ISSN 2250-3153

Application of Fuzzy If-Then Rule in Fuzzy Petersen Graph

Dr. G. Nirmala[*], S. Prabavathi[**]

[*]Head & Associate Professor, PG & Research Advisor, Dept. of Mathematics, K.N. Govt. Arts College for women (Autonomous), Thanjavur – 613007.
[**] Research Scholar, Dept. of Mathematics, K.N. Govt. Arts College,Thanjavru-613007, Tamil Nadu, India.

Abstract- In this work we illustrate how fuzzy if-then rules can be used for fuzzy Petersen graph. Basic results and its characteristics of fuzzy graphs are introduced. Further we introduced the results based on fuzzy if-then rule which is applied on fuzzy Petersen graph version of classical graph theory.

Index Terms- Fuzzy if-then rule, Fuzzy Petersen graph.

I. Introduction

Fuzzy systems based on fuzzy if-then rules have been successfully applied to various theorems in the field of fuzzy control. Fuzzy Rule based system has high comprehensibility because human users can easily understand the meaning of each fuzzy if-then rule through its linguistic interpretation.

Graph theory has numerous applications to problems in system analysis, operation research, transportation and economics. In many cases, however some aspects of a graph theoretic problem may be uncertain. For example the vehicle travel time or vehicle capacity on a road network may not be known exactly. In such cases, it is natural to deal with the uncertainty using fuzzy set theory.

The concept of a fuzzy graph is a natural generalization of crisp graphs, using fuzzy sets, and in many cases the extension principle. Our exposition of fuzzy graphs, given by Rosenfeld [Zadeh et al., 1975].

II. Basic Concepts

Definition 2.1: A graph G consists of a pair G:(V, E) where V(G) is a non-empty finite set whose elements are called points or vertices and E(G) is a set of unordered pairs of distinct elements of V(G). The elements of E(G) are called lines or edges of the graph G.

Example 2.1:

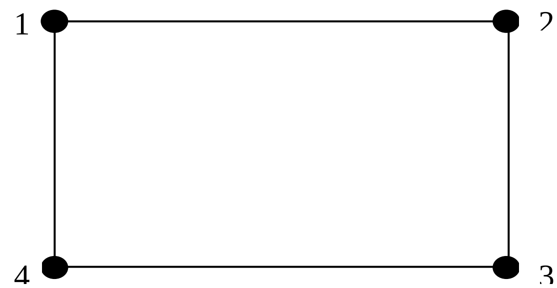

Fig 1: G

Let V={1,2,3,4} and
X={(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)}
G=(V, E) is a (4,6) graph.

Definition 2.2: The Petersen graph G is the simple graph with 10-verties and 15-edges. The Petersen graph is most commonly drawn as a pentagram inside with five spokes. It is called a Petersen graph.

Example 2.2:

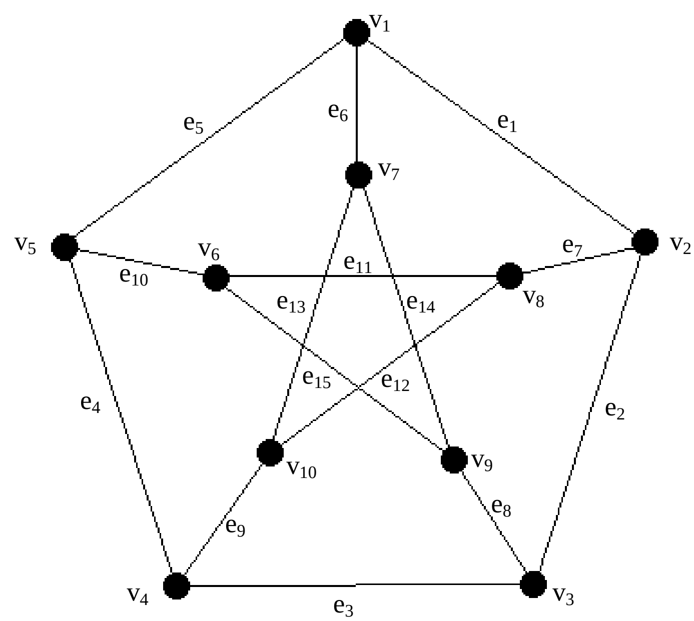

Fig 2: Petersen Graph

ISSN 2250-3153

It is set of vertices {v1,v2,v3,v4,v5,v6,v7,v8,v9,v10}
It is set of edges
{e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11,e12,e13,e14,e15}.

Definition 2.3: Let V be a non-empty set. A fuzzy graph is a pair of functions G:(σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ.
i.e. σ:V → [0,1] and μ:V×V → [0,1] such that μ(u,v) ≤ σ(u) ∧ σ(V) for all u,v in V.

