# Countabiliy of Infinite Countables and Applications to Quantitative Finance

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***Abstract-*** This paper is mainly concerned with the application of countability theory in Finance. The author undertakes to explain countability theory for infinite countables not only in a precise manner but also gives analytical examples to simplify the theory even further. Some of the ways of ascertaining countability of infinite countables as indicated by the author entail: finding a bijection, finding a surjection, through a listing of elements and also through the determination of whether the elements are injective. The paper amalgamates Finance and countability by determining main areas in which the theory of countability is applicable in Finance. The paper explores: countability of Financial Statements constitutes, profit countability, countability of required rate of return, countability of company financing, countability in determination of capital investment decisions, countability in working capital constituents and countability in projection of financial statements' data.

***Index Terms-*** Countable, Uncountable, Injective, bijection, surjection

## I. INTRODUCTION

### 1.1 Countable And Uncountables

Countability is an aspect of sets that relates to the number of elements in the set. A countable set is defined as being a set with the same cardinality and also one that is denumerable. A set with the same cardinality has same number of elements for that case as some subset of natural numbers. A set which is denumerable is said to mean one that is countably infinite. The opposite of non-denumerable is infinitely uncountable. A set that is not countable is said to be uncountable in countability laws [1].

The question is not whether elements in a finite set are countable or not since they are obviously countable by counting one at a time of those elements in any given finite set. However, one finite set may be said to be 'more crowded' than another when it contains more elements than the other finite set. The big question comes as to whether you can be able to confirm that countably infinite and uncountably infinite status of sets exist in countability of Financial Statements constitutes, profit countability, countability of required rate of return, countability of company financing, countability in determination of capital investment decisions, countability in working capital constituents and countability in projection of financial statements' data. This is the main question that this paper focuses to address.

[2] Researchers have come up with a less direct way of counting the rationals with the help of continued fraction, but theirs has the advantage of being constructive. The sequence of integers from the previous step is interpreted as a continued fraction such that,
e.g., {2, 4, 2, 2} is mapped onto $[2; 4, 2, 2] = 2 + 1/ (4 + 1/ (2 + 1/2)) = 49/22$.

A set X is called countably infinite if there is a bijection f that maps X onto the set N of rational numbers. X is called countable if it is either finite or countably infinite. X is called uncountable if it is not countable.

A set is finite if $/S/=n <\infty$. A set that is not finite is said to be infinite. All finite sets are countable since they are known and can be counted by the mare listing of elements.

A set is infinite if $/S/=n =\infty$. Infinite sets are countable if they are denumerable. A set is denumerable if there is a bijection (one to one function) such that $f_1\ s\rightarrow N$.

Here N = {1, 2, 3...} the set of natural numbers. The bijection establishes a one to one correspondence between the set elements S and the natural elements.

### 1.2 Determination of infinite countability

Infinite countable sets can be determined on the criteria below, that there exists either a bijection, surjection, listing of elements or an injection, and that can be confirmed as below shown:

**a) Injection**

An injection is a one to one function. If $x\neq y$ then $f(x) \neq f(y)$. Thus $F: s\rightarrow N$ is one to one if $f(x) =f(y)$ if and only if $x=y$. [1]

For instance, to determine whether Q of elements is countable by getting an injection, you can get two distinct primes say 5 and 7 and define $F:Q^+\rightarrow N$ by $f(m/n)=5^m7^n$. Suppose $f(m_1/n_2)=f(m_2/n_2)$, that will translate to $5^{m1}7^{n1}=5^{m2}7^{n2}$ and thus the existence of a unique decomposition $n_1=n_2$ and $m_1=m_2$. F is injective and thus $Q^+$ is countable [2].

Also, we can prove that ABD is infinitely countable by use of an injection as below:

Let f(a, b, d) = f(a', b', d'). Assume independent three primes say 2, 3, and 5.

This will translate to:
$2^{f(a)}\quad 3^{f(b)}\quad 5^{f(d)} = 2^{f(a')}\quad 3^{f(b')}\quad 5^{f(d')}$

By unique decomposition in N:
$2^{f(a)} = 2^{f(a')}$
$3^{f(b)} = 3^{f(b')}$
$5^{f(d)} = 5^{f(d')}$

But f1, f2f3 are injections where a=a', b=b' and d=d'. (a, b, d) = (a' b'd'). Hence f is injective and so ABD is countable.

**b) Surjection**

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A surjection gives the evaluative means as to whether infinite elements are countable or not. F:s→N is on surjective if F(s) =N i.e. for every n€$n_1$ there exists $s_o$€$s_1$ with every $f(s_o)$=n [1].

