^3} - \frac{\partial^2 u_1}{\partial y^2} + u_1 i\omega - R\frac{\partial u_1}{\partial y} + Mu_1 = T_1$$

$$-S_0 Ru_1^{111} - u_1^{11}(1 - S_0 i\omega) - Ru_1^1 + u_1(M + i\omega) = T_1 \qquad \ldots(13)$$

and

$$T(y,t) = T_0(y) + \varepsilon T_1(y) e^{iwt}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T_0}{\partial t} + \varepsilon T_1 i\omega e^{i\omega t}$$

$$\frac{\partial t}{\partial y} = \frac{\partial T_0}{\partial y} + \varepsilon e^{i\omega t}\,\frac{\partial T_1}{\partial y}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 T_0}{\partial y^2} + \varepsilon e^{i\omega t}\,\frac{\partial^2 T_1}{\partial y^2}$$

$$\frac{\partial^3 T}{\partial y^3} = \frac{\partial^3 T_0}{\partial y^3} + \varepsilon e^{i\omega t}\,\frac{\partial^3 T_1}{\partial y^3}$$

Substitute in equation (3),

$$\frac{\partial T}{\partial t} - R\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}$$

$$\left[\frac{\partial T_0}{\partial t} + \varepsilon(i\omega)e^{i\omega t}T_1\right] - R\left[\frac{\partial T_0}{\partial y} + \varepsilon e^{i\omega t}\,\frac{\partial T_1}{\partial y}\right] = \frac{1}{\Pr}\left[\frac{\partial^2 T_0}{\partial y^2} + \varepsilon e^{i\omega t}\,\frac{\partial^2 T_1}{\partial y^2}\right]$$

$$\left[\frac{\partial T_0}{\partial t} + \varepsilon(i\omega)e^{i\omega t}T_1\right] - R\frac{\partial T_0}{\partial y} - R\varepsilon e^{i\omega t}\,\frac{\partial T_1}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T_0}{\partial y^2} + \frac{1}{\Pr}\varepsilon e^{i\omega t}\,\frac{\partial^2 T_1}{\partial y^2}$$

Comparing harmonic and non harmonic terms,

$$-R\frac{\partial T_0}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T_0}{\partial y^2}$$

$$\frac{\partial^2 T_0}{\partial y^2} + PR\frac{\partial T_0}{\partial y} = 0$$

$$T_0^{11} + PRT_0^1 = 0 \qquad \ldots(14)$$

Equating the coefficient of $\varepsilon e^{i\omega t}$ on both sides,

$$\varepsilon e^{i\omega t} i\omega T_1 - R\varepsilon e^{i\omega t}\,\frac{\partial T_1}{\partial y} = \frac{1}{\Pr}\varepsilon e^{i\omega t}\,\frac{\partial^2 T_1}{\partial y^2}$$

$$P\varepsilon e^{i\omega t} i\omega T_1 - PR\varepsilon e^{i\omega t}\,\frac{\partial T_1}{\partial y} = \varepsilon e^{i\omega t}\,\frac{\partial^2 T_1}{\partial y^2}$$

$$Pi\omega T_1 - PR\frac{\partial T_1}{\partial y} = \frac{\partial^2 T_1}{\partial y^2}$$

$$\frac{\partial^2 T_1}{\partial y^2} + PR\frac{\partial T_1}{\partial y} - Pi\omega T_1 = 0$$

$$T_1^{11} + PRT_1^1 - PiwT_1 = 0 \qquad \ldots(15)$$

The boundary conditions (9a) and (9b) now reduces to,

$$u_0(0)=0=u_1(0)$$
$$T_0(0)=\theta_0,\ T_1(0)=\theta_1 \qquad \ldots(16a)$$
$$u_0(\infty)=0=u_1(\infty)$$
$$T_0(\infty)=0=T_1(\infty) \qquad \ldots(16b)$$

Assume the solution in the form

$$u_0=u_{01}+S_0u_{02}+0(S_0^2)$$
$$u_1=u_{11}+S_0u_{12}+0(S_0^2) \qquad \ldots(17a)$$
$$T_0=T_{01}+S_0T_{02}+0(S_0^2)$$
$$T_1=T_{11}+S_0T_{12}+0(S_0^2) \qquad \ldots(17b)$$

