Single Unreliable Server Interdependent Loss and Delay Queueing Model with Controllable Arrival Rate under N-Policy

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Abstract- This paper studies a loss and delay queueing model under the restriction of N-policy for the situations where arrival and service of customers are correlated and follows a bivariate Poisson process. When there is no customer present in the system then the server goes on vacation and returns back in the system whenever the specified N (>1) or more customers are accumulated. The server may breakdown only if it is working and is immediately sent to a repair facility to restore its capability as before failure. For steady state various operational characteristics have been derived. Sensitivity analysis has also been made to examine the effect of different parameters so as to facilitate optimal control policy.

Index Terms- N-Policy, Interdependent, Unreliable server, Loss and delay, Controllable, Bivariate Poisson process.

I. Introduction

Present investigation deals with a single unreliable server queue having interdependent and controllable rates. Such congestion situations arise in production and manufacturing processes, computer communication system, distribution and service sectors, etc. A queueing model in which arrivals and services are correlated is known as interdependent queueing model. A few works have been reported in literature regarding interdependent queueing models. Borst and Combe (1992) analysed the busy period of a correlated queue with exponential demand and service. Gray et al. have studied an M/G/1 type queueing model with service times depending on queue length. The M/M/1 interdependent queueing model with controllable arrival rates was studied by Rao et al. (2000). They observed that the mean dependence rate between the arrival and service processes can reduce the congestion in queues and delays in transmission. Begum and Maheswari (2002) developed the M/M/c interdependent queueing model with controllable arrival rates, which was the extended work of Rao et al. (2000). Jain and Sharma (2004 a) considered the controllable queue with balking and reneging. To reduce the balking behavior of the customers, the provision of additional removable servers was made by Jain and Sharma (2004 b) while studying controllable queue. The $M/M^{a,b}/C$ interdependent queueing model with controllable arrival rates was discussed by Sitrarasu et al. (2007). Estimation comparison on busy period for a controllable M/G/1 system with bicriterion policy was analysed by Ke et al. (2008). Yang et al. (2010) developed optimization and sensitivity analysis of controlling arrivals in the queueing system with single working vacation.

The loss and delay phenomena of the customers in the system is likely to bring about the understanding, that either the customers may like to wait in the queue to get service or may be lost when all the servers are busy. Jain et al. (2002) developed loss and delay queueing model for time-shared system with additional service positions and no passing. Performance indices of Markovian loss and delay queueing model with no passing and removable additional servers was studied by Jain and Singh (2003). User optimal state dependent routing in parallel tandom queues with loss was made by Spicer and Ziedins (2006). Fan (2007) developed a queueing model for mixed loss-delay systems with general inter arrival processes for wide-band calls. Kim et al. (2009) considered erlang loss queueing system with batch arrivals operating in a random environment. Network queue and loss analysis using histogram-based traffic models was analysed by Orallo and Carbo (2010).

In real time system, the server is unreliable and may breakdown when it is working; and is sent to be repaired at a repair facility of the system. It is therefore desirable to have information about the fact that in which manner server breakdown affects the performance of the system. A single server queue with arrival rate dependent on server breakdown was studied by Shogan (1979). Grey et al. (2000) considered a multiple vacation queueing model with breakdown. In his investigation, queue length distribution was obtained by using probability generating function method. Ke (2003) and Wang et al. (2005) have developed the models in different framework in this regards. An M/M/1 retrial queue with unreliable server was investigated by Serman and Kharoufeh (2006). A discrete-time retrial queue with negative customers and unreliable server was obtained by Wang and Zhang (2009). Wu and Ke (2010) developed computational algorithm and parameter optimization for a multi-server system with unreliable servers and impatient customers.

In N-policy system, the server turns on only when there are N (>1) or more customers present in the system, otherwise the server goes on vacation. Modified N-policy for M/G/1 queue was studied by Krishnamorthy and Deepak (2002). Jau (2003) considered the operating characteristic analysis on a general input queue with N-policy and a start up time. Optimal management of the N-policy $M/E_k/1$ queueing system with a removable service station was investigated by Pearn and Chang (2004). Chaudhary and Paul (2004) analysed a batch arrival queue with additional service channel under N-policy. The balking behavior of the customers has also been considered in this investigation according to which the customers may not like to join the system due to impatience. Optimal NT policies for M/G/1 system with a startup and unreliable server was analysed by Ke (2006). The N-policy for an unreliable server with delaying repair and two phases of service was obtaine by Choudhury et al. (2009). Comparison of two randomized policy M/G/1 queues with second optional service, server breakdown and startup was studied by Wang et al. (2010).

This paper studies optimal N-policy for a single server interdependent loss and delay queueing model with breakdowns, repairs and controllable arrival rate. The remaining part of the paper is organized as follows. Section 2 is devoted for the model description. Queue size distribution for different states has been obtained by using generating function method in section 3. Performance measures and optimal N-policy are given in sections 4 and 5, respectively. Special cases are deduced in section 6. Sensitivity analysis is carried out to explore the effect of different parameters on performance indices in section 7. The conclusion is drawn in section 8.

