

# Best Linear Unbiased Estimation of Location and Scale Parameters in Generalized Exponential Distribution under Type-II Censoring

A. Vasudeva Rao, S. Bhanu Prakash\* and Sd. Jilani

Department of Statistics, Acharya Nagarjuna University, Nagarjunanagar -522510,  
Guntur, Andhra Pradesh, India.

\* Corresponding author: prakash9293165890@gmail.com

**Abstract-** For standard generalized exponential distribution (GED) Raqab and Ahsanullah [1] have derived the exact forms of means, variances and covariances of order statistics. Using these expressions they have obtained the necessary coefficients for computing the BLUEs of location and scale parameters of GE distribution for known shape parameter  $\alpha = 0.5(0.5)5.0$  for complete samples of size up to 10. In this paper, using the formulae given by Raqab and Ahsanullah [1], we have developed R-program for computing the means of order statistics for samples of size up to 30; and the variances and covariances of order statistics for samples of size up to 20 for standard GE distribution for  $\alpha = 1.5(0.5)5.0$ . Using these means, variances and covariances, we have extended the computation of the BLUEs of the location and scale parameters of GED for both complete sample and Type-II censored samples of size up to  $n=20$ . We have tabulated the BLUE coefficients for all complete samples of size  $n=11(1)20$  for  $\alpha=1.5(0.5)2.0(1.0)5.0$ . Further, we have developed R-code for computing the coefficients of the BLUEs of location and scale parameters based on any type II censored sample (including a complete sample) of size up to  $n=20$  and for any choice of shape parameter  $\alpha$  in the interval  $[1.5, 6.0]$ . Finally, we demonstrate the computation of the BLUEs with two data sets.

**Index Terms-** Generalized exponential distribution; Means, variances and covariances of order statistics; BLUEs of location and scale parameters.

## I. INTRODUCTION

Gupta and Kundu [2] introduced the three-parameter generalized exponential (GE) distribution with the probability density function (pdf)

$$f(x; \alpha, \mu, \sigma) = \frac{\alpha}{\sigma} e^{-(x-\mu)/\sigma} \left(1 - e^{-(x-\mu)/\sigma}\right)^{\alpha-1}; \quad 0 \leq \mu < x < \infty, \alpha > 0 \text{ and } \sigma > 0 \quad (1.1)$$

and cumulative distribution function (cdf)

$$F(x; \alpha, \mu, \sigma) = \left(1 - e^{-(x-\mu)/\sigma}\right)^{\alpha} \quad (1.2)$$

Here  $\mu$ ,  $\sigma$  and  $\alpha$  are location, scale and shape parameters respectively. Some authors, later, in many articles replaced the scale parameter  $\sigma$  in GE distribution with its reciprocal  $\lambda = 1/\sigma$ . However, in this paper, we consider the above form of three-parameter GE distribution (which is originally introduced by Gupta and Kundu [2]) because one can compute the BLUEs only for location and scale parameters of a distribution using Lloyd [3] method. We denote the three parameter GE distribution as  $GE(\alpha, \mu, \sigma)$ . Using a real data set, Gupta and Kundu [2] have shown that the three-parameter GE model (1.1) fits better than the three-parameter gamma or three-parameter Weibull in some practical situations.

Since,  $\mu$  and  $\sigma$  are location and scale parameters, the distribution of  $Z=(X-\mu)/\sigma$  is the standard GE distribution, whose pdf and cdf are given as

$$f(z; \alpha) = \alpha e^{-z} \left(1 - e^{-z}\right)^{\alpha-1} \text{ and } F(z; \alpha) = \left(1 - e^{-z}\right)^{\alpha}, \quad z > 0, \alpha > 0 \quad (1.3)$$

It is observed that the hazard function is nondecreasing if  $\alpha > 1$ , it is decreasing if  $\alpha < 1$  and for  $\alpha = 1$ , it is constant. The GE has a unimodal and right-skewed density function. As the shape parameter increases the skewness gradually decreases. If the data come from a right-tailed distribution, then the GE can be used quite effectively for analyzing them.

Gupta and Kundu [4] considered different estimation procedures and compared their performances through numerical situations. Raqab and Ahsanullah[1] derived exact expressions for the single and product moments of order statistics from the standard GE distribution and hence obtained the coefficients of the best linear unbiased estimates (BLUEs) of the location and scale parameters of the GE model (1.1) for complete samples of size up to 10 for shape parameter  $\alpha = 0.5(0.5)5.0$ . Raqab [5] derived the exact expressions for means, variances and covariances of record statistics from the GE distribution and computed the coefficients of BLUEs of location and scale parameters based on record statistics. Raqab [6] established several recurrence relations satisfied by the single and the product moments of order statistics from the standard GE distribution in terms of polygamma and hypergeometric functions and heexplained the recursive procedure of computing the single and the product moments of all order statistics for all sample sizes. Khan and Kumar [7] studied the explicit expressions and some recurrences relations for single and product moments of lower generalized order statistics from GE distribution.

The main aim of this paper is to extend the work of Raqab and Ahsanullah[1] by providing the coefficients of the BLUEs of location and scale parameters of the GE distribution (1.1) for both complete and type-II censored samples of size up to 20. In Section 2, we have developed R-code for computing the means, variances and covariances of order statistics of standard GE distribution by evaluating the formulae given by Raqab and Ahsanullah[1]. Using these means, variances and covariances. In Section 3, we have tabulated the coefficients of the BLUEs of location and scale parameters for complete samples of size up to 20 for  $\alpha = 1.5(0.5)2.0(1.0)5.0$ . We have developed R-code for computing the coefficients of the BLUEs of location and scale parameters based on any Type-II censored sample (including a complete sample) of size up to  $n=20$  and for any choice of shape parameter  $\alpha$  in the interval  $[1.5, 6.0]$ . Finally, in section 4, we demonstrate the computation of the BLUEs with an illustration.

## II. MEANS, VARIANCES AND COVARIANCES OF STANDARD ORDER STATISTICS

In order to compute the coefficients of the BLUEs for location and scale parameters of a population using Lloyd [3] method, we require the means, variances and covariances of order statistics from the corresponding standard population. Therefore, in this section, we have computed the means, variances and covariances of order statistics for standard GE population (1.3) by evaluating the following exact and explicit expressions (derived by [1]) of first and second moments of  $i^{th}$  standard order statistic  $Z_{i:n}$  ( $i = 1, 2, \dots, n$ ) and the product moment of  $i^{th}$  and  $j^{th}$  ( $1 \leq i < j \leq n$ ) standard order statistics  $Z_{i:n}$  and  $Z_{j:n}$ .

