International Journal of Scientific and Research Publications, Volume 5, Issue 8, August 2015 1
ISSN 2250-3153

Two-stage fuzzy supply chain model with reduction in order cost and permissible delay in payments

J.Sujatha[*], P.Parvathi[**]

[*] Assistant Professor ,Department of Mathematics ,Quaid-E-Millath Government College for Women (Autonomous), Chennai-600 002 ,Tamil Nadu, India.

** Head & Associate Professor ,Department of Mathematics ,Quaid-E-Millath Government College for Women (Autonomous), Chennai-600 002 Tamil Nadu, India

Abstract- This paper focus on two stage supply chain model with a single vendor and a single buyer for a single product, taking into consideration the effect of deterioration and credit period incentives. We study and analyze the benefits of order cost reduction and credit period incentives in a co-ordinated supply chain . Shortage for the buyer is allowed and it is completely backlogged. The demand and shortage cost were taken as fuzzy parameters. Graded mean integration representation method is applied for defuzzification. This paper also includes a detailed numerical examples for more understanding of the proposed strategy.

Index Terms- Supply chain, order cost reduction, credit period, deteriorating items, fuzzy numbers and fuzzy concepts

I. Introduction

In conventional inventory models, uncertainties are treated as randomness and are handled by appealing to probability theory. However, in certain situations uncertainties are due to fuzziness and these cases the fuzzy set theory, originally introduced by Zadeh [23] is applicable. In decision making process, first, Bellman and Zadeh [2] introduced fuzzy set theory. Tanaka H, Okuda T and Asai K et.al.,[18] applied concept of fuzzy sets in decision making problems to consider the objectives as fuzzy goals over the α-cuts of a fuzzy constraints. Zimmerman [24] showed that the classical algorithms can be used in few inventory models.

Most of the inventory models only aimed at the determination of the optimum solutions that minimized cost or maximized profit from the buyer's or vendor's side. However, in the modern global competitive market, the buyer and the vendor should be treated as strategic patterns in the supply chain with a long-term cooperative relationship. For this reason, we consider a supply chain in which the vendor and buyer decides to invest in reducing ordering cost to streamline and speed up transactions via the application of information technology. Chang [4] studied a single vendor – single buyer integrated inventory models with controllable lead time and ordering cost reduction.

Chen and Kang [6] analyzed coordination between vendor and buyer considering trade credit and items of imperfect quality. Giannoccaro and Pontrandolfo [9] concentrated supply chain coordination by revenue sharing contracts. Goyal and Gupta [10] developed integrated inventory models; the buyer-vendor coordination. Wong et. al., [21] studied coordinating supply chains with sales rebate contracts and vendor-managed inventory. Many Researchers fairly documented the benefits of reduced order cost. For example, Porteus [16] and Uthayakumar and Parvathi [19] considered investment in reduced setups in the economic order quantity (EOQ) model. Billington [3] Kim et al. [12] , and Coates [8] developed EOQ models with setup cost reduction. However, these researchers investigated the benefit from order or setup cost reduction from a single party's viewpoint.

The efficiency of a supply chain management depends on active cooperation and close coordination between the vendor and the buyer. Jacker and Rosenblatt [11], Lee [13] et. al., Monahan [15], Weng [19] suggested that quantity discount is a coordination mechanism. Luo [14] proved that credit period is an effective mechanism for the buyer-vendor coordination. Many researchers consider deteriorating items in simple EOQ models. Perishable items, deteriorating items are considered by Sarkar [17] et..al., Yu [22] et. al., respectively in supply chains also. There is no single vendor, single buyer supply chain model which involves deteriorating items, order cost reduction and credit period incentives.Shortages refer to the inability to meet the demand at the required time schedule as preferred by the customer. Shortages are classified either completely backlogged shortages or partially backlogged shortages. Completely backlogged shortages are the shortages which are duly fulfilled by the vendor during the shortage period. Sometimes the shortages may not be completely backlogged and only a part of the demand can be met by the vendor during the shortage period, which is termed as partially backlogged.

Here, we propose a two stage supply chain model with single vendor, single buyer for deteriorating items. The vendor and the buyer decide upon an investment in ordering cost reduction and coordinate their inventory policies to minimize their joint average annual cost. The vendor requests the buyer to alter his current order size such that the vendor can benefit from lower ordering and inventory holding costs. To encourage the buyer to accept this strategy, the vendor must compensate the buyer for his increased inventory cost by offering an order size dependent credit period. Initially the buyer's behaviour is assumed to be captured by simple EOQ and the vendor's order size is an integer multiple of the buyer's such that his own inventory cost is minimized. If the buyer accepts to coordinate with the vendor, then the vendor's order size will be another integer multiple of the buyer's. Allowing shortages for buyer and demand rate are taken as fuzzy numbers. Graded mean integration representation method is applied for defuzzification.

Joint total cost for the supply chain with and without coordination are analysed through numeric examples.

II. FUZZY PRELIMINARIES

Definition 1: Let X denotes a universal set. Then the fuzzy subset $\widetilde{A}$ of X is defined by its membership function $\mu_{\widetilde{A}}(x): X \rightarrow [0,1]$ which assigns a real number $\mu_{\widetilde{A}}(x)$ in the interval [0,1], to each element x$\in$ X where the value of $\mu_{\widetilde{A}}(x)$ at x shows the grade of membership of x

Definition 2 : A fuzzy set $\widetilde{A}$ on R is convex if $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq min\left[\widetilde{A}(x_1), \widetilde{A}(x_2)\right]$ for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$.

Definition 3: A fuzzy set $\tilde{A}$ in the universe of discourse X is called as a fuzzy number in the universe of discourse X.