II. Model Descriptions

Consider a single unreliable server model wherein service time of customers, and life time and repair time of the server are assumed to be exponentially distributed with parameter μ, α and β respectively. There are two types of the customers in the system (i) loss customers (ii) delay customers. The customers who depart from the system, on finding the server busy on their arrival, are called the loss customers. On the other hand the customers who have patience to wait for their service if the server is busy with other customers are called the delay customers. Balking behavior of the customers is also considered due to which the customers may not like to join the queue on seeing it very long. The server starts service, whenever the specified N (>1) or more customers are accumulated in the system. Once the server is busy, he renders service till system becomes empty and after that he goes on vacation. The server may breakdown only when it is in working state. Let "i" denotes the status of server defined as follows:

$$i = \begin{cases} 0, & \text{server is idle.} \\ 1, & \text{server is turned on and in operation.} \\ 2, & \text{server is turned on and under repair.} \end{cases}$$

We also assume that the arrival and the service processes of system are correlated and follow a bivariate Poisson process having the joint probability mass function of the form:

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda+\mu-e)t} \sum_{j=0}^{\min(x_1,x_2)} \frac{(et)^j [(\lambda_i - e)t]^{(x_1-j)} [(\mu - e)t]^{(x_2-j)}}{j!(x_1-j)!(x_2-j)!}; \quad \lambda > 0,\ \mu > 0.$$

where $x_1,x_2=0,1,2,\ldots$ and $0 < e < \min(\lambda,\mu)$

The service is given to the customers in FIFO order with the same efficiency as before breakdown. The mean arrival rate of the customers depends upon the server's status and are given as follows:

$$\lambda = \begin{cases} \lambda_1 + \lambda_2, & i = 0 \\ \lambda_1 b_1 + \lambda_2 b_2, & i = 1 \\ \lambda_1 b_1, & i = 2 \end{cases}$$

where λ_1 and λ_2 are the arrival rates of the delay and loss customers, respectively. Here b_1 and b_2 are the joining probabilities of the delay (i.e type-1) and loss (i.e type-2) customers in the system when server is busy. Thus the balking probabilities of type 1 and 2 customers are $\bar{b}_1 = 1 - b_1$ and $\bar{b}_2 = 1 - b_2$, respectively.

Let $P_i(n)$ denote the probability that there are n customers present in the system when server is in state 'i', and let

E (I) the expected length of idle period.
E (B) the expected length of busy period.
E (D) the expected length of breakdown (i.e. under repair) period.
E (C) the expected cycle period.
P_I the long run fraction of time for which server is idle.
P_B the long run fraction of time for which server is busy.
P_D the long run fraction of time for which server is broken down and under repair.

The steady state equations governing the model are given as follows:

$$(\lambda_1 + \lambda_2 - e)P_0(0) = (\mu - e)P_1(1) \quad \ldots (1)$$

$$(\lambda_1 + \lambda_2 - e)P_0(n) = (\lambda_1 + \lambda_2 - e)P_0(n-1), \quad 1 \le n \le N-1 \quad \ldots (2)$$

$$\{(\lambda_1 b_1 + \lambda_2 b_2 - e) + \alpha + (\mu - e)\}P_1(1) = (\mu - e)P_1(2) + \beta P_2(1) \quad \ldots (3)$$

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$$\{(\lambda_1 b_1 + \lambda_2 b_2 - e) + \alpha + (\mu - e)\}P_1(n) = (\lambda_1 b_1 + \lambda_2 b_2 - e)P_1(n-1) + (\mu - e)P_1(n+1) + \beta P_2(n), \; 2 \le n \le N-1 \quad \text{…. (4)}$$

$$\{(\lambda_1 b_1 + \lambda_2 b_2 - e) + \alpha + (\mu - e)\}P_1(N) = (\lambda_1 b_1 + \lambda_2 b_2 - e)P_0(N-1) + (\lambda_1 b_1 + \lambda_2 b_2 - e)P_1(N-1) + (\mu - e)P_1(N+1) + \beta P_2(N) \quad \text{…. (5)}$$

$$\{(\lambda_1 b_1 + \lambda_2 b_2 - e) + \alpha + (\mu - e)\}P_1(n) = (\lambda_1 b_1 + \lambda_2 b_2 - e)P_1(n-1) + (\mu - e)P_1(n+1) + \beta P_2(n), \; n \ge N+1 \quad \text{…. (6)}$$

$$\{(\lambda_1 b_1 - e) + \beta\}P_2(1) = \alpha P_1(1) \quad \text{…. (7)}$$

$$\{(\lambda_1 b_1 - e) + \beta\}P_2(n) = (\lambda_1 b_1 - e)P_2(n-1) + \alpha P_1(n), \; n \ge 2 \quad \text{…. (8)}$$