Example 2.3:

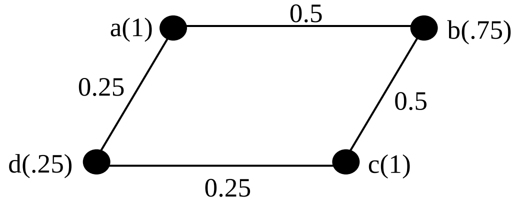

Fig 3: Fuzzy Graph

μ(a,b)=.5, μ(b,c)=.5, μ(c,d)=.25, μ(d,a)=.25 and σ ={1,.75,1,.25}.

Definition 2.4: The membership function values need not always be described by discrete values. Sometimes, these turn out to be as described by a continuous function. The most commonly used range of values of membership functions is the unit interval[0,1].

In this case, each membership function maps elements of a given universal set X, which is always a crisp set, into real numbers in [0,1]. The following two types of notations commonly used to denote the membership function.

1. The membership function of a fuzzy set Ã is denoted by $\mu_{\tilde{A}}$, i.e.,

$\mu_{\tilde{A}}$: X→**[0,1].**

2. The membership function of a fuzzy set Ã has the following form

Ã : X→[0,1].

Example 2.4:

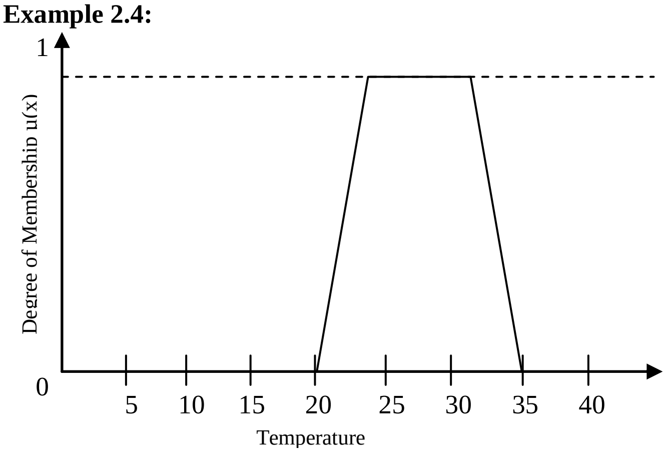

Fig 4: Continuous membership function for "COOL"

Definition 2.5: When a vertex $\sigma(u_i)$ is an end vertex of some edges μ(ui,vj) of any fuzzy graph G:(σ,μ), then $\sigma(u_i)$ and $\mu(u_i,v_j)$ are said to be incident to each other.

Example 2.5:

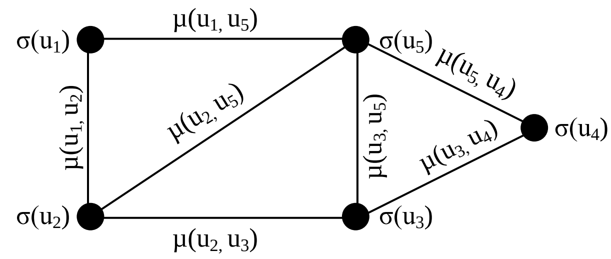

Fig 5: Incident Graph

(u_1,u_2), $\mu(u_2,u_3)$ and $\mu(u_2,u_5)$ are incident on $\sigma(u_2)$.

Definition 2.5: The degree of any vertex $\sigma(u_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on vertex $\sigma(u_i)$. And is denoted by $d(\sigma(u_i))$).

Example 2.5:

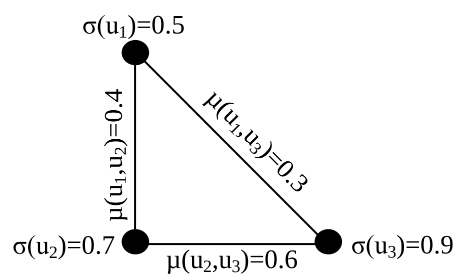

Fig 6: G

Degree of vertex $\sigma(u_2)$= degree of membership of all those edges which are incident on a vertex $\sigma(u_2)$

$$= \mu(u_1,u_2)+\mu(u_2,u_3)$$
$$= 0.4+0.6=1.0$$

i.e. $d[\sigma(u_2)] = 1.0$

Definition 2.6: A fuzzy rule is defined as a conditional statement in the form:
IF x is A THEN y is B

Where x and y are linguistic variables; A and B are linguistic values determined by fuzzy sets on the universe of discourse x and y respectively. The If-part of the rule "x is A" is called the antecedent or premise. Then-part of the rule "y is B" is called the consequence or conclusion.