Triangular fuzzy number

We consider the situation where fuzzy numbers are represented by triangular membership functions. The fuzzy number $\widetilde{A}$ is said to be triangular fuzzy number if it is fully determined by (a_1, a_2, a_3) of crisp numbers such that $(a_1 < a_2 < a_3)$ whose membership function, representing triangle, can be denoted by

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases}$$

The Function Principle

The function principle was introduced by Chen [23] to treat fuzzy arithmetical operations. This principle is used for the operation for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\widetilde{A} = (a_1, a_2, a_3)$ and $\widetilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\widetilde{B}$ is

$\widetilde{A} + \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\widetilde{B}$ is
$\widetilde{A} \times \widetilde{B} = (c_1, c_2, c_3)$ Where $T = (a_1b_1, a_1b_3, a_3b_1, a_3b_3), c_1 = \min T, c_2 = a_2b_2, c_3 = \max T$ if $a_1, a_2, a_3, b_1, b_2, b_3$ are all non zero positive real numbers, then $\widetilde{A} \times \widetilde{B} = (a_1b_1, a_2b_2, a_3b_3)$.

(iii) $-\widetilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of

$\tilde{A}$ and $\widetilde{B}$ is $\widetilde{A} - \widetilde{B} = (a_1 - b_3, a_1 - b_2, a_3 - b_1)$ where $a_1, a_2, a_3, b_1, b_2, b_3$ are any real numbers.

(iv) $\frac{1}{\widetilde{B}} = \widetilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$ where b_1, b_2, b_3 are all non zero positive real numbers, then the division of $\tilde{A}$ and $\widetilde{B}$ is
$\frac{\widetilde{A}}{\widetilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$

(v) For any real number K, $K\tilde{A} = (Ka_1, Ka_2, Ka_3) if K > 0$
$K\tilde{A} = (Ka_3, Ka_2, Ka_1) if K < 0$

Graded Mean Integration Representation Method

If $\tilde{A} = (a_1, a_2, a_3)$ is triangular fuzzy number then the graded mean integration representation of $\tilde{A}$ is given by $P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$

III. NOTATIONS AND ASSUMPTIONS

3.1 NOTATIONS

θ - Deterioration rate, a fraction of the on-hand inventory, $0 < \theta < 1$
F - A factor by which the buyer alters his order size on the vendor's request, where F>0, a decision variable.
d - Ordering cost improvement rate of the related investment, 0<d<1
M - Length of the credit period offered by the vendor to the buyer, a decision variable.
T_1 - Time at which shortage starts at the buyer.
G - Expenditure per unit time to operate the planned ordering system between the vendor and the buyer, an investment decision variable.
i_1, i_2 - The vendor and the buyer's cost of capital respectively.
h_1', h_2' - The vendor and the buyer's unit variable holding cost excluding the cost of capital respectively.
h_1, h_2 - The vendor and the buyer's unit variable holding cost including the cost of capital respectively.
ie., $h_1 = h_1' + P_1 i_1$ and $h_2 = h_2' + P_2 i_2$.
K_1, K_2 - Ordering cost of the vendor and the buyer per order, respectively.
P_1, P_2 - Delivered unit price paid by the vendor and the buyer per order, respectively.
T_v, T_b - Length of the replenishment cycle for the vendor and the buyer respectively.
TC_v, TC_b -Total average annual cost of the vendor and the buyer respectively.
JTC_0 - Joint total cost of the system without any coordination between the vendor and the buyer.
JTC - Joint total cost of the system in the presence of coordination between the vendor and the buyer.
ΔJTC - Relative improvement of the joint total cost.
$\tilde{R}$ - Fuzzy demand rate
$\tilde{S}$ - Fuzzy Shortage cost for buyer

3.2 ASSUMPTIONS

1. The supply chain involves only one item, single vendor and single buyer.
2. The replenishment occurs instantaneously at an infinite rate for both vendor and the buyer. ie., the lead time is zero.
3. There is no repair or replacement of deteriorated units.
4. Fuzzy demand rate is a known constant.
5. Shortages are allowed for buyer in fuzzy environment.
6. The planned ordering cost for the buyer is a decreasing function of the expenditure incurred per unit time on operating the new ordering system, which is given by $PO(G) = K_2 e^{-dG}$.
7. The vendor makes a decision on inventory for the buyer and on the investment amount in ordering cost reduction. That is the buyer adopts the VMI (Vendor managed inventory) policy.
8. The vendor's credit period begins at the time when the ordered quantity is delivered.
9. There is enough capacity in buyer's warehouse to store more products.
10. We explicitly split the holding cost into two components namely financing cost and variable holding cost.

IV. MODEL FORMULATION

In the absence of any coordination between the vendor and the buyer, the buyer's behaviour is assumed to be captured by simple EOQ.

The buyer's optimal ordering quantity is $Q_0 = \sqrt{\dfrac{2\tilde{R}K_2}{h_2}}$

and buyer's minimized cost is $\sqrt{2\tilde{R}K_2 h_2}$ and the fixed interval is $\dfrac{Q_0}{\tilde{R}}$. The vendor's order size is mQ_0 where m is a positive integer. Hence the average inventory held up by the vendor per year is

$$= \frac{[(m-1)Q_0 + (m-2)Q_0 + \ldots + 2Q_0 + Q_0]\dfrac{Q_0}{\tilde{R}}}{\left(\dfrac{mQ_0}{\tilde{R}}\right)}$$

$$= \frac{(m-1)Q_0}{2}.$$

Holding cost for the vendor $= \dfrac{(m-1)Q_0 h_1}{2}$ (1)

Ordering cost for the vendor $= \dfrac{K_1}{T_v}$ where $T_v = \dfrac{mQ_0}{\tilde{R}}$

$$= \frac{\tilde{R}K_1}{mQ_0} \qquad (2)$$

The differential equation that describes the instantaneous states of the inventory level of the vendor,

$I_1(t)$ over $(0, T_v)$ is $\dfrac{dI_1(t)}{dt} + \theta I_1(t) = -\tilde{R}$ if $0 \le t < T_v$ (3)

The solution is $I_1(t)e^{\theta t} = \dfrac{-\tilde{R}}{\theta}e^{\theta t} + c_1$, where c_1 is the constant of integration.

