The Stacked Semi-Groups and Fuzzy Stacked Systems on Transportation Models

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Abstract- This paper touched the concept of a new system of abstract algebra .Where we relied on a set of semi order, and the stack of elements in the form of a matrix. We proved that this system is a semi-group, after providing the fuzzy system to the system, and access to the algebraic operations determine the significance of any element in the set and reliance on the location element in the set . Identify new methods to resolve the issues of transportation models in Operations Research.

Index Terms- stacked set , stacked system , order fuzzy stacked , level stacked system , stacked-semi-group.

I. Introduction

The transportation model is actually a class of the linear programming models discussed in quantitative module . As it is for linear programming, software is available to solve transportation problems. To fully use such programs, though, you need to understand the assumptions that underlie the model. The basic idea in a transportation problem is that there are sites or sources of product that need to be shipped to destinations. Usually in issues transport models we are looking for reducing cost less as possible or increase (and a few of these exist). In this paper we tried to address a new concept which determine the cost to be accessible, and we dealt using fuzzy systems, range to the desired cost, even if we cannot get to the same cost . And the introduction of the concept of abstract algebra to make it semi-group and prove it.

Generally this study opens the door to new relationships and mathematical operations are very important, and will appear to the spotlight soon.

II. Preliminaries

2.1 Definition[6]

For a set A ,we define a membership function μ_A such as

$$\mu_A(x) = \begin{cases} 1 & iff \quad x \in A \\ 0 & iff \quad x \notin A \end{cases}$$

(" iff " is short for "if and only if ").

X be a classical set of objects, called the universe, whose generic elements are denoted x . Membership in a classical subset A of X is often viewed as a characteristic function , μ_A from X to {0,1} such that :

$\mu_A : X \to \{ 0 , 1 \}$.

2.2 Definition[8]

Let X be a space of points (objects),with a generic element of X denoted by x . Thus, X = {x}. A fuzzy set (class) A in X is characterized by a membership (characteristic) function $f_A(x)$ which associates with each point in X a real number in the interval [0, 1] , with the value of $f_A(x)$ at x representing the "grade of membership" of : x in A. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of 0 in A. When A is a set in the ordinary sense of the term, its membership function can take on only two values 0 and 1, with $f_A(x) = 1$ or 0 according as x does or does not belong to A . Thus, in this case $f_A(x)$ reduces to the familiar characteristic function of a set A . (When there is a need to differentiate between such sets and fuzzy sets, the sets with two-valued characteristic functions will be referred to as ordinary sets or simply sets).

2.3 Definition[2]

If Ã is a collection of objects denoted generically by x then a fuzzy set A in Ã is a set of ordered pairs:

$\tilde{A} = \{(x , \mu_{\tilde{A}}(x)) \mid x \in T \}$.

$\mu_{\tilde{A}}(x)$is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in A which maps X to the membership space M. (When M contains only the two points 0 and 1, Ã is nonfuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set.) The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

2.4 Definition[2]

The support of a fuzzy set Ã, S(Ã), is the crisp set of all x ∈ X such that :

$\mu_{\tilde{A}}(x) > 0$.

2.5 Definition[2]

The (crisp) set of elements that belong to the fuzzy set A at least to the degree a is called the α - level set :

$A_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \}$

$\tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha \}$ is called "strong α - level set" or "strong α -cut."

2.6 Definition[5]

A real function f defined on a real interval I is convex on I iff:

$\forall x_1,x_2,x_3 \in I: x_1 < x_2 < x_3 : [f(x_2)-f(x_1)] / [x_2-x_1] \leq [f(x_3)-f(x_2)] / [x_3-x_2]$.

or:

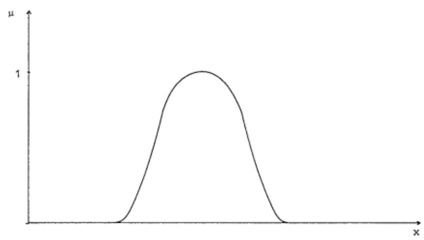
$\forall x_1, x_2, x_3 \in I : x_1 < x_2 < x_3 : [f(x_2) - f(x_1)] / [x_2 - x_1] \leq [f(x_3) - f(x_1)] / [x_3 - x_1]$.