The explicit expressions of the first and second moments of  $Z_{i:n}$  ( $i = 1, 2, \dots, n$ ) are

$$a_{i:n} = E(Z_{i:n}) = [B(i, n-i+1)]^{-1} \sum_{l=0}^{n-i} \frac{(-1)^l \binom{n-i}{l}}{i+1} [\psi((i+l)\alpha + 1) + \gamma] \quad (2.1)$$

$$a_{i:n}^{(2)} = E(Z_{i:n}^2) = [B(i, n-i+1)]^{-1} \sum_{l=0}^{n-i} \frac{(-1)^l \binom{n-i}{l}}{i+1} \left\{ [\psi((i+l)\alpha + 1) + \gamma]^2 - \psi'((i+l)\alpha + 1) + \frac{\pi^2}{6} \right\} \quad (2.2)$$

where  $B(\dots)$  is a beta function and  $\gamma = -\psi(1)$ . Here  $\psi(\square)$  and  $\psi'(\square)$  are the digamma and tri-gamma functions, given by

$$\psi(u) = \frac{d}{du} \Gamma(u), \quad \psi'(u) = \frac{d}{du} \psi(u) \text{ where } \Gamma(u) = \int_0^{\infty} x^{u-1} e^{-x} dx. \quad (2.3)$$

Similarly, the product moment of  $Z_{i:n}$  and  $Z_{j:n}$  is given by

$$a_{i,j:n} = E(Z_{i:n} Z_{j:n}) \\ = C_{i,j:n} \alpha^2 \sum_{k=1}^{\infty} \sum_{l=0}^{n-j} \sum_{m=0}^{j-i-1} (-1)^{l+m} \binom{j-i-1}{m} \binom{n-j}{l} \frac{[\psi((j+l)\alpha + k + 1) + \gamma]}{k[(j+l)\alpha + k][(i+m)\alpha + k]} \quad (2.4)$$

where  $C_{i,j:n} = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}$

Using Eq. (2.1), we can evaluate the means of order statistics of standard GE distribution. The variance ( $b_{i,i:n}$ ) of order statistic  $Z_{i:n}$  ( $i = 1, 2, \dots, n$ ) can be evaluated from the relation  $b_{i,i:n} = a_{i:n}^{(2)} - a_{i:n}^2$  using Eqs. (2.1) and (2.2). The covariance ( $b_{i,j:n}$ ) between order statistics  $Z_{i:n}$  and  $Z_{j:n}$  can be evaluated from the relation  $b_{i,j:n} = a_{i,j:n} - a_{i:n} a_{j:n}$  using Eqs. (2.1) and (2.4). For evaluation of these means, variances and covariances of order statistics, we have developed the necessary R-code. Using this R-program, one can compute the means for all samples of size up to 30; variances and covariances for all samples of size up to 20 for shape parameter  $\alpha = 1.5(0.5)5.0$ . This R-code is presented in the appendix as *R-program 1* along with sample output of the program; means of order statistics for  $n=25$  &  $30$ ; and variances and covariances of order statistics for  $n=15$  &  $20$ .

### III. BLUE's OF LOCATION AND SCALE PARAMETERS BASED ON TYPE II CENSORED SAMPLES

In this section, we discuss best linear unbiased estimation of location and scale parameter in three-parameter generalized exponential distribution. We construct the BLUEs of location and scale parameters, with known shape parameter for Type II censored samples.

Suppose  $X_{r+1:n} \leq X_{r+2:n} \leq \dots \leq X_{n-s:n}$  is a Type II doubly censored sample ( $r$  left most observations and  $s$  right most observations are censored from a planned sample of size  $n$ ) from the GE distribution (1.1). Since, the doubly censored sample includes left censored sample ( $s=0$ ), right censored sample ( $r=0$ ) and complete sample ( $r=s=0$ ), the following development includes all these cases.

Let  $Z_{i:n} = (X_{i:n} - \mu) / \sigma$ , ( $i = r+1, \dots, n-s$ ) so that  $Z_{r+1:n} \leq Z_{r+2:n} \leq \dots \leq Z_{n-s:n}$  is a Type II doubly censored sample from the standard GE distribution (1.3). Now, if the shape parameter  $\alpha$  is known then the BLUEs of  $\mu$  and  $\sigma$ , denoted by  $\mu^*$  and  $\sigma^*$ , are given by (pl. see Lloyd [3])

$$\mu^* = \frac{\mathbf{a}'\mathbf{B}^{-1}\mathbf{a}\mathbf{1}'\mathbf{B}^{-1} - \mathbf{a}'\mathbf{B}^{-1}\mathbf{1}\mathbf{a}'\mathbf{B}^{-1}}{\Delta} \mathbf{X} \quad \text{and} \quad \sigma^* = \frac{\mathbf{1}'\mathbf{B}^{-1}\mathbf{1}\mathbf{a}'\mathbf{B}^{-1} - \mathbf{1}'\mathbf{B}^{-1}\mathbf{a}\mathbf{1}'\mathbf{B}^{-1}}{\Delta} \mathbf{X} \quad (3.1)$$

Where  $\Delta = (\mathbf{a}'\mathbf{B}^{-1}\mathbf{a})(\mathbf{1}'\mathbf{B}^{-1}\mathbf{1}) - (\mathbf{a}'\mathbf{B}^{-1}\mathbf{1})^2$

Here  $\mathbf{a} = (a_{r+1:n}, \dots, a_{n-s:n})'$  and  $\mathbf{B} = ((b_{ij:n}))$ ,  $r+1 \leq i, j \leq n-s$  are respectively the mean vector and variance-covariance matrix of the order statistics  $Z_{r+1:n} \leq Z_{r+2:n} \leq \dots \leq Z_{n-s:n}$  from standard GE distribution and  $\mathbf{1}$  is a column vector of 1's of the same dimension of  $\mathbf{a}$ .

