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Implementation of One-Step Scheme Interpolation Function for Solving Singular Initial Value Problem (IVP) In Ordinary Differential Equation

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Abstract- In this research paper, we consider the implementation of one-step scheme interpolation function for solving singular initial value problem in ordinary differential equation of the form:
$f_{(x)} = a_o + a_1x + b\tan(\rho x + \sigma)$,where a_o, a_1 and b are real undetermined coefficient, ρ and σ are complex parameters. We consider the linear multistep method in the
$\sum_{j=0}^{K}\alpha_j\, y_{n+j} = h\sum_{j=0}^{K}B_j\, f_{n+j}$ where α_j and B_j are constants and $\alpha_k \neq 0$. The Taylor's series expansion of $y(x_{t+1})$ about $x = x_t$ is $y(x_{t+1}) = y(x_t) + \sum_{i=1}^{\infty}\frac{h^i y^i x_t}{1!}$ was also implemented to establish our result at k=5.

Index Terms- Interpolation Function, Stability, Consistency and Convergence

1.0 INTRODUCTION

In this research paper work, we shall consider one-step scheme interpolation function for solving Initial Value Problem (IVP) in ordinary differential equation in the form:

$$y^1 = f(x, y),\ y(x_o) = y_o, \quad a \leq x \leq b \qquad (1.1.1)$$

The development of numerical methods for singular IVP systems of ordinary differential equations (ODEs) have been attracting much attention due to their needs in the solutions of problems arising from the mathematical formulation of physical situations in singular perturbation, chemical kinetics population models, mechanical oscillation, process control and electrical circuit theory which often leads in initial value problems (IVPs) in ordinary differential equation that are singular.
Many researchers have done a great deal of work in this area. Worthy of note are those of [ref:Lambert (1980), Fatunla (1980, 1981,1982), Abhulimen and Otunta (2007), Carver, M. B., (1977), Conte, S. D. and Carl De Boor (1972), Conte, S. D. and Carl De Boor (1972), Evans, D. J. and Fatunla, S. O. (1975), Hay, J. L. Crosbie, R. E., and Chaplin, R. I. (1973), Henrici, P. D. (1992), Luke, Y. K., Fair, W. S., and Wimp, J (1975), and O'Regan, P. G. (1996)]. In their different approaches, they show the method of one-step technique of interpolation function for solving singular initial value problem (ivp) in ordinary differential equation.
The problem for this research work is to find a numerical solution to the (IVP) which is represented by $y^1 = f(x,y)$, $y(x_o) = y_o$, $a \leq x \leq b$ where f (x, y) is defined and continuous in a region $D \subset (a, b)$ that are singular and f (x, y) also satisfy a Lipschitz condition with respect to y.

However, our aim in this paper research work is to implement the problem represented by (1.1.1), where f (x, y) must satisfy a Lipschitz condition with respect to y. To achieve our aims, we therefore set the following objectives;

(i) To develop an interpolation function of singular ivp in the form.
$F_{(x)} = a_o + a_1x + b\tan(\rho x + \sigma)$.
(ii) To determine the performance, stability, characteristics, consistency and nature of convergence constructed in (i) above.
(iii) To develop a Fortran package for the implementation of our scheme.

1.2 SOME DEFINITIONS AND NOTATIONS

For the purpose of this research work, we have the following definitions.

(1) **Definition of Consistency : Lambert (1980)**
The general one-step method $F_{(x)} = a_o + a_1 x + b \tan(\rho_x + \sigma)$ 1.2.1

is said to be consistence with the initial value problem in equation (1.1.1) if $\emptyset(x, y, 0) = f(x, y)$. If the method in equation (1.2.1) is consistent with the initial-value problem, then $y(x + h) - y(x) - h\emptyset(x, y(x), 0) + 0(h^2) = 0(h^2)$.Since $y^1(x) = f(x, y(x)) = \emptyset(x, y(x), 0)$, by definition (1). Thus a consistent method has order of at least one. The only linear multi-step method which falls within the class in equation (1.2.1) is Euler's rule which is obtained by setting $\emptyset(x, y, h) = \emptyset_E(x, y, h) = f(x, y)$ (The subscript E denotes "Euler"). The consistency condition of definition (1) is then obviously satisfied and a simple calculation shows that the order according to definition (1) is one.