From eqs. (1) and (2), we get

$$P_1(1) = \left(\frac{\lambda_1 + \lambda_2 - e}{\mu - e}\right)P_0(0) \quad \text{…. (9)}$$

$$P_0(n) = P_0(0), \; 1 \le n \le N-1 \quad \text{…. (10)}$$

Using eqs. (7) and (8), we obtain $P_2(n)$ as

$$P_2(n) = \frac{\alpha}{\{(\lambda_1 b_1 - e) + \beta\}}\left[\sum_{k=1}^{n-1} B^{n-k} P_1(k) + P_1(n)\right], \; n \ge 2 \quad \text{…. (11)}$$

where

$$B = \frac{(\lambda_1 b_1 - e)}{(\lambda_1 b_1 - e) + \beta}$$

Eqs. (3) and (4) are used to get the value of $P_1(n)$, $(2 \le n \le N)$ as follows:

$$P_1(n) = P_1(1) + AP_1(n-1) + \frac{\alpha}{(\mu - e)}\sum_{k=1}^{n-2} B^{n-k} P_1(k), \; 2 \le n \le N \quad \text{…. (12)}$$

where

$$A = \frac{(\lambda_1 b_1 + \lambda_2 b_2 - e)(\lambda_1 b_1 - e) + \alpha(\lambda_1 b_1 - e) + \beta(\lambda_1 b_1 + \lambda_2 b_2 - e)}{(\mu - e)\{(\lambda_1 b_1 - e) + \beta\}}$$

Using eqs. (5) and (6), we get the value of $P_1(n)$, for $n \ge N+1$ as

$$P_1(n) = AP_1(n-1) + \frac{\alpha}{(\mu - e)}\sum_{k=1}^{n-2} B^{n-k} P_1(k) \quad \text{…. (13)}$$

It is not easy to use the recursive technique to obtain the expression for $P_0(0)$. We obtain the value of $P_0(0)$ by using the generating function approach in the next section.

III. The Generating Function

Define the following generating functions:

$$G_0(z)=\sum_{n=0}^{N-1} P_0(n)z^n \qquad \text{…. (14)}$$

$$G_1(z)=\sum_{n=1}^{\infty} P_1(n)z^n \qquad \text{…. (15)}$$

$$G_2(z)=\sum_{n=1}^{\infty} P_2(n)z^n \qquad \text{…. (16)}$$

Using eq. (10) in eq. (14), we get

$$G_0(z)=\frac{1-z^N}{1-z}P_0(0) \qquad \text{…. (17)}$$

On multiplying (1) and (3)-(6) with appropriate powers of z and summing over n, we find

$$\left[(\lambda_1 b_1+\lambda_2 b_2-e)z^2-\{(\lambda_1 b_1+\lambda_2 b_2-e)+\alpha+(\mu-e)\}z+(\mu-e)\right]G_1(z) + \beta z G_2(z)=(\lambda_1+\lambda_2-e)z(1-z^N)P_0(0) \qquad \text{…. (18)}$$

Similarly multiplying eqs. (7) and (8) by appropriate powers of z and summing, we have

$$\alpha G_1(z)+\left[(\lambda_1 b_1-e)z-\{(\lambda_1 b_1-e)+\beta\}\right]G_2(z)=0 \qquad \text{…. (19)}$$

From eqs. (18) and (19), we get

$$G_2(z)=\frac{\alpha(\lambda_1+\lambda_2-e)z(z^n-1)P_0(0)}{\left[(\lambda_1 b_1+\lambda_2 b_2-e)z^2-\{(\lambda_1 b_1+\lambda_2 b_2-e)+\alpha+(\mu-e)\}+(\mu-e)\right]\left[(\lambda_1 b_1-e)z-(\lambda_1 b_1-e)-\beta\right]-\alpha\beta z} \qquad \text{…. (20)}$$

$$G_1(z)=\frac{(\lambda_1+\lambda_2-e)z(1-z^n)\left[(\lambda_1 b_1-e)z-(\lambda_1 b_1-e)-\beta\right]P_0(0)}{\left[(\lambda_1 b_1+\lambda_2 b_2-e)z^2-\{(\lambda_1 b_1+\lambda_2 b_2-e)+\alpha+(\mu-e)\}z+(\mu-e)\right]\left[(\lambda_1 b_1-e)z-(\lambda_1 b_1-e)-\beta\right]-\alpha\beta z} \qquad \text{…. (21)}$$

The normalizing condition is given by

$G(1)=G_0(1) + G_1(1) + G_2(1)= 1$ …. (22)

Using eqs. (17), (20) and (21) in eq. (22) and applying the L-Hospital rule to get the limiting values when $z\rightarrow 1$,we obtain $P_0(0)$ as:

$$P_0(0)=\frac{\nu}{N\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} \qquad \text{…. (23)}$$

where $\nu=(\mu-e)\beta-(\lambda_1 b_1+\lambda_2 b_2-e)\beta-(\lambda_1 b_1-e)\alpha$

IV. Performance Measures

In order to derive expressions for various performance measures, we explore the complete cycle duration which is made of (i) idle period (ii) busy period (iii) down period, defined as follows. When the server is turned off, the corresponding length of time is called the idle period. The busy period (repair period) is the length of time when the server is turned on and in operation (under repair) and the customers are being served (waiting in queue to get service).