The variances and covariance of  $\mu^*$  and  $\sigma^*$  are given by

$$\text{var}(\mu^*) = \sigma^2 \left\{ \frac{\mathbf{a}'\mathbf{B}^{-1}\mathbf{a}}{\Delta} \right\}, \quad \text{var}(\sigma^*) = \sigma^2 \left\{ \frac{\mathbf{1}'\mathbf{B}^{-1}\mathbf{1}}{\Delta} \right\} \text{ and } \text{Cov}(\mu^*, \sigma^*) = -\sigma^2 \left\{ \frac{\mathbf{a}'\mathbf{B}^{-1}\mathbf{1}}{\Delta} \right\} \quad (3.2)$$

Using the above formulae, we extend the computation of the coefficients of the BLUE's  $\mu^*$  and  $\sigma^*$  for complete samples up to size  $n=11(1)20$  and are presented in **Table 1** along with the variances and covariance for shape parameter  $\alpha = 1.5(0.5)2.0(1.0)5.0$ .

We have developed R-program for computation of the coefficients of the BLUEs  $\mu^*$  and  $\sigma^*$  along with the variances and covariance of the estimators for any Type-II censored sample including complete samples up to size  $n=20$ ; with any choice of  $0 \leq r \leq n-2$  and  $0 \leq s \leq n-2$  such that  $r+s \leq n-2$ ; and for any choice of the shape parameter  $\alpha$  in the interval  $[1.5, 6.0]$ . This R-code is given in the appendix as *R-program 2*.

#### IV. ILLUSTRATIONS

In this section, we demonstrate the computation the BLUE's with two data sets. The first data set is the following ordered sample of size  $n=8$  from  $GE(\alpha, \mu, \sigma)$  with  $\alpha=2$ , which is simulated by Raqab and Ahsanullah[1].

8.5347 9.4276 9.4673 9.7726 9.7854 10.1109 11.5356 15.1027

For the above complete sample, Raqab and Ahsanullah[1] computed the BLUE's and are given below along with the variances and covariance of the estimators.

$$\mu^* = 7.93029 \text{ and } \sigma^* = 1.67994,$$

$$\text{Var}(\mu^*) = 0.078448\sigma^2, \text{ Var}(\sigma^*) = 0.108276\sigma^2, \text{ and } \text{Cov}(\mu^*, \sigma^*) = -0.055374\sigma^2.$$

We consider the above sample with the last observation (15.1027) censored and based on this right censored sample ( $r=0$  &  $s=1$ ), we compute the BLUE's along with the standard errors of the estimates. The necessary coefficients of the BLUE's are computed using *R-program 2* (given in the appendix) with the input of  $\alpha=2$ ,  $n=8$ ,  $r=0$  &  $s=1$ ; the output of the program is given below.

ENTER SHAPE PARAMETER OF STANDARD GE DISTRIBUTION (between 1.5 and 6.0)1: 2

2:

Read 1 item

ENTER SAMPLE SIZE (n between 2 and 20)1: 8

2:

Read 1 item

ENTER No. observations LEFT censored (r between 0 and n-2)1: 0

2:

Read 1 item

ENTER No. observations RIGHT censored (s between 0 and n-2)1: 1

2:

Read 1 item

Coefficients of the BLUE of Location parameter:

Coefficients of the BLUE of Scale parameter:

-0.7885 0.0397 0.0969 0.1210 0.1313 0.1373 0.2623

Variances and covariance of the BLUEs:

0.0822 0.1235 -0.0629

CHECKS ON BLUE COEFFICIENTS:

SUM OF the BLUE COEFFICIENTS OF LOCATION= 1.000000

SUM OF the BLUE COEFFICIENTS OF SCALE= 0.000000

Using the abovecomputed coefficients of the BLUE's, their variances and covariance; we compute the BLUE's  $\mu^*$  and  $\sigma^*$  along with the standard errors of the estimates based on the given right censored sample (last observation 15.1027 is censored) and are given below.

$$\mu^*=8.0480, \sigma^*=1.4435, S.E.(\mu^*)=\sqrt{0.0822}\sigma^*=0.4139 \text{ and } S.E.(\sigma^*)=\sqrt{0.1235}\sigma^*=0.5073$$

We also obtain the BLUE of the population mean  $T=E(Y)=\mu+(\Psi(3)+\gamma)\sigma$  and standard error of the estimate as

$$T^*=\mu^*+(\Psi(3)+\gamma)\sigma^*=10.2132, S.E.(T^*)=0.5976.$$

Further, we compute the BLUE's of  $\mu, \sigma$  and  $T$  based on the given sample with various choices of  $r$  and  $s$  and are presented below along with the standard errors of the estimates.

r	s	$\mu^*(S.E.(\mu^*))$	$\sigma^*(S.E.(\sigma^*))$	$T^*(S.E.(T^*))$
0	2	8.1424(0.3689)	1.2479(0.4765)	10.0143(0.5502)
1	0	8.5199(0.5175)	1.3192(0.4872)	10.4987(0.5208)
1	1	8.7959(0.3846)	0.9398(0.3784)	10.2056(0.3889)
1	2	9.0673(0.2413)	0.5576(0.2502)	9.9037(0.2467)

The second data set considered by us is the following type II right censored sample ( $r=0$  &  $s=5$ ) of size  $n=20$  from  $GE(\alpha, \mu, \sigma)$  with  $\alpha=3, \mu=0$  and  $\sigma=1$ , which is simulated by Raqab and Madi [8].

0.65306, 0.67631, 0.68341, 1.05645, 1.46194, 1.71555, 1.73903, 1.78940, 1.79847, 1.82522, 1.95587, 2.16530, 2.35033, 2.38706, 2.39005.

In this example also for computing the coefficients of the BLUE's, we run *R-program 2* with the input of  $\alpha=3, n=20, r=0$  &  $s=5$  and the output is given below.

ENTER SHAPE PARAMETER OF STANDARD GE DISTRIBUTION (between 1.5 and 6.0)1: 3

2:

Read 1 item

ENTER SAMPLE SIZE (n between 2 and 20)1: 20

2:

Read 1 item

ENTER No. observations LEFT censored (r between 0 and n-2)1: 0

2:

Read 1 item

ENTER No. observations RIGHT censored (s between 0 and n-2)1: 5

2:

Read 1 item

Coefficients of the BLUE of Location parameter:

0.8997 0.1804 0.1309 0.0244 0.0821 -0.0180 0.0287 -0.0102 -0.0176 0.0014 -0.0444 0.0005  
-0.0485 -0.0145 -0.1948

Coefficients of the BLUE of Scale parameter:

-0.5707 -0.0751 -0.0646 0.0506 -0.0495 0.0791 -0.0013 0.0537 0.0447 0.0303 0.0755 0.0186  
0.0789 0.0322 0.2976

