Bending Stress Analysis of High Contact Ratio Spur Gear

Khin Khin Thant*, Chaw Wint Yee Zaw**

* Department of Mechanical Engineering, Technological University (Thanlyin)
** Department of Mechanical Engineering, Technological University (Thanlyin)

DOI: 10.29322/IJSRP.9.08.2019.p9284
http://dx.doi.org/10.29322/IJSRP.9.08.2019.p9284

Abstract- Tooth gears are used to transmit the power with high velocity ratio. During this phase, they encounter high stress at the point of contact. Today gears are needed to increase load carrying capacity, reduce the transmission weight and fulfil noise requirements without increasing the manufacturing cost. The high performance gear which provide higher pressure or contact ratio. In this paper, gear pair for automotive gear box application was designed based on high performance gear approach.This paper presents a general approach for analyzing bending stresses using Lewis Equation from nine different positions of gear tooth flanks and it showcases how the contact ration governs the stress conditions. In this process comparison of bending stress was done of steel gear of high contact ratio (HCR) gear as a substitute in normal contact ratio (NCR) gear and the software programmed was performed in SOLIDWORKS and ANSYS Workbench to get the best result possible.This study will help to improve the performance of gear system.

Index Terms-bending stress, Lewis Equation, HCR, NCR, ANSYS

I. INTRODUCTION

This Gears are the most common types of transmitting power in the modern mechanical engineering world. They change the rate of rotation of machinery shaft and also the axis of rotation. For high speed machinery, such as an automobile transmission, gears are the optimal medium for low energy loss and high accuracy. A pair of teeth in action is generally subjected to two types of cyclic stresses:

1. Bending stresses inducing bending fatigue
2. Contact stresses causing contact fatigue [1].

Both these types of stresses may not attain their maximum value at the same point of contact. Bending stresses inducing bending fatigue Contact stress causing contact fatigue. Both these types of stresses may not attain their maximum values at the same point of contact. [1] However, combined action of both of them is the reason of failure of gear tooth leading to fracture at the root of a tooth under bending fatigue and surface failure, due to contact fatigue.S.C.Mohanty suggested an analytical method to calculated the individual tooth load during meshing cycle, also he referred to determination of the locations and sizes of contact zones along the path of contact for high contact ratio gearing (3<CR<2) [2]. A.H.Elkholy introduced a method to determine tooth load sharing especially for high contact ratio spur gear. [3].

High performance gears such as aerospace and automotive, for example – do not use standard basic rack for design. Instead, they rely on custom racks which provide higher pressure angles or contact ratio. Direct gear design method introduces an alternative gear design approach to maximize gear drive performance in custom gear applications. The gear tooth under consideration for NCR gearing is addendum of one unit module and a full depth of 2.25 times the module and the gear tooth for HCR gear is a full depth of 2.75 times the module and addendum of 1.25 times the module.The objective of this study focuses on the reduction of stresses occurs on HCR Spur gear by means of different contact ratio. The Bending stress has to be analyzed by means of analytical and FEA method.

II. SPUR GEAR

Spur gears are the most common type of gears. They have straight teeth, and are mounted on parallel shafts. Sometimes, many spur gears are used at once to create very large gear reductions.Spur gears are used in many devices like the electric screwdriver, dancing monster, oscillating sprinkler, wind up alarm clock, washing machine and dryer drive etc.

. Figure 1 Spur Gear

A. Contact Ratio

Contact ratio can defined as a number of teeth in contact at one time as these teeth pass through the contact zone. It is impractical to make contact ratio (CR) less than unity. If the contact ratio is one, then one pair of teeth leaves contact just as the next pair begins contact. This is undesirable, because slight error in the tooth spacing will cause oscillations in the velocity, vibration and noise. Gears should not generally be designed having contact ratios less than about 1.2, because inaccuracies in the mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level. Thus, even though a contact ratio 1.2 is acceptable, a minimum contact ratio of 1.4 is preferred and larger is better [4].High contact ratio can be achieved by different ways namely: increasing the number of teeth, lowering the pressure angle and increasing the addendum factor. A contact ratio of 2.0 means that there are always two pairs in contact, i.e. at the instant when one pair goes out of contact, a new pair comes in contact. When double pairs of teeth are engaged, the transmitted load will be divided between two meshing teeth.

$$CR = \frac{ARC}{P_c} = \frac{PTH}{P_b \cos\varphi} = \frac{PTH}{P_b} \quad (1)$$

where, CR = contact ratio;
ARC = arc of contact;
PTH = path of contact;
φ = pressure angle;
P_c =baser pitch
P_b = circular pitch

B. Teeth Pair Load Sharing

The pinion tooth diagram shown in Fig.2 is marked into seven different alphabetical points on the right side to show how the teeth pairs come into contact when they pass the mesh zone.