Substituting (17) in (12) to (15) and equating the coefficient of $S_{o,}$

$$\frac{\partial u_0}{\partial y}=u_{01}^1+S_0u_{02}^1$$
$$\frac{\partial^2 u_0}{\partial y^2}=u_{01}^{11}+S_0u_{02}^{11}$$
$$\frac{\partial^3 u_0}{\partial y^3}=u_{01}^{111}+S_0u_{02}^{111}$$
$$\frac{\partial u_1}{\partial y}=u_{11}^1+S_0u_{12}^1$$
$$\frac{\partial^2 u_1}{\partial y^2}=u_{11}^{11}+S_0u_{12}^{11}$$
$$\frac{\partial^3 u_1}{\partial y^3}=u_{11}^{111}+S_0u_{12}^{111}$$
$$\frac{\partial T_0}{\partial y}=T_{01}^1+S_0T_{02}^1$$
$$\frac{\partial^2 T_0}{\partial y^2}=T_{01}^{11}+S_0T_{02}^{11}$$
$$\frac{\partial T_1}{\partial y}=T_{11}^1+S_0T_{12}^1$$
$$\frac{\partial^2 T_1}{\partial y^2}=T_{11}^{11}+S_0T_{12}^{11}$$

From equation (13)

$$-S_0R\frac{\partial^3 u_1}{\partial y^3}-\frac{\partial^2 u_1}{\partial y^2}(1-S_0i\omega)-R\frac{\partial u_1}{\partial y}+u_1(M+i\omega)=T_1$$
$$-S_0R\left[u_{11}^{111}+S_0u_{12}^{111}\right]-(1-S_0iw)\left[u_{11}^{11}+S_0u_{12}^{11}\right]-R\left[u_{11}^1+S_0u_{12}^1\right]$$
$$+\left[u_{11}+S_0u_{12}+o\left(S_0^2\right)\right]\left[M+i\omega\right]=T_{11}+S_0T_{12}+o\left(S_0^2\right)$$
$$-S_0Ru_{11}^{11}-S_0^2Ru_{12}^{111}-u_{11}^{11}-S_0u_{12}^{11}+S_0i\omega u_{11}^{11}+S_0^2i\omega u_{12}^{11}-Ru_{11}^1-RS_0u_{12}^1$$
$$+Mu_{11}+MS_0u_{12}+i\omega u_{11}+i\omega S_0u_{12}=T_{11}+S_0T_{12}$$

Equating the coefficient of S_0 on both sides,

$$-S_0 Ru_{11}^{11} + S_0 i\omega u_{11}^{11} - S_0 u_{12}^{11} - RS_0 u_{12}^{1} + MS_0 u_{12} + i\omega S_0 u_{12} = S_0 T_2$$

$$-Ru_{11}^{111} + i\omega u_{11}^{11} - u_{12}^{11} - Ru_{12}^{1} + (M + i\omega)u_{12} = T_{12} \qquad \ldots(18)$$

Equating the constant terms,

$$-u_{11}^{11} - Ru_{11}^{1} + (M + iw)u_{11} = T_{11} \qquad \ldots(19)$$

From equation (12)

$$-S_0 R\frac{\partial^3 u_0}{\partial y^3} - \frac{\partial^2 u_0}{\partial y^2} - R\frac{\partial u_0}{\partial y} + Mu_0 = T_0$$

$$-S_0 R\left[u_{01}^{111} + S_0 u_{02}^{111}\right] - \left[u_{01}^{11} + S_0 u_{02}^{11}\right] - R\left[u_{01}^{1} + S_0 u_{02}^{1}\right] + M\left[u_{01} + S_0 u_{02} + o\left(S_0^2\right)\right]$$
$$= T_{01} + S_0 T_{02} + o\left(S_0^2\right)$$

$$-S_0 Ru_{01}^{111} - S_0^2 Ru_{02}^{111} - u_{01}^{11} - S_0 u_{02}^{11} - Ru_{01}^{1} - RS_0 u_{02}^{1} + Mu_{01} + MS_0 u_{02} + Mo\left(S_0^2\right)$$
$$= T_{01} + S_0 T_{02} + o\left(S_0^2\right)$$

Equating the coefficient of S_0 on both sides,

$$-S_0 Ru_{01}^{111} - S_0 u_{02}^{11} - RS_0 u_{02}^{1} + MS_0 u_{02} = S_0 T_{02}$$

$$-Ru_{01}^{111} - u_{02}^{11} - Ru_{02}^{1} + Mu_{02} = T_{02} \qquad \ldots(20)$$

Equating the constant terms,

$$-u_{01}^{11} - Ru_{01}^{1} + Mu_{01} = T_{01} \qquad \ldots(21)$$

From equation (14)

$$T_0^{11} + PRT_0^{1} = 0$$

$$\frac{\partial^2 T_0}{\partial y^2} + PR\frac{\partial T_0}{\partial y} = 0$$

$$\left(T_{01}^{11} + S_0 T_{02}^{11}\right) + PRT_{01}^{1} + PRS_0 T_{02}^{1} = 0$$