Variances and covariance of the BLUEs:

0.0468 0.0448 -0.0318

CHECKS ON BLUE COEFFICIENTS:

SUM OF the BLUE COEFFICIENTS OF LOCATION= 1.000000

SUM OF the BLUE COEFFICIENTS OF SCALE= 0.000000

Using the above coefficients of the BLUE's, we compute the BLUE's  $\mu^*$  and  $\sigma^*$  based on the given type II right censored sample and are given below along with the standard errors of the estimates.

$$\mu^*=0.2163 \text{ and } \sigma^*=1.0402, \text{ S.E.}(\mu^*)=0.2250 \text{ and S.E.}(\sigma^*)=0.2202.$$

## APPENDIX

### R-program 1: R-code for computing means, variances and covariances of ordered statistics of standard GED

```
#####  
# THIS R-CODE COMPUTES MEANS OF ORDER STATISTICS OF STANDARD GENERALIZED EXPONENTIAL DISTRIBUTION @  
# FOR DIFFERENT CHOICES OF SHAPE PARAMETER alpha FOR ALL SAMPLES OF SIZE UP TO 30 @  
#####  
sink("I:/banu/GEMDS");  
alphav=c(1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0)  
cat("\n MEANS OF SINGLE ORDER STATISTICS FOR STANDARD GENERALISED EXPONENTIAL DISTRIBUTION");  
cat("\n FOR DIFFERENT VALUES OF SHAPE PARAMETER (alpha) FOR SAMPLE SIZE n=1(1)30.");  
cat("\n\n ni alpha=");  
for (alpha in alphav) cat(" ", format(alpha, nsmall=1), " ");  
cat("\n\n");  
for (n in 1:30){  
for (r in 1:n){  
cat(format(n, width=4));  
cat(format(r, width=3), "\t");  
for (alpha in alphav){  
cat(format(round(GEMMI(n, r, alpha), digits=4), nsmall=4), "\t");}  
cat("\n");}  
cat("\n");}  
sink();  
#####  
# THIS R-CODE COMPUTES VARIANCES AND COVARIANCES OF ORDER STATISTICS OF STANDARD GENERALIZED EXPONENTIAL @  
# DISTRIBUTION FOR DIFFERENT CHOICES OF SHAPE PARAMETER alpha FOR ALL SAMPLES OF SIZE UP TO 20 @  
#####  
sink("I:/banu/GEVC");  
alphav=c(1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0);  
cat("\n VARIANCES AND COVARIANCES OF ORDER STATISTICS FOR STANDARD GENERALISED EXPONENTIAL DISTRIBUTION");  
cat("\n FOR DIFFERENT VALUES OF SHAPE PARAMETER (alpha) FOR SAMPLE SIZE n=2(1)20.");  
cat("\n\n ni j alpha=");  
for (alpha in alphav) cat(" ", format(alpha, nsmall=1), " ");  
cat("\n\n");  
for (n in 2:20)  
for (r in 1:n)  
for (s in r:n){cat(format(n, width=4));  
cat(format(r, width=5));  
cat(format(s, width=4), " ");  
for (alpha in alphav){  
if (r==s){cat(format(round(GEMM2(n, r, alpha) - GEMMI(n, r, alpha)**2, digits=6), nsmall=6), " ");}  
else { cat(format(round(GEPM(n, r, s, alpha) - GEMMI(n, r, alpha)*GEMMI(n, s, alpha), digits=6), nsmall=6), " ");}  
}
```

```

    }
    cat("\n");}
    sink();
    #####
    # THIS R-FUNCTION COMPUTES MEAN OF R-th ORDER STATISTICS OF STANDARD GENERALIZED @
    # EXPONENTIAL DISTRIBUTION @
    #####
    GEMOM1=function(n, r, al pha){
    GEMOM1=0;
    for (i in 0:(n-r)) {
    t=di gamma((r+i)*al pha+1)+0. 577215;
    GEMOM1=GEMOM1+(- 1)**i*comb(n-r, i)/(r+i)*t;
    }
    GEMOM1=GEMOM1/beta(r, n-r+1);
    return(GEMOM1);
    }
    #####
    # THIS R-FUNCTION COMPUTES SECOND ORDER MOMENT OF R-th ORDER STATISTIC OF STANDARD @
    # GENERALIZED EXPONENTIAL DISTRIBUTION @
    #####
    GEMOM2=function(n, r, al pha){
    GEMOM2=0;
    for (i in 0:(n-r)) {
    t=(di gamma((r+i)*al pha+1)+0. 577215)**2- tri gamma((r+i)*al pha+1)+pi**2/6;
    GEMOM2=GEMOM2+(- 1)**i*comb(n-r, i)/(r+i)*t;
    }
    GEMOM2=GEMOM2/beta(r, n-r+1);
    return(GEMOM2);
    }

    #####
    # THIS R-FUNCTION COMPUTES PRODUCT MOMENTS BETWEEN R-th AND S-th ORDER STATISTICS OF STANDARD @
    # GENERALIZED EXPONENTIAL DISTRIBUTION @
    #####
    GEPM=function(n, r, s, al pha){
    GEPM=0;
    GEPM1=0;
    crs=(factorial(n)/factorial(r-1)/factorial(s-r-1)/factorial(n-s));
    for (i in 0:(n-s)) {
    for (j in 0:(s-r-1)){
    GEPM1=0;
    for (k in 1:200) GEPM1=GEPM1+(di gamma((s+i)*al pha+k+1)+0. 5772157)/(k*((s+i)*al pha+k)*((r+j)*al pha+k));
    GEPM=GEPM+((- 1)**(i+j))*comb(s-r-1, j)*comb(n-s, i)*GEPM1;
    }
    }
    GEPM=GEPM*(al pha**2)*crs;
    return(GEPM);
    }
    #####
    # THIS FUNCTION COMPUTES COMBINATORIAL(n, r) @
    #####
    comb=function(n, r){
    comb=1;
    if (r==0) return(1);
    for (i in 1:r) comb=comb*(n-i+1)/i;
    return(comb);
    }
    
```

**Sample output of R-program 1:**

MEANS OF SINGLE ORDER STATISTICS FOR STANDARD GENERALISED EXPONENTIAL DISTRIBUTION FOR DIFFERENT VALUES OF SHAPE PARAMETER (alpha) FOR SAMPLE SIZE n=25 and 30.