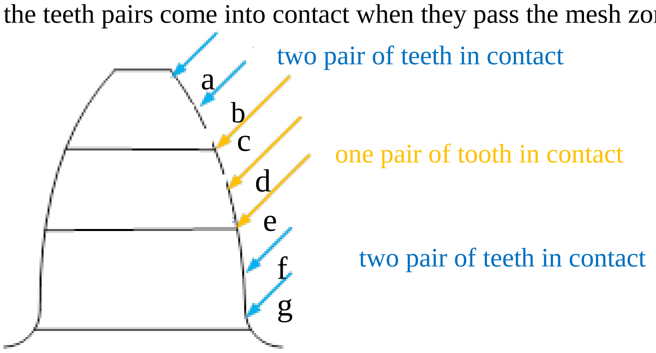

Figure 2. Pinion teeth

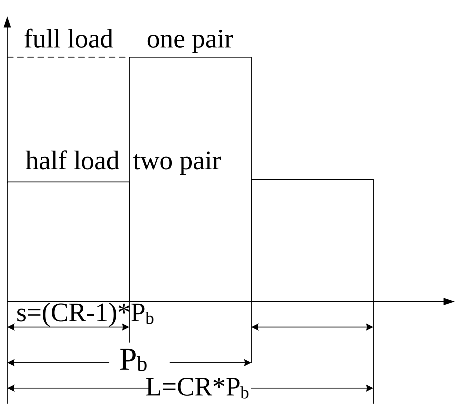

Figure 3.Loadingalong the path of contact

In Fig.2 and Fig 3, the points are located along the line of action within the interval of the length of action. Considering contact ratio 1.4 and above, the initial contact point for pinion tooth with the gear tooth is point 'g', and simultaneously for the second tooth of the same pinion, which is already in mesh will be at point 'c'. Therefore, the load will be shared between these two points similarly points 'f' and 'b', also 'e' and 'a' will have simultaneous action. Then point 'a' will goes out of contact, therefore the full load will be applied starting from point 'e', through point 'd' until the contact begin at point 'c', then new meshing tooth comes into contact [5].When double pairs of teeth are engaging, the transmitted load will divide between two meshing teeth. Load sharing depends on contact ratio and stiffness of meshing tooth at point of application of load. For this reason the load sharing at the point 'd', which is critical position for this case.

C. Material Properties

In this paper , the material of AISI-5160 alloy steel is used. AISI-5160 steel is alloy which containing magnesium, titanium and chromium.This material is used in vehical technology. This alloy steel can improved strength, toughness and corrosion resistance of gear.

Table 1. Material properties

Name	Value	units
Material	AISI-5160	-
Yield stress	1790	MPa
Modulus of elasticity	206.9	GPa
Poisson's ratio	0.3	-
condition	OQT- 400	
Brinell Hardness	684	

D. Gear Parameter

The solid model created two spur gears in meshedas shown in Fig 4 by using the SOLIDWORKS software, using the material properties are tabulated in table 1 and gear geometry is given in Table 2.

Table2. Spur Gear Parameter

Parameter	Pinion	Gear
profile	involute	involute
No: of teeth	21	51
module	4mm	4mm
face with	15mm	15mm
Pressure angle	20 degree	20 degree
Fillet radius	1mm	1mm
Maximum torque	540 Nm	
speed	350 rpm	
Addendum factor	1.3m	1.29m

Figure 4. Spur Gear Mesh

III. Force on Gear teeth

The power is transmitted from the input shaft to the output shaft. The teeth of pinion drive the teeth of gear and thus transmit the power to the gear. Gears are used to transmit mechanical power and this required applying mechanical torque that can be calculated.For SI unit;

$$\text{Power, } P = \frac{2\pi NT}{60} \qquad (2)$$

where , P is power transmitted by gears, Watt
N is speed of rotation,rpm
T is torque transmitted by gear, Nm

A. Transmitted Force

.

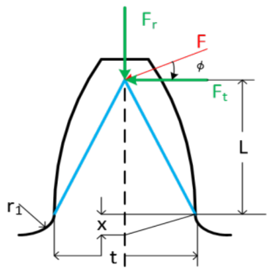

Figure 5. Load Transmitted on Spur Gear

The reaction force F between the mating teeth occur along the pressure line, and the power is transmitted by means of the force exerted by the tooth of the driving gear to the mesh of the driven gear. The reaction force F can be resolved into two components: a tangential force F_t and radial force F_r shown in Fig. 5.