By using the value of $P_0(n)$, $P_1(n)$ and $P_2(n)$, we compute the probabilities P_I, P_B and P_D respectively in the following manner:

$$P_I = \sum_{n=0}^{N-1} P_0(n) = G_0(1) \quad \text{.... (24)}$$

$$P_B = \sum_{n=1}^{\infty} P_1(n) = G_1(1) \quad \text{.... (25)}$$

$$P_D = \sum_{n=0}^{\infty} P_2(n) = G_2(1) \quad \text{.... (26)}$$

Using eq. (10) in eqs. (24)-(26), we have

$$P_I = NP_0(0) = \frac{\nu}{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu} \quad \text{.... (27)}$$

$$P_B = \frac{(\lambda_1+\lambda_2-e)\beta}{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu} \quad \text{.... (28)}$$

$$P_D = \frac{(\lambda_1+\lambda_2-e)\alpha}{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu} \quad \text{.... (29)}$$

The expected number of customers in the system when the server is turn off, turn on and operating and broken down state, respectively are as follows:

$$E(N_0) = G_0'(1) = \frac{(N-1)}{2} - \frac{(\lambda_1+\lambda_2-e)(N-1)(\alpha+\beta)}{2\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} \quad \text{.... (30)}$$

$$E(N_1) = G_1'(1) = \frac{(\lambda_1+\lambda_2-e)(N+1)\beta}{2\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} + \frac{(\lambda_1+\lambda_2-e)\{(\lambda_1 b_1-e)^2\alpha + (\lambda_1 b_1 - e)\alpha\beta + (\lambda_1 b_1+\lambda_2 b_2-e)\beta^2\}}{\nu\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} \quad \text{.... (31)}$$

$$E(N_2) = G_2'(1) = \frac{(\lambda_1+\lambda_2-e)(N+1)\alpha}{2\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} + \frac{(\lambda_1+\lambda_2-e)\alpha[(\lambda_1 b_1-e)\{(\lambda_1 b_1+\lambda_2 b_2-e)-\alpha-(\mu-e)\}-(\lambda_1 b_1-e)\beta]}{\nu\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}} \quad \text{.... (32)}$$

Now the expected number of customers in the system is:

$$E(N_s) = E(N_0)+E(N_1)+E(N_2)$$
$$= \frac{(N-1)}{2} + \frac{(\lambda_1+\lambda_2-e)[\beta(\mu-e)(\alpha+\beta)+\alpha(\lambda_1 b_1-e)\{(\lambda_1 b_1-e)-(\lambda_1 b_1+\lambda_2 b_2-e)+(\mu-e)\}]}{\nu\{(\lambda_1 b_1+\lambda_2 b_2-e)(\alpha+\beta)+\iota\}} \quad \text{.... (33)}$$

V. Cost Analysis

The expected idle period can be find using:

$$E(I)=\frac{N}{\lambda_1+\lambda_2} \quad \text{…. (34)}$$

Also E(C)=E(I)+E(B)+E(D) so that

P_I=E(I) / E(C), P_B=E(B) / E(C) and P_D=E(D) / E(C)
Thus,

$$E(B)=P_B E(C)=\frac{N\beta}{\nu} \quad \text{…. (35)}$$

$$E(D)=P_D E(C)=\frac{N\alpha}{\nu} \quad \text{…. (36)}$$

$$E(C)=P_C E(C)=\frac{N\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}}{(\lambda_1+\lambda_2-e)\nu} \quad \text{…. (37)}$$

To provide the optimal N-policy, we calculate the minimum expected total cost per unit time by considering the following cost elements:

$C_u(C_d)$ start up (shut down) cost for turning the server on (off)
$C_o(C_f)$ cost per unit time for keeping server on (off)
C_b cost per unit time for a break down server
C_h holding cost per customer per unit time present in the system

The expected total cost per unit time is given by:

$$E\{C(N)\}=(C_u+C_d)\frac{1}{E(C)}+C_bP_D+C_oP_B+C_fP_I+C_hE(N_S) \quad \text{…. (38)}$$

$$=(C_u+C_d)\frac{(\lambda_1+\lambda_2-e)\nu}{N\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}}+\frac{(\lambda_1+\lambda_2-e)(C_b\alpha+C_O\beta)+\nu C_f}{(\lambda_1+\lambda_2-e)(\alpha+\beta+\nu)}$$
$$+C_h\left[\frac{(N-1)}{2}+\frac{(\lambda_1+\lambda_2-e)[\beta(\mu-e)(\alpha+\beta)+\alpha(\lambda_1b_1-e)\{(\lambda_1b_1-e)-(\lambda_1b_1+\lambda_2b_2-e)+(\mu-e)\}]}{\nu\{(\lambda_1b_1+\lambda_2b_2-e)(\alpha+\beta)+\nu\}}\right] \quad \text{…. (39)}$$

The following inequality is used to obtain the optimal value of N so that the total expected cost could be minimized:

$$E\{C(N^*+1)\}>E\{C(N^*)\}<E\{C(N^*-1)\} \quad \text{…. (40)}$$

This provides

$$N^*(N^*-1)<\frac{2(\lambda_1+\lambda_2-e)\nu(C_u+C_d)}{C_h\{(\lambda_1+\lambda_2-e)(\alpha+\beta)+\nu\}}<N^*(N^*+1) \quad \text{…. (41)}$$

By considering the discrete parameter N as continuous one, an approximate optimal value of N can be obtained. So that using

$$\frac{dE\{C(N)\}}{dN}=0,$$

we have

$$N^* = \left[\frac{2(\lambda_1 + \lambda_2 - e)\nu(C_u + C_d)}{C_h \{(\lambda_1 + \lambda_2 - e)(\alpha + \beta) + \nu\}} \right]^{\frac{1}{2}} \quad \text{.... (42)}$$

VI. Special Cases

In this section, we deduce some special cases by setting appropriate parameters as follows:

Case I: Interdependent queueing model with balking and delay customers.

