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Comparison for Level Control of a Coupled-Tank using PI, PI-plus-Feedforward, and IMC Controllers

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Abstract- This paper investigates the performance of various control schemes for level control of a coupled tank process. The nonlinear dynamic model of the system was derived using the analytical and empirical approaches. To investigate the performance of the controllers, proportional plus integral control, proportional plus integral plus feedforward control and internal model control (IMC) have been proposed. The PI gains were determined using pole placement, Ciancone correlation, and Cohen-Coon tuning techniques. Time response specification and mean absolute error (MAE) are used to assess the level control performance of the designed controllers. Comparative MATLAB simulation assessments have shown that IMC with the least MAE value and fastest settling time has the best tracking performance as compared to other controllers.

Index Terms- coupled-tanks, Ciancone, Cohen-Coon, IMC, feedforward.

I. INTRODUCTION

Liquid level control is an integral aspect of many industrial processes. Some of the notable areas where the level control is essential include petrochemical industries, power generation, water treatment plants and food processing among others. A typical level control can be seen in a storage tank, where the desired level of water or other products is controlled at a specific set point. The liquids are processed by mixing or chemical treatment in the tanks. In many cases, the tanks are often coupled together such that an interaction between the levels and flows exist. In addition, various sensors exist for measuring these levels such as capacitive type sensor, float sensor and differential pressure sensor [1-2].

For safety, cost implication and other issues, simulation of the open loop and closed loop response is designed and analyzed before implementing on the real system. However, a reliable model which represents the system behavior must be derived using either the analytical approach or the empirical method before conducting the simulation. Depending on the system, empirical approach is employed where the system is difficult to model. Though, some dynamics of the system are ignored using this approach [3-4].

Interestingly, many researchers have employed different control schemes, mainly feedback techniques for level control of a nonlinear coupled tanks system. These includes comparative study of various tuning methods [5-6] and PI controller [3-4]. However, due to its highly nonlinear behavior, single feedback control is not sufficient to achieve good performance. Hence, modified or advanced control techniques have been introduced such as hybrid genetic-immune PI tuning [1], sliding mode control [2], nonlinear backstepping control [7], feedforward plus sliding mode control [8], internal model control [9] etc.

In this paper, a comparison for pole placement, Cohen Coon and Ciancone based proportional plus integral (PI) control, PI-plus-feedforward control and internal model control (IMC) is presented. A coupled tank system is considered. The nonlinear dynamic model of the coupled tanks is derived using analytical and empirical approaches. The designed controllers were simulated in the MATLAB environment. The performances of the designed controllers are investigated in time domain based on the mean absolute error and time response specifications. These results will provide useful information for the selection of appropriate control strategy for efficient control of liquid level.

The paper is organized into seven sections. These are:

1) Abstract
2) Introduction
3) Description of the coupled-tank system
4) Mathematical modeling of the coupled-tank
5) Design of controllers
6) Implementation and discussions of results
7) Conclusions.

II. THE COUPLED TANK SYSTEM

A lab scale coupled tank is one of the commonly used systems for control research and education at various institutions. Fig. 1 shows the schematic diagram of the coupled tanks considered for this study. As shown, it consists of a two-tank module with a pump connected to a water basin. The two tanks are configured such that the upper tank (tank 1) flows through an orifice outlet located at the bottom of the tank into the lower tank (tank 2). The outflow of the lower tank flows into the main water reservoir. The pump thrusts water upwards to the orifice "out1" before entering tank 1. The two tanks have identical parameters which are: tanks inside diameters (D_{t1}= D_{t2} = 4.445 cm), orifice diameters (D_{o1}= D_{o2} = 0.47625 cm), cross sectional areas (A_{t1}= A_{t2} = 15.5197 cm^2), outlet hole cross sectional areas (A_{o1}= A_{o2} = 0.1781 cm^2), pump constant (K_p=3.3 cm3/s/V), gravity (g=981cm/s^2), pump input voltage (V_p) and water level in tanks ($L_1$and L_2) in cm.