B.Bending Theory

Wilfred Lewis in 1893 provides a formula for estimating the bending stress in a tooth. A gear tooth was taken the full load at its tip as simple cantilever beam [6]. If a gear tooth for the rectangular beam, it can find the critical point in the root fillet of the gear by inscribing a parabola. In the Lewis Analysis some assumptions are to be considered which are as follows:

1. The Gear tooth act as a cantilever beam.
2. The Tangential component causes the bending moment about the base of the tooth.
3. The effect of Radial components is neglected.
4. The tangential component is uniformly distributed over the face width of tooth.
5. The effect of stress concentration is neglected.
6. It is assumed that at any time only one tooth will be in contact and takes the total load.

The weakest section of the gear tooth is at the section BC, where the parabola is tangent to the tooth profile. At the section BC the moment will be,

$$\sigma = \frac{M}{I/c} = \frac{6F_t \times h}{bt^2}$$

$$\sigma = \frac{F_t \times P_d}{bY} \quad \text{(U.S. Customary Unit)}$$

$$\sigma = \frac{F_t}{b \times Y \times m} \text{ (SI Unit)}$$

C.Numerical Analysis of Bending stress for NCR Gear

Firstly, the following calculation is started at $3°$ for NCR gear and other different position is expressed in Table 3.1

For $F_t = 6721.266$ N
m=4 mm
Y= 0.328
b= 15 mm

Table 3. Tangential load and bending stress of different position for normal contact ratio gear (NCR)

case	Tangential load ,	Lewis's bending stress (MPa)
1	6721.266	341.527
2	6605.666	335.653
3	6493.819	329.970
4	12771.392	648.953
5	12562.522	638.339
6	12360.090	628.053
7	6082.177	309.054
8	5987.227	304.229
9	5895.196	299.552

D.Numerical Analysis of Bending stress for HCR Gear

And then, to apply consider to formulation case of 4mm module, 21 teeth pinion and 51 teeth gear, pinion operating in 350 rpm, where the pressure angle is $20°$. But, the addendum factor 1.3 and profile shift factor is 0.4 is used for high contact ratio gear. The length of contact and contact ratio are computed and they are 18.893 mm and 1.6 respectively. While from Equation 2.11 the corresponding angle of contact is 0.4759 radian or $\approx 27°$, consider that the contact will start at angle $0°$ and end at angle $27°$. The select angular interval value is $3.8°$, so the progress of contact will be studied for each $3.8°$, which means there are 9 cases of contact under consideration. From high contact ratio gear, 9 different positions were calculated maximum contact stresses, these values expected as shown in below.

For $F_t = 5357.021$ N
m=4 mm
Y= 0.328
b= 15mm

Table 4. Tangential load and bending stress of different position for normal contact ratio gear (NCR).

case	Tangential load , F_t	Lewis's bending stress (MPa)
1	5375.021	272.206
2	6105.399	310.233
3	6835.778	347.346
4	7566.156	384.456

5	8296.535	421.572
6	7526.658	382.452
7	6796.279	345.339
8	6065.901	308.227
9	5335.523	271.114

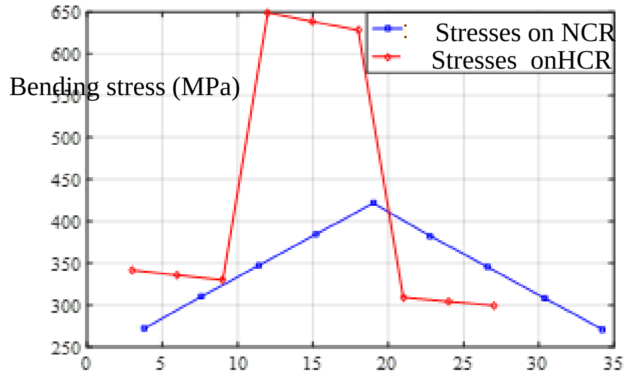

Figure 6. Lewis Bending Stress Vs Rotational Angle for NCR and HCR Gear

The normal contact ratio gearing Figure 6, the bending stress (Lewis) varies from 341.527 MPa at the root to 329.970 MPa at the start of single tooth contact, corresponding to less than 12.0°, then after sudden increase to 648.953 MPa at the degree rotation 12.0, the stress maintains to increase to 628.053 MPa till the rotation comes to 18.0°, then the stress goes to sudden decrease at the end of single pair tooth contact to 309.054 MPa at 21.0° rotation, gradually decrease to 299.552 MPa at the tip part of the tooth, in a manner to the load sharing pattern.When the high contact ratio gearing Figure 6, the maximum bending stress on the tooth is 421.572 MPa corresponding to 19.0° which is at the near pitch circle. At the root part and tip of tooth the stress goes decreasing to 272.206 MPa and 271.114 MPa respectively.

