A DEFACTO –DEJURE MODEL FOR POSITRONS, STELLAR NUCLEOSYNTHESIS, QUANTUM COHERENCE, SIMULATION, OBJECTIVE REALITY, QUANTUM DECOHERENCE, VIRTUAL PHOTONS, PHOTON TUNNELLING, ENZYMES, SPACE TIME, INCREASE IN SPEED OF CHEMICAL REACTIONS (SVERDUP AND SEMNOV’S NUMBER) AND QUANTUM TUNNELING

DR K N PRASANNA KUMAR, PROF B S KIRANAGI AND PROF C S BAGEWADI

ABSTRACT: There exists differential relations, contiguous similarities; in compassable anti generalities, presuppositional resemblances, dialectic transformation, portmanteau incompatibilities between different structures of stellar nucleosynthesis that occurs in stars, positrons, and Eulerian kinematic description thereof; calls for the attention of both simulation and Quantum coherence, Quantum Gravity and Objective reality. Despite the crying need for the same, and lack of comprehensive envelope of expression, there seems to be contential staticity and presciential dynamism in so far as these aspects are considered. Atrophied asseveration in stellar nucleosynthesis and environmental decoherence are probably most endearing attributions that call for both rational Leibnizism and Socratic subjectivity and of course discourse relativity. An evolutionist model is expounded with configurational entropy and morphological entity for these variables and we look at the system dispassionately without being disturbed by the state of the system. There is no clamor for participatory seriotological sermonisations or an orientation towards pedagogical pontification. We just state the facts and leave the rest to others to do the divergential affirmation, disjunctive synthesis. While we do resort to concept formulation, related phenomenological methodologies, transformational minimal conditions we neither resort to glorification or mortification of the thesis. Warts et al are presented without any hesitation, reservation, compunction or contrition, which probably is the testimony for the fact that human knowledge is limited and all the needs to be explored stretches in front like an ocean with all its cacophonous mendacious moorings and thromboses unbenedictory singularities with splashed contours and stigmatized boundaries.

Parameters taken in to consideration are:

1. Positrons
2. Stellar Nucleosynthesis
3. Simulation
4. Quantum Coherence
5. Quantum Gravity
6. Objective reality
7. Enzymes
8. Space-time
9. Virtual photons
10. Photonic tunneling (and visibility thereof)
11. Acceleration in Chemical reactions
12. Quantum Tunneling
POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE

NOTATION:

\[ G_{13} : \text{CATEGORY ONE OF STELLAR NUCLEOSYNTHESIS} \]
\[ G_{14} : \text{CATEGORY TWO OF STELLAR NUCLEOSYNTHESIS} \]
\[ G_{15} : \text{CATEGORY THREE OF STELLAR NUCLEOSYNTHESIS} \]
\[ T_{13} : \text{CATEGORY ONE OF POSITRONS} \]
\[ T_{14} : \text{CATEGORY TWO OF POSITRONS} \]
\[ T_{15} : \text{CATEGORY THREE OF POSITRONS} \]

SIMULATIONS AND QUANTUM COHERENCE: MODULE NUMBERED TWO:

Note: Every film is simulation. Every thought is simulation.

============================================================================

\[ G_{16} : \text{CATEGORY ONE OF SIMULATIONS} \]
\[ G_{17} : \text{CATEGORY TWO OF SIMULATIONS} \]
\[ G_{18} : \text{CATEGORY THREE OF SIMULATIONS} \]
\[ T_{16} : \text{CATEGORY ONE OF QUANTUM COHERENCE} \]
\[ T_{17} : \text{CATEGORY TWO OF QUANTUM COHERENCE} \]
\[ T_{18} : \text{CATEGORY THREE OF QUANTUM COHERENCE} \]

OBJECTIVE REALITY AND QUANTUM DECOHERENCE: MODULE NUMBERED THREE:

============================================================================

\[ G_{20} : \text{CATEGORY ONE OF OBJECTIVE REALITY} \]
\[ G_{21} : \text{CATEGORY TWO OF OBJECTIVE REALITY} \]
\[ G_{22} : \text{CATEGORY THREE OF OBJECTIVE REALITY} \]
\[ T_{20} : \text{CATEGORY ONE OF QUANTUM DECOHERENCE} \]
\[ T_{21} : \text{CATEGORY TWO OF QUANTUM DECOHERENCE} \]
\[ T_{22} : \text{CATEGORY THREE OF QUANTUM DECOHERENCE} \]
SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR:

\[ \begin{align*}
G_{24} : \text{CATEGORY ONE OF ENZYMES} \\
G_{25} : \text{CATEGORY TWO OF ENZYMES} \\
G_{26} : \text{CATEGORY THREE OF ENZYMES} \\
T_{24} : \text{CATEGORY ONE OF SPACE TIME} \\
T_{25} : \text{CATEGORY TWO OF SPACE TIME} \\
T_{26} : \text{CATEGORY THREE OF SPACETIME}
\end{align*} \]

PHOTONIC TUNNELING (VISIBILITY THEREOF) AND VIRTUAL PHOTONS: MODULE NUMBERED FIVE:

\[ \begin{align*}
G_{28} : \text{CATEGORY ONE OF PHOTONIC TUNNELING} \\
G_{29} : \text{CATEGORY TWO OF PHOTONIC TUNNELING} \\
G_{30} : \text{CATEGORY THREE OF PHOTONIC TUNNELING} \\
T_{28} : \text{CATEGORY ONE OF VIRTUAL PHOTONS} \\
T_{29} : \text{CATEGORY TWO OF VIRTUAL PHOTONS} \\
T_{30} : \text{CATEGORY THREE OF VIRTUAL PHOTONS}
\end{align*} \]

QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION: MODULE NUMBERED SIX:

\[ \begin{align*}
G_{32} : \text{CATEGORY ONE OF QUANTUM TUNNELING} \\
G_{33} : \text{CATEGORY TWO OF QUANTUM TUNNELING} \\
G_{34} : \text{CATEGORY THREE OF QUANTUM TUNNELING} \\
T_{32} : \text{CATEGORY ONE OF ACCELERATION IN CHEMICAL REACTIONS} \\
T_{33} : \text{CATEGORY TWO OF ACCELERATION IN CHEMICAL REACTIONS} \\
T_{34} : \text{CATEGORY THREE OF ACCELERATION IN CHEMICAL REACTIONS}
\end{align*} \]

\[ \begin{align*}
&= (a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{12})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}, (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}, \\
&\quad (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}, \\
&\quad (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)},
\end{align*} \]
\((a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}\)

are Accenctuation coefficients

\((a_{13}')^{(1)}, (a_{14}')^{(1)}, (a_{15}')^{(1)}, (b_{16}')^{(1)}, (b_{14}')^{(1)}, (b_{15}')^{(1)}, (a_{16}')^{(2)}, (a_{17}')^{(2)}, (a_{18}')^{(2)}, (b_{16}')^{(2)}, (b_{17}')^{(2)}, (b_{18}')^{(2)}, (a_{20}')^{(3)}, (a_{22}')^{(3)}, (b_{20}')^{(3)}, (b_{21}')^{(3)}, (b_{22}')^{(3)}, (a_{24}')^{(4)}, (a_{25}')^{(4)}, (a_{26}')^{(4)}, (b_{24}')^{(4)}, (b_{25}')^{(4)}, (b_{26}')^{(4)}, (b_{28}')^{(5)}, (b_{29}')^{(5)}, (b_{30}')^{(5)}, (a_{28}')^{(5)}, (a_{29}')^{(5)}, (a_{30}')^{(5)}, (a_{32}')^{(6)}, (a_{33}')^{(6)}, (a_{34}')^{(6)}, (b_{32}')^{(6)}, (b_{33}')^{(6)}, (b_{34}')^{(6)}\)

are Dissipation coefficients

**POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE**

The differential system of this model is now

\[
\frac{d\delta_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t)\right]G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}, t)\right]G_{14}
\]

\[
\frac{d\delta_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}, t)\right]G_{15}
\]

\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G, t)\right]T_{13}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G, t)\right]T_{14}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G, t)\right]T_{15}
\]

\(+ (a_{13}')^{(1)}(T_{14}, t) = \) First augmentation factor

\(- (b_{13}')^{(1)}(G, t) = \) First detritions factor

**SIMULATIONS AND QUANTUM COHERENCE: MODULE NUMBERED TWO**

The differential system of this model is now

\[
\frac{d\delta_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}, t)\right]G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17}, t)\right]G_{17}
\]

\[
\frac{d\delta_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}, t)\right]G_{18}
\]

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b_{16}')^{(2)} - (b_{16}'')^{(2)}(G_{19}, t)\right]T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b_{17}')^{(2)} - (b_{17}'')^{(2)}(G_{19}, t)\right]T_{17}
\]

\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b_{18}')^{(2)} - (b_{18}'')^{(2)}(G_{19}, t)\right]T_{18}
\]

\(+ (a_{16}')^{(2)}(T_{17}, t) = \) First augmentation factor

\(- (b_{16}')^{(2)}(G_{19}, t) = \) First detritions factor

**OBJECTIVE REALITY AND QUANTUM DECOHERENCE: MODULE NUMBERED THREE**
The differential system of this model is now

\[ \frac{dG_{20}}{dt} = (a_{20}^{(3)})G_{21} - [(a_{20}^{(3)}) + (a_{20}^{(3)}T_{21}, t)]G_{20} \]

\[ \frac{dG_{21}}{dt} = (a_{21}^{(3)})G_{20} - [(a_{21}^{(3)}) + (a_{21}^{(3)}T_{21}, t)]G_{21} \]

\[ \frac{dG_{22}}{dt} = (a_{22}^{(3)})G_{21} - [(a_{22}^{(3)}) + (a_{22}^{(3)}T_{21}, t)]G_{22} \]

\[ \frac{dT_{20}}{dt} = (b_{20}^{(3)})T_{21} - [(b_{20}^{(3)}) - (b_{20}^{(3)}G_{23}, t)]T_{20} \]

\[ \frac{dT_{21}}{dt} = (b_{21}^{(3)})T_{20} - [(b_{21}^{(3)}) - (b_{21}^{(3)}G_{23}, t)]T_{21} \]

\[ \frac{dT_{22}}{dt} = (b_{22}^{(3)})T_{21} - [(b_{22}^{(3)}) - (b_{22}^{(3)}G_{23}, t)]T_{22} \]

\[ + (a_{20}^{(3)}T_{21}, t) = \text{First augmentation factor} \]

\[ - (b_{20}^{(3)}G_{23}, t) = \text{First detritus factor} \]

**SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR**

The differential system of this model is now

\[ \frac{dG_{24}}{dt} = (a_{24}^{(4)})G_{25} - [(a_{24}^{(4)} + (a_{24}^{(4)}T_{25}, t)]G_{24} \]

\[ \frac{dG_{25}}{dt} = (a_{25}^{(4)})G_{24} - [(a_{25}^{(4)} + (a_{25}^{(4)}T_{25}, t)]G_{25} \]

\[ \frac{dG_{26}}{dt} = (a_{26}^{(4)})G_{25} - [(a_{26}^{(4)} + (a_{26}^{(4)}T_{25}, t)]G_{26} \]

\[ \frac{dT_{24}}{dt} = (b_{24}^{(4)})T_{25} - [(b_{24}^{(4)} - (b_{24}^{(4)}G_{27}, t)]T_{24} \]

\[ \frac{dT_{25}}{dt} = (b_{25}^{(4)})T_{24} - [(b_{25}^{(4)} - (b_{25}^{(4)}G_{27}, t)]T_{25} \]

\[ \frac{dT_{26}}{dt} = (b_{26}^{(4)})T_{25} - [(b_{26}^{(4)} - (b_{26}^{(4)}G_{27}, t)]T_{26} \]

\[ + (a_{24}^{(4)}T_{25}, t) = \text{First augmentation factor} \]

\[ - (b_{24}^{(4)}G_{27}, t) = \text{First detritus factor} \]

**PHOTONIC TUNNELING(VISIBILITY THEREOF) AND VIRTUAL PHOTONS: MODULE NUMBERED FIVE**

The differential system of this model is now

\[ \frac{dG_{28}}{dt} = (a_{28}^{(5)})G_{29} - [(a_{28}^{(5)} + (a_{28}^{(5)}T_{29}, t)]G_{28} \]

\[ \frac{dG_{29}}{dt} = (a_{29}^{(5)})G_{28} - [(a_{29}^{(5)} + (a_{29}^{(5)}T_{29}, t)]G_{29} \]

\[ \frac{dG_{30}}{dt} = (a_{30}^{(5)})G_{29} - [(a_{30}^{(5)} + (a_{30}^{(5)}T_{29}, t)]G_{30} \]
\[
\begin{align*}
\frac{dT_{28}}{dt} &= (b_{28})^{(5)}T_{29} - [(b_{28}')(5) - (b_{28}'')(5)](G_{31}, t)T_{28} \\
\frac{dT_{29}}{dt} &= (b_{29})^{(5)}T_{28} - [(b_{29}')(5) - (b_{29}'')(5)](G_{31}, t)T_{29} \\
\frac{dT_{30}}{dt} &= (b_{30})^{(5)}T_{29} - [(b_{30}')(5) - (b_{30}'')(5)](G_{31}, t)T_{30} \\
+ (a_{28}'')^{(5)}(T_{29}, t) &= \text{ First augmentation factor} \\
- (b_{28}'')^{(5)}(G_{31}, t) &= \text{ First detritions factor}
\end{align*}
\]

\textbf{QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION: MODULE NUMBERED SIX}

The differential system of this model is now
\[
\begin{align*}
\frac{dG_{32}}{dt} &= (a_{32})^{(6)}G_{33} - [(a_{32}')(6) + (a_{32}'')(6)](T_{33}, t)G_{32} \\
\frac{dG_{33}}{dt} &= (a_{33})^{(6)}G_{32} - [(a_{33}')(6) + (a_{33}'')(6)](T_{33}, t)G_{33} \\
\frac{dG_{34}}{dt} &= (a_{34})^{(6)}G_{33} - [(a_{34}')(6) + (a_{34}'')(6)](T_{33}, t)G_{34} \\
\frac{dT_{32}}{dt} &= (b_{32})^{(6)}T_{33} - [(b_{32}')(6) - (b_{32}'')(6)](G_{35}, t)T_{32} \\
\frac{dT_{33}}{dt} &= (b_{33})^{(6)}T_{32} - [(b_{33}')(6) - (b_{33}'')(6)](G_{35}, t)T_{33} \\
\frac{dT_{34}}{dt} &= (b_{34})^{(6)}T_{33} - [(b_{34}')(6) - (b_{34}'')(6)](G_{35}, t)T_{34} \\
+ (a_{32}'')^{(6)}(T_{33}, t) &= \text{ First augmentation factor} \\
- (b_{32}'')^{(6)}(G_{35}, t) &= \text{ First detritions factor}
\end{align*}
\]

\textbf{HOLISTIC CONCATENATE SYTEMAL EQUATIONS HENCEFORTH REFERRED TO AS “GLOBAL EQUATIONS”}

\textbf{POSITRONS AND STELLAR NUCLEOSYNTHESIS: MODULE NUMBERED ONE}

\textbf{OBJECTIVE REALITY AND QUANTUM DECOHERENCE: MODULE NUMBERED THREE}

\textbf{SIMULATIONS AND QUANTUM COHERENCE: MODULE NUMBERED TWO}

\textbf{SPACE-TIME AND ENZYMES: MODULE NUMBERED FOUR}

\textbf{PHOTONIC TUNNELING (VISIBILITY THEREOF) AND VIRTUAL PHOTONS: MODULE NUMBERED FIVE}

\textbf{QUANTUM TUNNELING AND ACCELERATED CHEMICAL REACTION: MODULE NUMBERED SIX}

\[
\begin{align*}
\frac{dG_{13}}{dt} &= (a_{13})^{(1)}G_{14} - \\
&\quad \left[ (a_{13}')^{(1)}(T_{14}, t) + (a_{13}'')^{(2,2)}(T_{17}, t) + (a_{20}'')^{(3,3)}(T_{21}, t) ight. \\
&\quad + (a_{24}'')^{(4,4,4)}(T_{25}, t) + (a_{29}'')^{(5,5,5,5)}(T_{29}, t) + (a_{32}'')^{(6,6,6,6)}(T_{33}, t) \right] G_{13}
\end{align*}
\]
\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \begin{vmatrix}
(a'_{14})^{(1)} + (a_{14})''^{(1)} (T_{14}, t) + (a_{17})''^{(2,2)} (T_{17}, t) + (a_{21})''^{(3,3)} (T_{21}, t) \\
+ (a_{25})^{(4,4,4)} (T_{25}, t) + (a_{29})^{(5,5,5,5)} (T_{29}, t) + (a_{33})^{(6,6,6,6,6)} (T_{33}, t)
\end{vmatrix}
\]
\[
G_{14}
\]
\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \begin{vmatrix}
(a'_{15})^{(1)} + (a_{15})''^{(1)} (T_{15}, t) + (a_{18})''^{(2,2)} (T_{18}, t) + (a_{22})''^{(3,3)} (T_{22}, t) \\
+ (a_{26})^{(4,4,4,4)} (T_{26}, t) + (a_{30})^{(5,5,5,5,5)} (T_{30}, t) + (a_{34})^{(6,6,6,6,6,6)} (T_{34}, t)
\end{vmatrix}
\]
\[
G_{15}
\]
Where \(G_{13})^{(1)}(T_{13}, t)\), \((a_{14})^{(1)}(T_{14}, t)\), \((a_{16})^{(1)}(T_{16}, t)\) are first augmentation coefficients for category 1, 2 and 3
\(+(a_{22})''^{(2,2)}(T_{22}, t)\), \(+(a_{23})''^{(3,3)}(T_{23}, t)\) are second augmentation coefficient for category 1, 2 and 3
\(+(a_{25})^{(4,4,4)}(T_{25}, t)\), \(+(a_{26})^{(4,4,4,4)}(T_{26}, t)\), \(+(a_{27})^{(4,4,4,4)}(T_{27}, t)\) are third augmentation coefficient for category 1, 2 and 3
\(+(a_{29})^{(5,5,5,5)}(T_{29}, t)\), \(+(a_{30})^{(5,5,5,5,5)}(T_{30}, t)\) are fourth augmentation coefficient for category 1, 2 and 3
\(+(a_{32})^{(6,6,6,6,6)}(T_{32}, t)\), \(+(a_{33})^{(6,6,6,6,6,6)}(T_{33}, t)\) are sixth augmentation coefficient for category 1, 2 and 3

\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \begin{vmatrix}
(b'_{13})^{(1)} - (b_{13})''^{(1)} (G_{13}, t) - (b_{16})''^{(2,2)} (G_{16}, t) - (b_{20})''^{(3,3)} (G_{20}, t) \\
- (b_{24})^{(4,4,4,4)} (G_{24}, t) - (b_{28})^{(5,5,5,5,5)} (G_{28}, t) - (b_{32})^{(6,6,6,6,6,6)} (G_{32}, t)
\end{vmatrix}
\]
\[
T_{13}
\]
\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \begin{vmatrix}
(b'_{14})^{(1)} - (b_{14})''^{(1)} (G_{14}, t) - (b_{17})''^{(2,2)} (G_{17}, t) - (b_{21})''^{(3,3)} (G_{21}, t) \\
- (b_{25})^{(4,4,4,4)} (G_{25}, t) - (b_{29})^{(5,5,5,5,5)} (G_{29}, t) - (b_{33})^{(6,6,6,6,6,6)} (G_{33}, t)
\end{vmatrix}
\]
\[
T_{14}
\]
\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{vmatrix}
(b'_{15})^{(1)} - (b_{15})''^{(1)} (G_{15}, t) - (b_{18})''^{(2,2)} (G_{18}, t) - (b_{22})''^{(3,3)} (G_{22}, t) \\
- (b_{26})^{(4,4,4,4,4)} (G_{26}, t) - (b_{30})^{(5,5,5,5,5,5)} (G_{30}, t) - (b_{34})^{(6,6,6,6,6,6,6)} (G_{34}, t)
\end{vmatrix}
\]
\[
T_{15}
\]
Where \(-(b_{13})''^{(1)}(G_{13}, t)\), \(-(b_{14})''^{(1)}(G_{14}, t)\) are first detriment coefficients for category 1, 2 and 3
\(+(b_{22})''^{(2,2)}(G_{22}, t)\), \(+(b_{23})''^{(3,3)}(G_{23}, t)\) are second detritum coefficients for category 1, 2 and 3
\(+(b_{25})^{(4,4,4,4)}(G_{25}, t)\), \(+(b_{26})^{(4,4,4,4,4)}(G_{26}, t)\) are third detritum coefficients for category 1, 2 and 3
\(+(b_{29})^{(5,5,5,5,5)}(G_{29}, t)\), \(+(b_{30})^{(5,5,5,5,5,5)}(G_{30}, t)\) are fourth detritum coefficients for category 1, 2 and 3
\(+(b_{32})^{(6,6,6,6,6,6,6)}(G_{32}, t)\), \(+(b_{33})^{(6,6,6,6,6,6,6,6)}(G_{33}, t)\) are fifth detritum coefficients for category 1, 2 and 3
\(+(b_{35})^{(6,6,6,6,6,6,6,6,6)}(G_{35}, t)\), \(+(b_{36})^{(6,6,6,6,6,6,6,6,6,6)}(G_{36}, t)\) are sixth detritum coefficients for category 1, 2 and 3
\[ \frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[ \frac{(a'_{18})^{(2)} + (a''_{18})^{(2)} (T_{17}, t) + (a''_{18})^{(1,1)} (T_{14}, t) + (a''_{22})^{(3,3,3)} (T_{21}, t) + (a''_{20})^{(5,5,5,5,5)} (T_{29}, t) + (a''_{30})^{(6,6,6,6,6)} (T_{33}, t)}{(a'_{20})^{(2)} + (a''_{20})^{(2)} (T_{25}, t) + (a''_{30})^{(5,5,5,5,5)} (T_{29}, t) + (a''_{30})^{(6,6,6,6,6)} (T_{33}, t)} \right] G_{18} \]