nialpha= 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

25 1	0.1121	0.1981	0.2867	0.3731	0.4555	0.5335	0.6069	0.6760
25 2	0.1937	0.3117	0.4249	0.5303	0.6277	0.7177	0.8010	0.8784
25 3	0.2669	0.4061	0.5344	0.6510	0.7569	0.8536	0.9423	1.0242
25 4	0.3368	0.4921	0.6315	0.7561	0.8681	0.9696	1.0621	1.1471
25 5	0.4054	0.5739	0.7222	0.8532	0.9699	1.0751	1.1705	1.2579
25 6	0.4739	0.6537	0.8094	0.9457	1.0664	1.1745	1.2724	1.3617
25 7	0.5432	0.7329	0.8951	1.0359	1.1599	1.2706	1.3705	1.4615
25 8	0.6140	0.8125	0.9804	1.1251	1.2521	1.3651	1.4667	1.5591
25 9	0.6868	0.8933	1.0663	1.2147	1.3443	1.4592	1.5624	1.6561
25 10	0.7624	0.9762	1.1539	1.3054	1.4373	1.5541	1.6587	1.7535
25 11	0.8413	1.0618	1.2438	1.3983	1.5323	1.6507	1.7566	1.8524
25 12	0.9242	1.1511	1.3369	1.4941	1.6301	1.7500	1.8571	1.9539

25 13	1.0120	1.2447	1.4343	1.5940	1.7318	1.8531	1.9612	2.0589
25 14	1.1057	1.3439	1.5369	1.6989	1.8385	1.9610	2.0702	2.1687
25 15	1.2064	1.4499	1.6461	1.8103	1.9515	2.0752	2.1853	2.2845
25 16	1.3158	1.5642	1.7635	1.9298	2.0724	2.1973	2.3083	2.4082
25 17	1.4359	1.6890	1.8912	2.0594	2.2035	2.3294	2.4412	2.5419
25 18	1.5695	1.8271	2.0320	2.2021	2.3475	2.4745	2.5871	2.6883
25 19	1.7207	1.9826	2.1901	2.3620	2.5086	2.6365	2.7499	2.8518
25 20	1.8953	2.1613	2.3714	2.5449	2.6928	2.8216	2.9357	3.0381
25 21	2.1030	2.3730	2.5854	2.7606	2.9096	3.0393	3.1541	3.2571
25 22	2.3605	2.6343	2.8490	3.0257	3.1759	3.3064	3.4218	3.5253
25 23	2.7010	2.9785	3.1954	3.3736	3.5248	3.6561	3.7722	3.8762
25 24	3.2080	3.4890	3.7081	3.8877	4.0399	4.1720	4.2887	4.3931
25 25	4.2148	4.4992	4.7204	4.9014	5.0546	5.1874	5.3046	5.4095

30 1	0.0987	0.1791	0.2634	0.3465	0.4263	0.5021	0.5737	0.6414
30 2	0.1697	0.2804	0.3883	0.4898	0.5842	0.6718	0.7531	0.8289
30 3	0.2328	0.3636	0.4860	0.5983	0.7011	0.7952	0.8820	0.9621
30 4	0.2924	0.4386	0.5717	0.6918	0.8004	0.8993	0.9897	1.0731
30 5	0.3502	0.5089	0.6508	0.7770	0.8906	0.9929	1.0862	1.1714
30 6	0.4076	0.5767	0.7258	0.8577	0.9743	1.0791	1.1751	1.2644
30 7	0.4644	0.6445	0.7989	0.9350	1.0554	1.1632	1.2615	1.3487
30 8	0.5238	0.7094	0.8706	1.0113	1.1349	1.2449	1.3432	1.4366
30 9	0.5783	0.7763	0.9413	1.0830	1.2099	1.3215	1.4235	1.5132
30 10	0.6380	0.8417	1.0124	1.1604	1.2867	1.3962	1.5037	1.5948
30 11	0.7034	0.9098	1.0828	1.2318	1.3641	1.4799	1.5827	1.6748
30 12	0.7629	0.9787	1.1588	1.3111	1.4416	1.5581	1.6640	1.7598
30 13	0.8349	1.0526	1.2334	1.3871	1.5221	1.6399	1.7458	1.8412
30 14	0.8983	1.1253	1.3107	1.4676	1.6026	1.7218	1.8302	1.9265
30 15	0.9729	1.2027	1.3906	1.5495	1.6862	1.8077	1.9148	2.0119
30 16	1.0472	1.2829	1.4742	1.6347	1.7733	1.8950	2.0041	2.1023
30 17	1.1273	1.3668	1.5609	1.7236	1.8636	1.9866	2.0960	2.1946
30 18	1.2123	1.4565	1.6532	1.8176	1.9589	2.0827	2.1931	2.2924
30 19	1.3035	1.5517	1.7509	1.9171	2.0597	2.1845	2.2955	2.3954
30 20	1.4020	1.6542	1.8558	2.0236	2.1674	2.2931	2.4048	2.5053
30 21	1.5093	1.7653	1.9692	2.1386	2.2835	2.4101	2.5225	2.6235
30 22	1.6275	1.8871	2.0933	2.2642	2.4102	2.5376	2.6506	2.7521
30 23	1.7593	2.0225	2.2308	2.4032	2.5502	2.6784	2.7920	2.8941
30 24	1.9088	2.1754	2.3858	2.5596	2.7076	2.8366	2.9508	3.0533
30 25	2.0819	2.3518	2.5642	2.7393	2.8883	3.0180	3.1328	3.2357
30 26	2.2882	2.5612	2.7756	2.9520	3.1019	3.2323	3.3476	3.4510
30 27	2.5443	2.8204	3.0366	3.2143	3.3652	3.4962	3.6120	3.7159
30 28	2.8835	3.1627	3.3807	3.5596	3.7113	3.8430	3.9594	4.0636
30 29	3.3893	3.6714	3.8912	4.0712	4.2238	4.3561	4.4730	4.5776
30 30	4.3949	4.6799	4.9014	5.0826	5.2359	5.3689	5.4862	5.5912

VARIANCES AND COVARIANCES OF ORDER STATISTICS FOR STANDARD GENERALISED EXPONENTIAL DISTRIBUTION FOR DIFFERENT VALUES OF SHAPE PARAMETER (alpha) FOR SAMPLE SIZE n=15 and 20.