The function f is strictly convex on I if, in the above inequalities, equality cannot hold.

2.7 Definition[2]

A fuzzy set Ã is convex if :

$\mu_{\tilde{A}} (\lambda x_1 + (1 - \lambda) x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, $x_1, x_2 \in X$, $\lambda \in [0, 1]$



Figures 1
Convex fuzzy set

2.8 Definition[7]

The mathematical systems is a set of interacting or interdependent components forming an integrated whole or a set of elements (often called 'components') and relationships which are different from relationships of the set or its elements to other elements or sets .

2.9 Example

(R , +) is a system , R is a set of all the reel numbers, and (+) the relation between the elements .

2.10 Definition[7]

A binary operation on a set S is a mapping of the Cartesian product S × S into S.

2.11 Definition[4]

Let S be a set and $\sigma : S \times S \rightarrow S$ a binary operation that maps each ordered pair (x , y) of S to an element σ(x , y) of S. The pair (S, σ) (or just S, if there is no fear of confusion) is called a groupoid .

2.12 Definition[3]

By groupoid (S, *) we shall mean a non-empty set S on which a binary operation * is defined. That is to say, we have a mapping *: S × S → S.

We shall say that (S, *) is a semigroup if * is associative, i.e. if

$$(\forall\ x, y, z \in S), ((x, y)*, z)* = (x, (y, z)*)*$$

2.13 Definition[3]

S is a finite semigroup if it has only a finitely many elements.

2.14 Definition[3]

A commutative semigroup is a semigroup S with property :
(∀ x, y ∈ S) (xy = yx).

2.15 Definition[3]

If there exists an element 1 of S such that (∀ x ∈ S) x1 = 1x = x .We say that 1 is an identity (element) of S and that S is a semigroup with identity.

2.16 Definition[1]

let T_α be a finite set , where T_α be a stacked set if and only if $a_\alpha \in T_\alpha$, $\alpha \in N/0$, α is the number of methods stacking elements in the set, and it is called paths ($P_1, P_2, \ldots, P_\alpha$).

2.17 Definition[1]

The system (T_α , τ) called stacked - system if and only if $a_\gamma\ \tau\ b_\beta = \min_0 (a_\gamma , b_\beta)$, and the system looking for (zero convergence) , and The system (T_α , Ʈ) called stacked - system if and only if a_γ Ʈ $b_\beta = \max_0 (a_\gamma , b_\beta)$, and the system looking for (zero spacing) . a_γ and $b_\beta \in T_\alpha$.

2.18 Definition[1]

The system (T_α , τ) called stacked - system if and only if $a_\gamma\ \tau\ b_\beta = \min_t(a_\gamma , b_\beta)$, and the system looking for (convergence of t) , and The system (T_α , Ʈ) called stacked - system if and only if a_γ Ʈ $b_\beta = \max_t(a_\gamma , b_\beta)$, and the system looking for (spacing of t) . a_γ and $b_\beta \in T_\alpha$, $t \in \mathcal{R}$.

2.19 Definition[1]

The order element on stacked system T_α ,where the system looking for zero convergence or zero spacing, is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ , ($O_0(a_\gamma)$) :

$$O_0(a_\gamma) = \lceil a_\gamma \rfloor_0 = \left[\frac{|a_{\gamma 1}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \frac{|a_{\gamma 2}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \ldots + \frac{|a_{\gamma \alpha}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha = \left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha .$$

2.20 Definition[1]

The order element on stacked system T_α ,where the system looking for convergence of t (or spacing of t), is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ ,($O_t(a_\gamma)$) :

$$O_t(a_\gamma) = \lceil a_\gamma \rfloor_t = \left[\frac{|a_{\gamma 1} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \frac{|a_{\gamma 2} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \ldots + \frac{|a_{\gamma \alpha} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha = \left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha .$$

2.21 Definition[1]

The order stacked set T_α in zero convergence system is set $O_0(T_\alpha) = \{x_1, x_2, \dots, x_n\}$ if $\lceil x_1 \rfloor_0 < \lceil x_2 \rfloor_0 < \dots < \lceil x_n \rfloor_0$ .And the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1}, \dots, x_1\}$ if $\lceil x_1 \rfloor_0 > \lceil x_2 \rfloor_0 > \dots > \lceil x_n \rfloor_0$.