Using the condition $I_1(T_v) = 0$, the solution of (3) is $I_1(t) = \dfrac{\tilde{R}}{\theta}[e^{\theta(T_v - t)} - 1]$ (4)

We know that $I(0) = Q$. In particular, for the vendor $I_1(0) = mQ_0$.

The number of deteriorated units during one cycle in vendor's place $= mQ_0 - \tilde{R}T_v$

Hence the average deterioration cost for the vendor $= \left\{\dfrac{\tilde{R}}{\theta}[e^{\theta T_v} - 1] - \tilde{R}T_v\right\}\dfrac{P_1}{T_v}$.

Vendor's total average annual cost is
TC_V = Ordering cost + Holding cost + Deterioration cost

$$= \frac{\tilde{R}K_1}{mQ_0} + \frac{(m-1)Q_0 h_1}{2} + \frac{P_1}{T_v}\left\{\frac{\tilde{R}}{\theta}[e^{\theta T_v} - 1] - \tilde{R}T_v\right\}, \text{ where } T_v = \frac{mQ_0}{\tilde{R}} \qquad (5)$$

Shortage cost for buyer is

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -\tilde{R} \quad \text{if } 0 \le t < T_1$$

$$\frac{dI_2(t)}{dt} = -\tilde{R} \quad \text{if} \quad T_1 \le t < T_b$$

$$I_2(t) = -\tilde{R}\,t$$

Shortage cost for buyer = $\tilde{S}\int_{T_1}^{T_b}(-\tilde{R}t)dt$

$$= -\frac{\tilde{S}\tilde{R}}{2}\left[(T_b)^2 - (T_1)^2\right], \text{ where } T_b = \frac{Q_0}{\tilde{R}} \quad (6)$$

we can find the buyer's total average annual cost as

$\tilde{TC}_b$ = Ordering cost + Holding cost + Deterioration cost + Shortage cost

$$= \frac{\tilde{R}K_2}{Q_0} + \frac{Q_0 h_2}{2} + \frac{P_2}{T_b}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta T_b} - 1\right] - \tilde{R}T_b\right\} - \frac{\tilde{S}\tilde{R}}{2}\left[(T_b)^2 - (T_1)^2\right], \text{ where } T_b = \frac{Q_0}{\tilde{R}} \quad (7)$$

Joint total cost of the vendor and the buyer without any coordination

$$= J\tilde{TC}_0 = \tilde{TC}_v + \tilde{TC}_b \quad (8)$$

$J\tilde{TC}_0$ is convex with respect to m (Appendix I).

Let m* be the optimum solution of $\underset{m \ge 1}{Min}\ J\tilde{TC}_0(m)$

$$m^* = \left\lceil \frac{\sqrt{h_1^2 + \frac{4K_1h_2}{K_2}(h_1 + 2P_1\theta)} - h_1}{2(h_1 + 2P_1\theta)} \right\rceil$$, where $\lceil x \rceil$ is the least integer greater than or equal to x. (Appendix II)

In order to achieve effective coordination, the vendor requests the buyer to alter his current order size by a factor, say F, (F>0) such that the vendor can benefit from lower setup, ordering and inventory holding costs. The buyer may be unwilling to accept this strategy due to the increase of inventory costs to him. Hence the vendor must compensate the buyer for his increased inventory costs and possibly provide an additional savings. We assume that the vendor offers a credit period M to the buyer and hence the buyer can earn interest from sales during the period M.

Buyer's new order size is fixed at FQ_0 and the vendor's new order size is nFQ_0 where n is a positive integer.

The cost incurred to the vendor in offering a credit period M is $P_2\tilde{R}Mi_1$.

Hence, in coordination, vendor's annual cost is

$$\frac{\tilde{R}K_1}{nFQ_0} + \frac{(n-1)FQ_0h_1}{2} + \frac{P_1}{T_v}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta T_v} - 1\right] - \tilde{R}T_v\right\} + P_2\tilde{R}Mi_1$$

$$= \frac{\tilde{R}K_1}{nFQ_0} + \frac{(n-1)FQ_0h_1}{2} + \frac{P_1\tilde{R}}{nFQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{n\theta FQ_0}{\tilde{R}}} - 1\right] - nFQ_0\right\} + P_2\tilde{R}Mi_1 \quad (9)$$

where $T_v = \frac{nFQ_0}{\tilde{R}}$.

Buyer's total interest earned from sales during the credit period is $P_2\tilde{R}Mi_2$. We assume that the vendor and the buyer also decide to reduce the ordering cost of the buyer through an investment. K_2 is the original ordering cost of the buyer.
$PO(G)$ is the planned ordering cost per order, which is a decreasing function of investment G and it is given by

$PO(G) = K_2 e^{-dG}$ where $0 < d < 1$ with

$PO(0) = K_2$ and $PO(G_0) = 0$.

Even with a great investment in ordering cost reduction, there must be some sort of operational cost for ordering. However it does not alter the conclusions in this chapter, therefore we assume that ordering cost can be reduced to zero with a maximum investment.

By considering the credit period and order cost reduction, the buyer's cost becomes

$$\frac{\tilde{R}K_2 e^{-dG}}{FQ_0} + \frac{FQ_0 h_2}{2} + \frac{P_2\tilde{R}}{FQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{\theta FQ_0}{\tilde{R}}} - 1\right] - FQ_0\right\}$$, where $T_b = \frac{FQ_0}{\tilde{R}}$

Joint total cost of the system with coordination is

$$\begin{aligned} JT\tilde{C}(F,G,M,n) = {} & \frac{\tilde{R}K_1}{nFQ_0} + \frac{(n-1)FQ_0 h_1}{2} + \frac{P_1\tilde{R}}{nFQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{n\theta FQ_0}{\tilde{R}}} - 1\right] - nFQ_0\right\} + P_2\tilde{R}Mi_1 \\ & + \frac{\tilde{R}K_2 e^{-dG}}{FQ_0} + \frac{FQ_0 h_2}{2} + \frac{P_2\tilde{R}}{FQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{\theta FQ_0}{\tilde{R}}} - 1\right] - FQ_0\right\} + G \end{aligned} \tag{10}$$

We have to minimize the joint total cost for the system which is a function of 4 variables F, G, M and n. First we can minimize the joint total cost ($JT\tilde{C}$) of the system for fixed n. $JT\tilde{C}$ is a function of three variables F, G and M.