ni j alpha= 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

15	1	1	0.013476	0.023858	0.033724	0.042503	0.050140	0.056748	0.062473	0.067458
15	1	2	0.011606	0.018833	0.025165	0.030501	0.034968	0.038725	0.041910	0.044635
15	1	3	0.010616	0.016425	0.021311	0.025324	0.028624	0.031363	0.033663	0.035615
15	1	4	0.009978	0.014949	0.019025	0.022317	0.024995	0.027199	0.029038	0.030592
15	1	5	0.009520	0.013929	0.017475	0.020306	0.022590	0.024459	0.026011	0.027318
15	1	6	0.009172	0.013170	0.016339	0.018845	0.020854	0.022491	0.023845	0.024982
15	1	7	0.008894	0.012576	0.015460	0.017724	0.019528	0.020993	0.022202	0.023214
15	1	8	0.008666	0.012096	0.014755	0.016829	0.018475	0.019807	0.020903	0.021819
15	1	9	0.008474	0.011697	0.014174	0.016094	0.017613	0.018838	0.019844	0.020684
15	1	10	0.008310	0.011359	0.013684	0.015478	0.016891	0.018029	0.018961	0.019739
15	1	11	0.008167	0.011067	0.013263	0.014950	0.016276	0.017340	0.018211	0.018935
15	1	12	0.008042	0.010813	0.012898	0.014493	0.015743	0.016745	0.017563	0.018243
15	1	13	0.007930	0.010588	0.012576	0.014092	0.015277	0.016224	0.016996	0.017638
15	1	14	0.007830	0.010388	0.012291	0.013737	0.014864	0.015763	0.016496	0.017104
15	1	15	0.007741	0.010208	0.012035	0.013419	0.014495	0.015353	0.016051	0.016629
15	2	2	0.021546	0.032373	0.041273	0.048479	0.054348	0.059188	0.063231	0.066651
15	2	3	0.019734	0.028296	0.035054	0.040387	0.044655	0.048130	0.051005	0.053419
15	2	4	0.018560	0.025786	0.031343	0.035657	0.039072	0.041829	0.044097	0.045992
15	2	5	0.017717	0.024044	0.028818	0.032480	0.035356	0.037665	0.039555	0.041129
15	2	6	0.017074	0.022745	0.026961	0.030166	0.032666	0.034664	0.036295	0.037649
15	2	7	0.016560	0.021728	0.025523	0.028386	0.030608	0.032376	0.033816	0.035009









ISSN 2250-3153

20	13	19	0.092813	0.097312	0.100101	0.101999	0.103371	0.104407	0.105212	0.105849
20	13	20	0.092050	0.096111	0.098619	0.100321	0.101549	0.102473	0.103188	0.103751
20	14	14	0.119578	0.127708	0.132834	0.136358	0.138929	0.140886	0.142427	0.143670
20	14	15	0.118199	0.125509	0.130097	0.133237	0.135516	0.137233	0.138561	0.139599
20	14	16	0.116935	0.123506	0.127611	0.130415	0.132443	0.133968	0.135141	0.136053
20	14	17	0.115774	0.121672	0.125342	0.127841	0.129644	0.130996	0.132031	0.132828
20	14	18	0.114701	0.119984	0.123259	0.125482	0.127081	0.128276	0.129185	0.129876
20	14	19	0.113706	0.118425	0.121337	0.123309	0.124723	0.125773	0.126565	0.127159
20	14	20	0.112780	0.116978	0.119558	0.121298	0.122540	0.123456	0.124136	0.124632
20	15	15	0.147586	0.155708	0.160785	0.164257	0.166781	0.168698	0.170203	0.171416
20	15	16	0.146034	0.153257	0.157739	0.160770	0.162923	0.164486	0.165616	0.166399
20	15	17	0.144606	0.151015	0.154975	0.157640	0.159523	0.160875	0.161833	0.162469
20	15	18	0.143285	0.148950	0.152433	0.154767	0.156403	0.157562	0.158359	0.158855
20	15	19	0.142059	0.147039	0.150087	0.152116	0.153524	0.154500	0.155143	0.155500
20	15	20	0.140917	0.145265	0.147909	0.149654	0.150844	0.151638	0.152115	0.152309
20	16	16	0.187801	0.195916	0.200949	0.204374	0.206855	0.208735	0.210208	0.211394
20	16	17	0.185982	0.193032	0.197291	0.200025	0.201774	0.202791	0.203203	0.203070
20	16	18	0.184316	0.190438	0.194104	0.196417	0.197842	0.198590	0.198763	0.198403
20	16	19	0.182766	0.188033	0.191149	0.193068	0.194179	0.194657	0.194574	0.193959
20	16	20	0.181319	0.185788	0.188382	0.189909	0.190686	0.190842	0.190426	0.189443
20	17	17	0.250503	0.258611	0.263603	0.266985	0.269428	0.271274	0.272719	0.273880
20	17	18	0.248043	0.254449	0.257753	0.259131	0.259052	0.257721	0.255230	0.251626
20	17	19	0.245988	0.251245	0.253768	0.254526	0.253889	0.252007	0.248941	0.244716
20	17	20	0.244037	0.248174	0.249873	0.249897	0.248512	0.245812	0.241821	0.236539
20	18	18	0.361803	0.369906	0.374861	0.378203	0.380610	0.382426	0.383845	0.384984
20	18	19	0.355213	0.357370	0.354819	0.348695	0.339504	0.327533	0.312983	0.296016
20	18	20	0.351871	0.351815	0.347198	0.338883	0.327259	0.312560	0.294965	0.274637
20	19	19	0.611981	0.620080	0.625000	0.628306	0.630681	0.632469	0.633863	0.634982
20	19	20	0.550267	0.516451	0.472497	0.420706	0.362544	0.299082	0.231159	0.159457
20	20	20	1.612150	1.620244	1.625133	1.628406	1.630750	1.632512	1.633884	1.634984