Equating the constant terms,

$$T_{01}^{11} + aT_{01}^{1} = 0 \qquad \ldots(22)$$

Equating the coefficient of S_0

$$T_{02}^{11} + aT_{02}^{1} = 0 \qquad \ldots(23)$$

From equation (15)

$$T_1^{11} + PRT_1^{1} - Pi\omega T_1 = 0$$

$$\frac{\partial^2 T_1}{\partial y^2} + PR\frac{\partial T_1}{\partial y} - Pi\omega T_1 = 0$$

$$\left[T_{11}^{11} + S_0 T_{12}^{11}\right] + PR\left[T_{11}^{1} + S_0 T_{12}^{1}\right] - Pi\omega\left[T_{11} + S_0 T_{12} + o\left(S_0^2\right)\right] = 0$$

$$T_{11}^{11} + S_0 T_{12}^{11} + PRT_{11}^{1} + PRS_0 T_{12}^{1} - Pi\omega T_{11} - Pi\omega S_0 T_{12} = 0$$

Equating the constant terms,

$$T_{11}^{11} + PRT_{11}^{1} - Pi\omega T_{11} = 0$$

a=Pr

$$T_{11}^{11} + aT_{11}^{1} - PiwT_{11} = 0 \qquad \ldots(24)$$

Equating the coefficient of S_0

$$T_{12}^{11} + aT_{12}^{1} - PiwT_{12} = 0 \qquad \ldots(25)$$

Where a=Pr.

In view of Equation (17), the boundary conditions (16) reduce to

$$\text{At} \quad y = 0 \quad \left.\begin{array}{l} u_{01} = 0 = u_{02} = u_{11} = u_{12} \\ T_{01} = \theta_0 \; T_{02} = 0 \\ T_{11} = \theta_1 \; T_{12} = 0 \end{array}\right\} \rightarrow \qquad \ldots(26a)$$

$$\text{As} \quad y \rightarrow \infty \quad \left.\begin{array}{l} u_{01} = 0 = u_{02} = u_{11} = u_{12} \\ T_{01} = 0 = T_{02} = T_{11} = T_{12} \end{array}\right\} \rightarrow \qquad \ldots(26b)$$

Solving the Equations (18) to (25) using the boundary conditions (26a) and (26b),

$$u = \frac{\theta_0}{q_1}\left[e^{-b_2 y} - e^{-ay}\right]\left(i + \frac{S_0 R a^3}{q_1}\right) + \frac{\varepsilon\theta_1}{4\left(c_1^2 + d_1^2\right)^3}$$

$$\left[\begin{array}{l} \left\{\left(c_1^3 - 3c_1 d_1^3\right)c_3 + d_3\left(3c_1^2 d_1 - d_1^3\right)\right\} \\ \left\{e^{-\frac{1}{2}(R+c_2)y}\cos\left(-\frac{1}{2}d_2 y + wt\right) - e^{-\frac{1}{2}(a+c)y}\cos\left(-\frac{1}{2}d_2 y + wt\right)\right\} \\ -\left\{d_3\left(c_1^3 - 3c_1 d_1^2\right) - c_3\left(3c_1^2 d_1 - d\right)\right\} \\ \left\{e^{-\frac{1}{2}(R+c_2)y}\sin\left(\frac{1}{2}d_2 y + wt\right) - e^{-\frac{1}{2}(a+c)y}\sin\left(-\frac{1}{2}dy + wt\right)\right\} \end{array}\right] \qquad \ldots(27)$$

$$T = \theta_0 e^{-ay} + \varepsilon\theta_1 e^{-\frac{1}{2}(a+c)y}\cos\left(-\frac{1}{2}dy + wt\right) \qquad \ldots(28)$$

Where,

$$b_2 = \frac{1}{2}\left[R + \sqrt{R^2 + 4M}\right] \qquad q_1 = a^2 - aR - M$$

$$c = \frac{1}{\sqrt{2}}\left[a^2 + \left(a^4 + 16P^2 w^2\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} \qquad d = \frac{1}{\sqrt{2}}\left[\left(a^4 + 16P^2 w^2\right)^{\frac{1}{2}} - a^2\right]^{\frac{1}{2}}$$

$$C_1 = \frac{1}{4}\left[(a+c)^2 - d^2 - 2R(a+c) - 4M\right]$$

$$d_1 = \frac{1}{2}(a+c)d - \frac{Rd}{2} - w$$

$$C_2 = \frac{1}{\sqrt{2}}\left[\left(R^2+4M\right)+\left\{\left(R^2+4M\right)^2+16w^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$d_2 = \frac{1}{\sqrt{2}}\left[\left\{\left(R^2+4M\right)^2+16w^2\right\}^{\frac{1}{2}}-\left(R^2+4M\right)\right]^{\frac{1}{2}}$$

$$C_3 = 4\left(c_1^2-d_1^2\right)+S_0\left\{\left(\left(a+c\right)^2-d^2\right)\frac{R}{2}\left(a+c\right)-2\left(a+c\right)d\left(\frac{Rd}{2}-w\theta_1\right)\right\}$$

$$d_3 = 8c_1d_1+S_0\left[\left(\left(a+c\right)^2-d^2\right)\left(\frac{Rd}{2}-w\theta_1\right)+\left(a+c\right)^2 dR\right]$$