Where \((a_{18})^{(2)}(T_{17}, t)\), \((a_{18})^{(1,1)}(T_{14}, t)\), and \((a_{18})^{(3,3,3)}(T_{21}, t)\) are first augmentation coefficients for category 1, 2 and 3

\((a_{22})^{(3,3,3)}(T_{21}, t)\), \((a_{22})^{(3,3,3,3)}(T_{25}, t)\), and \((a_{22})^{(5,5,5,5,5,5)}(T_{29}, t)\) are second augmentation coefficient for category 1, 2 and 3

\((a_{20})^{(5,5,5,5,5)}(T_{29}, t)\), \((a_{20})^{(6,6,6,6,6)}(T_{33}, t)\), \((a_{20})^{(7,7,7,7,7)}(T_{37}, t)\), \((a_{20})^{(8,8,8,8,8)}(T_{41}, t)\), \((a_{20})^{(9,9,9,9,9)}(T_{45}, t)\), \((a_{20})^{(10,10,10,10,10)}(T_{49}, t)\), \((a_{20})^{(11,11,11,11,11)}(T_{53}, t)\), \((a_{20})^{(12,12,12,12,12)}(T_{57}, t)\) are third augmentation coefficient for category 1, 2 and 3

\((a_{20})^{(5,5,5,5,5)}(T_{29}, t)\), \((a_{20})^{(6,6,6,6,6)}(T_{33}, t)\), \((a_{20})^{(7,7,7,7,7)}(T_{37}, t)\), \((a_{20})^{(8,8,8,8,8)}(T_{41}, t)\), \((a_{20})^{(9,9,9,9,9)}(T_{45}, t)\), \((a_{20})^{(10,10,10,10,10)}(T_{49}, t)\), \((a_{20})^{(11,11,11,11,11)}(T_{53}, t)\), \((a_{20})^{(12,12,12,12,12)}(T_{57}, t)\) are fourth augmentation coefficient for category 1, 2 and 3

\((a_{20})^{(5,5,5,5,5)}(T_{29}, t)\), \((a_{20})^{(6,6,6,6,6)}(T_{33}, t)\), \((a_{20})^{(7,7,7,7,7)}(T_{37}, t)\), \((a_{20})^{(8,8,8,8,8)}(T_{41}, t)\), \((a_{20})^{(9,9,9,9,9)}(T_{45}, t)\), \((a_{20})^{(10,10,10,10,10)}(T_{49}, t)\), \((a_{20})^{(11,11,11,11,11)}(T_{53}, t)\), \((a_{20})^{(12,12,12,12,12)}(T_{57}, t)\) are fifth augmentation coefficient for category 1, 2 and 3

\((a_{20})^{(5,5,5,5,5)}(T_{29}, t)\), \((a_{20})^{(6,6,6,6,6)}(T_{33}, t)\), \((a_{20})^{(7,7,7,7,7)}(T_{37}, t)\), \((a_{20})^{(8,8,8,8,8)}(T_{41}, t)\), \((a_{20})^{(9,9,9,9,9)}(T_{45}, t)\), \((a_{20})^{(10,10,10,10,10)}(T_{49}, t)\), \((a_{20})^{(11,11,11,11,11)}(T_{53}, t)\), \((a_{20})^{(12,12,12,12,12)}(T_{57}, t)\) are sixth augmentation coefficient for category 1, 2 and 3

\[ \frac{dG_{16}}{dt} = (b_{18})^{(2)} T_{17} \]

\[ \frac{dG_{17}}{dt} = (b_{17})^{(2)} T_{16} \]

\[ \frac{dG_{18}}{dt} = (b_{18})^{(2)} T_{17} \]

where \(-(b_{18})^{(2)}(G_{19}, t)\), \(-(b_{18})^{(1,1)}(G_{21}, t)\), \(-(b_{18})^{(1,1,1)}(G_{23}, t)\) are first derivetion coefficients for category 1, 2 and 3

\(-(b_{18})^{(1,1,1)}(G_{21}, t)\), \(-(b_{18})^{(1,1,1)}(G_{21}, t)\), \(-(b_{18})^{(1,1,1)}(G_{21}, t)\) are second derivetion coefficients for category 1, 2 and 3

\(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\) are third derivetion coefficients for category 1, 2 and 3

\(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\) are fourth derivetion coefficients for category 1, 2 and 3

\(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\) are fifth derivetion coefficients for category 1, 2 and 3

\(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\), \(-(b_{20})^{(1,1,1)}(G_{23}, t)\) are sixth derivetion coefficients for category 1, 2 and 3
\[ + (a_{29})^{5}(T_{25}, t) + (a_{22})^{5}(T_{32}, t) + (a_{33})^{5}(T_{33}, t) \] are first augmentation coefficients for category 1, 2 and 3

\[ + (a_{19})^{2.2}(T_{17}, t) + (a_{18})^{2.2}(T_{17}, t) \] are second augmentation coefficients for category 1, 2 and 3

\[ + (a_{17})^{1.1}(T_{15}, t) + (a_{16})^{1.1}(T_{15}, t) \] are third augmentation coefficients for category 1, 2 and 3

\[ + (a_{25})^{4.4.4.4.4}(T_{25}, t) + (a_{26})^{4.4.4.4.4}(T_{25}, t) + (a_{20})^{4.4.4.4.4}(T_{25}, t) \] are fourth augmentation coefficients for category 1, 2 and 3

\[ + (a_{29})^{5.5.5.5.5}(T_{29}, t) + (a_{30})^{5.5.5.5.5}(T_{29}, t) \] are fifth augmentation coefficients for category 1, 2 and 3

\[ + (a_{22})^{6.6.6.6.6}(T_{32}, t) + (a_{24})^{6.6.6.6.6}(T_{32}, t) \] are sixth augmentation coefficients for category 1, 2 and 3

\[ \frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} + B_{b_{20}}(G, T_{20}) \]

\[ \frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} + B_{b_{21}}(G, T_{21}) \]

\[ \frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} + B_{b_{22}}(G, T_{22}) \]

\[ - (a_{10})^{1.1}(G_{23}, t) - (a_{20})^{1.1}(G_{23}, t) - (b_{10})^{2.2}(G_{19}, t) \] are first detrition coefficients for category 1, 2 and 3

\[ - (a_{19})^{2.2.2}(G_{19}, t) - (a_{17})^{2.2.2}(G_{19}, t) - (b_{17})^{1.1.1}(G_{19}, t) \] are second detrition coefficients for category 1, 2 and 3

\[ - (a_{20})^{1.1.1}(G, t) - (b_{14})^{1.1.1}(G, t) - (b_{16})^{1.1.1}(G, t) \] are third detrition coefficients for category 1, 2 and 3

\[ - (a_{25})^{4.4.4.4.4}(G_{25}, t) - (a_{25})^{4.4.4.4.4}(G_{25}, t) - (a_{26})^{4.4.4.4.4}(G_{27}, t) \] are fourth detrition coefficients for category 1, 2 and 3

\[ - (a_{29})^{5.5.5.5.5}(G_{31}, t) - (a_{20})^{5.5.5.5.5}(G_{31}, t) - (a_{20})^{5.5.5.5.5}(G_{31}, t) \] are fifth detrition coefficients for category 1, 2 and 3

\[ - (a_{22})^{6.6.6.6.6}(G_{35}, t) - (a_{24})^{6.6.6.6.6}(G_{35}, t) \] are sixth detrition coefficients for category 1, 2 and 3

\[ \frac{dG_{24}}{dt} = (a_{24})^{4}G_{25} + (a_{24})^{4}(T_{25}, t) + (a_{25})^{5.5}(T_{25}, t) + (a_{25})^{6.6}(T_{25}, t) \]

\[ + (a_{14})^{1.1.1.1}(T_{14}, t) + (a_{16})^{2.2.2.2}(T_{14}, t) + (a_{17})^{3.3.3.3}(T_{14}, t) \]

\[ \frac{dG_{25}}{dt} = (a_{25})^{4}G_{24} + (a_{25})^{4}(T_{25}, t) + (a_{25})^{5.5}(T_{25}, t) + (a_{33})^{6.6}(T_{33}, t) \]

\[ + (a_{14})^{1.1.1.1}(T_{14}, t) + (a_{17})^{2.2.2.2}(T_{14}, t) + (a_{24})^{3.3.3.3}(T_{24}, t) \]

\[ \frac{dG_{26}}{dt} = (a_{26})^{4}G_{25} + (a_{26})^{4}(T_{25}, t) + (a_{26})^{5.5}(T_{25}, t) + (a_{34})^{6.6}(T_{33}, t) \]

\[ + (a_{15})^{1.1.1.1}(T_{15}, t) + (a_{17})^{2.2.2.2}(T_{15}, t) + (a_{24})^{3.3.3.3}(T_{24}, t) \]
Where \( (a_{24})^{4}(T_{24},t) \), \( (a_{25})^{4}(T_{25},t) \), \( (a_{26})^{4}(T_{26},t) \) are first augmentation coefficients for category 1, 2 and 3

\[ +(a_{28})^{3,5,5}(T_{28},t); +(a_{29})^{3,5,5}(T_{29},t); +(a_{30})^{3,5,5}(T_{30},t) \]

\( (a_{28})^{6,6,6}(T_{28},t); (a_{29})^{6,6,6}(T_{29},t); (a_{30})^{6,6,6}(T_{30},t) \) are second augmentation coefficients for category 1, 2 and 3

\[ +(a_{28})^{3,6,6}(T_{28},t); +(a_{29})^{3,6,6}(T_{29},t); +(a_{30})^{3,6,6}(T_{30},t) \]

\( (a_{28})^{5,5,5}(T_{28},t); (a_{29})^{5,5,5}(T_{29},t); (a_{30})^{5,5,5}(T_{30},t) \) are third augmentation coefficients for category 1, 2 and 3

\[ +(a_{28})^{1,1,1,1}(T_{28},t); +(a_{29})^{1,1,1,1}(T_{29},t); +(a_{30})^{1,1,1,1}(T_{30},t) \]

\( (a_{28})^{5,3,3,3}(T_{28},t); (a_{29})^{5,3,3,3}(T_{29},t); (a_{30})^{5,3,3,3}(T_{30},t) \) are fourth augmentation coefficients for category 1, 2, and 3

\[ +(a_{28})^{2,2,2,2}(T_{28},t); +(a_{29})^{2,2,2,2}(T_{29},t); +(a_{30})^{2,2,2,2}(T_{30},t) \]

\( (a_{28})^{6,6,6}(T_{28},t); (a_{29})^{6,6,6}(T_{29},t); (a_{30})^{6,6,6}(T_{30},t) \) are fifth augmentation coefficients for category 1, 2 and 3

\[ +(a_{28})^{3,3,3,3}(T_{28},t); +(a_{29})^{3,3,3,3}(T_{29},t); +(a_{30})^{3,3,3,3}(T_{30},t) \]