**R-program 2:** R-code for computing the coefficients of the BLUEs of Location and Scale Parameters of GED

```
#####
# THIS R-CODE COMPUTES COEFFICIENTS OF THE BLUEs OF LOCATION AND SCALE PARAMETERS OF GENERALIZED @
# EXPONENTIAL DISTRIBUTION FOR A SPECIFIED VALUE OF SHAPE PARAMETER (alpha) FOR A GIVEN COMPLETE @
# SAMPLE OR RIGHT CENSORED OR LEFT CENSORED OR DOUBLY CENSORED SAMPLE FOR ANY SAMPLE SIZE UP TO 20 @
#####
cat("\n ENTER SHAPE PARAMETER OF STANDARD GE DISTRIBUTION (between 1.5 and 6.0)");
alpha=scan();
cat("\n ENTER SAMPLE SIZE (n between 2 and 20)");n=scan();
cat("\n ENTER No. observations LEFT censored (r between 0 and n-2)");r=scan();
cat("\n ENTER No. observations RIGHT censored (s between 0 and n-2)");s=scan();
m=n-r-s;
I=rep(1, m); a=rep(0, m); B=a%*t(a);
for (i in 1:m) a[i]=GEMOMI(n, r+i, alpha);
for (i in 1:m)
for (j in i:m) { if (i==j) {B[i, i]=GEMOM2(n, r+i, alpha) - GEMOMI(n, r+i, alpha)**2;}
else { B[i, j]=GEPM(n, r+i, r+j, alpha) - GEMOMI(n, r+i, alpha)*GEMOMI(n, r+j, alpha); }
}
for (i in 1:(m-1)) for (j in (i+1):m) B[j, i]=B[i, j];
del ta=sum(a*solve(B, a))*sum(I*solve(B, I))-sum(a*solve(B, I))**2;
BI=solve(B);
A=BI%*(I%*t(a)-a%*t(I));
A=A%*BI;
BLMUC=(-t(a)%*A)/del ta;
BLSIGC=(t(I)%*A)/del ta;
cat("\n\nCoefficients of the BLUE of Location parameter:\n", " ", format(round(BLMUC, 4), nsmall=4));
cat("\n\nCoefficients of the BLUE of Scale parameter:\n", " ", format(round(BLSIGC, 4), nsmall=4));
vmu=sum(a*solve(B, a))/del ta;
vsi g=sum(I*solve(B, I))/del ta;
cov=-sum(I*solve(B, a))/del ta;
cat("\n\nVariances and covariance of the BLUEs: \n", " ",
format(round(vmu, 4), nsmall=4), format(round(vsi g, 4), nsmall=4), format(round(cov, 4), nsmall=4));
cat("\n");
cat("\n\nCHECKS ON BLUE COEFFICIENTS: \n");
cat("\n SUM OF the BLUE COEFFICIENTS OF LOCATION= ", format(round(sum(BLMUC), 6), nsmall=6));
cat("\n SUM OF the BLUE COEFFICIENTS OF SCALE= ", format(round(sum(BLSIGC), 6), nsmall=6));
cat("\n");
```

```
#####
# THIS R-FUNCTION COMPUTES MEAN OF R-th ORDER STATISTICS OF STANDARD GENERALIZED @
# EXPONENTIAL DISTRIBUTION
#####
GEMOMI=function(n, r, alpha){
GEMOMI=0;
for (i in 0:(n-r)) {
t=di gamma((r+i)*alpha+1)+0.577215;
GEMOMI=GEMOMI+(-1)**i*comb(n-r, i)/(r+i)*t;
}
GEMOMI=GEMOMI/beta(r, n-r+1);
return(GEMOMI);
}
```

```
#####
# THIS R-FUNCTION COMPUTES SECOND ORDER MOMENT OF R-th ORDER STATISTIC OF STANDARD @
# GENERALIZED EXPONENTIAL DISTRIBUTION
#
```

```

#####
GEMOM2=function(n, r, al pha){
GEMOM2=0;
for (i in 0:(n-r)) {
t=(di gamma((r+i)*al pha+1)+0. 577215)**2- tri gamma((r+i)*al pha+1)+pi**2/6;
GEMOM2=GEMOM2+(- 1)**i*comb(n-r, i)/(r+i)*t;
}
GEMOM2=GEMOM2/beta(r, n-r+1);
return(GEMOM2);
}
#####
# THIS R-FUNCTION COMPUTES PRODUCT MOMENTS BETWEEN R-th AND S-th ORDER STATISTICS OF STANDARD @
# GENERALIZED EXPONENTIAL DISTRIBUTION @
#####
GEPM=function(n, r, s, al pha){
GEPM=0;
GEPM1=0;
crs=(factorial(n)/factorial(r-1)/factorial(s-r-1)/factorial(n-s));
for (i in 0:(n-s)) {
for (j in 0:(s-r-1)){
GEPM1=0;
for (k in 1:200)GEPM1=GEPM1+(di gamma((s+i)*al pha+k+1)+0. 5772157)/(k*((s+i)*al pha+k)*((r+j)*al pha+k));
GEPM=GEPM+((- 1)**(i+j))*comb(s-r-1,j)*comb(n-s, i)*GEPM1;
}
}
GEPM=GEPM*(al pha**2)*crs;
return(GEPM);
}
#####
# THIS FUNCTION COMPUTES COMBINATORIAL(n, r) @
#####
comb=function(n, r){
comb=1;
if (r==0) return(1);
for (i in 1:r) comb=comb*(n-i+1)/i;
return(comb);
}

```

**Table 1:** Coefficients to get BLUEs of Location and Scale parameters in Generalized Exponential Distribution from compete samples of sizes n=11(1)20 for shape parameter  $\alpha = 1.5(0.5)2.0(1.0)5.0$ .

