Some Properties of Induced Intuitionistic Fuzzy Sets

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Abstract: In this paper we have proved some basic properties, related to union and intersection, of four different types of induced intuitionistic fuzzy sets.

Key words: Degree of membership, degree of non-membership, induced Intuitionistic fuzzy set.

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I. INTRODUCTION

The notion of Intuitionistic fuzzy set (IFS) was introduced by Atanassov in [1]. Various properties on intuitionistic fuzzy sets were discussed by many authors in [2-4,9]. The concept of induced intuitionistic fuzzy sets was introduced in [5]. A few relations between induced intuitionistic fuzzy sets and second order induced intuitionistic fuzzy sets were established in [6,7]. Some complement properties of induced intuitionistic fuzzy sets were discussed in [8]. Here, in this paper, we have proved some properties on union and intersection of four different types of induced intuitionistic fuzzy sets.

II. PRELIMINARIES

This section contains some basic definitions and notations which are used throughout the paper.

Definition 2.1 [1-3]: Let E be any non-empty set. An intuitionistic fuzzy set A of E is an object of the form

\[ A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \} \]

where the functions \( \mu_A: E \rightarrow [0,1] \) and \( \nu_A: E \rightarrow [0,1] \) denotes the degree of membership and the non-membership functions respectively and for every \( x \in E \), \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).

If A and B are two intuitionistic fuzzy sets of a non-empty set E then the following relations are valid [3]:

\[ A \subseteq B \text{ if and only if for all } x \in E, \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x); \]

\[ A = B \text{ if and only if for all } x \in E, \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x); \]

\[ A \cap B = \{ (x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))) : x \in E \}; \]

\[ A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))) : x \in E \}. \]
Considering the degree of membership $\mu_A(x), \mu_B(x)$ and the non-membership $v_A(x), v_B(x)$ for each element $x \in E$ of the intuitionistic fuzzy sets $A$ and $B$ respectively, of a non-empty set $E$, the four different types of induced intuitionistic fuzzy sets are defined as follows:

**Definition 2.2 [5]:** If $A$ and $B$ are two intuitionistic fuzzy sets of a non-empty set $E$ then

$$A^*(B) = A^* = \{ (x, \mu_A(x), \min(v_A(x), v_B(x)) : x \in E \};$$

$$A_*(B) = A_* = \{ (x, \mu_A(x), \max(v_A(x), v_B(x)) : x \in E \};$$

$$A^*(B) = A^* = \{ (x, \max(\mu_A(x), \nu_B(x)), v_A(x)) : x \in E \};$$

$$A_*(B) = A_* = \{ (x, \min(\mu_A(x), \nu_B(x)), v_A(x)) : x \in E \}.$$

**Note 2.3 [5]:** It is to be noted that

$$A^*(B) \neq B^*(A), A_*(B) \neq B_*(A), A^*(B) \neq B^*(A), A_*(B) \neq B_*(A).$$

### III. SOME PROPERTIES OF INDUCED INTUITIONISTIC FUZZY SETS

Let $A_1, A_2$ and $B$ be three intuitionistic fuzzy sets of $E.$ Then,

**Property 3.1:** i) $(A_1 \cup A_2)^*(B) = (A_1)^*(B) \cup (A_2)^*(B)$

ii) $(A_1 \cap A_2)^*(B) = (A_1)^*(B) \cap (A_2)^*(B)$

**Proof:** i) $(A_1 \cup A_2)^*(B) = \{ (x, \max(\mu_{A_1 \cup A_2}(x), \mu_B(x)), v_{A_1 \cup A_2}(x)) : x \in E \}$

$$= \{ (x, \max(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)), \min(v_{A_1}(x), v_{A_2}(x))) : x \in E \} \quad \ldots \quad (3.1.1)$$