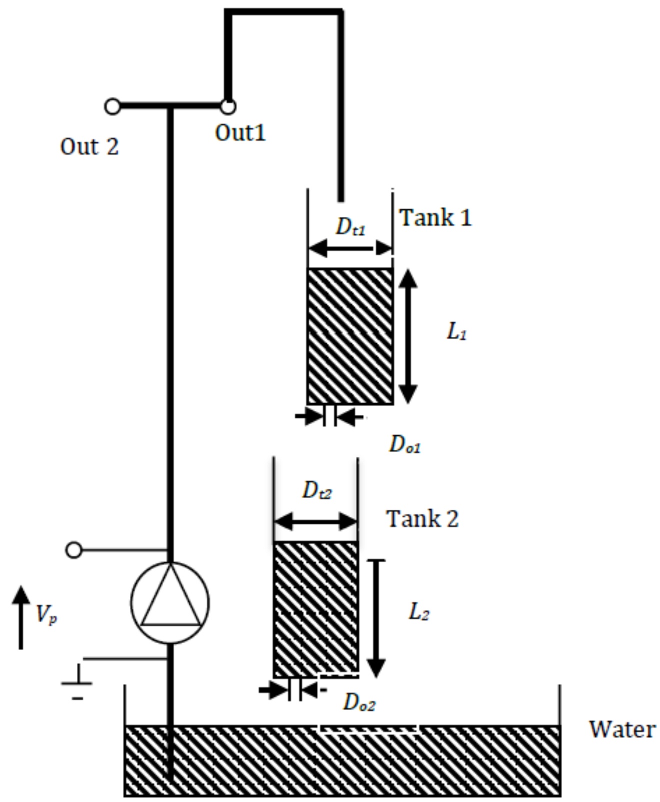


Fig. 1. Schematic diagram of coupled-tanks process.

III. MODELING OF THE COUPLED TANKS

In this section, the modeling of the coupled tanks is presented using analytical and empirical methods. The pump voltage and the level in tank 2 are the input and output of the system respectively. The main target is to maintain the level of tank 2 at a particular set point. The nonlinear model of the coupled tanks is expressed based on the defined parameters of section II.

A. The Analytical Model

By using the mass balanced law, the rate of change of the level in each tank can be obtained. The inflow to tank 1 can be expressed in Eq. 1 and by applying Bernouli's law for flow through the orifice at the bottom of tank 1, the outflow rate is given in Eq. 2.

$$Q_1^{in}(t) = K_p V_p \quad (1)$$

$$Q_1^{out}(t) = A_{o1}\sqrt{2gL_1} \quad (2)$$

Similarly, the inflow flow to tank 2 is the same as the outflow from tank 1 as given in Eq. 3. Therefore, the dynamic model for the coupled tanks can be written in Eq. 4 and Eq. 5.

$$Q_2^{in}(t) = Q_1^{out}(t) \quad (3)$$

$$\frac{\partial L_1}{\partial t} = -\frac{A_{o1}}{A_{t1}}\sqrt{2gL_1} + \frac{K_p}{A_{t1}}V_p \tag{4}$$

$$\frac{\partial L_2}{\partial t} = \frac{A_{o1}}{A_{t2}}\sqrt{2gL_1} - \frac{A_{o2}}{A_{t2}}\sqrt{2gL_2} \tag{5}$$

where the cross-sectional area of tank 1 and its outlet hole (similar representation as tank 2) are given as in Eq. 6.

$$A_{o1} = \frac{1}{4}\pi D_{o1}^2 \quad ; \quad A_{t1} = \frac{1}{4}\pi D_{o1}^2 \tag{6}$$

Due to the square roots function of L_1 and L_2 of Eq. 4 and 5, the two first order equations are nonlinear. In order to design a linear controller, the equations should be linearized about an operating point. A Taylor's series approximation is utilized for the linearization as in Eq. 7. The approximate square root of L1 about an operating point (L_{10}) of tank 1 and L_{20} for tank 2 can be obtained as in Eq. 8. Thus, the nonlinear models of Eq. 4 an 5 can be linearized into Eq. 9 and Eq. 10.

