Impulse Noise Removal Technique Using Variation Norm for Medical Images

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Abstract- This is a constrained optimization type of numerical algorithm for removing noise from images. The L1-norm of total variation of the image is minimized subject to constraints involving the statistics of the noise.

The constraints are imposed using Lagrange multipliers. The solution is obtained using the gradient-projection method. This amounts to solving a time dependent partial differential equation on a manifold determined by the constraints.

As $t \rightarrow \infty$ the solution converges to a steady state which is the denoised image.

The traditional L2-norm based regularization, which is known to remove high frequency noises in the reconstructed images and make them appear smooth. The recovered contrast in the reconstructed image in these type of methods are typically dependent on the iterative nature of the method employed, in which the non-linear iterative technique is known to perform better in comparison to linear techniques. The usage of non-linear iterative techniques in the real-time, especially in dynamical imaging, becomes prohibitive due to the computational complexity associated with them.

This new frame work along with the L1-norm based regularization can provide better robustness to noise and results in better contrast recovery compared to conventionalL2-based techniques. The proposed L1-based technique is computationally efficient.

Index Terms- Denoising, Medicalmages, Minimization, Variation Norm

I. Introduction

In many image processing problems, a denoising step is required to remove noise or spurious details from corrupted images. The Variational approaches have gained a wide popularity due to the possible addition of well-chosen regularity terms.

The presence of noise in images is unavoidable. It may be introduced by the image formation process, image recording, image transmission, etc. These random distortions make it difficult to perform any required image processing.

For example, the feature oriented enhancement is very effective in restoring blurry images, but it can be "frozen" by an oscillatory noise component. Even a small amount of noise is harmful when high accuracy is required, as in sub pixel image analysis.

In practice, to estimate a true signal in noise, the most frequently used methods are based on the least squares criteria. The rationale comes from the statistical argument that the least squares estimation is the best over an entire ensemble of all possible images. This procedure is L2 norm dependent. L2-norm based regularization, which is known to remove high frequency components in the reconstructed images and make them appear smooth. The recovered contrast in the reconstructed image will be poor.

The proper norm for images is the total variation (TV) norm. TV norms are essentially L1 norms of derivatives; hence L1 estimation procedures are more appropriate for the subject of image restoration.

We propose to de noise images by minimizing the total variation norm of the estimated solution. We derive a constrained minimization algorithm as a time dependent nonlinear PDE, where the constraints are determined by the noise statistics.

Traditional methods attempt to reduce/ remove the noise component prior to further image processing operations. However, the same TV/L1 philosophy can be used to design hybrid algorithms combining de noising with other noise sensitive image processing tasks.

II. DESCRIPTION

Let the observed intensity function uo(x, y) denote the pixel values of a noisy image

for x, y $\varepsilon\Omega$. Let u(x, y) denote the desired clean image, so

$$u_o(x,y)=u(x, y)+n(x, y) \qquad (1)$$

We wish to reconstruct u from uo. Most conventional variational methods involve a least squares L2 fit because this leads to linear equations. In our two dimensional continuous framework the constrained minimization problem is to minimize

$$\int (u_{xx}+u_{yy})^2 \qquad (2)$$

subject to constraints involving the mean

$$\int u = \int u_o \qquad (3)$$

and standard deviation

$$\int (u - u_o)^2 = \sigma^2 \qquad (4)$$

Where $\sigma > 0$ is given

The resulting linear system is solved using numerical linear algebra.

Such a minimization allows the recovery of a simple geometric description of the image *while* preserving boundaries. This framework is then very efficient when denoising images with flat zones but fails in preserving texture details.

It also fails in removing contrasted and isolated pixels in images corrupted by an impulse noise. Another drawback is that the minimizer presents a loss of contrast due to the L2 data fidelity term.

III. TOTAL VARIATION ALGORITHM

Our constrained minimization problem is to minimize

$$\int \sqrt{u_x^2+u_y^2}\, dx\, dy \qquad (5)$$

subject to constraints involving u_o.

$$\int u\, dx\, dy = \int u_o\, dx\, dy \qquad (6)$$

This constraint signifies the fact that the white noise *n(x, y)) is of zero mean and*

$$\int \frac{1}{2}(u-u_o)^2 dxdy = \sigma^2 \qquad (7)$$

The second constraint uses a priori information that the standard deviation of the noise *n(x, y) is sigma*

Thus we have one linear and one nonlinear constraint. We arrive at the Euler-Lagrange equations from equation

$$0=\frac{\partial}{\partial x}\left(u_x/\sqrt{u_x^2+u_y^2}\right)+\frac{\partial}{\partial y}\left(u_y/\sqrt{u_x^2+u_y^2}\right)-\lambda_1-\lambda_2(u-u_o) \qquad (8)$$

The solution procedure uses a parabolic equation with time as an evolution parameter, or equivalently, the gradient descent method. This means that we solve

$$u_t=\frac{\partial}{\partial x}\left(u_x/\sqrt{u_x^2+u_y^2}\right)+\frac{\partial}{\partial y}\left(u_y/\sqrt{u_x^2+u_y^2}\right)-(u-u_o),$$

for $t>0$, $x,y \in \Omega$ (9)

$$u_t=\left\{(u_{xx}u_y^2-2u_xu_yu_{xy}+u_{yy}u_x^2)\,/\,(u_x^2+u_y^2)^{3/2}\right\}-\lambda(u-u_o) \qquad (10)$$