2.1 Skin Friction

The expression for the skin friction on the plate is given by:

$$\tau = \frac{\mu^2 v_0}{L^2\rho}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\frac{\mu^2 v_0}{L^2\rho}\left[\begin{array}{l}\frac{\theta_0}{q_1}\left(a-b_2\right)\left(1+\frac{S_0Ra^3}{q_1}\right)t+\frac{\varepsilon\theta_1}{8\left(c_1^2+d_1^2\right)^3}\\ \left[\begin{array}{l}c_4\left\{\left(a+c-R-C_2\right)\cos wt+\left(d_2-d\right)\sin wt\right\}-d_4\\ \left\{\left(a+c-R-C_2\right)\sin wt+\left(d-d_2\right)\cos wt\right\}\end{array}\right]\end{array}\right] \qquad \ldots(29)$$

Where

$$C_4 = \left(c_1^3-3c_1d_1^2\right)C_3+d_3\left(3c_1^2d_1-d_1^3\right)$$
$$d_4 = d_3\left(c_1^3-3c_1d_1^2\right)-C_3\left(3c_1^2d_1-d_1^3\right)$$

2.2 Rate of Heat Transfer

From the point of view of applications in the technology, it is of interest to know the rate of heat transfer q between the fluid and the plate y=0.

The rate of heat transfer from the wall is given by

$$q = \frac{-KV_0\nu^2}{g\beta_1L^4}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$= \frac{-KV_0\nu^2}{g\beta_1L^4}\left[a\theta_0-\frac{\varepsilon\theta_1}{2}\left(d\sin wt-\left(a+c\right)\cos wt\right)\right]$$

III. RESULTS AND DISCUSSION

The velocity profiles for different values of magnetic parameter M (M=1, 2, 3, 4) with $S_0 = 0.2$ are shown in Figure 1. It is observed that velocity decreases as M increases. The velocity profiles for different fluid pressures (P=1, 2, 3, 4) with M=2 are shown in Figure 2. It is observed that an increase in P causes a decrease in velocity.

In Figure 3, we presented the velocity profiles for different values of suction parameter R (R=1, 2, 3, 4) with M=2. It is observed that velocity decreases as R increases. The velocity profiles for different values of visco-elastic parameter S_0 (S_0=0.10, 0.15, 0.20, 0.25) with M=2 are shown in Figure 4, and it is observed that an increase in S_0 leads to a decrease in velocity. From Figure 5, it is observed that C_f decreases with the increase in P for different values of M and P. A close observation reveals that C_f

decreases up to M=2, and then increases for values of M>2. Figure 6 shows the effect of S_0 on C_f. It is observed that C_f increases as S_0 increases. From Figure 7, it is noticed that C_f increases with the increase in R. From Figure 8, it is observed that the temperature decreases as P increases. From Figure 9, it is observed that the temperature decreases with the increase of R. Further, a close look at the figures reveals that the decrease in temperature is very sharp near the plate. The rate of heat transfer is presented in Figure 10. It is observed that Q increases with the increase in P or R.