$$f(x) = f(x_0) + (x - x_0)\, f'(x_0) + \ldots \tag{7}$$

$$\sqrt{L_1} = L_{10} + \frac{1}{2}L_{10}^{-1/2}(L_1 - L_{10}) \quad ; \quad \sqrt{L_2} = L_{20} + \frac{1}{2}L_{20}^{-1/2}(L_2 - L_{20}) \tag{8}$$

$$\frac{\partial \tilde{L}_1}{\partial t} = -\frac{A_{o1}}{A_{t1}}\sqrt{\frac{g}{2L_{10}}}\tilde{L}_1 + \frac{K_p}{A_{t1}}\tilde{V}_p \tag{9}$$

$$\frac{\partial \tilde{L}_2}{\partial t} = \frac{A_{o1}}{A_{t2}}\sqrt{\frac{g}{2L_{10}}}\tilde{L}_1 - \frac{A_{o2}}{A_{t2}}\sqrt{\frac{g}{2L_{20}}}\tilde{L}_2 \tag{10}$$

$$\tilde{L}_1 = L_1 - L_{10} \quad ; \quad \tilde{L}_1 = L_1 - L_{10} \quad ; \quad \tilde{V}_p = V_p - V_{pss} \tag{11}$$

Since the parameters of tank 1 and tank 2 are the same, $L_{10} = L_{20}$. In this study, the operating level is assumed to be 15 cm. To determine the steady state pump voltage (V_{pss}) that yields a steady state level (L10) in tank 1, the left-hand side of Eq. (9) is set to zero as obtained in Eq. 12.

$$\tilde{V}_{pss} = \frac{A_{o1}}{K_p}\sqrt{2gL_{10}} = 9.26V \tag{12}$$

The state space representation for the system and the transfer function of tank 1 and tank 2 of the system with respect to the input voltage using the system parameters can be respectively obtained Eq. 13 and Eq. 14.

$$A = \begin{bmatrix} -0.0656 & 0 \\ 0.0656 & -0.0656 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0.2127 \\ 0 \end{bmatrix} \quad ; \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad ; \quad D = \begin{bmatrix} 0 \end{bmatrix} \tag{13}$$

$$G_1(s) = \frac{\tilde{L}_1}{\tilde{V}_p} = \frac{3.24}{15.24s + 1} \quad ; \quad G_2(s) = \frac{\tilde{L}_2}{\tilde{L}_1} = \frac{1}{15.24s + 1} \tag{14}$$

B. The Empirical model

To obtain the dynamic model using the empirical approach, a steady state pump voltage is applied to the nonlinear dynamic models represented in Fig. 2. The output response of level in tank 2 is monitored. Note that before applying the empirical analysis, the following steps should be followed.

1) The process must be at steady state initially,
2) A step change is applied to the process as an input set point,
3) Finally, the process must attain a steady state for the applied set point input.

The empirical model is generally represented as first order system expressed in Eq. 15, where τ is the time constant, θ is the dead time and K_p is the process gain represented in Eq. 16.

$$G_p(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \tag{15}$$

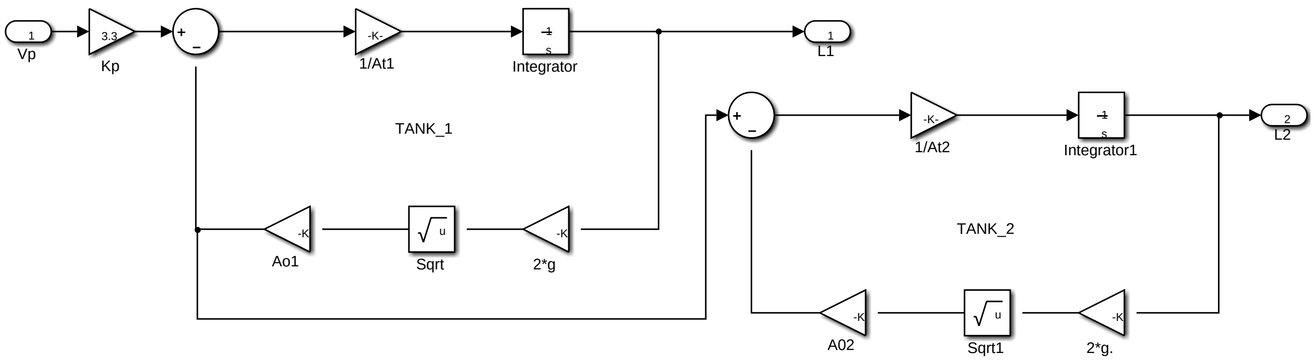


Fig. 2. Simulink block for the nonlinear coupled tank model.