In this case, there is no loss customers in the system so that $\lambda_2 = 0$.

(a) When $b_1 = b_2 = 1$, this case provides results for model without balking.

(b) Again setting $b_1 = b_2 = 1$ and $e = 0$, we get results which tally with those obtained by Jain (1997) for homogeneous arrival rate.

Case II: Queueing model without balking, constant arrival rates and delay customers.

In this case we substitute $b_1 = b_2 = 1$, $\lambda_1 = \lambda$, $\lambda_2 = 0$ and $e = 0$, so that our results coincide with that of Wang (1995).

Case III: On considering N=1, we get results for a single server controllable queue, with loss and delay.

VII. Sensitivity Analysis

In this section, sensitivity analysis is carried out to demonstrate the effect of different parameters on various performance indices. The graphs for the expected queue lengths are shown in figures 1 to 5. The default values of parameters are fixed as $\mu=1$, $\alpha=0.1$, $\beta=5$, $e=0.6$, $b_1=0.5$, $b_2=0.8$, $\lambda_1=0.9$, $\lambda_2=0.6$ and N=5.

Figures 1(a-c) shows the effects of N, α and β respectively on $E(N_s)$ by varying the arrival rate λ_1. From fig. 1(a), we examine that $E(N_s)$ increases slowly for increasing values of λ_1, upto $\lambda_1=0.5$, and then moderately upto $\lambda_1=0.9$; but beyond that it increases very sharply. By increasing the value of N, $E(N_s)$ also increases. The effect of α on $E(N_s)$ is shown in figure 1(b) where it is noted that as λ_1 increases, $E(N_s)$ increases slightly for lower values, but it increases sharply after $\lambda_1=0.9$. Also $E(N_s)$ decreases for increasing value of α. In figure 1(c), $E(N_s)$ has same trends with respect to λ_1 as observed in fig. 1(b). The increasing values of β do not show significant effect on $E(N_s)$ for lower values of λ_1 but as λ_1 grows, there appears visible increasing trends with respect to β. Concludingly lower arrival rate does not effect the queue length but it does have remarkable effect for higher rate, as we expect in real life situations. The failure rate α and repair rate β also do not affect queue length in the beginning but its affect is seen distinctly later.

Figures 2(a-c) depict $E(N_s)$ vs. λ_2 for different values of N, α and β respectively. We see that $E(N_s)$ increases initially gradually and then remarkably for higher value of λ_2 by increasing λ_2. In figure 2(a), $E(N_s)$ also increases as N increases. In figure 2(b) we observe that $E(N_s)$ decreases with the increasing values of α. Figure 2(c) demonstrates the effect of β on $E(N_s)$ and we notice that $E(N_s)$ increases with the increasing values of β.

Figures 3(a-c) visualize the effect of N on $E(N_s)$ by varying α, β and μ, respectively. In figure 3(a), it is observed that as α increases, $E(N_s)$ decreases, but $E(N_s)$ increases with N. In figure 3 (b), $E(N_s)$ increases significantly in initial stage with the increasing values of β, but tends to be constant value as β grows. If we increase the service rate, $E(N_s)$ decreases sharply upto $\mu=2$ and then after tends to a constant value as shown in figure 3(c). In figures 2(b) and 2(c), we observe the similar increasing effect of N as noted in fig. 3(a).

In figures 4(a-d), we examine the effect of parameter 'e' on $E(N_s)$ by varying different parameters. Initially $E(N_s)$ increases slowly for lower value of λ_1 and λ_2 and later on increases significantly for higher values, which is clear from figures 4(a) and 4(b), respectively. $E(N_s)$ decreases with the increasing value of 'e' in both the figures 4(a) and 4(b). It is noted that interdependence of rates affects the queue length reasonably. $E(N_s)$ increases with the increasing values of both α and β and decreases with the increasing value of 'e', as noticed from figures 4(c) and 4(d).

In figures 5(a-d), $E(N_s)$ is shown for different values of b_1. It is seen that $E(N_s)$ increases as b_1 increases. It is observed from figures 5(a) and 5(b) that as arrival rates λ_1 and λ_2 increase, the value of $E(N_s)$ slightly increases for lower rate but sharply for higher values of λ_1 and λ_2 as well as b_1. In figures 5(c) and 5(d), we notice that $E(N_s)$ decreases (increases) as α (β) increases; the effect of b_1 on $E(N_s)$ is more prevalent for lower (higher) value of α (β).

From the above sensitivity analysis, we conclude that by improving the grade of service, we can reduce the expected queue length to a certain extent as the service rate does not affect the expected queue length after a certain threshold value. The dependence parameter 'e' and balking parameter of delay customers also increase the expected queue length. The effect of higher arrival rate is more prevalent on queue length in comparison to lower rate.