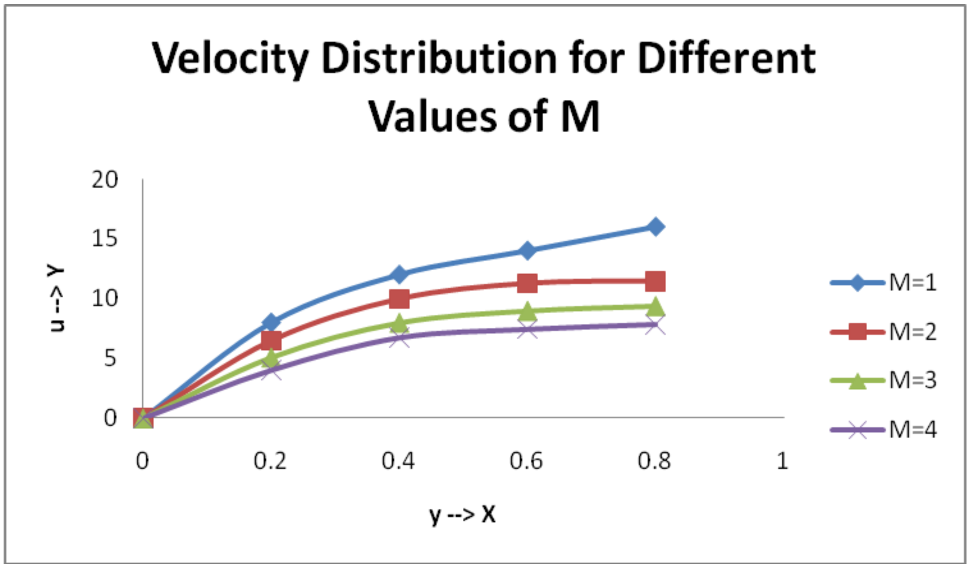

Figure: 1

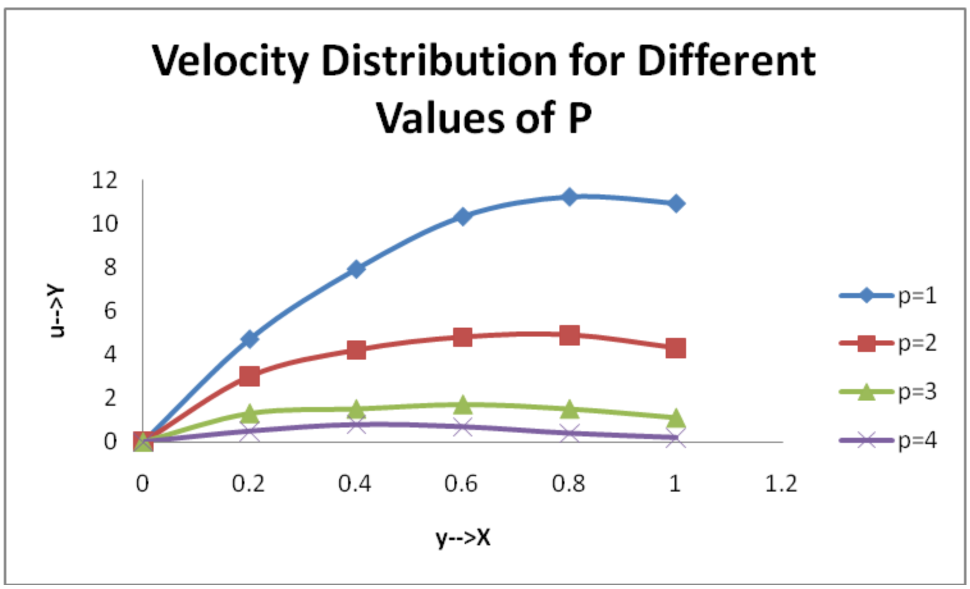

Figure: 2

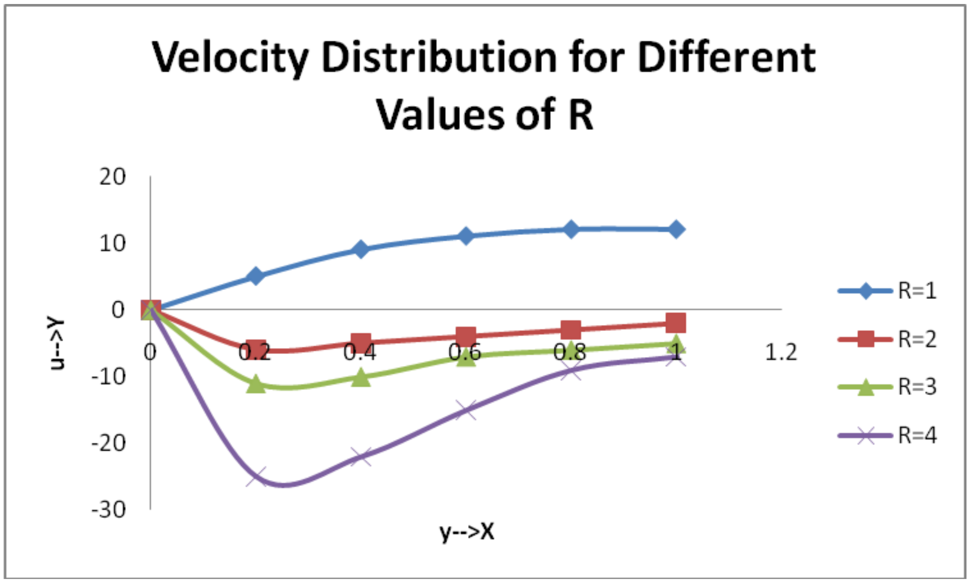