$$K_p = \frac{\Delta L_2}{\Delta V_p} \;\; ; \;\; \tau = 1.5(t_{63\%} - t_{28\%}) \;\; ; \;\; \theta = t_{63\%} - \tau \tag{16}$$

where ΔL_2 is the change of output level in tank 2, ΔV_p is the change of input voltage, $t_{63\%}$ and $t_{28\%}$ are the time taken by the response to reach 63% and 28% of the final settled value respectively. By using the output response of the tanks, the parameters of Eq. 16 can be obtained ang hence, Eq. 15 can express as the empirical model of the coupled tanks as in Eq. 17.

$$G_p(s) = \frac{1.62e^{-2.2s}}{22.82s+1} \tag{17}$$

IV. CONTROL DESIGN

In this section, various control strategies are presented. These include pole placement based proportional plus integral (PI) control, PI-plus-feedforward control and internal IMC control. To design a control scheme, the system must be controllable. To check for the controllability of the system, the determinant of the controllability matrix (G^c) must be nonsingular [10]. Thus, using Eq. 13, the controllability can be obtained as in Eq. 18.

$$G^C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0.2127 & -0.0139 \\ 0 & 0.0139 \end{bmatrix} \Rightarrow \left| G^C \right| \neq 0 \tag{18}$$

Therefore, the system is controllable and the proposed controllers can be designed. To simplify the control design, a constant flow is assumed from tank 1, thus, the steady state level of tank 1 is considered as the set point to tank 2.

A. Pole placement-based PI controller

The PID control families are most widely used control techniques in the industries due to their design simplicity and applicability to many processes. The proportional gain increases the system responses and improved the closed loop stability. However, it leaves a trace of an offset error and large proportional gain lead to instability of the system. The integral gain clears the steady state offset but larger integral time delayed the settling time of the process. In the presence of overshoots, the derivative gain is added to control. Though, it amplifies noise for noisy systems. Therefore, appropriate selection of the control depends on the system dynamics. For any of the PID controls, the error value is calculated by taking the difference between the set-point and the measured controlled variable. The control tries to make the as close to zero as possible.

To design pole placement control for tank 2, certain design specifications must be outline. In this work, percentage overshoot (PO) of 10% and settling time (ts) of 20 s are considered. Consider a standard feedback characteristics equation with negligible sensor and valve dynamics given in Eq. 19, where $G_p(s)$ is the plant transfer function and $G_c(s)$ is controller given in Eq. 20.

$$1 + G_c(s)G_p(s) \tag{19}$$

$$G_c(s) = K_c + \frac{K_i}{s} \tag{20}$$

where K_c is the proportional gain and K_i is the integral gain. Thus, substituting for Eq. 14 and Eq. 20 in Eq. 19 yields the characteristics equation in Eq. 21.

$$s^2 + \frac{(1+K_pK_c)}{\tau}s + \frac{K_iK_p}{\tau} \tag{21}$$

In addition, consider the standard second order characteristic equation given in Eq. 22, where ζ is the damping ratio and ω_n is the natural frequency of the plant as in /

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \quad (22)$$

$$\zeta = \frac{\left|\ln \frac{1}{100} PO\right|}{\sqrt{\left(\ln \frac{1}{100} PO\right)^2 + \pi^2}} \quad ; \quad \omega_n = \frac{4}{\zeta t_s} \quad (23)$$

Finally, comparing Eq. 21 and Eq. 22 yields the PI control gains in Eq. 24 and by substituting the design specifications in Eq. 23 and solving for Eq. 24 gives the control gains as in Eq. 25.

$$K_c = \frac{(2\zeta\omega_n\tau - 1)}{K_p} \quad ; \quad K_i = \frac{\omega_n^2\tau}{K_p} \quad (24)$$

$$K_c = 5.1 \quad ; \quad K_i = 1.7 \quad (25)$$

B. Cohen-Coon PID Tuning

In this type of control gains tuning, an open loop response is considered. As an empirical approach, similar procedures as outline above are followed. The Cohen – Coon formulations requires process with dead time as shown in Table I [11]. In this work, PI gains are considered and based on the open loop response of Eq. 17 and using the Table I formulations, the PI control gains are obtained as in Eq. 26.