The order stacked set T_α , where the system looking for convergence of t (or spacing of t), is set $O_0(T_\alpha) = \{x_1, x_2, \dots, x_n\}$ if $\lceil x_1 \rfloor_t < \lceil x_2 \rfloor_t < \dots < \lceil x_n \rfloor_t$ .And the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1}, \dots, x_1\}$ if $\lceil x_1 \rfloor_t > \lceil x_2 \rfloor_t > \dots > \lceil x_n \rfloor_t$.

2.22 Definition[1]

- If (T_α, τ) or (T_α, Υ) is stacked – system then $\forall a_\alpha, b_\beta \in T_\alpha$:

$Max_t(a_\alpha, b_\beta) = a_\alpha \Upsilon b_\beta$, and $Min_t(a_\alpha, b_\beta) = a_\alpha \tau b_\beta$

- If (T_α, τ) or (T_α, Υ) is stacked – system then $\forall a_\alpha, b_\beta \in T_\alpha$:

$$\max_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha : if \lceil a_\alpha \rfloor_t > \lceil b_\beta \rfloor_t . \\ b_\beta : if \lceil b_\beta \rfloor_t > \lceil a_\alpha \rfloor_t . \end{cases}$$

$$\min_t(a_\alpha, b_\beta) = \begin{cases} a_\alpha : if \lceil a_\alpha \rfloor_t < \lceil b_\beta \rfloor_t . \\ b_\beta : if \lceil b_\beta \rfloor_t < \lceil a_\alpha \rfloor_t . \end{cases}$$

- If $a_\alpha = t$,(in $\lceil a_\alpha \rfloor_t$) then we suppose that $|a_\alpha - t| = \Delta t$, and where $\sum_i^\alpha |a_i - t|$, $\alpha \in \{1, ..i\}$ then, we compensate $|a_\alpha - t| = 0$.
- If $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil a_\alpha \rfloor_t \neq \lceil b_\beta \rfloor_t$ the system is type-1 .

2.23 Definition[1]

A stacked-semigroup is a stacked-system T_α , with associative binary operation .

2.23 Theorem [1]

(i) If the systems (T_α, τ) is a stacked-system (type – 1), then (T_α, τ) is a semigroup and called a stacked-semigroup .

(ii) If the systems $(T_\alpha, \Upsilon)$is a stacked-system (type – 1), then (T_α, Υ) is a semigroup and called a stacked-semigroup.

Prove this theorem earlier in paper [1]

III. STACKED SET AND FUZZY SYSTEMS ON TRANSPORTATION MODELS

Let $T_\alpha = \{x_{11}, x_{12}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{nn}\}$ be a set of the cost in the transportation model , from sales centers to distribution centers, $n \in N / 0$

Or : T_α =

x_{11}	x_{12}	…..	x_{1n}
x_{21}	x_{22}	…..	x_{2n}
..∴	..∴	..∴	⋮
x_{n1}	x_{n2}	…..	x_{nn}

Table 1

Such that rows means selling centers , and columns means the distribution centers , or the opposite.

Then T_α be a set with some relations between an element such that we stacks this relations vertically and horizontally, then T_α is called a stacked set .

And this set becomes a system if there is some operation on it , like looking for minimums or maximums , as in transport models .then T_α is called a stacked system.

3.1 Definition

Let $T_\alpha = \{x_{11}, x_{12}, \dots, x_{1n}, x_{21}, \dots, x_{2n}, \dots, x_{nn}\}$ be called a stacked set if the elements in T_α are stacked in terms of the place (horizontally and vertically).