Also, $(A_1)^*(B) \cup (A_2)^*(B)$

$$= \{ (x, \max(\mu_{A_1}(x), \mu_B(x)), v_{A_1}(x)) : x \in E \} \cup \{ (x, \max(\mu_{A_2}(x), \mu_B(x)), v_{A_2}(x)) : x \in E \}$$

$$= \{ (x, \max(\mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x)), \min(v_{A_1}(x), v_{A_2}(x))) : x \in E \} \quad \ldots \quad (3.1.2)$$

From (3.1.1) and (3.1.2) we have,

$$(A_1 \cup A_2)^*(B) = (A_1)^*(B) \cup (A_2)^*(B).$$

ii) $(A_1 \cap A_2)^*(B) = \{ (x, \max(\mu_{A_1 \cap A_2}(x), \mu_B(x)), v_{A_1 \cap A_2}(x)) : x \in E \}$

$$= \{ (x, \max(\min(\mu_{A_1}(x), \mu_{A_2}(x)), \mu_B(x)), \max(v_{A_1}(x), v_{A_2}(x))) : x \in E \}$$
Therefore, for each \( x \in E \),
\[
\mu_{(A_1 \cap A_2)^*(B)}(x) = \begin{cases} 
\max (\mu_{A_1}(x), \mu_B(x)) & \text{for } \mu_{A_1}(x) \leq \mu_{A_2}(x) \\
\max (\mu_{A_2}(x), \mu_B(x)) & \text{for } \mu_{A_2}(x) < \mu_{A_1}(x) 
\end{cases} 
\]
\[
= \begin{cases} 
\mu_B(x) & \text{for } \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\
\min (\mu_{A_1}(x), \mu_{A_2}(x)) & \text{for } \mu_B(x) \leq \mu_{A_1}(x), \mu_{A_2}(x) \\
\mu_B(x) & \text{for } \mu_B(x) \in [\mu_{A_1}(x), \mu_{A_2}(x)] \text{ or } [\mu_{A_2}(x), \mu_{A_1}(x)] 
\end{cases} 
\quad \ldots \ldots \quad (3.1.3)
\]

Also, \((A_1)^*(B) \cap (A_2)^*(B)\)
\[
= \left\{ x, \max (\mu_{A_1}(x), \mu_B(x)), \nu_{A_1}(x) : x \in E \right\} \cap \left\{ x, \max (\mu_{A_2}(x), \mu_B(x)), \nu_{A_2}(x) : x \in E \right\} 
\]
\[
= \left\{ x, \min \left( \max (\mu_{A_1}(x), \mu_B(x)), \max (\mu_{A_2}(x), \mu_B(x)) \right), \max (\nu_{A_1}(x), \nu_{A_2}(x)) : x \in E \right\} 
\]

Therefore, for each \( x \in E \),
\[
\mu_{(A_1 \cap A_2)^*(B)}(x) = \begin{cases} 
\mu_B(x) & \text{for } x \in E \text{ and } \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\
\min (\mu_{A_1}(x), \mu_{A_2}(x)) & \text{for } x \in E \text{ and } \mu_B(x) \leq \mu_{A_1}(x), \mu_{A_2}(x) \\
\mu_B(x) & \text{for } x \in E \text{ and } \mu_B(x) \in [\mu_{A_1}(x), \mu_{A_2}(x)] \text{ or } [\mu_{A_2}(x), \mu_{A_1}(x)] 
\end{cases} 
\quad \ldots \ldots \quad (3.1.4)
\]

Also, for each \( x \in E \),
\[
\nu_{(A_1 \cap A_2)^*(B)}(x) = \max (\nu_{A_1}(x), \nu_{A_2}(x)) = \nu_{(A_1)^*(B) \cap (A_2)^*(B)}(x) 
\quad \ldots \ldots \quad (3.1.5)
\]

From (3.1.3), (3.1.4) and (3.1.5) we have,
\[
(A_1 \cap A_2)^*(B) = (A_1)^*(B) \cap (A_2)^*(B).
\]

**Property 3.2:**

i) \((A_1 \cup A_2)^*(B) = (A_1)^*(B) \cup (A_2)^*(B)\)

ii) \((A_1 \cap A_2)^*(B) = (A_1)^*(B) \cap (A_2)^*(B)\)