\( (a_{28})^{1,1,1,1}(T_{28},t); (a_{29})^{1,1,1,1}(T_{29},t); (a_{30})^{1,1,1,1}(T_{30},t) \) are sixth augmentation coefficients for category 1, 2 and 3

\[ \frac{dT_{24}}{dt} = (b_{24})^{4}(T_{24}) - (b_{13})^{3}(1,1,1,1)(G_{1},t) - (b_{16})^{2,2,2,2}(G_{19},t) - (b_{20})^{3,3,3,3}(G_{23},t) \]

\[ \frac{dT_{25}}{dt} = (b_{25})^{4}(T_{25}) - (b_{14})^{3}(1,1,1,1)(G_{1},t) - (b_{17})^{2,2,2,2}(G_{19},t) - (b_{21})^{3,3,3,3}(G_{23},t) \]

\[ \frac{dT_{26}}{dt} = (b_{26})^{4}(T_{26}) - (b_{15})^{3}(1,1,1,1)(G_{1},t) - (b_{18})^{2,2,2,2}(G_{19},t) - (b_{22})^{3,3,3,3}(G_{23},t) \]

Where \( -(b_{24})^{5}(G_{24},t) \), \( -(b_{25})^{5}(G_{25},t) \), \( -(b_{26})^{5}(G_{26},t) \) are first detrition coefficients for category 1, 2 and 3

\[ -(b_{28})^{5,5,5}(G_{28},t); -(b_{29})^{5,5,5}(G_{29},t); -(b_{30})^{5,5,5}(G_{30},t) \]

\( -(b_{28})^{6,6,6}(G_{28},t); -(b_{29})^{6,6,6}(G_{29},t); -(b_{30})^{6,6,6}(G_{30},t) \) are second detrition coefficients for category 1, 2 and 3

\[ -(b_{28})^{5,5,5}(G_{28},t); -(b_{29})^{5,5,5}(G_{29},t); -(b_{30})^{5,5,5}(G_{30},t) \] are third detrition coefficients for category 1, 2 and 3

\[ -(b_{28})^{1,1,1,1}(G_{28},t); -(b_{29})^{1,1,1,1}(G_{29},t); -(b_{30})^{1,1,1,1}(G_{30},t) \] are fourth detrition coefficients for category 1, 2 and 3

\[ -(b_{28})^{2,2,2,2}(G_{28},t); -(b_{29})^{2,2,2,2}(G_{29},t); -(b_{30})^{2,2,2,2}(G_{30},t) \] are fifth detrition coefficients for category 1, 2 and 3

\[ -(b_{28})^{3,3,3,3}(G_{28},t); -(b_{29})^{3,3,3,3}(G_{29},t); -(b_{30})^{3,3,3,3}(G_{30},t) \] are sixth detrition coefficients for category 1, 2 and 3

\[ \frac{dG_{28}}{dt} = (a_{28})^{5}(G_{28}) - (a_{28})^{5}(T_{28},t) + (a_{29})^{5}(T_{29},t) + (a_{30})^{5}(T_{30},t) + (a_{13})^{3,3,3,3}(T_{21},t) \]

\[ \frac{dG_{29}}{dt} = (a_{29})^{5}(G_{29}) - (a_{29})^{5}(T_{29},t) + (a_{30})^{5}(T_{30},t) + (a_{14})^{3,3,3,3}(T_{21},t) \]

\[ \frac{dG_{30}}{dt} = (a_{30})^{5}(G_{30}) - (a_{30})^{5}(T_{30},t) + (a_{20})^{3,3,3,3}(T_{21},t) \]

Where \( +(a_{28})^{5}(T_{28},t) \), \( +(a_{29})^{5}(T_{29},t) \), \( +(a_{30})^{5}(T_{30},t) \) are first augmentation coefficients for category 1, 2 and 3

\[ +(a_{24})^{4,4,4}(T_{24},t); +(a_{25})^{4,4,4}(T_{25},t); +(a_{26})^{4,4,4}(T_{26},t) \]

\( +(a_{13})^{4,4,4}(T_{14},t); +(a_{16})^{2,2,2,2}(T_{17},t); +(a_{20})^{3,3,3,3}(T_{21},t) \) are second augmentation coefficient for category 1, 2 and 3

\www.ijsrp.org
\[ \frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \frac{[b_{29}^{(5)}G_{31}^{(5)}(G_{31}, t)] + [b_{24}^{(4,4)}(G_{27}, t)] - [b_{32}^{(6,6,6)}(G_{35}, t)] - [b_{13}^{(1,1,1,1)}(G, t)] - [b_{16}^{(2,2,2,2,2)}(G_{19}, t)] - [b_{20}^{(3,3,3,3)}(G_{23}, t)]}{T_{28}} \]
\[ \frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \frac{[b_{29}^{(5)}G_{31}^{(5)}(G_{31}, t)] + [b_{25}^{(4,4)}(G_{27}, t)] - [b_{33}^{(6,6,6)}(G_{35}, t)] - [b_{14}^{(1,1,1,1,1)}(G, t)] - [b_{17}^{(2,2,2,2,2)}(G_{19}, t)] - [b_{21}^{(3,3,3,3)}(G_{23}, t)]}{T_{29}} \]
\[ \frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \frac{[b_{30}^{(5)}G_{31}^{(5)}(G_{31}, t)] + [b_{26}^{(4,4)}(G_{27}, t)] - [b_{34}^{(6,6,6)}(G_{35}, t)] - [b_{15}^{(1,1,1,1,1)}(G, t)] - [b_{18}^{(2,2,2,2,2)}(G_{19}, t)] - [b_{22}^{(3,3,3,3,3)}(G_{23}, t)]}{T_{30}} \]

where \[-(b_{29})^{(5)}G_{31}^{(5)}(G_{31}, t) \] are first detrition coefficients for category 1, 2, and 3.

\[-(b_{29})^{(5)}G_{31}^{(5)}(G_{31}, t) \] are second detrition coefficients for category 1, 2, and 3.

\[-(b_{30})^{(5)}G_{31}^{(5)}(G_{31}, t) \] are third detrition coefficients for category 1, 2, and 3.

\[-(b_{30})^{(5)}G_{31}^{(5)}(G_{31}, t) \] are fourth detrition coefficients for category 1, 2, and 3.

\[-(b_{20})^{(5)}G_{31}^{(5)}(G_{31}, t) \] are sixth detrition coefficients for category 1, 2, and 3.

\[ \frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \frac{[a_{32}^{(5)} + (a_{32})^{(6)}(T_{33}, t)] + [a_{28}^{(5,5,5)}(T_{29}, t)] + [a_{24}^{(4,4,4)}(T_{25}, t)] + [a_{14}^{(1,1,1,1,1)}(T_{14}, t)] + [a_{16}^{(2,2,2,2,2)}(T_{17}, t)] + [a_{20}^{(3,3,3,3,3)}(T_{21}, t)]}{G_{32}} \]
\[ \frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \frac{[a_{33}^{(5)} + (a_{33})^{(6)}(T_{33}, t)] + [a_{29}^{(5,5,5)}(T_{29}, t)] + [a_{25}^{(4,4,4)}(T_{25}, t)] + [a_{14}^{(1,1,1,1,1)}(T_{14}, t)] + [a_{17}^{(2,2,2,2,2)}(T_{17}, t)] + [a_{21}^{(3,3,3,3,3)}(T_{21}, t)]}{G_{33}} \]
\[ \frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \frac{[a_{34}^{(5)} + (a_{34})^{(6)}(T_{33}, t)] + [a_{30}^{(5,5,5)}(T_{29}, t)] + [a_{26}^{(4,4,4)}(T_{25}, t)] + [a_{15}^{(1,1,1,1,1)}(T_{14}, t)] + [a_{19}^{(2,2,2,2,2)}(T_{17}, t)] + [a_{22}^{(3,3,3,3,3)}(T_{21}, t)]}{G_{34}} \]

\[ + (a_{29}^{(5)}(T_{30}, t) + (a_{30}^{(5)}(T_{30}, t) + (a_{31}^{(5)}(T_{31}, t) + (a_{32}^{(5)}(T_{32}, t) + (a_{33}^{(5)}(T_{33}, t) + (a_{34}^{(5)}(T_{34}, t) \] are first augmentation coefficients for category 1, 2, and 3.

\[ + (a_{29}^{(5,5,5)}(T_{29}, t) + (a_{30}^{(5,5,5)}(T_{29}, t) + (a_{31}^{(5,5,5)}(T_{29}, t) + (a_{32}^{(5,5,5)}(T_{29}, t) + (a_{33}^{(5,5,5)}(T_{29}, t) + (a_{34}^{(5,5,5)}(T_{29}, t) \] are second augmentation coefficients for category 1, 2, and 3.

\[ + (a_{29}^{(4,4,4)}(T_{29}, t) + (a_{30}^{(4,4,4)}(T_{29}, t) + (a_{31}^{(4,4,4)}(T_{29}, t) + (a_{32}^{(4,4,4)}(T_{29}, t) + (a_{33}^{(4,4,4)}(T_{29}, t) + (a_{34}^{(4,4,4)}(T_{29}, t) \] are third augmentation coefficients for category 1, 2, and 3.

\[ + (a_{14}^{(1,1,1,1,1)}(T_{14}, t) + (a_{15}^{(1,1,1,1,1)}(T_{14}, t) + (a_{16}^{(1,1,1,1,1)}(T_{14}, t) + (a_{17}^{(1,1,1,1,1)}(T_{14}, t) + (a_{18}^{(1,1,1,1,1)}(T_{14}, t) + (a_{19}^{(1,1,1,1,1)}(T_{14}, t) \] are fourth augmentation coefficients for category 1, 2, and 3.

\[ + (a_{14}^{(2,2,2,2,2)}(T_{17}, t) + (a_{15}^{(2,2,2,2,2)}(T_{17}, t) + (a_{16}^{(2,2,2,2,2)}(T_{17}, t) + (a_{17}^{(2,2,2,2,2)}(T_{17}, t) + (a_{18}^{(2,2,2,2,2)}(T_{17}, t) + (a_{19}^{(2,2,2,2,2)}(T_{17}, t) \] are fifth augmentation coefficients.
They satisfy  Lipschitz condition:

\[|a_i''(t)|, |b_i''(t)| < \tilde{K}_{13} |T_{14} - t| e^{-(\tilde{R}_{13})} \]

\[|a_i''(G(t)) - a_i''(G(t'))| < \tilde{K}_{13} |G(t) - G(t')| e^{-(\tilde{R}_{13})} \]

Where we suppose

(A) \((a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0,\)

\[i, j = 13, 14, 15\]

(B) The functions \((a_i'')^{(1)}, (b_i'')^{(1)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(1)} , \ (r_i)^{(1)} :\)

\[(a_i')^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}\]

\[(b_i')^{(1)}(G, t) \leq (r_i)^{(1)} \leq (\hat{B}_{13})^{(1)}\]

(C) \(\lim_{T_{14} \to \infty} (a_i'')^{(1)}(T_{14}, t) = (p_i)^{(1)}\)

\(\lim_{G \to \infty} (b_i'')^{(1)}(G, t) = (r_i)^{(1)}\)

**Definition of** \((\hat{A}_{13})^{(1)}, \ (\hat{B}_{13})^{(1)} :\)

Where \((\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}\) are positive constants and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\[|a_i''(T_{14}, t) - a_i''(T_{14}, t')| \leq (\tilde{K}_{13})^{(1)}|T_{14} - T_{14}'| e^{-(\tilde{R}_{13})} \]

\[|b_i''(G', t) - b_i''(G', t')| < (\tilde{K}_{13})^{(1)}|G' - G'| e^{-(\tilde{R}_{13})}\]
With the Lipschitz condition, we place a restriction on the behavior of functions 
\((a''_i)^{(1)}(T_{14}, t)\) and \((a''_i)^{(1)}(T_{14}, t)\) \((T_{14}, t)\) and \((T_{14}, t)\) are points belonging to the interval \([\bar{M}_{13}]\). It is to be noted that \((a''_i)^{(1)}(T_{14}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\bar{M}_{13})^{(1)} = 1\) then the function \((a''_i)^{(1)}(T_{14}, t)\), the first augmentation coefficient WOULD be absolutely continuous.