(2) **Definition of Local Truncation Error : Lambert (1980)**

The local truncation error at x_{n+1} of the general explicit one-step method in equation (1.2.1) above is defined to be T_{n+1} where $T_{n+1} = y(x_{n+1}) — h\emptyset(x_n, y(x_n),h)$ and (x) is the theoretical solution of the initial—value problem. Then the local truncation error at xn+k of an explicit linear K—step method satisfies $T_{n+k}= y(x_{n+k}) — y_{n+k.}$ The relationship between global and local truncation error is $e_{n+1} \leq K\ T_{n+1}$ where K is a constant. The local truncation error is directly proportional to the global error introduce at each step mostly when the derivation and computation of local truncation error is rigorous arid all previous solutions are exacts.

(3) **Definition of Convergence : Lambert (1980)**

The general one-step method equation (1.2.1) above is said to be convergent to the initial value problem in equation (1.1.1) if the corresponding approximation y_n satisfies $y_n \to y(x_n)$as $n \to \infty$

(4) The function f (x, y) in (1.1.1) is said to satisfy a **Lipschitz condition** in y, over a region D, if there exists a constant L such that

$$\| f(x, y_1) - f(x, y_2) \| \leq L \|y_1 - y_2\| \qquad (1.1.3)$$

In this case, L is called the Lipschitz constant and f (x, y) is said to be Lipschitzian.

(5) A point x_n in the interval (a, b) at which y_n is being computed in called a **mesh point**.

(6) The difference, $h = x_{n+1} - x_n$ between the current point x_n and the next point x_{n+1} is called the **mesh size** of the subinterval $(x_n, x_{n+1}) \subset (a, b)$.

(7) **Region of absolute stability (RAS):**

A region D of the complex plane is said to be a region of absolute stability of a given method, if the method is absolutely stable for $\bar{h}$ ЄD for a one-step integrator, D is RAS if $|s(\bar{h})| < |$ for every $\bar{h}$ ЄD.

(8) **Interval of absolute stability (IAS):**

An interval (α, B) of the real line in said to be an interval of absolute stability of the method is absolutely stable for all $\bar{h} \in (\alpha, B)$. If the method is obviously unstable for all $\bar{h}$ it is said to have no interval of absolute stability (IAS). The interval of absolute stability is determined by the coefficients of the method.

1.3 THE DERIVATION OF INTERPOLATION FUNCTION

In this research paper work, we shall develop an interpolation function of singular initial value problem in (1.1.1) by using a function.

$$F_{(x)} = a_o + a_1 x + b\tan(\rho_x + \sigma) \qquad (1.3.1)$$

where $a_{o,}$ a_1 and b are real undetermined coefficient, ρ and σ are complex parameters.

We define $\theta_x = \rho_x + \sigma$ (1.3.2)

Substitute (1.3.2) in (1.3.1), we have

$$F_{(X)} = a_o + a_1 x + b\tan\theta_{(X)} = y \qquad (1.3.3)$$

$$F_{(Xt)} = a_o + a_1 x_t + b\tan\theta_t = y_t \qquad (1.3.4)$$

$$F_{(Xt+1)} = a_o + a_1 x_{t+1} + b\tan\theta_{t+1} = y_{t+1} \qquad (1.3.5)$$

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Set $\theta_{t+1} = \theta_t + \rho h$

Therefore (1.3.5) becomes

$F_{(Xt+1)} = a_o + ax_{t+1} + b \tan(\theta_t + \rho h)$ (1.3.6)

Differentiating (1.3.4), we have

$F^1(x_t) = a_1 + b(\sec^2 \theta_t)\rho = f_t$ (1.3.7)

$= a_1 + \rho h(1 + \tan^2 \theta_t) = ft$ (1.3.7b)

Differentiating (1.3.7) we have,

$F^{11}{}_{(Xt)} = 2\rho^2 b \tan\theta_t \sec^2\theta_t = f_t^{(1)}$ (1.3.8)