The buyer will accept the new strategy if his increased cost is less than or equal to the interest earned in the credit period. Buyer's total inventory cost without any co-ordination, deterioration and order cost reduction is $\sqrt{2\tilde{R}K_2h_2}$.

The buyer will accept the new strategy if

$$\begin{aligned} & \frac{\tilde{R}K_2 e^{-dG}}{FQ_0} + \frac{FQ_0 h_2}{2} + \frac{P_2\tilde{R}}{FQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{\theta FQ_0}{\tilde{R}}} - 1\right] - FQ_0\right\} - \sqrt{2\tilde{R}K_2h_2} \\ & - \frac{\tilde{S}R}{2}\left[\left(\frac{FQ_0}{\tilde{R}}\right)^2 - (T_1)^2\right] \le P_2\tilde{R}Mi_2 \end{aligned}$$

Putting $Q_0 = \sqrt{\frac{2\tilde{R}K_2}{h_2}}$ in the above equation,

$$\begin{aligned} \tilde{M} \ge {} & \frac{e^{-dG}}{FP_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}} + \frac{F}{P_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}} + \frac{1}{Fi_2}\sqrt{\frac{h_2}{2\tilde{R}K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] - F\sqrt{\frac{2\tilde{R}K_2}{h_2}}\right\} \\ & - \frac{\tilde{S}\tilde{R}}{2P_2\tilde{R}i_2}\left[\frac{2F^2\tilde{R}K_2}{\tilde{R}^2h_2} - T_1^2\right] - \frac{1}{P_2i_2}\sqrt{\frac{2K_2h_2}{\tilde{R}}} \end{aligned}$$

$$\tilde{M} \ge \frac{1}{P_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}}\left[F + \frac{e^{-dG}}{F}\right] + \frac{1}{\theta Fi_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] - \frac{1}{i_2} - \frac{2}{P_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}} - \frac{\tilde{S}}{2P_2i_2}\left[\frac{2F^2K_2}{\tilde{R}h_2} - T_1^2\right]$$

$$\tilde{M} \ge \frac{1}{P_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}}\left[F + \frac{e^{-dG}}{F} - 2\right] + \frac{1}{\theta Fi_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] - \frac{1}{i_2} - \frac{\tilde{S}}{2P_2i_2}\left[\frac{2F^2K_2}{\tilde{R}\,h_2} - T_1^2\right] \tag{11}$$

Consider the equality sign in result (11) and use M in equation (10) we get

$$JT\tilde{C}(F,G)=\frac{\tilde{R}K_1}{nFQ_0}+\frac{(n-1)FQ_0h_1}{2}+\frac{P_1\tilde{R}}{nFQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{\theta nFQ_0}{\tilde{R}}}-1\right]-nFQ_0\right\}$$
$$+P_2\tilde{R}i_1\left\{\frac{1}{P_2i_2}\sqrt{\frac{K_2h_2}{2\tilde{R}}}\left[F+\frac{e^{-dG}}{F}-2\right]+\frac{1}{\theta Fi_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]-\frac{1}{i_2}-\frac{\tilde{S}}{2P_2i_2}\left[\frac{2F^2K_2}{\tilde{R}\ h_2}-T_1^{\ 2}\right]\right\}$$
$$+\frac{\tilde{R}K_2e^{-dG}}{FQ_0}+\frac{FQ_0h_2}{2}+\frac{P_2\tilde{R}}{FQ_0}\left\{\frac{\tilde{R}}{\theta}\left[e^{\frac{\theta FQ_0}{\tilde{R}}}-1\right]-FQ_0\right\}+G-\frac{\tilde{S}\tilde{R}}{2}\left[\left(\frac{FQ_0}{\tilde{R}}\right)^2-(T_1)^2\right]$$

Here also putting $Q_0=\sqrt{\frac{2\tilde{R}K_2}{h_2}}$.

$$JT\tilde{C}(F,G)=\frac{K_1}{nF}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+(n-1)Fh_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1}{nF}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}$$
$$+\frac{i_1}{i_2}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left[F+\frac{e^{-dG}}{F}-2\right]+\frac{P_2(i_1+i_2)}{Fi_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}$$
$$+\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left[F+\frac{e^{-dG}}{F}\right]-\tilde{R}\left(P_1+P_2\left(\frac{i_1+i_2}{i_2}\right)\right)+G-\frac{\tilde{S}\tilde{R}}{2}\left(\frac{i_1+i_2}{i_2}\right)\left[\frac{2F^2K_2}{\tilde{R}\ h_2}-T_1^{\ 2}\right] \quad (12)$$

$JT\tilde{C}(F,G)$ is convex with respect to F and G for any given positive integer n (Appendix III).