3.2 Definition

If T_α is a stacked system of element denoted generically by x then a fuzzy stacked system T_μ in T_α is a system of ordered pairs:

$T_\mu = \{(x, \mu_T(x)) \mid x \in T_\alpha\}$.

μ_T (x) is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in T_μ which maps T_α to the membership space M . (When M contains only the two points 0 and 1, T_μ is nonfuzzy and $\mu_T(x)$ is identical to the characteristic function of a nonfuzzy stacked set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite . Elements with a zero degree of membership are normally not listed .

$T_\mu = \{(x, \mu_T(x)) \mid x \in T\}$.

3.3 Theorem

If T_α is a stacked system, $\forall x, a \in T_\alpha$:

$\mu_{T1}(x) = (1 + |a - x|)^{-1}$.

$\mu_{T2}(x) = (1 + (a - x)^2)^{-1}$.

$\mu_{Tn}(x) = (1 + (|a - x|)^n)^{-1}$

are types of a function such that $\mu_{Ti}(x) \in [0, 1], i \in \{1, 2, \dots, n\}$.

proof :let T_α is a stacked system, $\forall x, a \in T_\alpha$**:**

$|a - x| \geq 0 \Rightarrow (1 + |a - x|) \geq 1 \Rightarrow 0 \leq (1 + |a - x|)^{-1} \leq 1$

$\Rightarrow \mu_{T1}(x) = (1 + |a - x|)^{-1} \in [0, 1]$.

$(a - x)^2 \geq 0 \Rightarrow (1 + (a - x)^2) \geq 1 \Rightarrow 0 \leq (1 + (a - x)^2)^{-1} \leq 1$

$\Rightarrow \mu_{T1}(x) = (1 + (a - x)^2)^{-1} \in [0, 1]$, so $\mu_{Tn}(x) = (1 + (|a - x|)^n)^{-1}) \in [0, 1], i \in \{1, 2, \dots, n\}$.

3.4 Theorem

If T_α is a stacked system, $\forall x \in T_\alpha$, $\mu_T(a) \geq \mu_T(x)$ then :

$\mu_T(x) = (1 + (a - x)^n)^{-1}$, $n \in N/0$,is convex function .

Proof : Let $x_1 < a < x_2$,and $\forall$ x $\in$ T_α , $\mu_T(a) \geq \mu_T(x)$ then :
$\mu_T(a) \geq \mu_T(x_1)$, and $\mu_T(a) \geq \mu_T(x_2)$
So :$[\mu_T(a) - \mu_T(x_1)] / [a - x_1] < 0 < [\mu_T(x_2) - \mu_T(a)] / [x_2 - a]$. From definition 2.6 :$[\mu_T(a) - \mu_T(x_1)] / [a - x_1] \geq [\mu_T(x_2) - \mu_T(a)] / [x_2 - a]$,then $\mu_T(x) = (1 + (|a - x|)^n)^{-1}$ is convex function .

3.5 Definition

If T_α is a stacked system, then the fuzzy stacked system T_μ in T_α is a system of ordered pairs:
$T_\mu = \{(x, \mu_T(x)) \mid x \in T\}$.
Such that :

$$\mu_T(x) = (1 + (|x - a|)^n)^{-1}, n \in N/0 \text{ and } x \in T_\alpha$$

3.6 Remark

During this paper we take into account the system is always looking for the nearly values of a .

3.7 Definition

For a finite fuzzy stacked set T_μ the column(row) cardinality $\backslash T_{\mu(C \text{ or } R)} \backslash$ is defined as :

$$\backslash T_{\mu(C \text{ or } R)} \backslash = \sum_{x \in (CorR)} \mu_{T(C \text{ or } R)}(x).$$

3.8 Definition

Let $\mu(x_{\gamma\beta}) \in T_\mu$, γ is a row and β is column then the order fuzzy stacked of $\mu(x_{\gamma\beta})$ is :
$O(x) = \| \mu(x_{\gamma\beta}) \| = [\mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash + \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash]/2$.