TABLE I. COHEN COON TUNING FORMULATIONS

Type	K_c	K_i	K_d
P	$\left(\frac{\tau}{K_p\theta}\right)\left(1+\frac{\theta}{3\tau}\right)$	-	-
PI	$\left(\frac{\tau}{K_p\theta}\right)\left(0.9+\frac{\theta}{12\tau}\right)$	$\theta\left(\frac{30+\frac{3\theta}{\tau}}{9+\frac{20\theta}{\tau}}\right)$	-
PID	$\left(\frac{\tau}{K_p\theta}\right)\left(\frac{4}{3}+\frac{\theta}{4\tau}\right)$	$\theta\left(\frac{32+\frac{6\theta}{\tau}}{13+\frac{8\theta}{\tau}}\right)$	$\theta\left(\frac{4}{11+\frac{2\theta}{\tau}}\right)$

$$K_c = 4.06 \quad ; \quad K_i = 7.75 \quad (26)$$

C. Ciancone correlation-based PI controller

Ciancone based tuning is also an empirical method and it is applied to processes with dead time using the Ciancone correlation graph. The aforementioned steps for the empirical model analysis are also followed for this tuning as well. By using the parameters of Eq. 17, the fraction dead time is obtained as in Eq. 27 and by tracing the value of Eq. 27 on the Ciancone correlation graph yields the PI control gains as in Eq. 28.

$$\frac{\theta}{(\theta+\tau)} \quad (27)$$

$$K_c = 0.9259 \quad ; \quad K_i = 18.51 \quad (28)$$

D. PI plus Feedforward control

Feedback plus feedforward technique is one of the advance control schemes commonly used in the process industries. This scheme significantly improves the performance of a process in the presence of a measured disturbance. The feedback control takes care of the measured variable with respect to the set point while the feedforward eliminates the effect of external disturbance to the process. In this study, the out flow from tank 1 is considered as the disturbance to tank 2. Thus, the feedforward control can be obtained by solving tank 2 nonlinear model of Eq. 10 at static equilibrium. Hence, the feedforward control gain can be obtained as in Eq. 29.

$$K_{ff}(s) = \frac{A_{o2}}{A_{o1}} = 1 \quad (29)$$

E. Internal Model Controller (IMC)

The main idea of IMC scheme is to obtain a good closed loop response from the open loop dynamic model. The internal model law states that acceptable control can be achieved if and only if the closed loop control encapsulates some dynamics of the process. Thus, the controller depends on the accuracy of the derived model, because the controller would have the inverse dynamics of the plant in order to perfectly track the reference input [12]. Fig. 3 shows the general block diagram of an IMC scheme. The IMC design is in two phases. First, the process model (G_m) is factored into the invertible part (G_{m-}) and the non-invertible part (G_{m+}). The G_{m+} contains the time delays and the right half plane zeros. The controller (G_c) is given in Eq. 29, where G_f is a low pass filter with the general form shown in Eq. 30 for N represents the order of the model in order to get a perfect poles zero cancellation, τ_c is the controller time constant which is the critical design parameter of IMC scheme. Depending upon the system, the filter time constant can have the value of the dead time of the system. After block reduction technique, the closed loop controller (G_{cc}) can be expressed in Eq. 31 with negligible valve dynamics ($G_v \approx 0$).

$$G_c(s) = \frac{G_f(s)}{Gm^-} \quad (29)$$

$$G_f(s) = \frac{1}{(\tau_c s + 1)^N} \quad (30)$$

$$G_{cc}(s) = \frac{G_c(s)}{1 - G_c(s) G_m(s)} \quad (31)$$

By applying Pade approximation of the dead time, Eq. 17 can express in terms of PID control gains known as IMC-PID based tuning as shown in Table II.