Making b the subject of the formula we have,

$$b = \frac{f_t^{(1)}}{2p^2 \tan\theta_t \sec^2\theta_t} \quad (1.3.9)$$

From (1.3.7b) $a_1 = f_t - \rho_h(1 + \tan^2\theta_t)$ (1.3.9a)

Putting (1.3.9) into (1.3.9a), we have

$$a_1 = f_t - \frac{f_t^{(1)}\rho(1+\tan^2\theta)}{2\rho^2\tan\theta_t\sec^2\theta_t}$$

$$a_1 = f_t - \frac{f_t^{(1)}}{2\rho^2\tan\theta_t} \quad (1.3.10)$$

Substituting (1.3.5) from (1.3.4), we have

$F_{(Xt+1)} - F_{(Xt)} = a_1 h + b(\tan\theta_{t+1} - \tan\theta_t)$ (1.3.10a)

Putting (1.3.10) into (1.3.10a), we have

$$F_{(Xt+1)} - F_{(Xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho\tan\theta_t}\right\} + b\{\tan\theta_t + \tan(\rho h) - \tan\theta_t\}$$

$$F_{(Xt+1)} - F_{(Xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho\tan\theta_t}\right\} + b\left\{\frac{\tan\rho h + \tan^2\theta\tan(\rho h)}{1 - \tan\theta_t + \tan(\rho h)} - \tan\theta_t\right\}$$

$$F_{(Xt+1)} - F_{(Xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho\tan\theta_t}\right\} + b\left\{\frac{\tan\theta_t + \tan\rho h - \tan\theta_t + \tan^2\theta(\rho h)}{1 - \tan\theta_t + \tan(\rho h)}\right\}$$

$$F_{(Xt+1)} - F_{(Xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho tan\theta_t}\right\} + b\left\{\frac{\tan\rho h + \tan^2\theta\tan\rho h}{1 - \tan\theta_t + \tan(\rho h)}\right\} \quad [1.3.11]$$

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Substituting b = $\dfrac{f_t^{(1)}}{2\rho^2 \tan\theta_t \sec^2\theta_t}$ in the above expression, we have

$$F_{(xt+1)} - F_{(xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho \tan_t}\right\} + - \frac{f_t^{(1)}}{2\rho^2 \tan\theta_t \sec^2\theta_t}\,\frac{(1+\tan^2\theta_t)\tan(\rho h)}{(1+\tan\theta_t \tan(\rho h)}$$

$$F_{(Xt+1)} - F_{(Xt)} = h\left\{f_t - \frac{f_t^{(1)}}{2\rho \tan_t}\right\} + \frac{f_t^{(1)}\tan(h)}{2\rho^2 \tan\theta_t + (1-\tan\theta_t \tan\rho h}$$

$$\therefore y_{t+1} - y_t = h\left\{f_t - \frac{f_t^{(1)}}{2\rho \tan\theta_t}\right\} + \frac{f_t^{(1)}\tan(h)}{2\rho^2 \tan\theta_t + (1-\tan\theta_t \tan\rho h\,)} \qquad (1.3.12)$$

ρ and θ_t are determined by using the definition of local truncated error denoted by

$$T_{t+1} = \quad y(x_{t+1}) - y_{t+1} \qquad (1.3.13)$$

Where $y(x_{t+1})$ is the theoretical solution at $x = x_{t+1}$ and y_{t+1} is the numerical solution obtained by adopting (1.3.12). Taylor's series expansion of $y(x_{t+1})$ about $x = x_t$ is

$$y(x_{t+1}) = y(x_t) + \sum_{i=1}^{\infty} \frac{h^i y^i x_t}{1!} \qquad (1.3.14)$$