Figure: 3

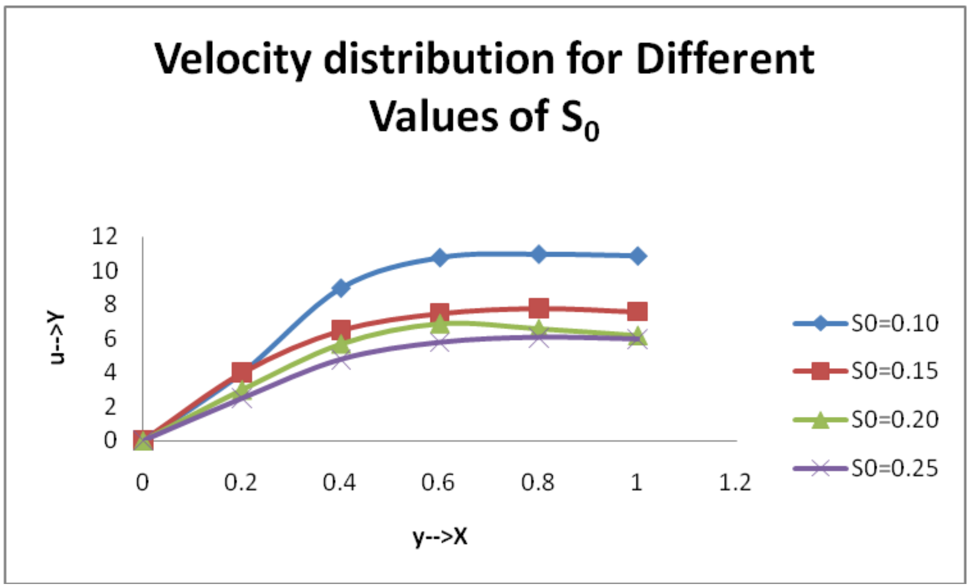

Figure:4

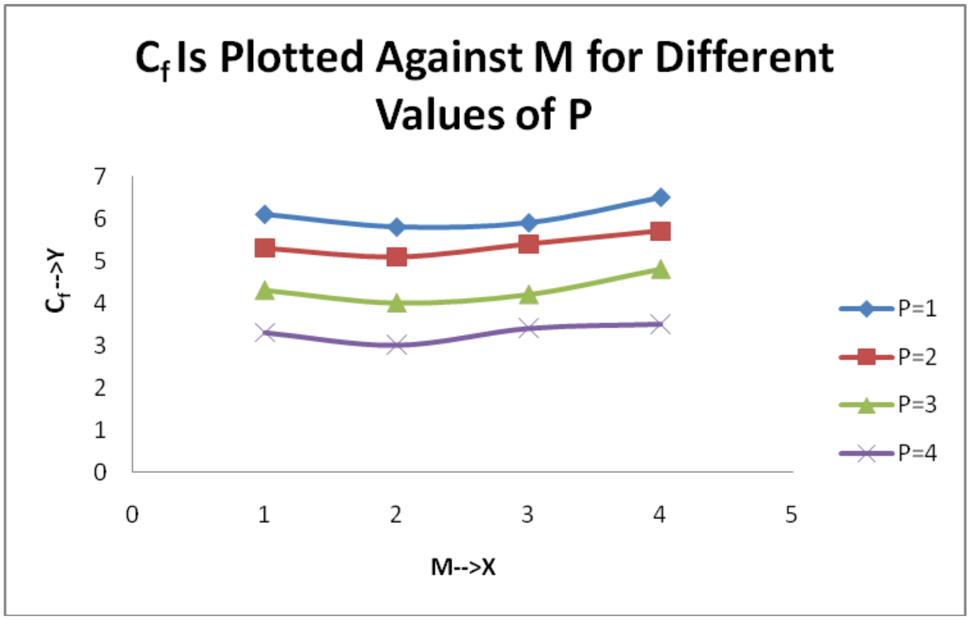

Figure: 5

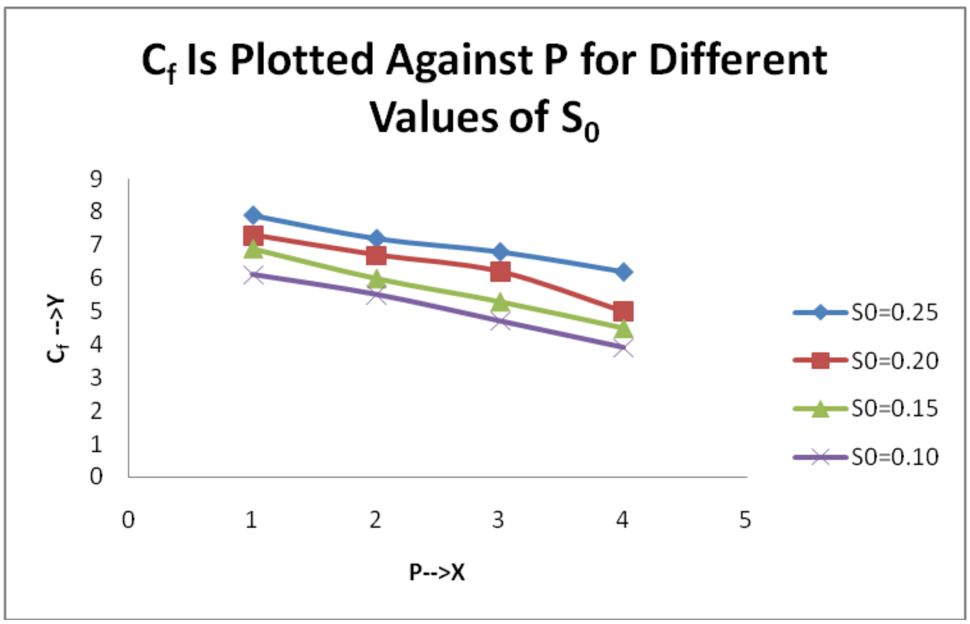