TABLE II. IMC-PI BASED TUNING RULES

Type	K_pK_c	K_i	K_d
PI	$\frac{\tau}{\tau_c}$	τ	-

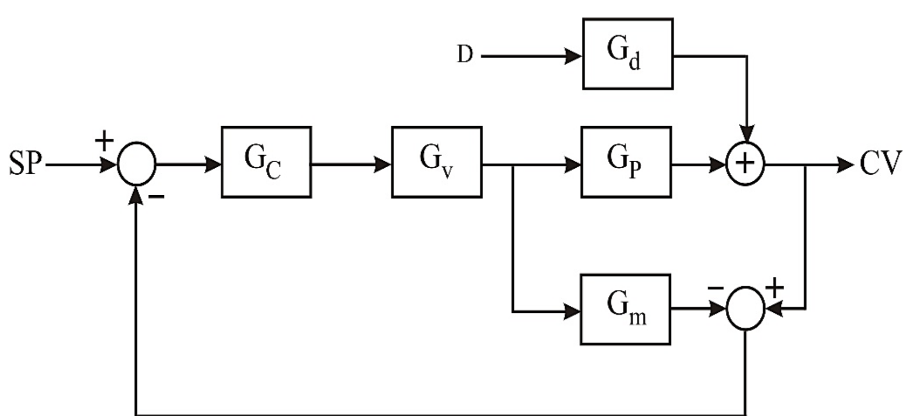
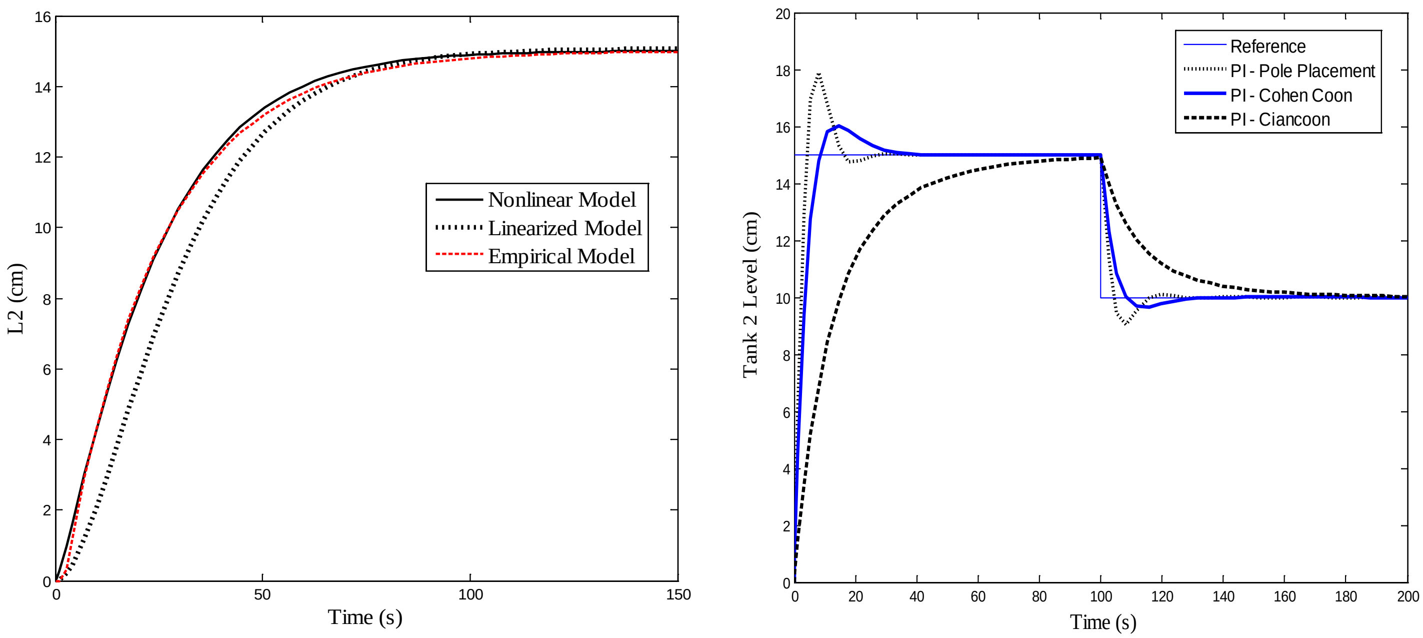