For instance, to confirm whether Q of integers is countably infinite by getting a surjection of g: N→$Q^+$,we assume any two fixed primes say 5 and 7.

Define g by g (n) ={m/n if n=$5^m7^n$}.
Let t=k/r €$Q^+$, k, r €n.
Take n=$5^k7^n$€n.
g(n)=g($5^k7^r$)=k/r.

g is onto *(surjective) and hence $Q^+$ is countable and so $Q^-$ as well shall be countable.

Also assume ABD is countable and you are proving by aid of a surjection g: n→ A B D. You assume three primes say 2, 3, and 5 and define g by:
g(n)=m/n/o if n=$2^m3^n5^o$.

Let t=k/l/r€ A B D, k, l, r €n,

Take n=$2^k3^l5^r$€n.

The resulting will be g(n)=g($2^k3^l5^r$)=k/l/r. Thus ABD of integers is infinitely countable.

### c) One to one correspondence (Bijection)

We can "count" the members of a countable set just like we could count the natural numbers. An infinite set like X= {$x_1$, $x_2$...} is thus countable, for we can put the members of X into the one-to-one correspondence with the natural numbers:

$x_1$→1 $x_2$→2 $x_m$ →m

[1] To see this more clearly, let f be a bijection from X onto N. Then f is invertible, and the inverse function $f^{-1}$ is a bijection from N onto X. But this means that we must have: X= {$F^{-1}$ (1), $F^{-1}$ (2)...}.

Thus, if we let $x_i$= $f^{-1}$ (1), i=1,2,... we may write X={$x_1$,$x_2$,...}.
For instance, to determine whether M of integers is countable we get a bijection F: n→M by: 0 if n=1
f (n)= k if n=2k
-k if n=2k+1

N=1, 2, 3, 4, 5, 6, 7, 8, 9...
↕ ↕ ↕ ↕ ↕ ↕ ↕ ↕ ↕...
M=0, 1, -1, 2, -2, 3,-3, 4,-4....

The function is a bijection since the sets agree in their cardinality.

Also, you can define g:m→N where g(n)=2k if k>0, 2k+1 if k≤0 and k=0 if g(0)=1, g(1)=2(1)=2 and g(-1)=2(1-1)+1=3.

| | | | | |
|---|---|---|---|---|
| N=1 | 2 | 2 | 4 | 5 |
| ↕ | ↕ | ↕ | ↕ | ↕ |
| g= 0 | -1 | 1 | -2 | 2 |

Thus there exists a bijection and hence the set M is countably infinite. It is worth noting that if A϶B, and B is countable, then A is countable too.

### d) Giving a listing of elements

Infinite countability can also be determined by giving a listing of elements [1]. For instance, to determine whether an infinite element P is countable by giving a listing of elements where $P^+$ is a series of +ve rationals and $P^-$ as a series of –ve rationals, it shall be enough to show an array in $P^+$ with row k containing all elements with denominator k.

| | | | | |
|---|---|---|---|---|
| 1: 1/1 | 2/1 | 3/1 | 4/1 | 5/1........ |
| 2: ½ | 2/2 | 3/2 | 4/2 | 5/2........ |
| 3: 1/3 | 2/3 | 3/3 | 4/3 | 5/3........ |
| 4: ¼ | 2/4 | ¾ | 4/4 | 5/4........ |

By getting a listing on sum depending on sum of numerator and denominator as below;

| | | | | |
|---|---|---|---|---|
| 1/1→ | 2/1→ | 3/1→ | 4/1→ | 5/1........ |
| ↕ | ↕ | ↕ | ↕ | ↕.............................. |
| ½→ | 2/2→ | 3/2→ | 4/2→ | 5/2........ |
| ↕ | ↕ | ↕ | ↕ | ↕.............................. |
| 1/3→ | 2/3→ | 3/3→ | 4/3→ | 5/3........ |
| ↕ | ↕ | ↕ | ↕ | ↕.............................. |
| ¼→ | 2/4→ | ¾→ | 4/4→ | 5/4........ |

Thus 1, 2, 1/2, 1/3, 3/4, 3/2...we can list all elements and hence $P^+$ is countable and hence $P^-$shall be countable as well.

## II. Countable sets verses Uncountable sets

The following sets are countable:

a) Q and all its subsets
b) Any countable union of countable sets
c) Any countable direct product of countable set i.e. countable times countables equals countable
d) Any extension (field) of countable set

The following sets are uncountable:

a) Any open/closed interval in R is uncountable set
b) (0,1)<R since 0,1 is uncountable, R is uncountable
c) Any subset of an interval in R is uncountable. i.e. A€C if A is uncountable and thus B
d) Countable times uncountable set gives an uncountable set.