**Definition of \((\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}\):**

\[(\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}, \text{ are positive constants}\]

\[\frac{(a''_i)^{(1)}(\bar{M}_{13})^{(1)}}{(b''_i)^{(1)}(\bar{M}_{13})^{(1)}} < 1\]

**Definition of \((\bar{P}_{13})^{(1)}, (\bar{Q}_{13})^{(1)}\):**

\[\text{There exists two constants } (\bar{P}_{13})^{(1)} \text{ and } (\bar{Q}_{13})^{(1)} \text{ which together} \]

\[\text{with } (\bar{M}_{13})^{(1)}, (\bar{k}_{13})^{(1)}, (\bar{A}_{13})^{(1)} \text{ and } (\bar{B}_{13})^{(1)} \text{ and the constants}\]

\[(a''_i)^{(1)}, (a'_i)^{(1)}, (b''_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, t = 13, 14, 15,\]

satisfy the inequalities

\[\frac{1}{(\bar{M}_{13})^{(1)}}[(a''_i)^{(1)} + (a'_i)^{(1)} + (\bar{A}_{13})^{(1)} + (\bar{P}_{13})^{(1)}(\bar{k}_{13})^{(1)}] < 1\]

\[\frac{1}{(\bar{M}_{13})^{(1)}}[(b''_i)^{(1)} + (b'_i)^{(1)} + (\bar{B}_{13})^{(1)} + (\bar{Q}_{13})^{(1)}(\bar{k}_{13})^{(1)}] < 1\]

Where we suppose

\[(a''_i)^{(2)}, (a'_i)^{(2)}, (b''_i)^{(2)}, (b'_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)} > 0, \text{ } i, j = 16, 17, 18\]

**The functions \((a''_i)^{(2)}, (b''_i)^{(2)}\) are positive continuous increasing and bounded.**

**Definition of \((p_i)^{(2)}, (r_i)^{(2)}\):**

\[(a''_i)^{(2)}(T_{17}, t) \leq (p_i)^{(2)} \leq (\bar{A}_{16})^{(2)}\]

\[(b''_i)^{(2)}(G_{19}, t) \leq (r_i)^{(2)} \leq (\bar{B}_{16})^{(2)}\]

\[\lim_{T_{12} \to \infty} (a''_i)^{(2)}(T_{17}, t) = (p_i)^{(2)}\]

\[\lim_{G_{19} \to \infty} (b''_i)^{(2)}(G_{19}, t) = (r_i)^{(2)}\]

**Definition of \((\bar{A}_{16})^{(2)}, (\bar{B}_{16})^{(2)}\):**

Where \((\bar{A}_{16})^{(2)}, (\bar{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}\) are positive constants and \(i = 16, 17, 18\)

They satisfy Lipschitz condition:

\[|a''_i^{(2)}(T_{17}, t) - a''_i^{(2)}(T_{17}, t)| \leq (\bar{M}_{16})^{(2)}|T_{17} - T_{12}|e^{-a_{16}^{(2)}t}\]

\[|b''_i^{(2)}(G_{19}, t) - b''_i^{(2)}(G_{19}, t)| \leq (\bar{M}_{16})^{(2)}|G_{19} - G_{19}'|e^{-a_{16}^{(2)}\ell}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a''_i)^{(2)}(T_{17}, t)\)

\[\text{and} (a''_i)^{(2)}(T_{17}, t), (T_{17}, t) \text{ and } (T_{12}, t) \text{ are points belonging to the interval } [\bar{M}_{16}]^{(2)}, (\bar{M}_{16})^{(2)}\]. It is to be noted that \((a''_i)^{(2)}(T_{17}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\bar{M}_{16})^{(2)} = 1\) then the function \((a''_i)^{(2)}(T_{17}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of \((\bar{M}_{16})^{(2)}, (\bar{k}_{16})^{(2)}\):**
(I) \((M_{16})^{(2)}, (k_{16})^{(2)}\), are positive constants

\[
\frac{(a_2)^{(2)}}{(M_{16})^{(2)}}, \frac{(b_2)^{(2)}}{(M_{16})^{(2)}} < 1
\]

**Definition of \((\tilde{P}_{13})^{(2)}, (\tilde{Q}_{13})^{(2)}\):**

There exists two constants \((\tilde{P}_{16})^{(2)}\) and \((\tilde{Q}_{16})^{(2)}\) which together with \((M_{16})^{(2)}, (k_{16})^{(2)}\), \(A_{16}^{(2)}\) and \((\tilde{Q}_{16})^{(2)}\) and the constants \((a_i)^{(2)}, (b_i)^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16, 17, 18\),

satisfy the inequalities

\[
\frac{1}{(M_{16})^{(2)}} \left[ (a_i)^{(2)} + (a_i)'^{(2)} + (\tilde{A}_{16})^{(2)} + (\tilde{P}_{16})^{(2)} (\tilde{k}_{16})^{(2)} \right] < 1
\]

\[
\frac{1}{(M_{16})^{(2)}} \left[ (b_i)^{(2)} + (b_i)'^{(2)} + (\tilde{B}_{16})^{(2)} + (\tilde{Q}_{16})^{(2)} (\tilde{k}_{16})^{(2)} \right] < 1
\]

Where we suppose

\((j)\) \((a_j)^{(3)}, (a_j)'^{(3)}, (a_j)^{''}(3), (b_j)^{(3)}, (b_j)'^{(3)}, (b_j)^{''}(3) > 0, \quad i, j = 20, 21, 22\)

The functions \((a_j)^{''}(3), (b_j)^{''}(3)\) are positive continuous increasing and bounded.

**Definition of \((p_i)^{(3)}, (r_i)^{(3)}\):**

\((a_i)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (A_{20})^{(3)}\)

\((b_i)^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (B_{20})^{(3)}\)

\(\lim_{T_{21} \to 0} (a_i)^{(3)}(T_{21}, t) = (p_i)^{(3)}\)

\(\lim_{G_{23} \to 0} (b_i)^{(3)}(G_{23}, t) = (r_i)^{(3)}\)

**Definition of \((\tilde{A}_{20})^{(3)}, (\tilde{B}_{20})^{(3)}\):**

Where \((\tilde{A}_{20})^{(3)}, (\tilde{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}\) are positive constants and \(i = 20, 21, 22\)

They satisfy Lipschitz condition:

\[|(a_i)^{(3)}(T_{21}', t) - (a_i)^{(3)}(T_{21}, t)| \leq (\tilde{A}_{20})^{(3)}|T_{21} - T_{21}'| e^{-((\tilde{A}_{20})^{(3)} t)}\]

\[|(b_i)^{(3)}(G_{23}', t) - (b_i)^{(3)}(G_{23}, t)| \leq (\tilde{B}_{20})^{(3)}|G_{23} - G_{23}'| e^{-((\tilde{B}_{20})^{(3)} t)}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i)^{(3)}(T_{21}', t)\) and \((a_i)^{(3)}(T_{21}, t)\). \((T_{21}', t)\) are points belonging to the interval \([\tilde{k}_{20}(3), (\tilde{M}_{20})^{(3)}]\). It is to be noted that \((a_i)^{(3)}(T_{21}, t)\) is uniformly continuous. In the eventualty of the fact, that if \((\tilde{M}_{20})^{(3)} = 1\) then the function \((a_i)^{(3)}(T_{21}, t)\), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of \((\tilde{M}_{20})^{(3)}, (\tilde{k}_{20})^{(3)}\):**

\((K)\) \((\tilde{M}_{20})^{(3)}, (\tilde{k}_{20})^{(3)}\), are positive constants

\[
\frac{(a_j)^{(3)}}{(\tilde{M}_{20})^{(3)}}, \frac{(b_j)^{(3)}}{(\tilde{M}_{20})^{(3)}} < 1
\]

There exists two constants \((\tilde{P}_{20})^{(3)}\) and \((\tilde{Q}_{20})^{(3)}\) which together with \((\tilde{M}_{20})^{(3)}, (\tilde{k}_{20})^{(3)}, (\tilde{A}_{20})^{(3)}\) and \((\tilde{B}_{20})^{(3)}\) and the constants \((a_i)^{(3)}, (a_j)^{(3)}, (b_i)^{(3)}, (b_j)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20, 21, 22,\)

www.ijsrp.org
satisfy the inequalities
\[
\frac{1}{(M_{20})^3} [(a_i)^{(3)} + (a'_i)^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{B}_{20})^{(3)}] < 1
\]
\[
\frac{1}{(M_{20})^3} [(b_i)^{(3)} + (b'_i)^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}] < 1
\]

Where we suppose
\[
(a_i)^{(4)}, (a'_i)^{(4)}, (a''_i)^{(4)}, (b_i)^{(4)}, (b'_i)^{(4)}, (b''_i)^{(4)} > 0, \quad i, j = 24, 25, 26
\]

(M) The functions \((a_i)^{(4)}, (b_i)^{(4)}\) are positive continuous increasing and bounded.

**Definition of** \((p_i)^{(4)}, (r_i)^{(4)}\):
\[
(a_i)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}
\]
\[
(b_i)^{(4)}((G_{27}), t) \leq (r_i)^{(4)} \leq (b_i)^{(4)} \leq (\hat{B}_{24})^{(4)}
\]

(N) \(\lim_{T_{25} \rightarrow t}(a_i)^{(4)}(T_{25}, t) = (p_i)^{(4)}\)
\(\lim_{G \rightarrow e}(b_i)^{(4)}((G_{27}), t) = (r_i)^{(4)}\)

**Definition of** \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}\):

Where \((\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}\) are positive constants and \(\{i = 24, 25, 26\}\)

They satisfy Lipschitz condition:
\[
| (a_i)^{(4)}(T_{25}, t) - (a_i)^{(4)}(T_{25}, t) | \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}|e^{-((\hat{A}_{24})^{(4)} t)} e^{-((\hat{A}_{24})^{(4)} t)}
\]
\[
| (b_i)^{(4)}((G_{27}), t) - (b_i)^{(4)}((G_{27}), t) | \leq (\hat{k}_{24})^{(4)}|((G_{27}) - (G_{27}) | e^{-((\hat{B}_{24})^{(4)} t)}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i)^{(4)}(T_{25}, t)\) and \((a_i)^{(4)}(T_{25}, t) \cdot (T_{25}, t) \) and \((T_{25}, t)\) are points belonging to the interval \([\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)}\]. It is to be noted that \((a_i)^{(4)}(T_{25}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{M}_{24})^{(4)} = 4\) then the function \((a_i)^{(4)}(T_{25}, t)\), the FOURTH augmentation coefficient \(\text{WOULD}\) be absolutely continuous.