Figure: 6

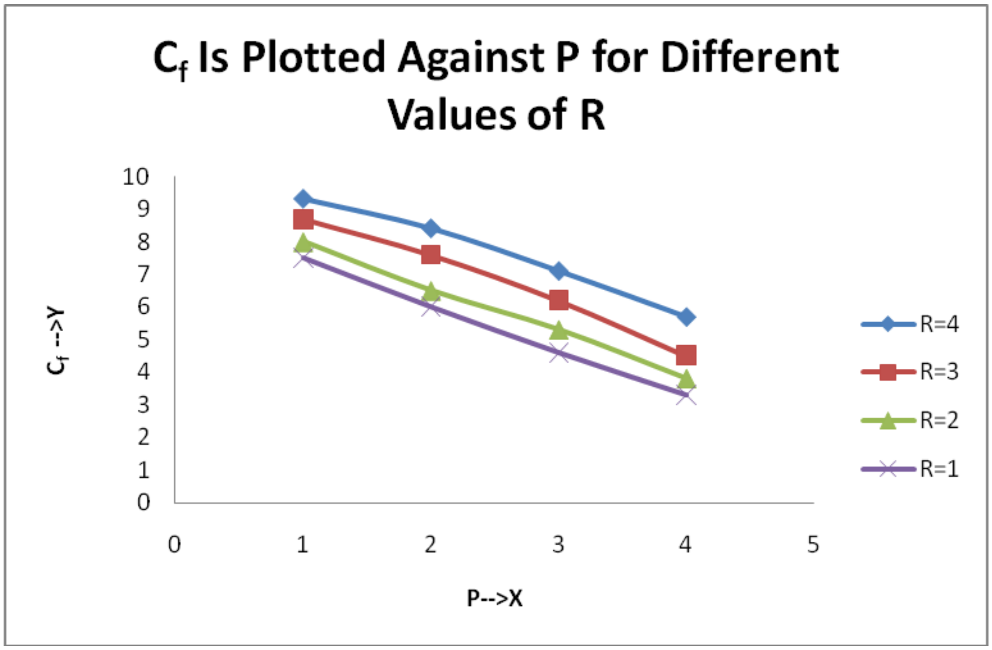

Figure: 7

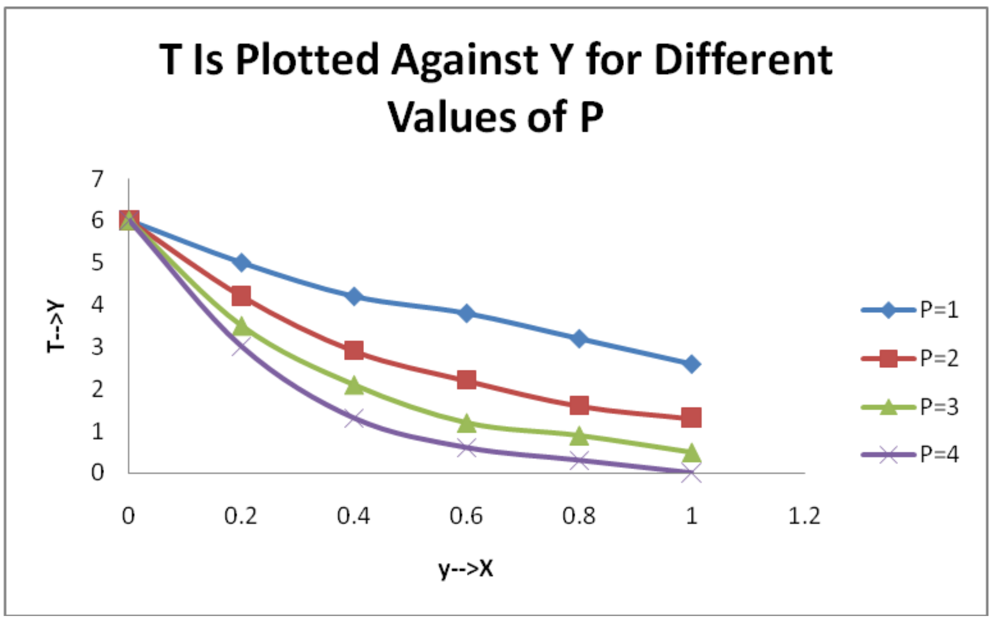

Figure: 8