## III. Application of Countability theory to Finance

Finance refers to the art and science of managing money. Countability theory finds a number of financial applications as below argued:

### a) Countability of Profits

Profit cannot just be determined from a vacuum. There must be substantive counting of total expenses and total revenues such that the difference between total revenues and total expenses gives rise to the profit realized an important phenomenon in all

businesses. The profit objective has remained the main and key objective since time memorial due to the fact that organizations need to survive and their survival is depended upon their profitability. The focus of profit maximization objective is to achieve the highest possible profits during the year and hence organizations must plan well their revenues and expenses such that when counted they yield higher profitability.
i.e. Profit =Revenue – Expenses

### b) Countability to determine required rate of return

A firm's required rate of return refers to the minimum rate of return that a project must generate if it has to receive funds. It is mostly abbreviated as ($R_j$). $R_j$ is therefore the opportunity cost of capital or returns expected from the second best alternative [3].
Required Rate of Return = Risk free rate + Risk premium

$$R_j = R_f + \beta\,(R_m - R_f)$$

Where
$R_j$: represents the expected rate of return
$R_f$: represents the risk free rate of return
$\beta$: represents the risk measurement, also called beta
$R_m$ represents the risk market risk
Thus, $\beta\,(R_m - R_f)$ represents the risk premium

In other words, the required rate of return can therefore be expressed as follows:
$R_f = Rf + (R_j1 - R_f)\,\beta 1 + (R_j2 - Rf)\,\beta 2 + (R_j3 - R_f)\,\beta 3$
Where;

1. $R_j1$, $R_j2$… $R_jn$ = Expected return of security j under macro-economic factors 1, 2 ….n.
2. $\beta 1$, $\beta 2$….. $\beta n$ = Beta of security j/ Sensitivity of security j returns to change in factors 1, 2, ….n.
3. $R_j$ for every factor is computed on assumption that there is unit sensitivity to a given economic factor and a zero sensitivity to all other factors.

Firms can determine their required rates of return by countably knowing the two main components involved: the risk free rate and the premium and counting the sum of the two to determine the resultant solution.

### c) Countability of Financial Statements components

The most common Financial Statements entail: the statement of financial position, the statement of equity, the statement of income and the statement of cash flows. [3]. Countability applies in all this since all the elements of these statements must be counted and appropriate treatment is taken for the purpose of each statement to be ascertained. For instance, in order to realize the fruits of the statement of income, the elements from the statement, say, incomes and expenses must be countably determined and subtracted there-with, to realize the net income or the net loss.

### d) Countability of Company Financing

A company can be financed by either equity, debt or a mix of the two. A company must count to determine the best mix that will yield higher leveraging benefits as compared to the contrary. Therefore, the different sources of finances must be counted in regard to the category they fall, whether equity or debt, and thus find the relevant financing policy for the company to adopt [4].
The Capital structure of a company explains the financing aspects and attributes of the company. Pecking order theory explains a distinct preference in the use of internal finance over external finance countably from retained earnings to debt, then external equity and finally internal equity. This order is countably consistent.

### e) Other Application areas

There are zillions of areas as may be explained under which the discipline of countability theory lies in Finance, the most common are as explained above. Other areas include: countability in determination of capital investment decisions, countability in working capital constituents and countability in projection of financial statements' data among others.

## IV. Conclusion

Countability theory cannot be separated from Finance theory since they have a lot in common. However, there should be a proper understanding of the two fields since countability bases its root from the Arithmetic discipline and another is a Finance discipline. It is worth noting that there are a number of ways of determining element countability verses its countability as discussed in the context of this paper. Finance merges with Countability theory from an argument that all its elements can be determined and are either finite (known) or infinitely countable (as projected) and hence denumerable by the financial experts using previous years' data. From the works of this paper, there is nothing to indicate that the author's objectives have not been met.

## References

[1] Cantor, Georg (1874),"Über eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen", Journal für die Reine und Angewandte Mathematik 77: 258–262
[2] J. Czyz and W. Self, (2003). The Rationals Are Countable: Euclid's Proof, The College Mathematics Journal, Vol. 34, No. 5, pp. 367-369
[3] Ok, Efe A.2007, Real Analysis with economic application.. Pg 82-107. Princeton University
[4] R.Hill (2013), Working capital Management: Theory and Strategy, Bookboon
[5] R.Hill (2008), Strategic Financial Management, Bookboon

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