**Definition of** \((\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}\):
\[
(\hat{M}_{24})^{(4)}(\hat{k}_{24})^{(4)}, \text{ are positive constants}
\]
\[
\frac{(a_j)^{(4)}}{(M_{24})^{(4)}} \frac{(b_j)^{(4)}}{(M_{24})^{(4)}} \leq 1
\]

**Definition of** \((\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}\):

(Q) There exists two constants \((\hat{P}_{24})^{(4)}\) and \((\hat{Q}_{24})^{(4)}\) which together with \((\hat{M}_{24})^{(4)}(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}(\hat{A}_{24})^{(4)}\) and \((\hat{B}_{24})^{(4)}\) and the constants \((a_i)^{(4)}, (a_i)^{(4)}, (b_i)^{(4)}, (b_i)^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}\), \(i = 24, 25, 26,\) satisfy the inequalities
\[
\frac{1}{(M_{24})^{(4)}} [(a_i)_{(4)} + (a_j)_{(4)} + (\dot{A}_{24})_{(4)} + (\dot{\beta}_{24})_{(4)} (\dot{K}_{24})_{(4)}] < 1
\]
\[
\frac{1}{(M_{24})^{(4)}} [(b_i)_{(4)} + (b_j)_{(4)} + (\dot{B}_{24})_{(4)} + (\dot{Q}_{24})_{(4)} (\dot{K}_{24})_{(4)}] < 1
\]

Where we suppose

\[ (a_i)_{(5)}, (a_j)_{(5)}, (a_k)_{(5)}, (b_i)_{(5)}, (b_j)_{(5)}, (b_k)_{(5)} > 0, \ i, j = 28, 29, 30 \]

\textbf{Definition of } \((p_i)_{(5)}, (r_j)_{(5)}\):

\[
(a_i)_{(5)} (T_{29}, t) \leq (p_i)_{(5)} \leq (\dot{A}_{28})_{(5)}
\]
\[
(b_i)_{(5)} (G_{31}, t) \leq (r_j)_{(5)} \leq (\dot{B}_{28})_{(5)}
\]

\textbf{(T)} \[
\lim_{T_2 \rightarrow 0} (a_i)_{(5)} (T_{29}, t) = (p_i)_{(5)}
\]
\[
\lim_{G \rightarrow 0} (b_i)_{(5)} (G_{31}, t) = (r_j)_{(5)}
\]

\textbf{Definition of } \([\dot{A}_{28}]_{(5)}, (\dot{B}_{28})_{(5)}\):

Where \([\dot{A}_{28}]_{(5)}, (\dot{B}_{28})_{(5)}, (p_i)_{(5)}, (r_j)_{(5)}\) are positive constants and \(t = 28, 29, 30\)

They satisfy Lipschitz condition:

\[
|(a_i)_{(5)} (T_{29}, t) - (a_i)_{(5)} (T_{29}, t)| \leq (\dot{K}_{28})_{(5)} |T_{29} - T_{29}'| e^{-((\dot{A}_{28})_{(5)}) t}
\]
\[
|(b_i)_{(5)} (G_{31}, t) - (b_i)_{(5)} (G_{31}, t)| \leq (\dot{K}_{28})_{(5)} |(G_{31} - (G_{31})')| e^{-((\dot{B}_{28})_{(5)}) t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i)_{(5)} (T_{29}, t)\) and \((a_i)_{(5)} (T_{29}, t)\)'s \((T_{29}, t)\) and \((T_{29}, t)\) are points belonging to the interval \([\dot{K}_{28}]_{(5)}, (\dot{M}_{28})_{(5)}\]. It is to be noted that \((a_i)_{(5)} (T_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\dot{M}_{28})_{(5)} = 5\) then the function \((a_i)_{(5)} (T_{29}, t)\), the \textit{Fifth augmentation coefficient} attributable would be absolutely continuous.

\textbf{Definition of } \([\dot{M}_{28}]_{(5)}, (\dot{K}_{28})_{(5)}\):

\[
([\dot{M}_{28}]_{(5)}, (\dot{K}_{28})_{(5)}) \text{ are positive constants}
\]
\[
\frac{(a_i)}{((\dot{M}_{28})_{(5)}}, \frac{(b_i)}{((\dot{M}_{28})_{(5)} < 1
\]

\textbf{Definition of } \([\dot{P}_{28}]_{(5)}, (\dot{Q}_{28})_{(5)}\):

There exists two constants \((\dot{P}_{28})_{(5)}\) and \((\dot{Q}_{28})_{(5)}\) which together with \([\dot{M}_{28}]_{(5)}\), \((\dot{K}_{28})_{(5)}\), \((\dot{A}_{28})_{(5)}\), \((\dot{B}_{28})_{(5)}\) and \(t = 28, 29, 30\), satisfy the inequalities

\[
\frac{1}{(\dot{M}_{28})_{(5)}} [(a_i)_{(5)} + (a_j)_{(5)} + (\dot{A}_{28})_{(5)} + (\dot{P}_{28})_{(5)} (\dot{K}_{28})_{(5)}] < 1
\]
\[
\frac{1}{(\dot{M}_{28})_{(5)}} [(b_i)_{(5)} + (b_j)_{(5)} + (\dot{B}_{28})_{(5)} + (\dot{Q}_{28})_{(5)} (\dot{K}_{28})_{(5)}] < 1
\]

Where we suppose
\((a_i)^{(6)}, (a'^{a}_{i})^{(6)}, (a^{'^{a}}_{i})^{(6)}, (b_i)^{(6)}, (b'^{a}_{i})^{(6)}, (b^{'^{a}}_{i})^{(6)} > 0, \quad i, j = 32,33,34\)

(\text{W}) The functions \((a_i)^{(6)}, (b_i)^{(6)}\) are positive continuous increasing and bounded.

**Definition of** \((p_{i})^{(6)}, (r_{i})^{(6)}\):

\[ (a_i)^{(6)} (T_{33}, t) \leq (p_{i})^{(6)} \leq (A_{32})^{(6)} \]
\[ (b_i)^{(6)} ((G_{35}), t) \leq (r_{i})^{(6)} \leq (B_{32})^{(6)} \]

\((X)\)
\[ \lim_{T_{33} \to 0} (a_i)^{(6)} (T_{33}, t) = (p_{i})^{(6)} \]
\[ \lim_{G \to 0} (b_i)^{(6)} ((G_{35}), t) = (r_{i})^{(6)} \]

**Definition of** \((\tilde{A}_{32})^{(6)}, (\tilde{B}_{32})^{(6)}\):

Where \((\tilde{A}_{32})^{(6)}, (\tilde{B}_{32})^{(6)}, (p_{i})^{(6)}, (r_{i})^{(6)}\) are positive constants and \( i = 32,33,34\).

They satisfy Lipschitz condition:
\[ |(a_i)^{(6)} (T_{33}, t) - (a_i)^{(6)} (T_{33}, t)| \leq (\tilde{A}_{32})^{(6)} |T_{33} - T_{33}| e^{-(\tilde{A}_{32})^{(6)} t} \]
\[ |(b_i)^{(6)} ((G_{35}), t) - (b_i)^{(6)} ((G_{35}), t)| \leq (\tilde{B}_{32})^{(6)} |(G_{35}) - (G_{35})| e^{-(\tilde{B}_{32})^{(6)} t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i)^{(6)} (T_{33}, t)\)
and \((a_i)^{(6)} (T_{33}, t)\) \cdot \(T_{33} \) and \((T_{33}, t)\) are points belonging to the interval \([ \tilde{A}_{32}^{(6)}, \tilde{B}_{32}^{(6)} ]\). It is to be noted that \((a_i)^{(6)} (T_{33}, t)\) is uniformly continuous. In the eventuality of the fact, if \((\tilde{M}_{32})^{(6)} = 6\)
then the function \((a_i)^{(6)} (T_{33}, t)\) , the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of** \((\tilde{M}_{32})^{(6)}, (\tilde{K}_{32})^{(6)}\):

\((\tilde{M}_{32})^{(6)}, (\tilde{K}_{32})^{(6)}\) are positive constants
\[ \frac{(a_{i})^{(6)}}{(\tilde{M}_{32})^{(6)}} - \frac{(b_{i})^{(6)}}{(\tilde{B}_{32})^{(6)}} < 1 \]

**Definition of** \((\tilde{P}_{32})^{(6)}, (\tilde{Q}_{32})^{(6)}\):

There exists two constants \((\tilde{P}_{32})^{(6)}\) and \((\tilde{Q}_{32})^{(6)}\) which together with \((\tilde{M}_{32})^{(6)}, (\tilde{K}_{32})^{(6)}, (\tilde{A}_{32})^{(6)}\) and \((\tilde{B}_{32})^{(6)}\) and the constants \((a_{i})^{(6)}, (a'^{a}_{i})^{(6)}, (b_{i})^{(6)}, (b'^{a}_{i})^{(6)}, (p_{i})^{(6)}, (r_{i})^{(6)}\), \( i = 32,33,34\), satisfy the inequalities

\[ \frac{1}{(\tilde{M}_{32})^{(6)}} [(a_{i})^{(6)} + (a'^{a}_{i})^{(6)} + (\tilde{A}_{32})^{(6)} + (\tilde{P}_{32})^{(6)} (\tilde{K}_{32})^{(6)}] < 1 \]
\[ \frac{1}{(\tilde{B}_{32})^{(6)}} [(b_{i})^{(6)} + (b'^{a}_{i})^{(6)} + (\tilde{B}_{32})^{(6)} + (\tilde{Q}_{32})^{(6)} (\tilde{K}_{32})^{(6)}] < 1 \]