3.9 Theorem

If T is a stacked system, $\forall$ x , a $\in$ T :
$O(x) = \| \mu(x_{\gamma\beta}) \| = [\mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash + \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash]/2$
is a type of a function such that $O(x) \in [0, 1]$,

Proof :from definitions (3.5) , (3.7) , (3.8).
$0 \leq \mu(x_{\gamma\beta}) \leq \backslash T_{\mu(\gamma)} \backslash \Rightarrow 0 / \backslash T_{\mu(\gamma)} \backslash \leq \mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash \leq \backslash T_{\mu(\gamma)} \backslash / \backslash T_{\mu(\gamma)} \backslash \Longrightarrow 0 \leq \mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash \leq 1$ (A) . And so $0 \leq \mu(x_{\gamma\beta}) \leq \backslash T_{\mu(\beta)} \backslash \Rightarrow 0 / \backslash T_{\mu(\beta)} \backslash \leq \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash \leq \backslash T_{\mu(\beta)} \backslash / \backslash T_{\mu(\beta)} \backslash \Longrightarrow 0 \leq \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash \leq 1$(B).
Then (A) +(B) $\Rightarrow 0+0 = 0 \leq \mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash + \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash \leq 1 + 1 = 2 \Longrightarrow 0 \leq [\mu(x_{\gamma\beta}) / \backslash T_{\mu(\gamma)} \backslash + \mu(x_{\gamma\beta}) / \backslash T_{\mu(\beta)} \backslash]/ 2 \leq 1 \Longrightarrow O(x)) \in [0, 1]$

3.10 Definition

If T_α is a stacked system, then the fuzzy level stacked system $l(T_\mu)$ in T_α is a system :
$l(T_\mu) = \{(x, \mu_T(x), O(x)) \mid x \in T_\alpha\}$.

3.11 Definition

Let $l(T_\mu)$ in T_α is a fuzzy level stacked system $[l(T_\mu) = \{(x, \mu_T(x), O(x)) | x \in T_\alpha\}]$, then :
$\max_{t}1[O(T)] = \{x_1, x_2, \ldots, x_n\}$ .That's where :
$\max[O(T)] = x_1$, $\max[O(T)/\{R_{x_1}, C_{x_1}\}] = x_2$,… , $\max[O(T)/\{R_{x_1}, C_{x_1}, R_{x_2}, C_{x_2}, \ldots, R_{x_{n-1}}, C_{x_{n-1}}\}] = x_n$, C mean column and R mean row ,and if $x_\beta \in \max_{t1}(O(T))$, then $R_\beta \cap \max_{t1}(O(T)) = \{x_\beta\}$, and $C_\beta \cap \max_{t1}(O(T)) = \{x_\beta\}$,$|C_i| = |R_i| = |\mathrm{Max}_{t1}(O(T))| = n$.

3.12 Example

Suppose that a product is transferred from four stores $\{C_1, C_2, C_3, C_4\}$, to four center sales $\{R_1, R_2, R_3, R_4\}$ at cost price shown in the following table , and is the transfer of four units of the cost or price close to this, to Centers of sale

T_α =

	C_1	C_2	C_3	C_4
R_1	1	8	3.5	2
R_2	6	7	4	4.5
R_3	1	5	6	8.5
R_4	3	1	2	0

Table 2

$$\mu_\alpha(x) = (1 + (x - a)^2)^{-1} \; : \; x \in T_\alpha$$

1- Where a = 4 , so $\mu_4(1) = (1 + (1 - 4)^2)^{-1} = 0.1$, $\mu_4(0) = 0.059$, $\mu_4(2) = 0.2$, $\mu_4(3) = 0.5$, $\mu_4(3.5) = 0.8$, $\mu_4(4) = 1$, $\mu_4(4.5) = 0.8$, $\mu_4(5) = 0.5$, $\mu_4(6) = 0.2$, $\mu_4(7) = 0.1$, $\mu_4(8) = 0.059$, and $\mu_4(9) = 0$ ($9 \notin T_\alpha$) .
2- Since $T_\mu = \{(x, \mu_\alpha(x)) \mid x \in T_\alpha\}$, then the fuzzy stacked system T_μ in T_α is