VIII. Conclusion

The main purpose of this study is to obtain the optimal N-policy of a single unreliable server interdependent loss and delay queueing model with controllable arrival rate. Queue size distribution, the expected number of customers in the system and optimal N-policy are established by using the generating function method. The expected queue length can be reduced by increasing the service rate up to a certain level. The optimal control policy for the queue length by selecting suitable parameters examined by sensitivity analysis, may be helpful to decision makers in designing appropriate service facility while reducing loss and delay of customers.

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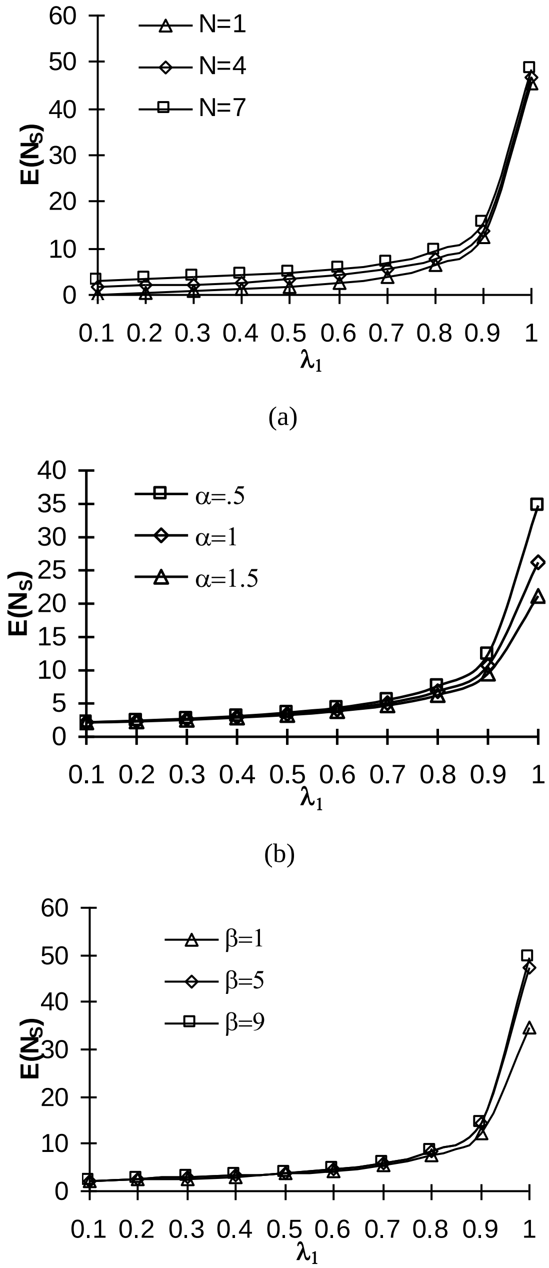

(c)

Fig. 1: Expected number of customers in the system $E(N_s)$ by varying λ_1 for different value of (a) N (b) α, and (c) β

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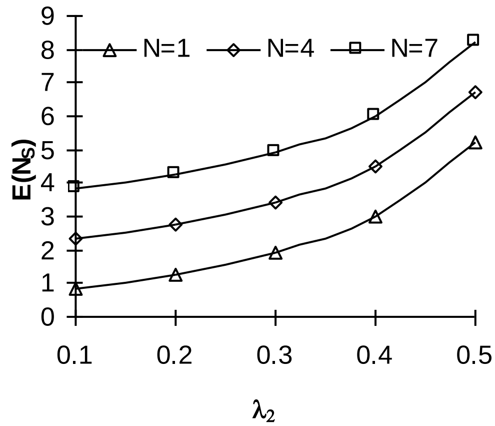
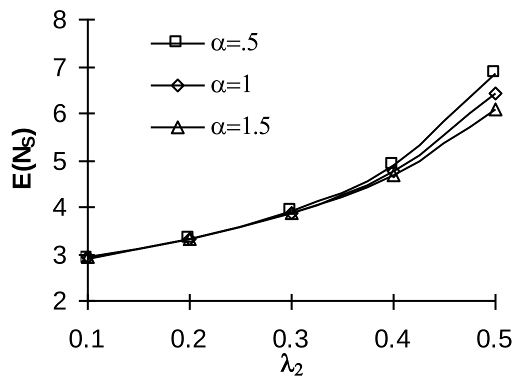


(a)



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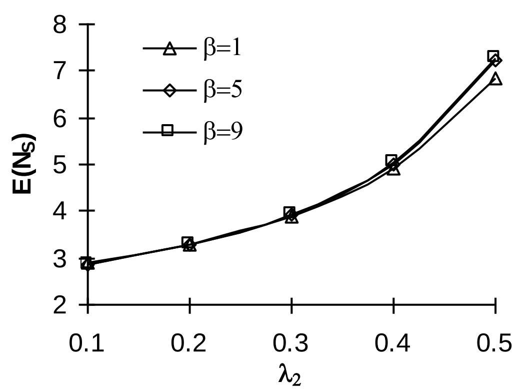

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Fig. 2: Expected number of customers in the system $E(N_s)$ by varying λ_2 for different value of (a) N (b) α, and (c) β