**Theorem 1:** If the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of** \(G_{1}(0), T_{1}(0)\):

\[ G_{1}(t) \leq (\tilde{P}_{13})^{(6)} e^{(\tilde{R}_{13})^{(6)} t}, \quad G_{1}(0) = G_{1}^{0} > 0 \]
\[ T_i(t) \leq (\hat{Q}_{13})^{(1)}e^{(\hat{R}_{13})^{(1)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) \)

\[ G_i(t) \leq (\hat{P}_{16})^{(2)}e^{(\hat{R}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{16})^{(2)}e^{(\hat{R}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0 \]

\[ G_i(t) \leq (\hat{P}_{20})^{(3)}e^{(\hat{R}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{20})^{(3)}e^{(\hat{R}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) \)

\[ G_i(t) \leq (\hat{P}_{24})^{(4)}e^{(\hat{R}_{24})^{(4)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{24})^{(4)}e^{(\hat{R}_{24})^{(4)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) \)

\[ G_i(t) \leq (\hat{P}_{32})^{(5)}e^{(\hat{R}_{32})^{(5)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{32})^{(5)}e^{(\hat{R}_{32})^{(5)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Definition of** \( G_i(0), T_i(0) \)

\[ G_i(t) \leq (\hat{P}_{32})^{(6)}e^{(\hat{R}_{32})^{(6)}t}, \quad G_i(0) = G_i^0 > 0 \]

\[ T_i(t) \leq (\hat{Q}_{32})^{(6)}e^{(\hat{R}_{32})^{(6)}t}, \quad T_i(0) = T_i^0 > 0 \]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)}e^{(\hat{R}_{13})^{(1)}t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)}e^{(\hat{R}_{13})^{(1)}t} \]
By
\[ \tilde{g}_{13}(t) = G_{13}^{0} + \int_{0}^{t} \left[ (a_{13}(1) G_{14}(s_{(13)})) - \left( (a'_{13}(1) + a''_{13}(1))(T_{14}(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]
\[ \tilde{g}_{14}(t) = G_{14}^{0} + \int_{0}^{t} \left[ (a_{14}(1) G_{13}(s_{(13)})) - \left( (a'_{14}(1) + a''_{14}(1))(T_{14}(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]
\[ \tilde{g}_{15}(t) = G_{15}^{0} + \int_{0}^{t} \left[ (a_{15}(1) G_{13}(s_{(13)})) - \left( (a'_{15}(1) + a''_{15}(1))(T_{14}(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]
\[ \tilde{T}_{13}(t) = T_{13}^{0} + \int_{0}^{t} \left[ (b_{13}(1) T_{14}(s_{(13)})) - \left( (b'_{13}(1) - b''_{13}(1))(G(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]
\[ \tilde{T}_{14}(t) = T_{14}^{0} + \int_{0}^{t} \left[ (b_{14}(1) T_{13}(s_{(13)})) - \left( (b'_{14}(1) - b''_{14}(1))(G(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]
\[ \tilde{T}_{15}(t) = T_{15}^{0} + \int_{0}^{t} \left[ (b_{15}(1) T_{13}(s_{(13)})) - \left( (b'_{15}(1) - b''_{15}(1))(G(s_{(13)}), s_{(13)}) \right) \right] ds_{(13)} \]

Where \( s_{(13)} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_{i}, \ T_{i} : \mathbb{R}_{+} \to \mathbb{R}_{+} \) which satisfy
\[
G_{i}(0) = G_{i}^{0}, \quad T_{i}(0) = T_{i}^{0}, \quad G_{i}^{0} \leq (\tilde{P}_{16})^{(2)}, \quad T_{i}^{0} \leq (\tilde{Q}_{16})^{(2)},
\]
\[
0 \leq G_{i}(t) - G_{i}^{0} \leq (\tilde{P}_{16})^{(2)} e^{(\mathcal{B}_{16})^{(2)} t},
\]
\[
0 \leq T_{i}(t) - T_{i}^{0} \leq (\tilde{Q}_{16})^{(2)} e^{(\mathcal{B}_{16})^{(2)} t}.
\]

By
\[ \tilde{g}_{16}(t) = G_{16}^{0} + \int_{0}^{t} \left[ (a_{16}(2) G_{17}(s_{(16)})) - \left( (a'_{16}(2) + a''_{16}(2))(T_{17}(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]
\[ \tilde{g}_{17}(t) = G_{17}^{0} + \int_{0}^{t} \left[ (a_{17}(2) G_{16}(s_{(16)})) - \left( (a'_{17}(2) + a''_{17}(2))(T_{17}(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]
\[ \tilde{g}_{18}(t) = G_{18}^{0} + \int_{0}^{t} \left[ (a_{18}(2) G_{17}(s_{(16)})) - \left( (a'_{18}(2) + a''_{18}(2))(T_{17}(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]
\[ \tilde{T}_{16}(t) = T_{16}^{0} + \int_{0}^{t} \left[ (b_{16}(2) T_{17}(s_{(16)})) - \left( (b'_{16}(2) - b''_{16}(2))(G(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]
\[ \tilde{T}_{17}(t) = T_{17}^{0} + \int_{0}^{t} \left[ (b_{17}(2) T_{16}(s_{(16)})) - \left( (b'_{17}(2) - b''_{17}(2))(G(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]
\[ \tilde{T}_{18}(t) = T_{18}^{0} + \int_{0}^{t} \left[ (b_{18}(2) T_{16}(s_{(16)})) - \left( (b'_{18}(2) - b''_{18}(2))(G(s_{(16)}), s_{(16)}) \right) \right] ds_{(16)} \]

Where \( s_{(16)} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(3)} \) defined on the space of sextuples of continuous functions \( G_{i}, \ T_{i} : \mathbb{R}_{+} \to \mathbb{R}_{+} \) which satisfy
\[
G_{i}(0) = G_{i}^{0}, \quad T_{i}(0) = T_{i}^{0}, \quad G_{i}^{0} \leq (\tilde{P}_{20})^{(3)}, \quad T_{i}^{0} \leq (\tilde{Q}_{20})^{(3)},
\]
\[
0 \leq G_{i}(t) - G_{i}^{0} \leq (\tilde{P}_{20})^{(3)} e^{(\mathcal{B}_{20})^{(3)} t},
\]

www.ijsrp.org
0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{20})^{(3)}e(\theta_{20})^{(3)}t

By
\begin{align*}
\tilde{g}_{20}(t) &= G_{20}^0 + \int_0^t \left[(a_{20})^{(3)}G_{21}(s_{20}) - \left((a_{20})^{(3)} + a_{20}^{(3)}(T_{21}(s_{20}), s_{20})\right)G_{20}(s_{20})\right] ds_{20} \\
\tilde{g}_{21}(t) &= G_{21}^0 + \int_0^t \left[(a_{21})^{(3)}G_{20}(s_{20}) - \left((a_{21})^{(3)} + (a_{21})^{(3)}(T_{21}(s_{20}), s_{20})\right)G_{21}(s_{20})\right] ds_{20} \\
\tilde{g}_{22}(t) &= G_{22}^0 + \int_0^t \left[(a_{22})^{(3)}G_{21}(s_{20}) - \left((a_{22})^{(3)} + (a_{22})^{(3)}(T_{21}(s_{20}), s_{20})\right)G_{22}(s_{20})\right] ds_{20} \\
\tilde{T}_{20}(t) &= T_{20}^0 + \int_0^t \left[(b_{20})^{(3)}T_{21}(s_{20}) - \left((b_{20})^{(3)} - (b_{20})^{(3)}(G(s_{20}), s_{20})\right)T_{20}(s_{20})\right] ds_{20} \\
\tilde{T}_{21}(t) &= T_{21}^0 + \int_0^t \left[(b_{21})^{(3)}T_{20}(s_{20}) - \left((b_{21})^{(3)} - (b_{21})^{(3)}(G(s_{20}), s_{20})\right)T_{21}(s_{20})\right] ds_{20} \\
\tilde{T}_{22}(t) &= T_{22}^0 + \int_0^t \left[(b_{22})^{(3)}T_{21}(s_{20}) - \left((b_{22})^{(3)} - (b_{22})^{(3)}(G(s_{20}), s_{20})\right)T_{22}(s_{20})\right] ds_{20}
\end{align*}

Where $s_{20}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions $G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy
\begin{align*}
G_i(0) &= G_i^0, \ T_i(0) = T_i^0, \ G_i^0 \leq (P_{24})^{(4)}, T_i^0 \leq (\tilde{Q}_{24})^{(4)}, \\
0 &\leq G_i(t) - G_i^0 \leq (P_{24})^{(4)}e(\theta_{24})^{(4)}t, \\
0 &\leq T_i(t) - T_i^0 \leq (\tilde{Q}_{24})^{(4)}e(\theta_{24})^{(4)}t
\end{align*}

By
\begin{align*}
\tilde{g}_{24}(t) &= G_{24}^0 + \int_0^t \left[(a_{24})^{(4)}G_{25}(s_{24}) - \left((a_{24})^{(4)} + a_{24}^{(4)}(T_{25}(s_{24}), s_{24})\right)G_{24}(s_{24})\right] ds_{24} \\
\tilde{g}_{25}(t) &= G_{25}^0 + \int_0^t \left[(a_{25})^{(4)}G_{24}(s_{24}) - \left((a_{25})^{(4)} + (a_{25})^{(4)}(T_{25}(s_{24}), s_{24})\right)G_{25}(s_{24})\right] ds_{24} \\
\tilde{g}_{26}(t) &= G_{26}^0 + \int_0^t \left[(a_{26})^{(4)}G_{25}(s_{24}) - \left((a_{26})^{(4)} + (a_{26})^{(4)}(T_{25}(s_{24}), s_{24})\right)G_{26}(s_{24})\right] ds_{24} \\
\tilde{T}_{24}(t) &= T_{24}^0 + \int_0^t \left[(b_{24})^{(4)}T_{25}(s_{24}) - \left((b_{24})^{(4)} - (b_{24})^{(4)}(G(s_{24}), s_{24})\right)T_{24}(s_{24})\right] ds_{24} \\
\tilde{T}_{25}(t) &= T_{25}^0 + \int_0^t \left[(b_{25})^{(4)}T_{24}(s_{24}) - \left((b_{25})^{(4)} - (b_{25})^{(4)}(G(s_{24}), s_{24})\right)T_{25}(s_{24})\right] ds_{24} \\
\tilde{T}_{26}(t) &= T_{26}^0 + \int_0^t \left[(b_{26})^{(4)}T_{25}(s_{24}) - \left((b_{26})^{(4)} - (b_{26})^{(4)}(G(s_{24}), s_{24})\right)T_{26}(s_{24})\right] ds_{24}
\end{align*}

Where $s_{24}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions $G_i, T_i : \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy
\begin{align*}
G_i(0) &= G_i^0, \ T_i(0) = T_i^0, \ G_i^0 \leq (P_{28})^{(5)}, T_i^0 \leq (\tilde{Q}_{28})^{(5)}, \\
0 &\leq G_i(t) - G_i^0 \leq (P_{28})^{(5)}e(\theta_{28})^{(5)}t
\end{align*}
0 ≤ T_i(t) − T_i^0 ≤ (Q_{28})^5 e(β_{28})^5 t

By

\[\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28}^{(5)}) G_{28}(s_{(28)}) - \left( (a_{28}^{(5)} + a_{28}^{(5)}) (T_{28}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

\[\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29}^{(5)}) G_{29}(s_{(28)}) - \left( (a_{29}^{(5)} + a_{29}^{(5)}) (T_{29}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

\[\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30}^{(5)}) G_{30}(s_{(28)}) - \left( (a_{30}^{(5)} + a_{30}^{(5)}) (T_{30}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

\[\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28}^{(5)}) T_{28}(s_{(28)}) - \left( (b_{28}^{(5)} + b_{28}^{(5)}) (G_{28}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

\[\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29}^{(5)}) T_{29}(s_{(28)}) - \left( (b_{29}^{(5)} + b_{29}^{(5)}) (G_{29}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

\[\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30}^{(5)}) T_{30}(s_{(28)}) - \left( (b_{30}^{(5)} + b_{30}^{(5)}) (G_{30}(s_{(28)}), s_{(28)}) \right) \right] ds_{(28)}\]