E. Simulation of Bending Stress Analysis

In the procedure for generating a FEA model for bending stress analyses, the equations used to generate the gear tooth profile curve were the same. When meshing the teeth is ANSYS, if “SMART SIZE” is used the number of elements near the roots ofthe teeth are automatically smaller than in other places. Figure 7 shows the mesh 3D model of NCR gear tooth and Figure 10 shows the mesh 3D model of HCR gear tooth.

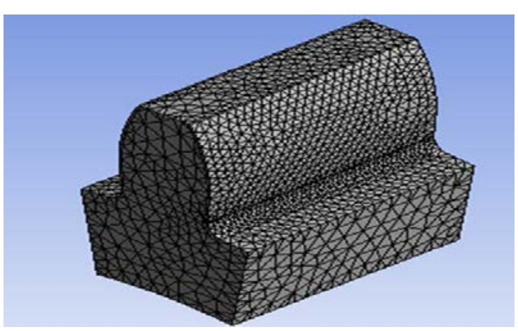

Figure 7. Mesh of NCR Gear Tooth

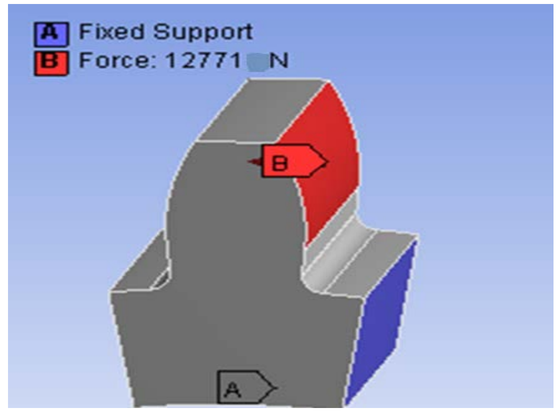

Figure 8. Boundary Condition and Force Applied

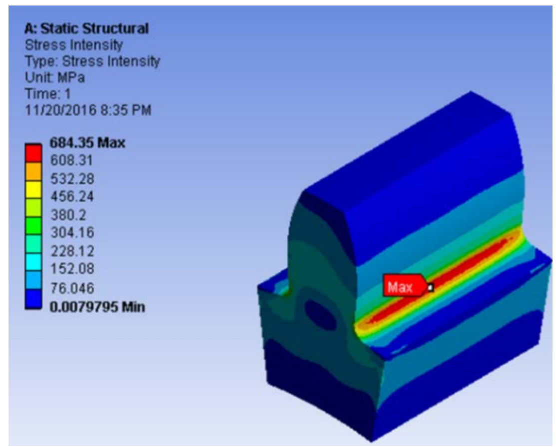

Figure 9. Maximum Stress at the Root of Tooth for NCR Gear

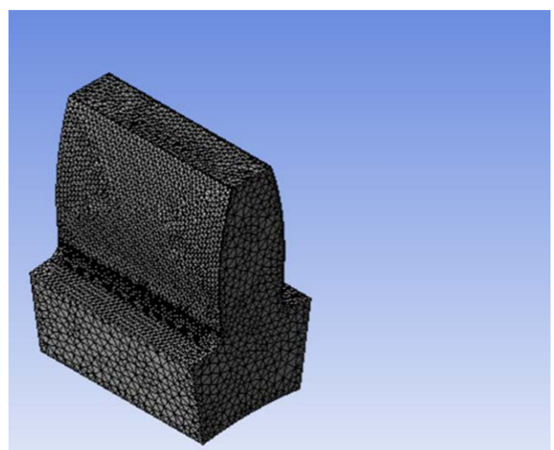

Figure 10. Mesh of HCR Gear Tooth

Figure 8 and Figure 11 are showed the boundary condition and applied force for NCR and HCR gears. Figure 9 and Figure 12 showed that the maximum tensile stress on the tensile side and maximum compressive stresses on other side of the tooth, respectively. It also indicated that only one tooth is enough for the bending stress analysis for the 3D model.