The optimal values of F and G can be found by solving the equations $\frac{\partial JT\tilde{C}(F,G)}{\partial F}=0$ and $\frac{\partial JT\tilde{C}(F,G)}{\partial G}=0$ simultaneously. By substituting these optimal values in equation (12), the joint total cost for the system becomes a function of n.

$$JT\tilde{C}(n)=\frac{k_1}{nF}\sqrt{\frac{\tilde{R}h_2}{2k_2}}+(n-1)Fh_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1}{nF}\sqrt{\frac{\tilde{R}h_2}{2k_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta nF\sqrt{\frac{2k_2}{\tilde{R}h_2}}}-1\right]\right\}$$
$$+\frac{i_1}{i_2}\sqrt{\frac{\tilde{R}k_2h_2}{2}}\left[F+\frac{e^{-dG}}{F}-2\right]+\frac{P_2(i_1+i_2)}{Fi_2}\sqrt{\frac{\tilde{R}h_2}{2k_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2k_2}{\tilde{R}h_2}}}-1\right]\right\}+\sqrt{\frac{\tilde{R}k_2h_2}{2}}\left[F+\frac{e^{-dG}}{F}\right]$$
$$-\tilde{R}\left(P_1+P_2\left(\frac{i_1+i_2}{i_2}\right)\right)-\frac{\tilde{S}\tilde{R}}{2}\left[\frac{2F^2k_2}{\tilde{R}h_2}-(T_1)^2\right]\left(\frac{i_1+i_2}{i_2}\right)+G$$

Thus our problem is concluded as the following optimization problem . $\underset{n\geq 1}{\text{Minimize}}\ JT\tilde{C}(n)$ (13)

The optimum value of n can be obtained as

$$\therefore n^*=\begin{cases}\left\lceil \frac{1}{F}\sqrt{\frac{k_1}{k_2}\left(\frac{h_2}{h_1+P_1\theta}\right)+\frac{1}{4}}-\frac{1}{2}\right\rceil & \text{if} \quad \frac{k_1h_2}{k_2F^2(h_1+P_1\theta)}\geq 2\\ 1 & \text{otherwise}\end{cases} \quad (14)$$

where $\lceil x \rceil$ is the least integer greater than or equal to x. (Appendix IV)
We propose the following algorithm to find the solution for the above optimization problem.

V. ALGORITHM

Step 1. Put G = 0 in the equation $\frac{\partial JT\tilde{C}(F,G)}{\partial F}=0$ and find F.

Step 2. Substitute F in the equation $\frac{\partial JT\tilde{C}(F,G)}{\partial G}=0$ and find G

Step 3. Put G in the equation $\frac{\partial JT\tilde{C}(F,G)}{\partial F} = 0$ to find F

Step 4.

Step 5. Repeat steps 2 and 3 until there is no change in the successive values of F and G.

Step 6.

Step 7. If $\frac{k_1 h_2}{k_2 F^2 (h_1 + P_1\theta)} \geq 2$ then $n^* = \left[\frac{1}{F} \sqrt{\frac{K_1}{K_2} \left(\frac{h_2}{h_1 + P_1\theta} \right)} - \frac{1}{2} \right]$ otherwise $n^* = 1$.

Step 8. Find m*

$$\tilde{M} = \frac{1}{P_2 i_2} \sqrt{\frac{K_2 h_2}{2\tilde{R}}} \left[F + \frac{e^{-dG}}{F} - 2 \right] + \frac{1}{\theta F i_2} \sqrt{\frac{\tilde{R} h_2}{2K_2}} \left[e^{\theta F \sqrt{\frac{2K_2}{\tilde{R} h_2}}} - 1 \right] - \frac{1}{i_2}$$

Step 9. Compute $- \frac{\tilde{S}}{2P_2 i_2} \left[\frac{2FK_2}{\tilde{R} h_2} - (T_1)^2 \right]$

Step 10. Compute $JT\tilde{C}(n^*)$ and $JT\tilde{C}_0(m^*)$.

Step 11. Compute ΔJTC where $\Delta JTC = \left(\frac{JTC_0 - JTC}{JTC_0} \right) \times 100\%$

VI. NUMERICAL EXAMPLES

Example 1- Consider the following data $\tilde{R}$ = (900,1000,1100) units/year, P_1=\$300 / unit, P_2=\$400 / unit, h_1' =\$100 /unit/year, h_2' =\$150 /unit/year, T_1=0.1, $\tilde{S} = (40,50,60)$ θ = 0.07, d = 0.002, K_1= \$300 /order, K_2=\$200 /order, , i_1= 0.1, i_2= 0.1. By applying the above proposed algorithm we find the optimum values. JTC_0 =\$16502, JTC = \$13501, ΔJTC = 18.185%. Thus by the effective coordination between the vendor and the buyer in a supply chain, the joint total cost of the system can be reduced. Hence the relative improvement of the joint total cost (ΔJTC) by the investment in reducing ordering cost is also high.

Example 2- First nine datas in example 1 are kept fixed while the remaining data are changed. The results are shown in Table (1). From this table we observe that when i_2 increases joint total cost also increases and hence the relative improvement of the joint total cost decreases. The investment in reducing the ordering cost of the buyer also decreases when i_2 increases.

Table (1)
Changes in the relative improvement of the joint total cost with the cost of capital of the buyer

K_1	K_2	i_1	i_2	JTC_0	JTC	ΔJTC	G
300	200	0.05	0.15	16992	14347	15.566	210.3
300	200	0.05	0.2	17474	14684	15.969	210.08
300	300	0.05	0.1	18167	13689	24.649	215.72
300	300	0.05	0.15	18785	15975	14.958	192.7
500	200	0.05	0.15	19563	14678	24.970	610.75

500	200	0.05	0.2	20050	17720	11.620	560.21

Table (2)
Analysis on relative improvement of the joint total cost with the ordering cost of the vendor

K_1	K_2	i_1	i_2	JTC_0	JTC	ΔJTC
300	200	0.05	0.1	16482	13524	17.946
400	200	0.05	0.1	17841	15756	11.686
460	200	0.05	0.1	18579	16456	11.426
500	200	0.05	0.1	19046	17200	9.692

Table (3)
Analysis on relative improvement of the joint total cost with the ordering cost of the buyer

K_1	K_2	i_1	i_2	JTC_0	JTC	ΔJTC
300	200	0.05	0.15	16992	14323	15.707
300	250	0.05	0.15	17931	14994	16.379
300	300	0.05	0.15	18785	15884	15.443

Example 3

We change the values of the parameters K_1, K_2, i_1, and i_2 while the other parameters are kept fixed. The computational values are presented in Table (2). From Table (2) we come to know that when K_1 increases the joint total cost of the system increases. Consequently the relative improvement of the joint total cost decreases.