Fig. 3. Block diagram of IMC scheme

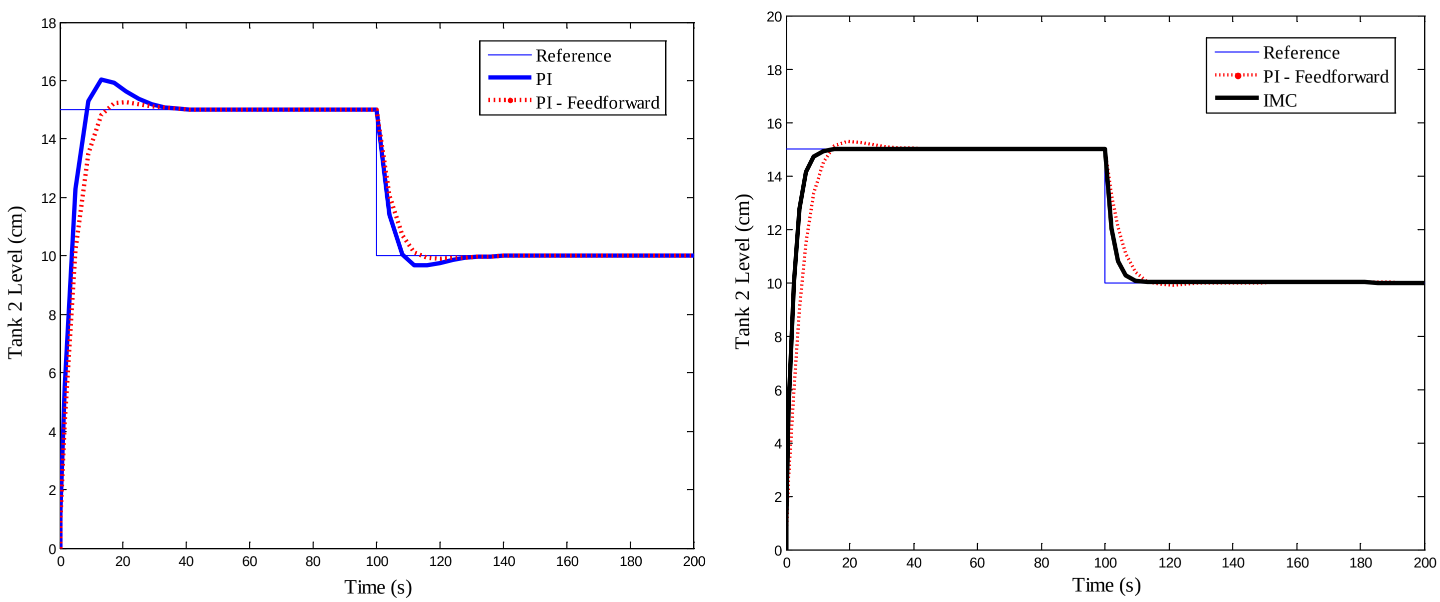
V. IMPLEMENTATION OF RESULTS AND DISCUSSIONS

In this section, MATLAB simulation analysis of the results from the designed controllers on a coupled tank process is presented. A reference level starting from zero position to settle at 15 cm and then decrease to 10 cm is used as the tracking indices for the controllers. Fig. 4(a) shows the comparison of the nonlinear model, empirical and linearized model subjected to an input voltage. It can be observed that the empirical model represents the system more closely. Fig. 4(b) shows the set-point tracking performance for the PI controller tuned using pole placement, Ciancone correlation and Cohen-Coon methods. It can be seen that Cohen-Coon correlation gives the best performance in terms of least maximum overshoot and fast settling time while Pole placement demonstrated faster response and settling time but with higher overshoot. However, Ciancone has the poorest performance in terms of response time. In addition, comparing the best feedback control with the PI plus feedforward based on Cohen-Coon shows that adding the feedforward action improves the tracking performance of the of the single PI control as shown in Fig. 5(a). Also, Fig. 6(b) shows the comparison of IMC-PI based on the PI plus feedforward control schemes. The summary of the time response specifications and MAE values for all the controllers are shown in Table III. The smaller the MAE value, the better the controller, it is noted that IMC as the advance technique gives the best tacking performance as compared to the remaining designed controllers.