With the assumption that there was no previous error $y_t = y(x_t)$ as well as using Macularin series expansion of $\tan(\rho h)$ and binomial expansion of $(1 - \tan\theta \tan^3\rho h)^{-1}$. we have

$$(1-\tan\theta_t \tan\rho h)^{-1} = 1 + \tan\theta_t \tan(\rho h) + \tan^2\theta \tan^2(\rho h) + \tan^3\theta \tan^3(\rho h)$$

$$\tan(\rho h) = h + \frac{(\rho h)^3}{3} + \frac{2(\rho h)^5}{15} + 17\frac{(\rho h)^7}{315} +$$

$$\tan^2(\rho h) = (\rho h)^2 + \frac{2}{3}(\rho h)^4 + \frac{17(\rho h)^6}{45} + \frac{2790(\rho h)^5}{14175} + \frac{4810(\rho h)^{10}}{7875} \qquad (1.3.15)$$

$$\tan^3(\rho h) = (\rho h)^3 + (\rho h)^5 + \frac{11}{15}(\rho h)^7 + ...$$

$$(1 - \tan\theta_t \tan(\rho h)^{-1} = 1 + (\rho \tan\theta_t)h + (\rho^2 \tan^2\theta_t)h^2 + (\frac{\rho^3}{3}\tan\theta + \rho^3 \tan\theta_t)$$

$$h^3 + \left(\frac{2}{3}\rho^4 \tan\theta_t\right)h^4 + \left(\frac{2}{15}\rho^5 \tan\theta_t + \rho^5 \tan^2\theta_t\right)h^5 + \frac{17}{45}\rho^6 \tan^2\theta_t + ... \qquad (1.3.16)$$

$$\therefore y_{t+1} - y_t = h\left\{f_t - \frac{f_t^{(1)}}{2\rho\rho\tan_t}\right\} + f_t^{(1)}\{\rho h + \rho^2\tan\theta_t\}h^2 + \left(\frac{\rho^3}{3} + \rho^3\tan\theta_t\right)h^3 + \left(\frac{2}{3}\rho^4\tan\theta_t + \rho^4\tan^3\theta\right)h^4$$

$$+\left(\frac{2}{15}\rho^5+\rho^5\tan^2\theta\right)h^5+\frac{\left(\frac{17}{45}\rho^6\tan\theta_t+\frac{4}{3}\rho^6\right)}{2\rho^2\tan\theta_t}h^6 \quad (1.3.17)$$

$$y_{t+1}=y_t+hy_t^{(1)}+\frac{h^2y_t^{(1)}}{2!}+\frac{h^3y_t^{(3)}}{3!}+\ldots \quad (1.3.18)$$

Equating coefficient of h's in (1.3.17) and (1.3.18) we have,

$$f_t^{(1)}\frac{\left(\frac{1}{3}\rho^3+\rho^3+\tan\theta_t\right)}{2\rho^2\tan\theta_t}=\frac{f_t^{(2)}}{3!}$$

$$f_t^{(2)}\rho\left(\frac{1}{3!}+\tan^2\theta_t\right)=\frac{f_t^{(2)}\tan\theta_t}{3} \quad (1.3.19)$$

$$f_t^{(1)}\frac{\left(\frac{2}{3}\rho^4\tan\theta_t+\rho^4+\tan^3\theta_t\right)}{2\rho^2\tan\theta_t}=\frac{f_t^{(3)}}{4!}$$

$$f_t^{(1)}\rho^2\left(\frac{2}{3}\tan^2\theta_t\right)=\frac{f_t^{(3)}}{12} \quad (1.3.20)$$

$$T_{n+1}=h^5\frac{\left(\frac{2}{15}\rho^5+\rho^5+\tan^2\theta_t\right)}{2\rho^2\tan\theta_t}f_t^{(1)}-\frac{f^{(4)}}{5!}h^5$$

$$=h^5\rho^3\frac{\left(\frac{2}{15}+\tan^2\theta_t\right)}{2\tan\theta_t}f_t^{(1)}-\frac{f_t^{(4)}}{120}h^5 \quad (1.3.21)$$

1.4 NUMERICAL EXPERIMENT, RESULTS AND CONCLUSION

1.4.1 Selection of Initial Value Problems

We now apply the formula derived to solve some tested initial value problems. The idea is to enable us see its level of performance and compare the results with those of the existing interpolation function.