		Coefficient of										$\frac{Var(\mu^*, \sigma^*)}{\sigma^2}$		
$\alpha$	n	$X_{1:n}$	$X_{2:n}$	$X_{3:n}$	$X_{4:n}$	$X_{5:n}$	$X_{6:n}$	$X_{7:n}$	$X_{8:n}$	$X_{9:n}$	$X_{10:n}$	$\frac{Var}{\sigma^2}$	$\frac{Cov}{\sigma^2}$	
		$X_{11:n}$	$X_{12:n}$	$X_{13:n}$	$X_{14:n}$	$X_{15:n}$	$X_{16:n}$	$X_{17:n}$	$X_{18:n}$	$X_{19:n}$	$X_{20:n}$			
1.50	11	1.0505	0.0568	0.0173	0.0002	-0.0089	-0.0143	-0.0175	-0.0195	-0.0198	-0.0231			
		-0.0217											0.0265	
		-0.7711	0.0318	0.0628	0.0760	0.0824	0.0858	0.0873	0.0870	0.0830	0.0915		0.0811	-0.0214
12	12	1.0339	0.0595	0.0209	0.0039	-0.0050	-0.0104	-0.0137	-0.0159	-0.0171	-0.0171			
		-0.0202	-0.0188										0.0229	
		-0.7643	0.0233	0.0537	0.0668	0.0733	0.0769	0.0787	0.0795	0.0787	0.0744		0.0734	-0.0185
0.0835	13	1.0193	0.0615	0.0237	0.0069	-0.0019	-0.0073	-0.0106	-0.0129	-0.0144	-0.0152			
		-0.0149	-0.0179	-0.0165									0.0200	
		-0.7581	0.0163	0.0460	0.0592	0.0657	0.0694	0.0714	0.0725	0.0727	0.0716		0.0670	-0.0162
0.0672	14	1.0064	0.0631	0.0259	0.0095	0.0006	-0.0047	-0.0081	-0.0104	-0.0119	-0.0131			
		-0.0135	-0.0132	-0.0161	-0.0146								0.0177	
		-0.7524	0.0104	0.0396	0.0527	0.0592	0.0630	0.0652	0.0664	0.0670	0.0670		0.0617	-0.0143
0.0656	15	0.9949	0.0642	0.0278	0.0115	0.0027	-0.0025	-0.0061	-0.0082	-0.0100	-0.0110			
		-0.0119	-0.0121	-0.0117	-0.0145	-0.0130							0.0158	
		-0.7472	0.0055	0.0340	0.0473	0.0537	0.0575	0.0599	0.0611	0.0620	0.0622		0.0571	-0.0128
0.0620	16	0.9845	0.0653	0.0292	0.0132	0.0044	-0.0007	-0.0043	-0.0065	-0.0083	-0.0093			
		-0.0103	-0.0108	-0.0109	-0.0105	-0.0133	-0.0117						0.0142	
		-0.7424	0.0013	0.0292	0.0425	0.0489	0.0527	0.0552	0.0565	0.0575	0.0578		0.0531	-0.0115
0.0580	17	0.9750	0.0661	0.0304	0.0146	0.0057	0.0010	-0.0031	-0.0046	-0.0071	-0.0076			
		-0.0089	-0.0095	-0.0099	-0.0099	-0.0094	-0.0122	-0.0106					0.0129	
		-0.7379	-0.0023	0.0249	0.0385	0.0447	0.0484	0.0513	0.0522	0.0538	0.0538		0.0496	-0.0104
0.0544	18	0.9665	0.0666	0.0315	0.0159	0.0069	0.0027	-0.0028	-0.0019	-0.0071	-0.0056			
		-0.0080	-0.0083	-0.0087	-0.0092	-0.0090	-0.0085	-0.0113	-0.0097				0.0118	
		-0.7338	-0.0052	0.0208	0.0351	0.0411	0.0444	0.0485	0.0474	0.0513	0.0497		0.0466	-0.0095
0.0512	19	0.9588	0.0665	0.0330	0.0165	0.0076	0.0056	-0.0051	0.0038	-0.0105	-0.0018			
		-0.0076	-0.0075	-0.0072	-0.0084	-0.0083	-0.0083	-0.0077	-0.0105	-0.0089			0.0108	
		-0.7302	-0.0071	0.0166	0.0323	0.0381	0.0397	0.0480	0.0480	0.0404	0.0518	0.0444		0.0439
0.0486	20	0.9534	0.0609	0.0430	0.0083	0.0138	0.0079	-0.0133	0.0189	-0.0214	0.0028			
		-0.0016	-0.0160	0.0017	-0.0119	-0.0058	-0.0084	-0.0074	-0.0071	-0.0098	-0.0082	0.0099		
		-0.7282	-0.0039	0.0051	0.0376	0.0304	0.0359	0.0526	0.0261	0.0586	0.0389		0.0415	-0.0080
		0.0410	0.0545	0.0376	0.0498	0.0436	0.0451	0.0421	0.0386	0.0516	0.0429			





**Note:** Against each  $\alpha$  and  $n$  the figures in the first two rows are corresponding to location parameter and the figures in the second two rows are corresponding to scale parameter.

## REFERENCES

- [1]. M. Z.Raqab and M. Ahsanullah, "Estimation of location and scale parameters of generalized exponential distribution based on order statistics". *J. Statist. Computat. Simul.*, vol. 69, 2001, pp.109-124.
- [2]. R. D.Gupta and D.Kundu, "Generalized exponential distribution", *Austral. N. Z. Statist.*, vol. 41, 1999, pp.173-188.
- [3]. E. H. Lloyd (1952), "Least squares estimation of location and scale parameters using order statistics", *Biometrika*, vol. 39, 1952, pp.88-95.
- [4]. R. D.Gupta and D. Kundu, "Generalized exponential distributions: different methods of estimation", *J. Statist. Comput. Simul.*, vol. 69, 2001, pp.315-338.
- [5]. M. Z.Raqab, "Inferences for generalized exponential based on record statistics". *J. Statist. Plann. Infer.*, vol.104, 2002, pp.339-350.
- [6]. M. Z.Raqab, "Generalized Exponential Distribution: Moments of Order Statistics", *Statistics*, vol. 38, 2004, pp.29-41.
- [7]. R. U.Khan and D. Kumar, "Exponential Distribution of Lower generalized Order Statistics from Generalized Exponential Distribution and a Characterization". *Mathematical methods of Statistics*, vol.20, 2011, pp.150-157.
- [8]. M. Z.Raqab and M. T. Madi, "Bayesian inference for the generalized exponential distribution". *J. Statist. Computat. Simul.*, vol. 75, 2005, pp.541-852.

## AUTHORS

**First Author** –Dr.A. Vasudeva Rao, M.Sc., M.Phil., Ph.D., Professor and Head, Department of Statistics, Acharya Nagarjuna University, Nagarjunanagar-522501, Guntur, Andhra Pradesh, India, Email: [profavrao@gmail.com](mailto:profavrao@gmail.com).

**Second Author**–S. Bhanu Prakash, M.Sc., Ph.D. student, Department of Statistics, Acharya Nagarjuna University, Nagarjunanagar-522501, Guntur, Andhra Pradesh, India, Email: [prakash9293165890@gmail.com](mailto:prakash9293165890@gmail.com)

**Third Author**– Sd. Jilani, M.Sc., Ph.D. student, BSR-RFSMS, Department of Statistics, Acharya Nagarjuna University, Nagarjunanagar-522501, Guntur, Andhra Pradesh, India, Email: [jilanisyed1992@gmail.com](mailto:jilanisyed1992@gmail.com)

**Correspondence Author**–S. Bhanu Prakash, Email: [prakash9293165890@gmail.com](mailto:prakash9293165890@gmail.com), contact number:9293165890.