Example 4

Here we analyze the joint total cost with the increase of the ordering cost of the buyer (K_2). We consider the same data as in example 1 except K_1, K_2, i_1 and i_2. The results are shown in Table (3). From this Table we see that joint total cost of the system (with or without coordination) increases with the increase of K_2 and hence the relative improvement of the joint total cost decreases.

VII. CONCLUSION

The efficient management of inventories in a supply chain is achieved through better coordination and more cooperation between the vendor and the buyer. Here, a two-stage fuzzy supply chain model with single vendor and single buyer was considered. Deterioration of items was also taken shortage for the buyer was allowed which was completely backlogged. Shortage is very practical in day-to-day life and hence we can't omit shortage of items. To encourage the buyer, the vendor offer credit period incentives to the buyer and invest for order cost reduction in the supply chain. The demand and shortage cost were taken as fuzzy parameters. The joint total cost with coordination is lesser than the joint total cost without coordination. This is illustrated through the numerical examples. This paper may be extended by considering partial backlogging shortages for the vendor and the buyer and the fuzzy nature of order cost, holding cost and deterioration rate.

APPENDIX I Proof of $J\tilde{TC}_0$ is convex with respect to m.

$$J\tilde{TC}_0 = \frac{K_1}{m}\sqrt{\frac{\tilde{R}h_2}{2K_2}} + (m-1)h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}} + \frac{P_1\tilde{R}}{m\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] - P_1\tilde{R}$$

$$+\sqrt{2\tilde{R}K_2h_2} + \frac{P_2\tilde{R}}{\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] - P_2\tilde{R} - \frac{\tilde{S}\tilde{R}}{2}\left[\frac{2K_2}{Rh_2} - (T_1)^2\right]$$

$$\frac{\partial J\tilde{TC}_0}{\partial m} = \frac{-K_1}{m^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}} + h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}} - \frac{P_1\tilde{R}}{m^2\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}} - 1\right] + \frac{P_1\tilde{R}}{m}e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}$$

$$\frac{\partial^2 JT\tilde{C}_0}{\partial m^2}=\frac{2K_1}{m^3}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+\frac{2P_1\tilde{R}}{m^3\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]-\frac{2P_1\tilde{R}}{m^2}e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{P_1\tilde{R}\theta}{m}\sqrt{\frac{2K_2}{\tilde{R}h_2}}e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}$$

$$\frac{\partial JT\tilde{C}_0}{\partial m}=0\Rightarrow\frac{-K_1}{m^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}-\frac{P_1\tilde{R}}{m^2\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]+\frac{P_1\tilde{R}}{m}e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}=0$$

$$\therefore\frac{\partial^2 JT\tilde{C}_0}{\partial m^2}=\frac{2h_1}{m}\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1\tilde{R}\theta}{m}\sqrt{\frac{2K_2}{\tilde{R}h_2}}e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}>0$$

$\therefore JT\tilde{C}_0$ is convex with respect to m.

APPENDIX II

Let m* be the optimum solution of $\underset{m\ge 1}{Min}\, JT\tilde{C}_0(m)$.

$$\therefore m^*=Max\left\{\min\left\{m/JT\tilde{C}_0(m+1)\ge JT\tilde{C}_0(m)\right\},0\right\}$$

Now $JT\tilde{C}_0(m+1)\ge JT\tilde{C}_0(m)$ implies

$$\frac{K_1}{m+1}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+mh_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1\tilde{R}}{(m+1)\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta(m+1)\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]$$

$$\ge\frac{K_1}{m}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+(m-1)h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1\tilde{R}}{m\theta}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left[e^{\theta m\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]$$

Hence we get

$$m^*=Max\left\{\min\left\{m/h_1m(m+1)+2P_1\theta m^2\ge\frac{2\tilde{R}K_1}{Q_0^2}\right\},0\right\}$$

Also by taking equality sign

$$h_1m^2+h_1m+2P_1\theta m^2=\frac{K_1h_2}{K_2}$$

$$m=\frac{\sqrt{h_1^2+\frac{4K_1h_2}{K_2}(h_1+2P_1\theta)}-h_1}{2(h_1+2P_1\theta)}$$

since m cannot be negative.

$$\therefore m^*=\left\lceil\frac{\sqrt{h_1^2+\frac{4K_1h_2}{K_2}(h_1+2P_1\theta)}-h_1}{2(h_1+2P_1\theta)}\right\rceil$$

where $\lceil x\rceil$ is the least integer greater than or equal to x.

APPENDIX III Proof of $JT\tilde{C}$ (F, G) is convex with respect to F and G.

$$\frac{\partial JT\tilde{C}(F,G)}{\partial F}=\frac{-K_1}{nF^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+(n-1)h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}-\frac{P_1}{nF^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}$$

$$+\frac{P_1}{nF}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\frac{\tilde{R}}{\theta}\theta n\sqrt{\frac{2K_2}{\tilde{R}h_2}}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{i_1}{i_2}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left[1-\frac{e^{-dG}}{F^2}\right]$$

$$-\frac{P_2(i_1+i_2)}{F^2i_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}+\frac{P_2(i_1+i_2)}{Fi_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\frac{\tilde{R}}{\theta}\theta\sqrt{\frac{2K2}{\tilde{R}h_2}}e^{\theta F\sqrt{\frac{2K2}{\tilde{R}h_2}}}$$

$$+\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left[1-\frac{e^{-dG}}{F^2}\right]-\frac{2\tilde{S}FK_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)$$