To obtain the numerical solution y_{t+1} at $x = x_{t+1}$ the function f(x, y) and its higher derivatives are evaluated at $x = x_t$. The values obtained are used in (1.3.12) and (1.3.13)

Problem 1: Our method, consider $y^1 = 1 + y^2$, with mesh size h = 0.05

Table 1: Experimental Result

X	H	Theoretical solution Y_t	Numerical Solution Y_{t+1}	Error
0.05	0.05	1.10535551	1.1054390	$.763367369\text{x}10^{-4}$
0.10	0.05	1.22304879	1.2232060	$.128508053\text{x}10^{-3}$
0.15	0.05	1.35608774	1.3563164	$.168551889\text{x}10^{-3}$
0.20	0.05	1.50849752	1.5087907	$.200243872\text{x}10^{-3}$
0.25	0.05	1.68579627	1.6861813	$.228325494\text{x}10^{-3}$
0.30	0.05	1.89576495	1.8962732	$.268042910\text{ x}10^{-3}$

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0.35	0.05	2.14974943	2.1504447	.324247923 x10^{-3}
0.40	0.05	2.46496249	2.4659612	.405001097 x10^{-3}
0.45	0.05	2.86888368	2.8708886	.524302857 x10^{-3}
0.50	0.05	3.40822297	3.4106376	.707985416 x10^{-3}
0.55	0.05	4.16936335	4.1735691	.100771160 x10^{-2}
0.60	0.05	5.33185411	5.3400701	.153856182 x10^{-2}
0.65	0.05	7.34043450	7.3595575	.259838399 x10^{-2}
0.70	0.05	11.6813686	11.741792	.514599444 x10^{-1}
0.75	0.05	28.2382227	28.592778	.124001812 x10^{-1}
0.80	0.05	68.4798454	66.498083	.298017906 x10^{-1}

Table 2: Lambert and Shaw (1965, 1966)

X	Theoretical solution Y	- A (n)	N (n)	Initial Solution Error	Improved solution
0.1	1.223048,880	0.871,0524	-1.459,5387	2x10^{-7}	2x10^{-8}
0.2	1.508,497,647	0.818,6067	-1.2095811	5x10^{-7}	3x10^{-8}
0.3	1.895,765,123	0.797,0428	-1.03901411	1x10^{-6}	6x10^{-8}
0.4	2.464,962,757	0.7887937	-1.032,812,1	3x10^{-6}	1x10^{-7}
0.5	3.408,223,442	0.7861141	-1.009,3671	5x10^{-6}	2x10^{-7}
0.6	5.331,855,223	0.7854784	-1.001, 6126	1x10^{-5}	5x10^{-5}
0.65	7.354,436,575	0.7854150	-1.000,4736	3x10^{-5}	1x10^{-6}
0.70	11.681,373,800	0.785,4002	-1.0000712	7x10^{-5}	3x10^{-6}
0.75	28.238,232,850	0.785,3987	-1.000020	4x10^{-4}	4x10^{-5}

Table 3: Shaw (1966) h = 0.05

X	Theoretical solution Y	Error in numerical solutions	
		Rational (k=1)	Rational (k=2)
0.1	1.223048,888	8x10^{-8}	1x10^{-4}
0.2	1.508,497,647	2x10^{-7}	3x10^{-4}
0.3	1.895,765,123	4x10^{-7}	6x10^{-4}
0.4	2.464,962,757	7x10^{-7}	1x10^{-4}
0.5	3.408,223,442	1x10^{-6}	3x10^{-3}
0.6	5.331,855,223	4x10^{-6}	7x10^{-2}
0.65	7.354,436,575	8x10^{-6}	1x10^{-2}
0.70	11.681,373,800	2x10^{-5}	4x10^{-2}
0.75	28.238,252,830	1x10^{-4}	3x10^{-1}