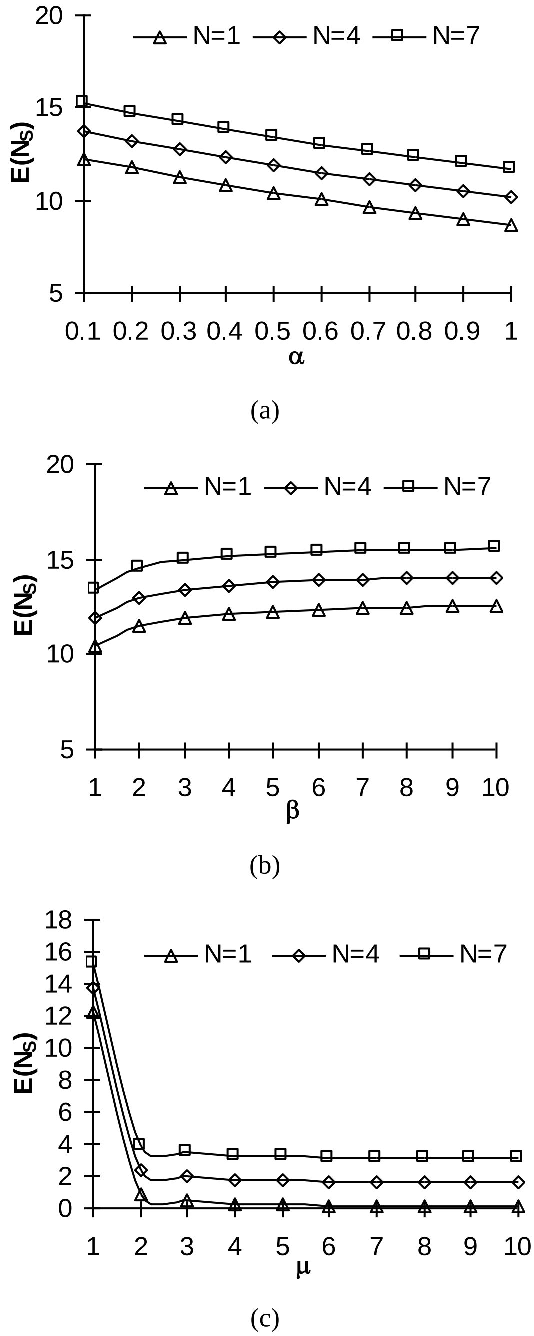

Fig. 3: Expected number of customers in the system $E(N_s)$ by varying (a) α (b) β and (c) μ for different values of N.

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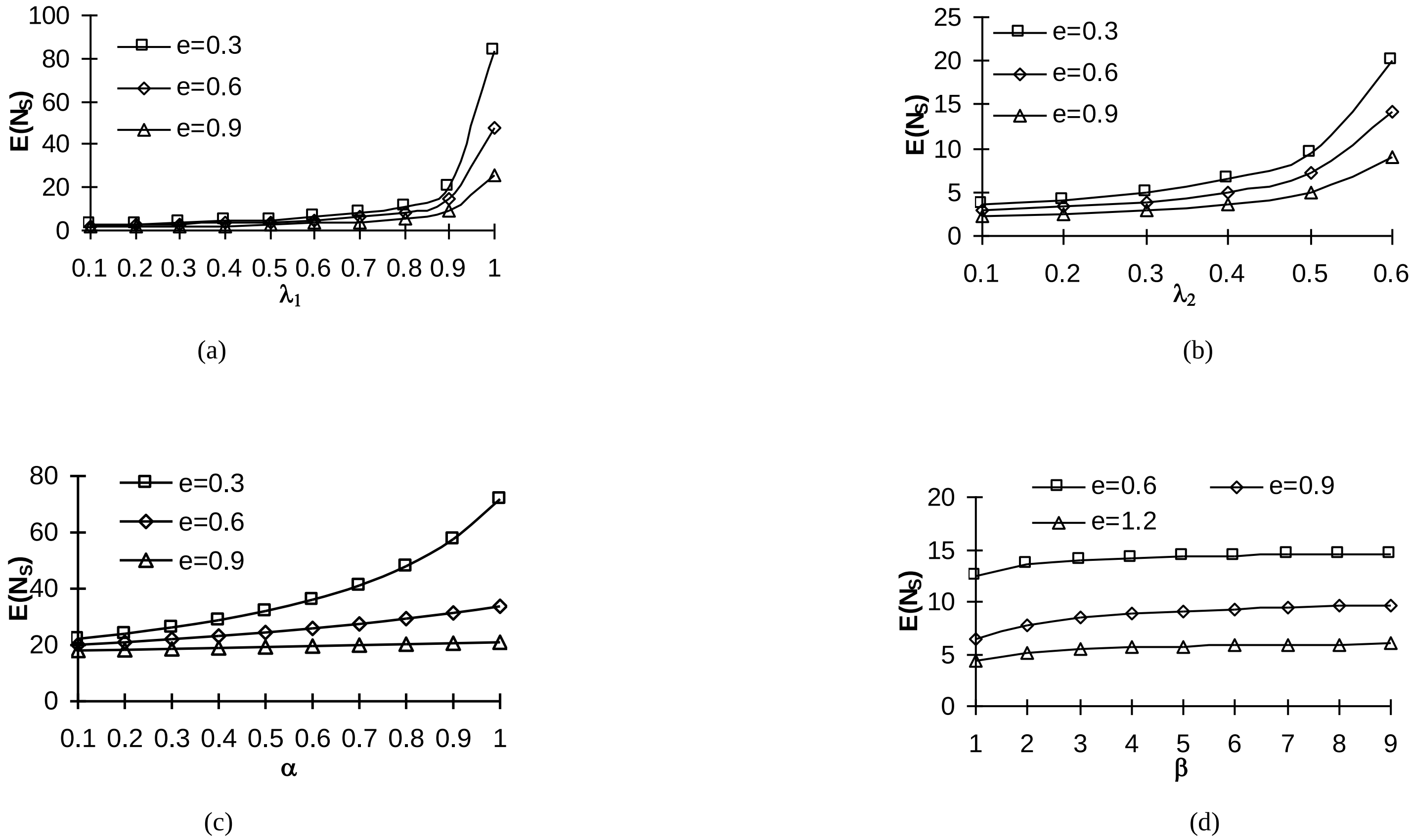