$$\frac{\partial^2 JT\tilde{C}(F,G)}{\partial F^2}=\frac{K_1}{nF^3}\sqrt{\frac{2\tilde{R}h_2}{K_2}}+\frac{P_1}{nF^3}\sqrt{\frac{2\tilde{R}h_2}{K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}-\frac{2P_1\tilde{R}}{F^2}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}$$

$$+\sqrt{2\tilde{R}K_2h_2}\left(\frac{i_1+i_2}{i_2}\right)\left(\frac{e^{-dG}}{F^3}\right)+\frac{P_2(i_1+i_2)}{F^3 i_2}\sqrt{\frac{2\tilde{R}h_2}{K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}$$

$$-\frac{2P_2(i_1+i_2)\tilde{R}}{F^2 i_2}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{P_2(i_1+i_2)\theta}{Fi_2}\sqrt{\frac{2\tilde{R}K_2}{h_2}}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{2\tilde{S}K_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)$$

$$\frac{\partial JT\tilde{C}(F,G)}{\partial G}=\frac{-de^{-dG}}{F}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left(\frac{i_1}{i_2}+1\right)+1.$$

$$\frac{\partial^2 JT\tilde{C}(F,G)}{\partial G^2}=\frac{d^2e^{-dG}}{F}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left(\frac{i_1}{i_2}+1\right)>0.$$

$$\frac{\partial JT\tilde{C}(F,G)}{\partial G}=0\Rightarrow G=\frac{1}{d}\ln\left[\frac{(i_1+i_2)d}{i_2F}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\right].$$

$$\frac{\partial^2 JT\tilde{C}(F,G)}{\partial F\partial G}=\frac{1}{F},\qquad \frac{\partial^2 JT\tilde{C}(F,G)}{\partial G^2}=d$$

$$\frac{\partial JT\tilde{C}(F,G)}{\partial F}=0\Rightarrow\frac{K_1}{nF^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}+\frac{P_1}{nF^2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}+\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left(\frac{i_1+i_2}{i_2}\right)\frac{e^{-dG}}{F^2}$$

$$+\frac{P_2(i_1+i_2)}{F^2i_2}\sqrt{\frac{\tilde{R}h_2}{2K_2}}\left\{\frac{\tilde{R}}{\theta}\left[e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-1\right]\right\}=(n-1)h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1\tilde{R}}{F}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}$$

$$+\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left(\frac{i_1+i_2}{i_2}\right)+\frac{P_2(i_1+i_2)\tilde{R}}{Fi_2}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{2SFK_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)$$

$$\frac{\partial^2 JT\tilde{C}(F,G)}{\partial F^2}=\frac{2}{F}\left\{(n-1)h_1\sqrt{\frac{\tilde{R}K_2}{2h_2}}+\frac{P_1\tilde{R}}{F}e^{n\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\sqrt{\frac{\tilde{R}K_2h_2}{2}}\left(\frac{i_1+i_2}{i_2}\right)\right.$$

$$\left.+\frac{P_2(i_1+i_2)\tilde{R}}{Fi_2}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{2\tilde{S}FK_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)\right\}-\frac{2P_1\tilde{R}}{F^2}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{2P_2(i_1+i_2)\tilde{R}}{F^2i_2}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}$$

$$+\frac{P_1n\theta}{F}\sqrt{\frac{2\tilde{R}K_2}{h_2}}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{P_2(i_1+i_2)\theta}{Fi_2}\sqrt{\frac{2\tilde{R}K_2}{h_2}}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{2\tilde{S}K_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)$$

$$=\frac{2(n-1)h_1}{F}\sqrt{\frac{\tilde{R}K_2}{h_2}}+\frac{2(i_1+i_2)}{Fi_2}\sqrt{\frac{\tilde{R}K_2h_2}{2}}$$

$$+\frac{P_1n\theta}{F}\sqrt{\frac{2\tilde{R}K_2}{h_2}}e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{P_2(i_1+i_2)\theta}{Fi_2}\sqrt{\frac{2\tilde{R}K_2}{h_2}}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}-\frac{6\tilde{S}K_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)$$

$$\frac{\partial^2 JT\tilde{C}(F,G)}{\partial F^2}\cdot\frac{\partial^2 JT\tilde{C}(F,G)}{\partial G^2}-\left[\frac{\partial^2 JT\tilde{C}(F,G)}{\partial F\partial G}\right]^2$$

$$=\frac{1}{F^2}\left\{Fd\left\{2\sqrt{\frac{\tilde{R}K_2}{2h_2}}\left[(n-1)h_1+P_1n\theta e^{\theta nF\sqrt{\frac{2K_2}{\tilde{R}h_2}}}+\frac{P_2(i_1+i_2)\theta}{i_2}e^{\theta F\sqrt{\frac{2K_2}{\tilde{R}h_2}}}\right]-\frac{6\tilde{S}FK_2}{h_2}\left(\frac{i_1+i_2}{i_2}\right)\right.\right.$$

$$+\frac{2(i_1+i_2)}{i_2}\sqrt{\frac{\tilde{R}K_2h_2}{2}}\Bigg\}-1\Bigg\}>0 .$$

APPENDIX IV

Let n* be the optimal solution of $\underset{n\geq 1}{Minimize}\ JT\tilde{C}(n)$

Then $JT\tilde{C}(n^*)-JT\tilde{C}(n^*-1)\leq 0$ and $JT\tilde{C}(n^*)-JT\tilde{C}(n^*+1)\leq 0$

Now $JT\tilde{C}(n^*)-JT\tilde{C}(n^*-1)\leq 0$ implies

$$n^*(n^*-1)\leq \frac{\frac{k_1}{F}\sqrt{\frac{\tilde{R}h_2}{2k_2}}}{Fh_1\sqrt{\frac{\tilde{R}k_2}{2h_2}}+\frac{P_1\theta F}{2}\sqrt{\frac{2\tilde{R}k_2}{h_2}}}$$

$$n^*(n^*-1)\leq \frac{k_1h_2}{k_2F^2\{h_1+P_1\theta\}},\quad \left(n^*-\frac{1}{2}\right)^2\leq \frac{k_1h_2}{k_2F^2\{h_1+P_1\theta\}}+\frac{1}{4} \qquad (14)$$