**Proof:**

i) \((A_1 \cup A_2)^*(B) = \left\{ x, \mu_{A_1 \cup A_2}(x), \min (\nu_{A_1 \cup A_2}(x), \nu_B(x)) : x \in E \right\} 
\]
\[
= \left\{ x, \max (\mu_{A_1}(x), \mu_{A_2}(x)), \min (\nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x)) : x \in E \right\} 
\quad \ldots \ldots \quad (3.2.1)
\]

Also,
\[
(A_1)^*(B) \cup (A_2)^*(B) = \left\{ x, \mu_{A_1}(x), \min (\nu_{A_1}(x), \nu_B(x)) : x \in E \right\} \cup \left\{ x, \mu_{A_2}(x), \min (\nu_{A_2}(x), \nu_B(x)) : x \in E \right\} 
\]
\[
= \left\{ x, \max (\mu_{A_1}(x), \mu_{A_2}(x)), \min (\nu_{A_1}(x), \nu_B(x)), \min (\nu_{A_2}(x), \nu_B(x)) : x \in E \right\}
\]
\[
\{ (x, \max (\mu_{A_1}(x), \mu_{A_2}(x)), \min (v_{A_1}(x), v_{A_2}(x), v_B(x))) : x \in E \} 
\]

From (3.2.1) and (3.2.2) we have,
\[
(A_1 \cup A_2)'(B) = (A_1)'(B) \cup (A_2)'(B).
\]

ii) \( (A_1 \cap A_2)'(B) = \left\{ (x, \mu_{A_1 \cap A_2}(x), \min (v_{A_1 \cap A_2}(x), v_B(x))) : x \in E \right\} \]

\[
= \left\{ (x, \min (\mu_{A_1}(x), \mu_{A_2}(x)), \min (v_{A_1}(x), v_{A_2}(x))) : x \in E \right\}.
\]

Therefore, for each \( x \in E \),
\[
v_{(A_1 \cap A_2)'(B)}(x) = \left\{ \begin{array}{ll}
v_B(x) & \text{if } v_B(x) \leq \max (v_{A_1}(x), v_{A_2}(x)) \\
\max (v_{A_1}(x), v_{A_2}(x)) & \text{if } v_B(x) > \max (v_{A_1}(x), v_{A_2}(x))
\end{array} \right\}
\]

\( (A_1)'(B) \cap (A_2)'(B) = \left\{ (x, \mu_{A_1}(x), \min (v_{A_1}(x), v_B(x))) : x \in E \right\} \cap \left\{ (x, \mu_{A_2}(x), \min (v_{A_2}(x), v_B(x))) : x \in E \right\} \]

\[
= \left\{ (x, \min (\mu_{A_1}(x), \mu_{A_2}(x)), \min (v_{A_1}(x), v_B(x))) : x \in E \right\}
\]

Therefore, for each \( x \in E \),
\[
v_{(A_1)'(B) \cap (A_2)'(B)}(x) = \left\{ \begin{array}{ll}
v_B(x) & \text{if } v_B(x) \leq \max (v_{A_1}(x), v_{A_2}(x)) \\
\max (v_{A_1}(x), v_{A_2}(x)) & \text{if } v_B(x) > \max (v_{A_1}(x), v_{A_2}(x))
\end{array} \right\}
\]

Hence from (3.2.3) and (3.2.4) we have, for each \( x \in E \),
\[
v_{(A_1 \cap A_2)'(B)}(x) = v_{(A_1)'(B) \cap (A_2)'(B)}(x)
\]

Also, for each \( x \in E \),
\[
\mu_{(A_1 \cap A_2)'(B)}(x) = \min (\mu_{A_1}(x), \mu_{A_2}(x)) = \mu_{(A_1)'(B) \cap (A_2)'(B)}(x)
\]

So, by (3.2.5) and (3.2.6) we have,
\[
(A_1 \cap A_2)'(B) = (A_1)'(B) \cap (A_2)'(B).
\]