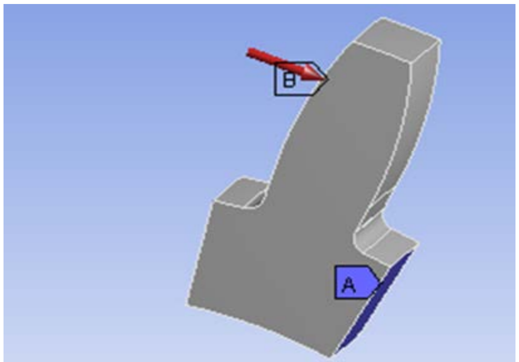

Figure11. Boundary Condition and Force Applied

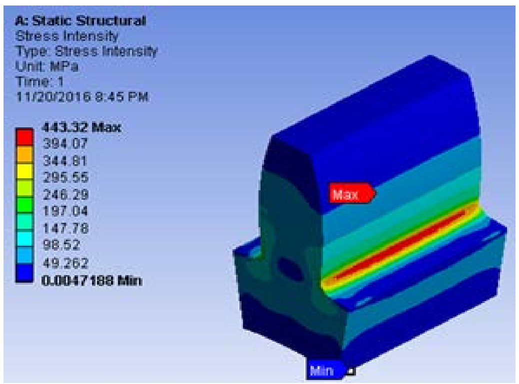

Figure12. Maximum Stress at the Root of Tooth for NCR Gear

F.Comparison of Theoretical and Simulation

For determining the stresses at any stage during the design of gears face width, number of teeth and root fillet radius are important parameters. To determine the stress variations with the addendum factor relative to gear set weight models of spur gear are made by keeping constant other parameter i.e. pressure angle, tooth thickness etc. The combined effect of these parameters on bending stress also indicated in Figure 5.5 and Figure 5.8. Then NCR gear tooth is changed HCR gear tooth, the different bending stress value obtained. The following table is showed Lewis bending stress, AGMA bending stress and ANSYSsoftware resulted for 21 teeth of (NCR and HCR) spur gear.

Table.5. Comparison of Lewis Bending Stress and ANSYS Software

case	Lewis bending stress (MPa)	ANSYS software (MPa)	% different
NCR	648.953	684.350	5.1
HCR	421.572	443.320	4.9

This table showcases a general view of geometry of spur gear under static loading. It states the Lewis equation which is conventionally used for the stress calculation of a gear tooth further it gives Finite element analysis method for the same using ANSYS 15.0. The results shows the effect of high contact ratio on the bending stresses of the gear tooth and compare the normal contact ratio.

IV. CONCLUSION

In this paper load sharing of normal contact ratio and high contact ratio values using MATLAB code and Lewis bending equation have been used to find the bending stresses and compare finite element analysis (ANSYS).It allows reducing the maximum bending stress in the high contact ratio (HCR) gear tooth area 35%. However load carrying capacity can be increased by reducing the bending stress using the high contact ratio gear which in turn can be reduce the contact stress.

ACKNOWLEDGMENT

The author wishes to express the deepest thanks and gratitude to her supervisor Dr. Than ThanHtike, Professor,Mechanical Engineering Department of Yangon Technological University The author special thanks go to Dr .KhinThu Zar, Demonster, Mechanical Engineering Department of Technological University (Thanlyin), for her invaluable advice and effective suggestion throughout the study. The author would like to express her thanks to her member Daw Chaw Wint Yee Zaw, Lecture, Mechanical Engineering Department of Technological University (Thanlyin), for her valuable comments and indispensable guidance during this study. Finally, her special thanks to all who helped her towards the successful completion of this study.

REFERENCES

[1] M.V. Vamis, S.B., A.Sriram, "Spur Gear Tooth Stress Analysis and Stress Reduction", B-Tech Project Report, 2013.

[2] S.C, M., "Tooth Load Sharing and Contact Stress Analysis of High Contact Ratio Spur Gears in Mesh", National Convention of Mechanical Engineers, Roukela, 2002.

[3] A.H, E., "Tooth Load Sharing in High Contact Ratio Spur Gears", Journal of Mechanisms, Transmissions and Automation in Design, 1985. **107**(1): p. 11-16..

[4] E. H. Miller, "A note on reflector arrays (Periodical style—Accepted for publication)," *IEEE Trans. Antennas Propagat.*, to be published.

[5] R.L, N., "Design of Machinery", Mc.Graw-Hill Companies Inc, U.S.A, New York, 2001.

[6] Maitra, G.M., "Handbook of Gear Design". Second Edition, 2001. Chapter-(2).

AUTHORS

First Author – khinkhinthant, ME, Technological university (Thanluin) and khinkhinthant05@gmail.com.
Second Author – Chaw Wint Yee Zaw, ME, Technological University (Thanlyin) and chawzaw7@gmail.com.

Correspondence Author – Author name, email address, alternate email address (if any), contact number