Fig. 4: $E(N_s)$ by varying (a) λ_1 (b) λ_2 (c) α and (d) β for the different values of 'e'.

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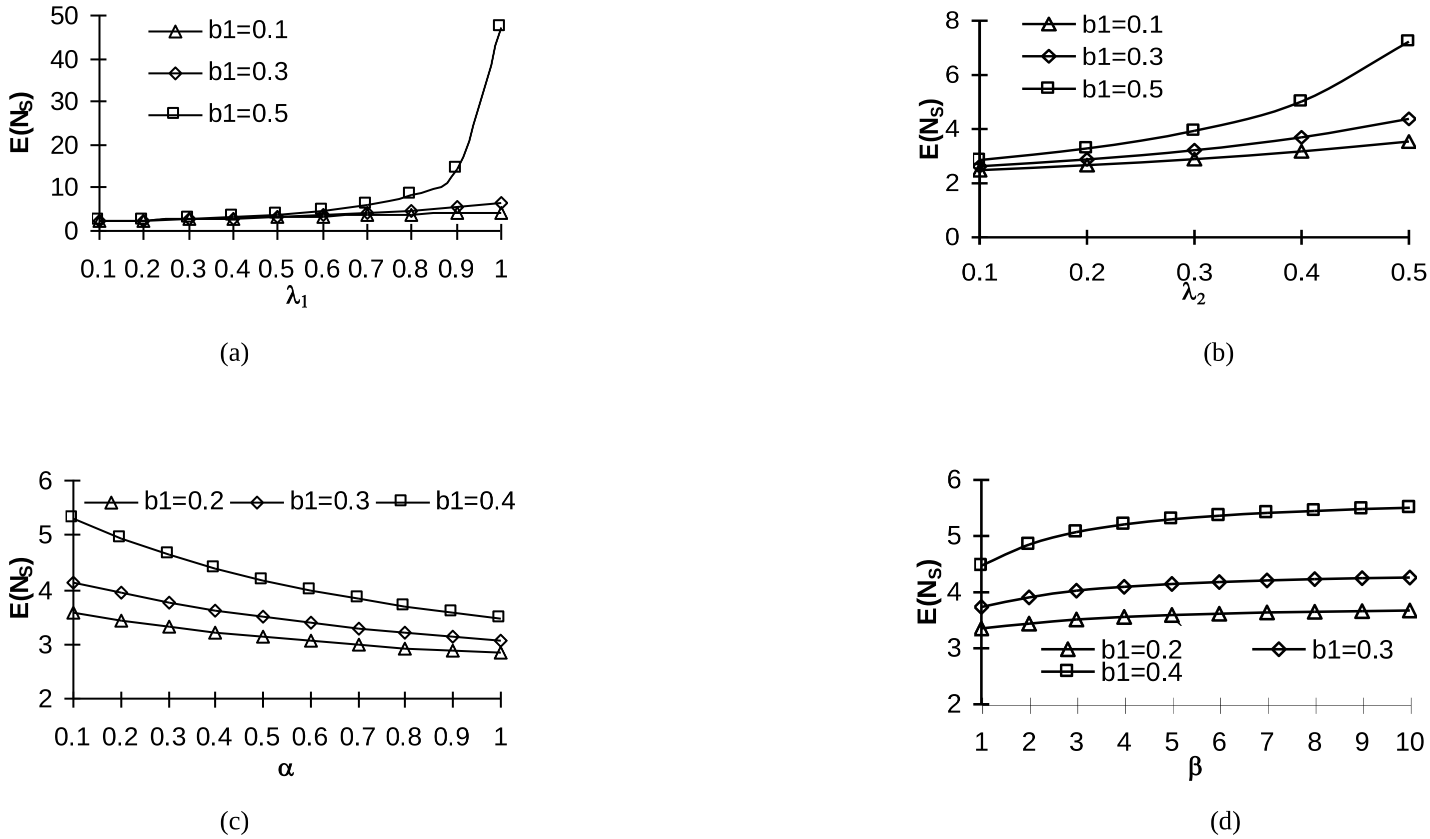

Fig. 5: Expected number of customers in the system $E(N_s)$ by varying (a) λ_1 (b) λ_2 (c) α and (d) β for the different values of 'b_1'.