Example 2.6:

a) If pressure is high, then volume is small.
b) If the road is slippery, then driving is dangerous.
c) If an apple is red, then it is ripe.
d) If the speed is high, then apply the brake a little.

I. Application of Fuzzy If-Then Rule

Theorem 3.1: Let £ be a residuated lattice on [0,1] and a structure S be given by (X,Y,$\{A_i, B_i\}_{i=1,\ldots,n}$, £ , °). Let,

ISSN 2250-3153

moreover, the t-norm * be a continuous Archimedean t-norm with a continuous additive generator g. A fuzzy function F_R: $\mathcal{F}(X) \to \mathcal{F}(Y)$ realizes GMP in the structure S with respect to fuzzy IF-THEN rules if and only if

$D_g(B_i, F_R(A)) \leq D_g(A_i, A)$
for each i and each fuzzy set $A \in \mathcal{F}(X)$.

Theorem 3.2 (Main Theorem): Let S = $(X, Y, \{A_i, B_i\}_{i=1,\ldots,n}, £, \circ)$ be a structure. A fuzzy function F_R: $\mathcal{F}(X) \to \mathcal{F}(Y)$ is a model of fuzzy IF-THEN rules

(x,y) are R
x is A
y is A∘R

in the structure S if and only if it realizes GMP in S with respect to all fuzzy IF-THEN rules.

Proposition3.3: The rule "Single-input-single-output approximate reasoning model:
Rule: If X, then Y.

Premise: X',

Conclusion: Y'

where X, X', Y, Y' $\in L^{Fp}$. If Y⊆X and X is $\tilde{\tau}$ -i type consistent with respect to (α, β, J), then the above rule is (α, β, τ, J)-i type representable in L_{vpl}.

Theorem 2.1: In fuzzy Petersen graph
$\sum_{i=1}^{n} d[\sigma(v_i)] = 2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$ = Twice the sum of degree of membership of (u_i, v_{i+1}).where $\sigma(v_i)$ is degree of all vertices v_i and $\mu(u_i, v_{i+1})$ is degree of membership $u_i, v_{i+1} \in E$.

Proof: Since G:(σ,μ) is a fuzzy Petersen graph of the graph G:(V,E), then consider 10-verties $\sigma(v_1)$, $\sigma(v_2)$,…, $\sigma(v_{10})$ of fuzzy Petersen graph G:(σ,μ).

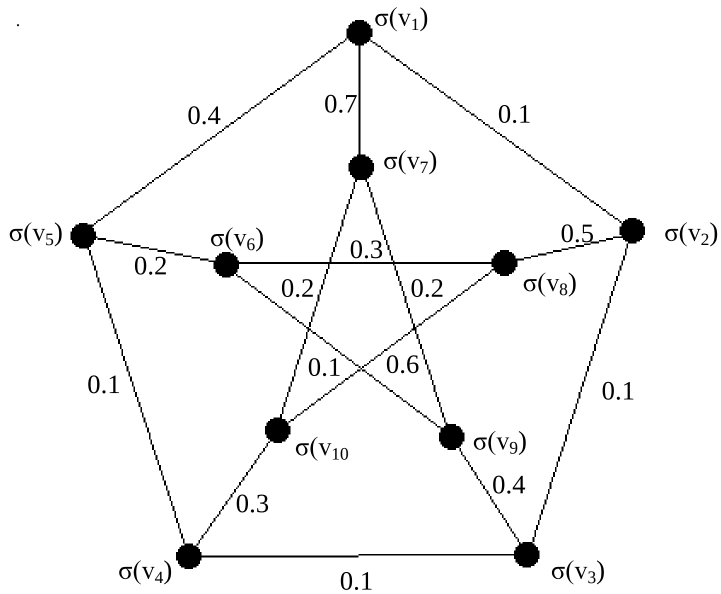

Fig 7: Fuzzy Petersen Graph

Rule1:
If add the membership grade of edges which are incident on any degree of vertex $\sigma(u_i)$, **then** the corresponding membership values of vertices vary.