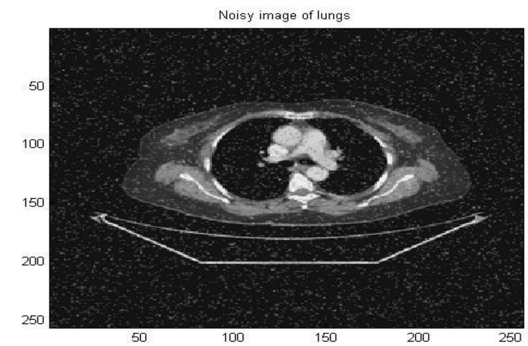
We have dropped the first constraint because it is automatically enforced by our evolution procedure if the mean of u(x, y, 0) is the same as that of u0(x, y).

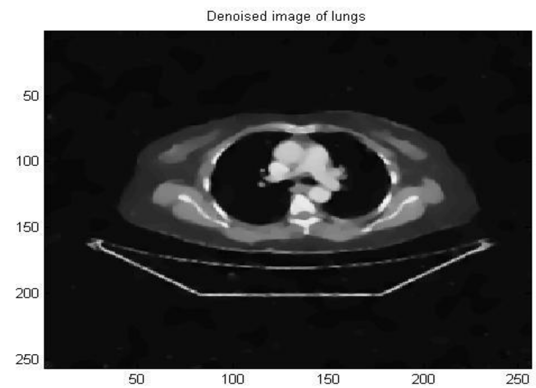
As t increases, we approach a denoised version of our image. We compute λ (t) by multiplying equation (9) by (u - u0) and integrating by parts over Ω. If steady state has been reached, the left side of (9) vanishes. We then have

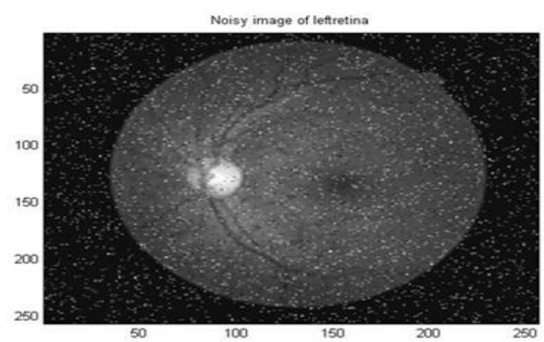
$$\lambda=1/\sigma^2\int\left\{(u_{xx}u_y^2-2u_xu_yu_{xy}+u_{yy}u_x^2)\,/\,(u_x^2+u_y^2)^{3/2}\right\}(u-u_o)\, dx\, dy \qquad (11)$$

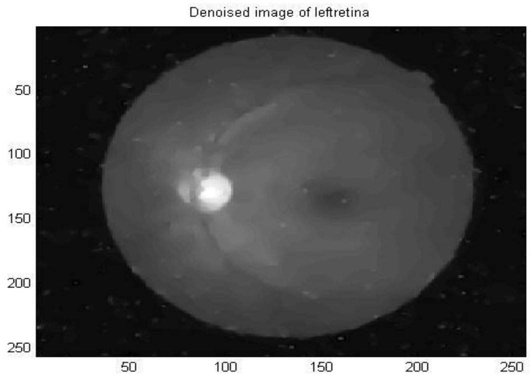
This gives us a dynamic value $\lambda(t)$, which appears to converge as $t \to \infty$. The theoretical justification for this approach comes from the fact that it is merely the gradient-projection method.

IV. RESULTS









V. CONCLUSION AND DISCUSSIONS

This is a constrained optimization type of numerical algorithm for removing noise from images. The total variation of the image is minimized subject to constraints involving the statistics of the noise. Traditional methods attempt to reduce/ remove the noise component prior to further image processing operations. However, the same TV/L1 philosophy can be used to design hybrid algorithms combining de noising with other noise sensitive image processing tasks.

L2-norm based regularization, which is known to remove high frequency noises in the reconstructed images and make them appear smooth. The recovered contrast in the reconstructed image in these type of methods are typically dependent on the iterative nature of the method employed, in which the non-linear iterative technique is known to perform better in comparison to linear techniques. The usage of non-linear iterative techniques in the real-time, especially in dynamical imaging, becomes prohibitive due to the computational complexity associated with them. As shown in the figures, this new frame work along with the L1-norm based regularization can provide better robustness to noise and results in better contrast recovery compared to conventional L2-based techniques. The proposed L1-based technique is computationally efficient compared to its counterpart L2-based one.

L1-Minimization frame work can be applied in image reconstruction methods in diffuse optical tomography. Modern diffuse optical imaging systems are multi-modal in nature, where diffuse optical imaging is combined with traditional imaging modalities such as MRI, CT, and Ultrasound. A novel approach that can more effectively use the structural information provided by the traditional imaging modalities can be introduced, which is based on prior image constrained-L1 minimization scheme.

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