Table 4 : $Y = 1 + Y^2$, Y(0) =1, Theoretical Solution Y(X) = TAN (X + π/4)
Error in Non Linear Multistep Method Fatunla (1981)

x	Theoretical solution Y	K = 1	K = 2	K = 3	K = 4	K = 5
0	1.000.000,000	-	-	-	-	-
0.1	1.223048,880	1.22816 (-2)	-8.2774 (-0)	-	-	-
0.2	1.508,497,647	2.91873 (-2)	-1.89419 (-3)	-2.29807 (-4)	-5.95328 (-5)	-
0.3	1.895,765,123	5.57996 (-1)	-3.51966 (-4)	-5.43415 (-5)	-5.43415 (-5)	2.30964 (-5)
0.4	2.464,962,757	1.04486 (-1)	-6.47338 (-3)	-1.68543 (-4)	-1.68543 (-4)	2.40806 (-5)
0.5	3.408,223,442	2.13434 (-1)	-1.31241 (-2)	-2.23773 (-4)	-2.23773 (4)	7.23361 (-3)
0.6	5.331,855,223	5.56258 (-1)	-3.40542 (-2)	-7.54129 (-4)	-7.54129 (-4)	2.27200 (-4)
0.65	7.354,436,575	1.10611 (0)	-6.63368 (-2)	-1.50951 (-3)	-1.50951 (-3)	2.33836 (-4)
0.70	11.681,373,800	3.09171 (0)	-1.76210 (-2)	-3.32208 (-3)	-3.32208 (-3)	7.84943 (-3)
0.75	28.238,232,850	2.90303 (1)	-1.07376 (0)	-1.89317 (-1)	-1.89317 (-1)	4.90205 (-5)

When we take a look at tables 1, 2, 3 and 4 results, it shows that our result obtained in table 1 compares favorably with that of Lambert and Shaw (1965,1966), Fatunla (1981) in table 2, 3 and 4 above. The comparism is with a relative low level error.

Problem 2: Consider $y^1 = y$, $y(0) = 1$ $0 \leq x \leq 1$ with exact solution $y = \exp(x)$, $h = 0.1$

Xn	H	Y_t	Y_{t+1}	e_n
0.1	0.1	1.105170965	1.105170919	0.000000047
0.2	0.1	1.221402764	1.210341837	0.011060930
0.3	0.1	1.349858880	1.315512763	0.034346110
0.4	0.1	1.491824746	1.420683674	0.071141070
0.5	0.1	1.648721218	1.525854586	0.122866600
0.6	0.1	1.822118878	1.631025527	0.191093400
0.7	0.1	2.013752699	1.736196409	0.277556300
0.8	0.1	2.225541115	1.841367348	0.384173800
0.9	0.1	2.459603310	1. 94 6538288	0.513065000
1.0	0.1	2.718282223	2.051709171	0. 666573000

Problem 2: Aashikpelokhai (1991) $y^1 = y$, $y(0) = 1$, $0 \leq x \leq 1$
with exact solution $y = \exp(x)$, $h = 0.1$

Xn	Y_t	Y_{t+1}	En	Order 5	Order 4	Order 3
0.1	1.10517092	1.10517	9.2 x10-7	-0.15622	0.60450	1.57761
0.2	1.22140028	1.22140	2.8 x10-7	-0.17265	0.48289	1.74353
0.3	1.34985881	1.34985	4.7 x10-6	-0.19081	0.38781	1.92690
0.4	1.49182470	1.49182	1.2 x10-6	-0.21085	0.36697	2.12955
0.5	1.64872127	1.64872	8.8 x10-6	-0.23305	0.321056	2.35355
0.6	1.82211880	1.82211	2.7 x10-6	-0.25756	0.02256	2.60104
0.7	2.01375271	2.01375	9.3 x10-7	-0.28465	0.35937	2.87460
0.8	2.22554093	2.22554	3.1 x10-6	-0.31459	0.34473	3.17692
0.9	2.45960311	2.45960	1.8 x10-6	-0.34765	0.30697	3.51104
1.0	2.71828183	2.71828	1.8 x10-6	-0.38424	0.03830	3.88030

The above table confirms the improved performance of our new scheme interpolation function for solving ODE problem can cope favorably with the numerical integrator of Aashikpelokhai (1991) with the solution $y = e^x$

1.5 CONCLUSION AND RECOMMENDATION

From the result in tables 1, 2, 3, and 4 it becomes very clear that our proposed new linear multistep method of order 5 is more efficient and accurate when compared with the existing methods of Lambert, Shaw (1965, 1966) and Fatunla of (1982).
This package has facilitates easy computation, we say that users of this our method will find it very helpful in solving initial value problems. We therefore recommended it for users whose are currently working in this area of research.

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