Now $JT\tilde{C}(n^*)-JT\tilde{C}(n^*+1)\leq 0$ implies , $n^*(n^*+1)\geq \frac{k_1h_2}{k_2F^2\{h_1+P_1\theta\}}$

$$\left(n^*+\frac{1}{2}\right)^2\geq \frac{k_1h_2}{k_2F^2\{h_1+P_1\theta\}}+\frac{1}{4} \qquad (15)$$

From (14) , $n^*\leq \sqrt{\frac{k_1}{k_2F^2}\left(\frac{h_2}{h_1+P_1\theta}\right)+\frac{1}{4}}+\frac{1}{2}$ (16) From (15) ,

$$n^*\geq \sqrt{\frac{k_1}{k_2F^2}\left(\frac{h_2}{h_1+P_1\theta}\right)+\frac{1}{4}}-\frac{1}{2} \qquad (17)$$

From (16) and (17) , $\therefore n^*=\left\lceil\sqrt{\frac{k_1}{k_2F^2}\left(\frac{h_2}{h_1+P_1\theta}\right)+\frac{1}{4}}-\frac{1}{2}\right\rceil$

If $2>\frac{k_1}{k_2}\left(\frac{h_2}{F^2(h_1+P_1\theta)}\right)>0$ then $n^*=1$.

$$\therefore n^*=\begin{cases}\left\lceil\sqrt{\frac{k_1}{k_2F^2}\left(\frac{h_2}{h_1+P_1\theta}\right)+\frac{1}{4}}-\frac{1}{2}\right\rceil & \text{if } \frac{k_1h_2}{k_2F^2(h_1+P_1\theta)}\geq 2\\ 1 & \text{otherwise.}\end{cases}$$

REFERENCES

[1] Banerjee A joint economic-lot-size model for purchase and vendor, Decision Sciences, 17(1986) 292-311.

[2] Bellman R and Zadeh LA(1970), " Decision-making in a fuzzy environment". Management science,17, B141-B164.

[3] Billington P (1987) The classic economic production quantity model with setup cost as a function of capital expenditure. Decision Sciences 18:25-42.

[4] Chang HC, Ouyang LY, Wu KS, Ho CH (2006) Integrated vendor buyer cooperative inventory models with controllable lead time and ordering cost reduction. European Journal of Operational Research Vol.170, pp. 481-495.

[5] Chen JM, Chen LT, 2004. Pricing and Lot sizing for a deteriorating item in a periodic review inventory system with shortages, Journal of the Operational Research Society 55, 892-901.

[6] Chen LH, Kang FS, Coordination between vendor and buyer considering trade credit and items of imperfect quality, International Journal of Production Economics, 123(2010)52-56.

[7] Chen SH, Operations of fuzzy numbers with function principle, Tamkang Journal of Management Sciences, 6(1), (1985), pp 13-26.

[8] Coates ER (1996) Manufacturing setup cost reduction, Comput Ind Eng 31:111-114.

[9] Giannoccaro L and Pontrandolfo P supply chain coordination by revenue sharing contracts, International Journal of Production Economics,89(2004)131-139.

[10] Goyal SK, Gupta YP (1989) Integrated inventory models: the buyer-vendor coordination. European Journal of Operational Research, 14:261-269.

[11] Jacker JV, Rosenblatt MJ (1985) Single period inventory models with demand uncertainly and quantity discounts: behavioural implication and a solution procedure. Nav Res Logist 32:537- 550.

[12] Kim S, Hayya J, Hong J (992) Setup reduction in the economic production quantity. Decision Sciences 23:500-508.

[13] Lee HL, Rosenblatt MJ (1986) A generalized quantity discount pricing method to increase supplier's profits. Management Science 32:1177-1185.

[14] Luo J (2007) Buyer-vendor inventory coordination with credit period incentives. International Journal of Production Economics 108:143-152.

[15] Monahan JP (1984) A quantity discount pricing model to increase profits. Manage Sci 30:720-726.

[16] Porteus EL (1985) Investing in reduced setups in the EOQ model. Manage Sci 31:998-1010.

[17] Sarkar BR, Jamal, Wang S (2000) Supply chain model for perishable products under inflation and permissible delay in payment. Comput Oper Res 27:380-390.

[18] Tanaka H, Okuda T and Asai K (1974) "On fuzzy mathematical programming " Journal of cybernetics,3(4) , 37-46.

[19] Uthayakumar R, Parvathi P (2008) Inventory models with mixture of backorders involvingreducible lead time and setup cost. Opsearch 45(1):12-33.

[20] Weng ZK (1995) Channel coordination and quantity discounts. Manage Sci 41:1509-1522.

[21] Wong K, Qi .J and S.Y.S. Leung, Coordinating supply chain with sales rebate contracts and vendor-managed inventory, International Journal of Production Economics 120(2009)151- 161.

[22] Yu TG, Liang L, Wang C, Wang ZQ (2004) An integrated inventory vendor-managed inventory model for deteriorating items. Chin J Manage Sci 12:32-37.

[23] Zadeh LA (1965) "Fuzzy sets inform and Control" 8,338-353.

[24] Zimmerman HJ (1976), "Description and optimization of fuzzy system", International journal of General systems, 2,209-215

[25] Zimmerman HJ (1978), "fuzzy linear programming with several objective functions", fuzzy sets and systems, 1,46-55.

AUTHORS

First Author – J.Sujatha, Assistant Professor ,Department of Mathematics ,Quaid-E-Millath Government College for Women (Autonomous), Chennai-600 002 ,Tamil Nadu, India.

Second Author – P.Parvathi, Head & Associate Professor ,Department of Mathematics ,Quaid-E-Millath Government College for Women (Autonomous), Chennai-600 002 Tamil Nadu, India