Example:

$d[\sigma(v_1)] = 0.4+0.5+0.1 = 1.2$
$d[\sigma(v_2)] = 0.1+0.4+0.1 = 0.7$
$d[\sigma(v_3)] = 0.1+0.1+0.3 = 0.6$
$d[\sigma(v_4)] = 0.1+0.1+0.2 = 0.5$
$d[\sigma(v_5)] = 0.1+0.4+0.7 = 1.7$
$d[\sigma(v_6)] = 0.7+0.3+0.1 = 0.6$
$d[\sigma(v_7)] = 0.5+0.2+0.2 = 1.1$
$d[\sigma(v_8)] = 0.6+0.3+0.4 = 1.4$
$d[\sigma(v_9)] = 0.1+0.2+0.3 = 0.7$
$d[\sigma(v_{10})] = 0.2+0.2+0.6 = 1.1$

Rule2:
If sum of the membership grade of the degrees added, **then** we get the sum of the membership grade of the edges.
Example:
$\sum_{i=1}^{10} d[\sigma(v_i)] = 1.2+0.7+0.6+0.5+1.7+0.6+1.1+1.4+0.7+1.1$
$= 8.6$

Since each edges are incident on two vertices then by the definition of degree of any vertex of fuzzy Petersen graph, the sum of the degree of vertices in fuzzy Petersen graph G:(σ,μ) is twice the degree of membership of edges in G :(V,E).

Rule3:
If add all the membership grade of edges increased, **then** the sum of membership values of edges increased.

Example:
$\sum_{i=1}^{10} \mu(u_i, v_{i+1})$ = 0.1+0.1+0.1+0.1+0.4+0.7+0.5+0.4+0.3+0.2+ 0.3+0.6+0.2+0.2+0.1
= 4.3

Rule4:

If the sum of the degree of vertices occurs, **then** it is equal to twice the degree of membership values of edges.

Example:
$\sum_{i=1}^{10} d[\sigma_{(v_i)}] = 2 \sum_{i=1}^{10} \mu_{(u_i, v_{i+1})}$

Hence, proved the theorem.

III. Conclusion

We have discussed the Petersen graph and degree of every vertex of fuzzy Petersen graph. The fuzzy If-Then Rules are also discussed. Finally fuzzy if-then rule applied on fuzzy Petersen graph with different characteristics.

References

[1] Y.Vaishnaw and S.Sharma "Some analoges on fuzzy graph" International Journal of Computational and Mathematical Science 2:4 2008.

[2] "Fuzzy If-Then Rules In Computational Intelligence" Theory and applications edited by Da Ruan and Etienne E.Kerre.

[3] "Invitation to Graph Theory" S.Arumugam and S.Ramachandran.

[4] A.Nagoor Gani and K.Radha "The Degree of a vertex in some Fuzzy Graphs".

[5] Nagoor Gani and S. R. Latha; "On Irregular fuzzy graphs" Applied Mathematical Sciences,vol. 6, 2012, No. 11, 517-523.

[6] Ramaswamy and Poornima B. "Product fuzzy graphs" IJCSNS International Journal of computer science and network security, vol.9 No.1,2009.

[7] Dr.G.Nirmala and N.Vanitha "Risk Analysis of River-Type Hydro-Power Plants" Aryabhatta Journal of Mathematics & Informatics Vol.6, No. 1, Jan-July, 2014.

[8] Dr.G.Nirmala and K.Uma, "Fuzzy Graph-Estimations of strongest weight", The PMU Journal of Humanities and Sciences.

[9] Dr.G.Nirmala and N.Vanitha [2010], "Risk of Construction Project with Fuzzy Characteristics", Naros Publication.

[10] Y. Vaishnaw and A.S. Ranadive "On isomorphism between fuzzy graphs" Chhattisgarh Journal of science and technology, volume 3 & 4 (2007), 41-52.

[11] Bershtein L.S. and Bozhenyuk A.V.; " Estimation of Isomorphism Degree of Fuzzy Graphs" .

[12] Nagoor Gani and J. Malarvizhi "Isomorphism on Fuzzy Graphs" International Journal of Computational and Mathematical Sciences 2:4 2008.

[13] Dr.G.Nirmala and G.Suvitha "Implication Relations In Fuzzy Propositions" Aryabhatta Journal of Mathematics & Informatics Vol.6, No. 1, Jan-July, 2014.

[14] Dr.G.Nirmala and G.Suvitha "Comparision of 3 valued Logic using Fuzzy Rule-Fuzzy" International Journal of Scientific and Research Publications, Volume 3, Issue 8, (August 2013) 1 ISSN 2250-3153.

Authors

First Author - Dr. G. Nirmala, Head & Associate Professor, PG & Research, Dept. of Mathematics, K.N. Govt. Arts College for women (Autonomous), Thanjavur – 613007, Tamil Nadu, India. nirmalamanokar11@ yahoo.com
Second Author – Mrs. S. Prabavathi, Research Scholar, Dept. of Mathematics, K.N. Govt. Arts College, Thanjavru-613007, Tamil Nadu, India. nhpraba@gmail.com