$T_\mu = \{ (1_{11}, 0.1), (8_{12}, 0.059), (3.5_{13}, 0.8), (2_{14}, 0.2), (6_{21}, 0.2), (7_{22}, 0.1), (4_{23}, 1), (4.5_{24}, 0.8), (1_{31}, 0.1), (5_{32}, 0.5), (6_{33}, 0.2), (8_{34}, 0.059), (3_{41}, 0.5), (1_{42}, 0.1), (2_{43}, 0.2), (0_{44}, 0.059) \}$.
Or

T_μ =

	C_1	C_2	C_3	C_4
R_1	0.1	0.059	0.8	0.2
R_2	0.2	0.1	1	0.8
R_3	0.1	0.5	0.2	0.047
R_4	0.5	0.1	0.2	0.059

Table 3

3- $O_{11}(0.1) = [(0.1/(0.1+0.059+0.8+0.2))+(0.1/(0.1+0.2+0.1+0.5))]/2 = 0.0987$.

And so : $O_{12}(0.059) = 0.0643$, $O_{13}(0.8) = 0.527$, $O_{14}(0.2) = 0.1767$, $O_{21}(0.2) = 0.15873$, $O_{22}(0.1) = 0.08969$, $O_{23}(1) = 0.465$, $O_{24}(0,8) = 0.552$, $O_{31}(0.1) = 0.1146$, $O_{32}(0.5) = 0.62$, $O_{33}(0.2) = 0.16$, $O_{34}(0.047) = 0.049$, $O_{41}(0.5) = 0.5688$, $O_{42}(0.1) = 0{,}124$, $O_{43}(0.2) = 0.16$, $O_{44}(0.1) = 0.052$.

$O(T) = \{ (1_{11}, 0.0987), (8_{12}, 0.0643), (3.5_{13}, 0.527), (2_{14}, 0.1767), (6_{21}, 0.1583), (7_{22}, 0.08969), (4_{23}, 0.465), (4.5_{24}, 0.552), (1_{31}, 0.1146), (5_{32}, 0.62), (6_{33}, 0.16), (8.5_{34}, 0.047), (3_{41}, 0.5688), (1_{42}, 0.124), (2_{43}, 0.16), (1_{44}, 0.052) \}$.

Or

T_O =

	C_1	C_2	C_3	C_4
R_1	0.0987	0.0643	0.527	0.1767
R_2	0.1583	0.08969	0.465	0.552
R_3	0.1146	0.62	0 .16	0.047
R_4	0.5688	0.124	0.16	0.052

Table 4

4- $l(T_4)$ = {(x , μ_T(x), O(x)) | x ∈ T_α } = { (1_{11} , 0.1, 0.0987), (8_{12} , 0.059 , 0.0643) , (3.5_{13} , 0.8 , 0.527) , (2_{14} , 0.2 , 0.1767) , (6_{21} , 0.2 , 0.1583) , (7_{22} , 0.1 , 0.08969) , (4_{23} , 1 , 0.465) , (4.5_{24} , 0.8 , 0.552) ,(1_{31} , 0.1 , 0.1146) , (5_{32} , 0.5 , 0.62) , (6_{33} , 0 .2 , 0 .16) , (8.5_{34} , 0.059 , 0.047) , (3_{41} , 0.5 , , 0.5688) , (1_{42} , 0.1 , 0.124) , (2_{43} , 0.2 , 0.16) , (1_{44} , 0.1 , 0.052)}.

5- Max_{t1} [$l(T_4)$] = { (5_{32} , 0.5 , 0.62) , (3_{41} , 0.5 , 0.5688) , (4.5_{24} , 0.8 , 0.552) , (3.5_{13} , 0.8 , 0.527) }.

6- Max_{t1} [$O(T_4)$]= { 0.62_{32} , 0.5688_{41} , 0.552_{24} , 0.527_{13} }.

Or Max_{t1} [$O(T_4)$]= { (5_{32} , 0.62) , (3_{41} , 0.5688) , (4.5_{24} ,0.552) , (3.5_{13} , 0.527) }.