(a) (b)

Fig. 4: (a) Open loop response of the tank 2 level (b) Comparison for PI level control using Pole placement, Cohen Coon and Ciancone tunings



(a) (b)

Fig. 5: (a) Comparison for PI and PI plus feedforward level controllers (b) Comparison for PI plus feedforward with IMC scheme for level control

TABLE III. PERFORMANCE ANALYSIS OF THE CONTROLLERS

Controller	***Settling Time (s)***	***Max. Overshoot (%)***	***MAE Value***
PI-Pole Placement	30	20%	2.50
PI-Ciancone	100	0	3.65
PI-Cohen Coon	33	7%	2.70
PI plus Feedforward	29	3%	2.61
IMC	12	0	2.40

VI. CONCLUSION

Investigations into level control techniques for a coupled tank process using the PI controller, PI plus feedforward and IMC scheme have been presented. The nonlinear dynamic model of the system was derived using the analytical and empirical approaches. Simulations of the dynamic model of a coupled tank have been performed to study the effectiveness of the controllers. The results of the proposed controllers showed a significant tracking performance using all the controllers. The performances of the controllers demonstrated that IMC scheme provides the best level tracking followed by PI plus feedforward control as compared to the single PI controller.

REFERENCES

[1] S. M. Nawi, A. N. Abdalla, and M. S. Ramli, "Improved Coupled Tank Liquid Levels System Based on Hybrid Genetic-Immune Adaptive tuning of PI Controller," *Int. Conf. Electr. Control Comput. Eng.*, pp. 247–252, 2011.

[2] A. Hur, "Sliding Mode Control for Coupled-tank Liquid Level Control," *10th Int. Conf. Front. Inf. Technol.*, pp. 325–331, 2012.

[3] A. Nurnsomrarr, T. Suksri, and M. Thumma, "Design of 2-DOF PI Controller with Decoupling for Coupled-Tank Process," *Int. Conf. Control , Autom. Syst.*, pp. 339–344, 2007.

[4] I. R. Danica, "Robust PI Controller Design for Coupled-Tank Process : LMI Approach," *2013 Int. Conf. Process Control*, pp. 147–151, 2013.

[5] V. L. R. Haneema, "Comparative Study of Various Level Control Techniques for a Two Tank System," *2nd Int. Conf. Innov. Inf. Embed. Commun. Syst.*, pp. 1–6, 2015.

[6] R. Kumar, S. K. Singla, and V. Chopra, "Comparison among some well known control schemes with different tuning methods," *J. Appl. Res. Technol.*, vol. 13, pp. 409–415, 2015.

[7] R. B. Enayache, L. C. H. Laoui, X. D. Ovifaaz, and P. B. Ussy, "Real-time Nonlinear Adaptive Backstepping Liquid Level Control nonlinear adaptive backstepping liquid level control for a for a State Coupled Three Tank System state coupled three tank system," *Proc. Eur. Control Conf.*, pp. 1011–1016, 2009.

[8] B. B. Musmade and B. M. Patre, "Feedforward-plus-sliding mode controller design with experimental application of coupled tank system," *Trans. Inst. Meas. Control*, vol. 35, no. 8, pp. 1058–1067, 2015.

[9] U. M. Nath, "Centralized Auto-tuned IMC-PI Controllers for Industrial Coupled Tank Process with Stability Analysis," *2015 IEEE 2nd Int. Conf. Recent Trends Inf. Syst.*, pp. 296–301, 2015.

[10] A. B. Alhassan, B. B. Muhammad, K. A. Danapalasingam, and Y. Sam, "Optimal Analysis and Control of 2D Nonlinear Gantry Crane System," *IEEE Int. Conf. Smart Sensors Appl.*, pp. 30–35, 2015.

[11] D. Seborg, T. Edgar, *Process Dynamics and Control*, 2nd ed. Hamilton Printing, 2003.

[12] T. Tham, *Internal Model Control for Chemical and Process Eng.*, University of Newcaslte upon Tyne, 2002.

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