**Property 3.3:** i) \( (A_1 \cup A_2)'(B) = (A_1)'(B) \cup (A_2)'(B) \)

ii) \( (A_1 \cap A_2)'(B) = (A_1)'(B) \cap (A_2)'(B) \)

**Proof:** i) For each \( x \in E \),
\begin{align*}
\mu_{(A_1 \cup A_2), (B)}(x) &= \min \left( \mu_{A_1 \cup A_2}(x), \mu_B(x) \right) = \min \left( \max \left( \mu_{A_1}(x), \mu_{A_2}(x) \right), \mu_B(x) \right) \\
&= \begin{cases} 
\max \left( \mu_{A_1}(x), \mu_{A_2}(x) \right) & \text{when } \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\
\mu_B(x) & \text{elsewhere}
\end{cases} \quad \ldots \ldots (3.3.1)
\end{align*}

and \( \mu_{(A_1 \cap A_2), (B)}(x) = \max \left( \min \left( \mu_{A_1}(x), \mu_B(x) \right), \min \left( \mu_{A_2}(x), \mu_B(x) \right) \right) \)

\begin{align*}
&= \begin{cases} 
\max \left( \mu_{A_1}(x), \mu_{A_2}(x) \right) & \text{when } \mu_B(x) \geq \mu_{A_1}(x), \mu_{A_2}(x) \\
\mu_B(x) & \text{elsewhere}
\end{cases} \quad \ldots \ldots (3.3.2)
\end{align*}

Hence from (3.3.1) and (3.3.2) we have, for each \( x \in E \),

\[ \mu_{(A_1 \cup A_2), (B)}(x) = \mu_{(A_1 \cap A_2), (B)}(x) \] \quad \ldots \ldots (3.3.3)

Also for each \( x \in E \),

\[ v_{(A_1 \cup A_2), (B)}(x) = v_{A_1 \cup A_2}(x) = \min \left( v_{A_1}(x), v_{A_2}(x) \right) = v_{(A_1 \cup A_2), (B)}(x) \] \quad \ldots \ldots (3.3.4)

So, from (3.3.3) and (3.3.4) we have,

\( (A_1 \cup A_2), (B) = (A_1), (B) \cup (A_2), (B) \).

ii) For each \( x \in E \),

\[ \mu_{(A_1 \cap A_2), (B)}(x) = \min \left( \mu_{A_1 \cap A_2}(x), \mu_B(x) \right) = \min \left( \min \left( \mu_{A_1}(x), \mu_{A_2}(x) \right), \mu_B(x) \right) \]

\[ = \min \left( \mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x) \right) \] \quad \ldots \ldots (3.3.5)

and \( \mu_{(A_1 \cap A_2), (B)}(x) = \min \left( \min \left( \mu_{A_1}(x), \mu_B(x) \right), \min \left( \mu_{A_2}(x), \mu_B(x) \right) \right) \)

\[ = \min \left( \mu_{A_1}(x), \mu_{A_2}(x), \mu_B(x) \right) \] \quad \ldots \ldots (3.3.6)

Therefore from (3.3.5) and (3.3.6), we have,

\[ \mu_{(A_1 \cap A_2), (B)}(x) = \mu_{(A_1 \cap A_2), (B)}(x) \] \quad \ldots \ldots (3.3.7)

Also for each \( x \in E \),

\[ v_{(A_1 \cap A_2), (B)}(x) = v_{A_1 \cap A_2}(x) = \max \left( v_{A_1}(x), v_{A_2}(x) \right) = v_{(A_1 \cap A_2), (B)}(x) \] \quad \ldots \ldots (3.3.8)

Hence from (3.3.7) and (3.3.8), we have,

\[ (A_1 \cap A_2), (B) = (A_1), (B) \cap (A_2), (B) \].
Property 3.4: i) \((A_1 \cup A_2)\cdot (B) = (A_1)\cdot (B) \cup (A_2)\cdot (B)\)

\[ \text{ii) } (A_1 \cap A_2)\cdot (B) = (A_1)\cdot (B) \cap (A_2)\cdot (B) \]