7- Max_{t1} [T]= {5_{32} , 3_{41} , 4.5_{24} , 3.5_{13} } .

IV. STACKED SEMI-GROUPS ON TRANSPORTATION MODELS

4.1 Definition

Let $l(T_\mu)$ in T_α be a fuzzy level stacked system [$l(T_\mu)$={(x,μ_T(x),O(x))|x ∈ T_α }] , then :

The best level of T_μ is b.l(T_μ) = {(x , μ_T(x) , b.l(x)) | x ∈ T_α , b.l(x) = O(x) if b.l(x)∈ Max_{t1}[O(T_α)] , or b.l(x) = 0 if b.l(x) ∉ Max_{t1}[O(T_α)] } .

4.2 Definition

Let (T_0 ,τ^1) is stacked system (0 ∈ T_0 , and 0 = min[O(T)]) ,∀ a ,b∈ T_0 , then a τ^1 b = b τ^1 a = τ^1 (a , b) = max_{t1} (a , b) = a, if b.l(a) > b.l(b) , if b.l(a) = b.l(b) = 0 ,then a τ^1 b = 0 .

4.3 Remark

if b.l(a) = b.l(b) ≠ 0 , or O(a) = O(b) then there is two stacked systems one of them called $T_{[a]}$,this system takes b.l.(a) > b.l(b) , and $T_{[b]}$ takes b.l.(b) > b.l(a) .

4.4 Example

From example(3-12) above :

1- b.l(T_4) = {(x , μ_T(x), b.l(x)) | x ∈ T } = { (1_{11} , 0.1, 0), (8_{12} , 0.059 , 0) , (3.5_{13} , 0.8 , 0.527) , (2_{14} , 0.2 , 0) , (6_{21} , 0.2 , 0) , (7_{22} , 0.1 , 0) , (4_{23} , 1 , 0) , (4.5_{24} , 0.8 , 0.548) ,(1_{31} , 0.1 , 0) , (5_{32} , 0.5 , 0.62) , (6_{33} , 0 .2 , 0) , (8_{34} , 0.059 , 0) , (3_{41} , 0.5 , , 0.5688) , (1_{42} , 0.1 , 0) , (2_{43} , 0.2 , 0.5657) , (1_{44} , 0.1 , 0)}.

Or

b.l(T_4) =

	C_1	C_2	C_3	C_4
R_1	0	0	0.527	0
R_2	0	0	0	0.548
R_3	0	0.62	0	0
R_4	0.5688	0	0	0

Table 5

2- 3.5_{13} τ^1 5_{32} = 5_{32} , 4_{23} τ^1 3_{41} = 3_{41} , 6_{33} τ^1 1_{31} = 0_{44} , 4_{23} τ^1 0_{44} = 0_{44} , 0_{44} τ^1 5_{32} = 5_{32} .

4.5 Theorem

The operation τ^1 is binary operation in T^0 .

Proof : Let a , b ∈ T_α and T_α is stacked set so 0 ∈ T_α and O(0)= min[O(T_α)], from definition 4-2 above a τ^1 b = τ^1 (a , b) = max_{t1}(a, b) = a or b or 0 = (a ∨ b ∨ 0) ∈ T_α , so ∀ a , b ,0 ∈ T_α ⟹ a τ^1 b ∈ T_α ⟹ τ^1 is cartesian product T_α × T_α into T_α , and from definition 2-10 , τ^1 is binary operation .

4.6 Theorem

The operation τ^1 is associative .

Proof : Let a , b , c ∈ T_α , then ((a τ^1 b) τ^1 c) = ((a ∨ b∨ 0) τ^1 c) =((a ∨ b ∨ 0) ∨ c ∨ 0)=(a ∨ 0 (b ∨ d) ∨ 0) =(a ∨ (b τ^1 c) ∨ 0) =(a τ^1 (b τ^1 c)) , then τ is associative relation .

4.7 Theorem

If the system (T_α , τ^1) is a stacked-system then (T_α , τ^1) is a semigroup and called a stacked-semigroup .

proof : From theorems (4-5) ,(4-6) and definition (2-12) (about the semigroup) we proof that (T_α , τ^1) is a semigroup.