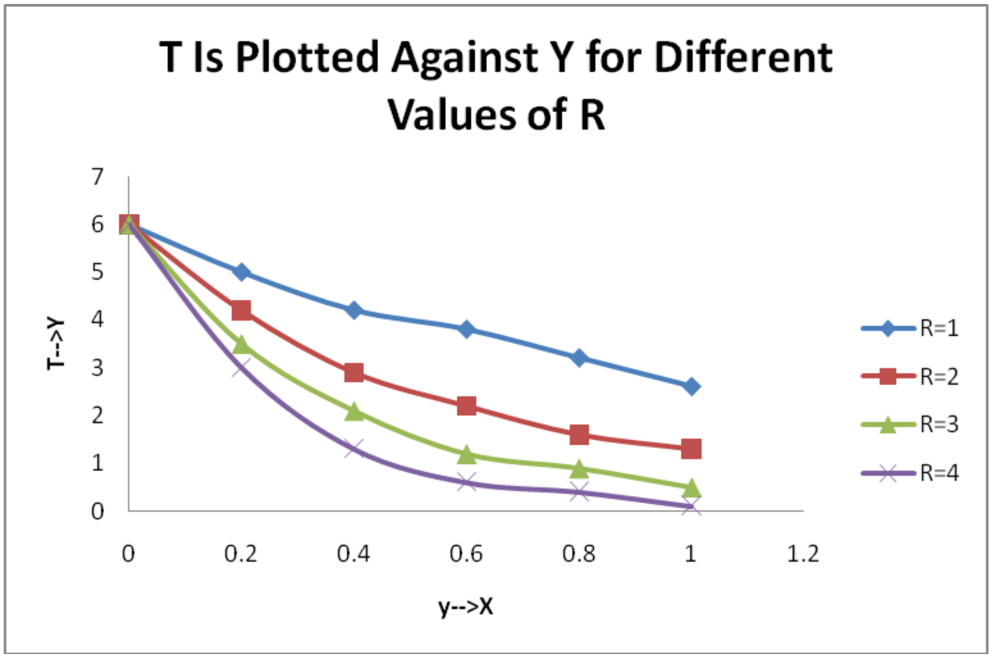

Figure: 9

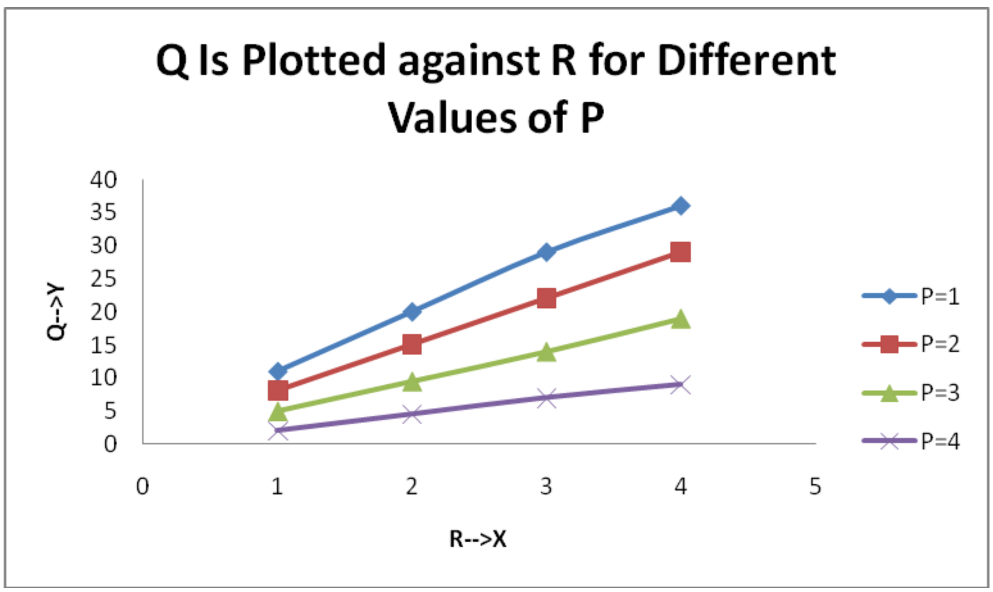

Figure: 10

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