Proof: i) For each \(x \in E\),

\[ \mu_{(A_1 \cup A_2)\cdot (B)}(x) = \mu_{A_1 \cup A_2}(x) = \max \left( \mu_{A_1}(x), \mu_{A_2}(x) \right) = \mu_{(A_1)\cdot (B) \cup (A_2)\cdot (B)}(x) \]  \hspace{1cm} \ldots \ldots (3.4.1) \]

Also, for each \(x \in E\),

\[ \nu_{(A_1 \cup A_2)\cdot (B)}(x) = \max \left( \nu_{A_1 \cup A_2}(x), \mu_B(x) \right) = \max \left( \min \left( \nu_{A_1}(x), \nu_{A_2}(x) \right), \mu_B(x) \right) \]

\[ = \begin{cases} 
\min \left( \nu_{A_1}(x), \nu_{A_2}(x) \right) & \text{when } \mu_B(x) \leq \nu_{A_1}(x), \nu_{A_2}(x) \\
\mu_B(x) & \text{elsewhere}
\end{cases} \]  \hspace{1cm} \ldots \ldots (3.4.2) \]

\[ \nu_{(A_1)\cdot (B) \cup (A_2)\cdot (B)}(x) = \min \left( \max \left( \nu_{A_1}(x), \nu_B(x) \right), \max \left( \nu_{A_2}(x), \nu_B(x) \right) \right) \]

\[ = \begin{cases} 
\min \left( \nu_{A_1}(x), \nu_{A_2}(x) \right) & \text{when } \mu_B(x) \leq \nu_{A_1}(x), \nu_{A_2}(x) \\
\mu_B(x) & \text{elsewhere}
\end{cases} \]  \hspace{1cm} \ldots \ldots (3.4.3) \]

From (3.4.2) and (3.4.3) we have, for each \(x \in E\),

\[ \nu_{(A_1 \cup A_2)\cdot (B)}(x) = \nu_{(A_1)\cdot (B) \cup (A_2)\cdot (B)}(x) \]  \hspace{1cm} \ldots \ldots (3.4.4) \]

So from (3.4.1) and (3.4.4) we have,

\[ (A_1 \cup A_2)\cdot (B) = (A_1)\cdot (B) \cup (A_2)\cdot (B). \]

ii) For each \(x \in E\),

\[ \mu_{(A_1 \cap A_2)\cdot (B)}(x) = \mu_{A_1 \cap A_2}(x) = \min \left( \mu_{A_1}(x), \mu_{A_2}(x) \right) = \mu_{(A_1)\cdot (B) \cap (A_2)\cdot (B)}(x) \]  \hspace{1cm} \ldots \ldots (3.4.5) \]

Also for each \(x \in E\),

\[ \nu_{(A_1 \cap A_2)\cdot (B)}(x) = \max \left( \nu_{A_1 \cap A_2}(x), \nu_B(x) \right) = \max \left( \max \left( \nu_{A_1}(x), \nu_{A_2}(x) \right), \nu_B(x) \right) \]

\[ = \max \left( \nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x) \right) \]  \hspace{1cm} \ldots \ldots (3.4.6) \]

and \( \nu_{(A_1)\cdot (B) \cap (A_2)\cdot (B)}(x) = \max \left( \nu_{(A_1)\cdot (B)}(x), \nu_{(A_2)\cdot (B)}(x) \right) \)

\[ = \max \left( \max \left( \nu_{A_1}(x), \nu_B(x) \right), \max \left( \nu_{A_2}(x), \nu_B(x) \right) \right) \]

\[ = \max \left( \nu_{A_1}(x), \nu_{A_2}(x), \nu_B(x) \right) \]  \hspace{1cm} \ldots \ldots (3.4.7) \]

Hence, from (3.4.5), (3.4.6) and (3.5.7) we have,

\[ (A_1 \cap A_2)\cdot (B) = (A_1)\cdot (B) \cap (A_2)\cdot (B). \]
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