4.8 Theorem

If the semigroup (T_α , τ^1) is a stacked-semigroup , then (T_α , τ^1) is a commutative semigroup.

Proof : Let x , y ∈ T_α, then x τ^1 y = x ∨ y∨ 0 = y ∨ x∨ 0 = y τ^1 x ∈ T_α ⟹ x τ^1 y = y τ^1 x ⟹ (T_α , τ^1) is a commutative semigroup.

4.9 Theorem

If the semigroup (T_α , τ^1)is a stacked-semigroup , then (T_α , τ^1) is a finite semigroup

proof : From definition of the stacked set T is a finite ,then from definition(2-13) (T_α , τ^1) is a finite semigroup .

4.10 Theorem

If (T_α , τ^1) is a stacked-semigroup (τ^1 is operation defined in definition (4-2)), then there is no an identity element in (T_α , τ^1).

proof : From definition (4-2). Let (T ,τ^1) is stacked system , a ,b∈ T_α , then a τ^1 b = b τ^1 a = τ^1 (a , b) = max_{t1} (a , b) = a, if

$O(a) = O(b) = 0$, then $a\ \tau^1\ b = b\ \tau^1\ a = 0 \in T$. let e is identity element ,then $O(e) \geq 0$.

1- If $O(e)= 0$, $O(a)=0$, then $a\ \tau^1\ e = e\ \tau^1\ a = 0$, it is not necessary to be $0 = a$, so there is no $e \in T_\alpha$ such that $O(e)= 0$, and $\forall a \in T_\alpha$, $a\ \tau^1 e = e\ \tau^1\ a = e$.
2- Now let $O(e) > 0$, $O(a)=0$, then $a\ \tau^1 e = e\ \tau^1\ a = e$. This is contrary to the concept of an identity element (such that $\forall a \in T$, $a\ \tau^1 e = e\ \tau^1\ a = a$) .

From (1 and 2) there is no an identity element in (T , τ^1), such that $e \in T$, $\max_{t1} (x , e) = \max_{t1} (e , x) = x$ ($\forall x \in T$) .

4.11 Theorem

If (T_α , τ^1) is a stacked-semigroup (τ^1 is operation defined in definition (4-2)), then there is a zero element (x_{zero})in (T , τ^1), such that ($\forall x , x_{zero} \in T_\alpha$): $x_{zero}\ \tau^1\ x = x\ \tau^1\ x_{zero} = x_{zero}$.

Proof : From definition (4-2). Let (T_α ,τ^1) is stacked system , a ,b$\in T_\alpha$, then $a\ \tau^1\ b = \tau (a , b) = \max_{t1} (a , b) = a$, if b.l(a) > b.l(b) . When $O(a) = O(b) = 0$, then $a\ \tau^1\ b = \tau (a , b) = \max_{t1} (a , b) = 0_\sigma$, now let $O(x_{zero}) \geq O(x)$ ($\forall x , x_{zero} \in T$) , then $x_{zero}\ \tau^1\ x = x\ \tau^1\ x_{zero} = x_{zero}$ for all $x \in T_\alpha$, so x_{zero} is a zero element , when $O(x_{zero}) \geq O(x)$, ($\forall x , x_{zero} \in T_\alpha$) .

V. RESULTS AND PAPER TARGETS

1- There is a new shape of the sets, and the elements order is not larger or smaller, but by placement the elements in the set .This set make a different system of known mathematical systems, through some mathematical operations.
2- Apply this concept to some mathematical models such as in transport models (Operations Research) and others, and through conversion solutions transport models known to search for minimize the cost to determine the cost if we wish this.
3- Insert new mathematical meanings of some words that already exist, but not in common use such as semi-order set , stacked sets and stacked system and other.
4- Prove the stacked system is a semi-group.
5- Finally .This paper is submitted to a series of papers that will reveal some new concepts in the same system reached researcher . The expansion of the concept of the sets is very necessary to expand the research and development of the pattern previously.

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