Where \(s_{(28)}\) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \(\mathcal{A}^{(6)}\) defined on the space of sextuples of continuous functions \(G_t, T_t: \mathbb{R}_+ \rightarrow \mathbb{R}_+\) which satisfy

\[G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 ≤ (P_{32})^{(6)}, T_i^0 ≤ (Q_{32})^{(6)},\]

\[0 ≤ G_i(t) - G_i^0 ≤ (P_{32})^{(6)} e(β_{32})^6 t\]

\[0 ≤ T_i(t) - T_i^0 ≤ (Q_{32})^{(6)} e(β_{32})^6 t\]

By

\[\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[ (a_{32}^{(6)}) G_{32}(s_{(32)}) - \left( (a_{32}^{(6)} + a_{32}^{(6)}) (T_{32}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

\[\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[ (a_{33}^{(6)}) G_{33}(s_{(32)}) - \left( (a_{33}^{(6)} + a_{33}^{(6)}) (T_{33}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

\[\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[ (a_{34}^{(6)}) G_{34}(s_{(32)}) - \left( (a_{34}^{(6)} + a_{34}^{(6)}) (T_{34}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

\[\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[ (b_{32}^{(6)}) T_{32}(s_{(32)}) - \left( (b_{32}^{(6)} + b_{32}^{(6)}) (G_{32}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

\[\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[ (b_{33}^{(6)}) T_{33}(s_{(32)}) - \left( (b_{33}^{(6)} + b_{33}^{(6)}) (G_{33}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

\[\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[ (b_{34}^{(6)}) T_{34}(s_{(32)}) - \left( (b_{34}^{(6)} + b_{34}^{(6)}) (G_{34}(s_{(32)}), s_{(32)}) \right) \right] ds_{(32)}\]

Where \(s_{(32)}\) is the integrand that is integrated over an interval \((0, t)\)

(a) The operator \(\mathcal{A}^{(1)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that
\[ G_{13}(t) \leq G_{13}^0 + \int_0^t \left[ (a_{13})^{(1)} \left( G_{14}^0 + (p_{13})^{(1)} e^{(p_{13})^{(1)} t} \right) \right] ds_{13} = \\
\left( 1 + (a_{13})^{(1)} t \right) G_{14}^0 + \frac{(a_{13})^{(1)} (p_{13})^{(1)}}{(M_{13})^{(1)}} \left( e^{(p_{13})^{(1)} t} - 1 \right) \]

From which it follows that

\[ (G_{13}(t) - G_{13}^0) e^{-(p_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(M_{13})^{(1)}} \left[ \left( (p_{13})^{(1)} + G_{14}^0 e^{(p_{13})^{(1)} t} - 1 \right) \right] \]

\( (G_i^0) \) is as defined in the statement of theorem 1

Analogous inequalities hold also for \( G_{14}, G_{15}, T_{13}, T_{14}, T_{15} \)

(b) The operator \( A^{(2)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[ G_{16}(t) \leq G_{16}^0 + \int_0^t \left[ (a_{16})^{(2)} \left( G_{17}^0 + (p_{16})^{(2)} e^{(p_{16})^{(2)} t} \right) \right] ds_{16} = \\
\left( 1 + (a_{16})^{(2)} t \right) G_{17}^0 + \frac{(a_{16})^{(2)} (p_{16})^{(2)}}{(M_{16})^{(2)}} \left( e^{(p_{16})^{(2)} t} - 1 \right) \]

From which it follows that

\[ (G_{16}(t) - G_{16}^0) e^{-(p_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(M_{16})^{(2)}} \left[ \left( (p_{16})^{(2)} + G_{17}^0 \right] e^{(p_{16})^{(2)} t} - 1 \right) \]

Analogous inequalities hold also for \( G_{17}, G_{19}, T_{16}, T_{17}, T_{18} \)

(a) The operator \( A^{(3)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[ G_{20}(t) \leq G_{20}^0 + \int_0^t \left[ (a_{20})^{(3)} \left( G_{21}^0 + (p_{20})^{(3)} e^{(p_{20})^{(3)} t} \right) \right] ds_{20} = \\
\left( 1 + (a_{20})^{(3)} t \right) G_{21}^0 + \frac{(a_{20})^{(3)} (p_{20})^{(3)}}{(M_{20})^{(3)}} \left( e^{(p_{20})^{(3)} t} - 1 \right) \]

From which it follows that

\[ (G_{20}(t) - G_{20}^0) e^{-(p_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ \left( (p_{20})^{(3)} + G_{21}^0 \right] e^{(p_{20})^{(3)} t} - 1 \right) \]

Analogous inequalities hold also for \( G_{21}, G_{22}, T_{20}, T_{21}, T_{22} \)

(b) The operator \( A^{(4)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[ G_{24}(t) \leq G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)} \left( G_{25}^0 + (p_{24})^{(4)} e^{(p_{24})^{(4)} t} \right) \right] ds_{24} = \\
\left( 1 + (a_{24})^{(4)} t \right) G_{25}^0 + \frac{(a_{24})^{(4)} (p_{24})^{(4)}}{(M_{24})^{(4)}} \left( e^{(p_{24})^{(4)} t} - 1 \right) \]

From which it follows that

\[ (G_{24}(t) - G_{24}^0) e^{-(p_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(M_{24})^{(4)}} \left[ \left( (p_{24})^{(4)} + G_{25}^0 \right] e^{(p_{24})^{(4)} t} - 1 \right) \]

\( (G_i^0) \) is as defined in the statement of theorem 1
(c) The operator \( \mathcal{A}^{(5)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that
\[
G_{28}(t) \leq G_{28}^{0} + \int_{0}^{t} \left[ (a_{28})^{(5)} \left( G_{28}^{0} + (\hat{P}_{28})^{(5)} e^{(\hat{Q}_{28})^{(5)} t} \right) \right] ds_{(28)} = \\
(1 + (a_{28})^{(5)} t)G_{28}^{0} + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(M_{28})^{(5)}} \left( e^{(\hat{Q}_{28})^{(5)} t} - 1 \right)
\]
From which it follows that
\[
(G_{28}(t) - G_{28}^{0}) e^{-((\hat{Q}_{28})^{(5)} t)} \leq \frac{(a_{28})^{(5)} ((\hat{P}_{28})^{(5)} + G_{28}^{0}) e^{-\frac{(\hat{P}_{28})^{(5)} + G_{28}^{0}}{c_{28}^{(5)}}} + (\hat{P}_{28})^{(5)}}
\]
\( (G_{i}^{0}) \) is as defined in the statement of theorem 1

(d) The operator \( \mathcal{A}^{(6)} \) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that
\[
G_{32}(t) \leq G_{32}^{0} + \int_{0}^{t} \left[ (a_{32})^{(6)} \left( G_{32}^{0} + (\hat{P}_{32})^{(6)} e^{(\hat{Q}_{32})^{(6)} t} \right) \right] ds_{(32)} = \\
(1 + (a_{32})^{(6)} t)G_{32}^{0} + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(M_{32})^{(6)}} \left( e^{(\hat{Q}_{32})^{(6)} t} - 1 \right)
\]
From which it follows that
\[
(G_{32}(t) - G_{32}^{0}) e^{-((\hat{Q}_{32})^{(6)} t)} \leq \frac{(a_{32})^{(6)} ((\hat{P}_{32})^{(6)} + G_{32}^{0}) e^{-\frac{(\hat{P}_{32})^{(6)} + G_{32}^{0}}{c_{32}^{(6)}}} + (\hat{P}_{32})^{(6)}}
\]
\( (G_{i}^{0}) \) is as defined in the statement of theorem 6

Analogous inequalities hold also for \( G_{25}, G_{26}, T_{24}, T_{25}, T_{26} \)

It is now sufficient to take \( \frac{(a_{i})^{(1)}}{(M_{i})^{(1)}} < 1 \) and to choose \( (\hat{P}_{13})^{(1)} \) and \( (\hat{Q}_{13})^{(1)} \) large to have
\[
\frac{(a_{i})^{(1)}}{(M_{i})^{(1)}} \left[ (\hat{P}_{13})^{(1)} + (\hat{P}_{13})^{(1)} + G_{j}^{0} e^{-\frac{(\hat{P}_{13})^{(1)} + G_{j}^{0}}{c_{j}^{(1)}}} \right] \leq (\hat{P}_{13})^{(1)}
\]
\[
\frac{(b_{i})^{(1)}}{(M_{i})^{(1)}} \left[ (\hat{Q}_{13})^{(1)} + T_{j}^{0} e^{-\frac{(\hat{Q}_{13})^{(1)} + T_{j}^{0}}{r_{j}^{(1)}}} \right] + (\hat{Q}_{13})^{(1)} \leq (\hat{Q}_{13})^{(1)}
\]
In order that the operator \( \mathcal{A}^{(1)} \) transforms the space of sextuples of functions \( G_{i}, T_{i} \) satisfying GLOBAL EQUATIONS into itself

The operator \( \mathcal{A}^{(1)} \) is a contraction with respect to the metric
\[
d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) =
\]
www.ijsrp.org
\begin{align*}
\sup_{i \in \mathbb{R}_+} \max_{t \in [a_i, b_i]} |G_i^{(1)}(t)| - G_i^{(2)}(t) e^{-(\beta_{13})^{(1)} t} \max_{i \in \mathbb{R}_+} |T_i^{(1)}(t)| - T_i^{(2)}(t) e^{-(\beta_{13})^{(1)} t}.
\end{align*}

Indeed if we denote

**Definition of \( \tilde{G}, \tilde{T} \):**

\[
( \tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T)
\]

It results

\[
|\tilde{G}_{13}^{(1)} - \tilde{G}_{13}^{(2)}| \leq \int_0^t (a_{13}(s))^{(1)} |G_{14}^{(1)} - G_{14}^{(2)}| e^{-(\beta_{13})^{(1)} s} e^{(\beta_{13})^{(1)} s} \, ds_{13} + \\
\int_0^t (a_{13}(s))^{(1)} |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\beta_{13})^{(1)} s} e^{(\beta_{13})^{(1)} s} + \\
(a_{13}^{(1)}(T_{14}^{(1)}, s_{13})) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\beta_{13})^{(1)} s} e^{(\beta_{13})^{(1)} s} + \\
G_{13}^{(2)}(a_{13}^{(1)}(T_{14}^{(1)}, s_{13}) - (a_{13}^{(1)}(T_{14}^{(1)}, s_{13})) |G_{13}^{(1)} - G_{13}^{(2)}| e^{-(\beta_{13})^{(1)} s} e^{(\beta_{13})^{(1)} s}) \, ds_{13}
\]

Where \( s_{13} \) represents integrand that is integrated over the interval \([0, t]\).

From the hypotheses it follows

\[
|G^{(1)} - G^{(2)}| e^{-(\beta_{13})^{(1)} t} \leq \frac{1}{(\beta_{13})^{(1)}} (a_{13}(1)) + (a_{13}^{(1)}(1) + (\beta_{13})^{(1)}(\beta_{13})^{(1)} + (\beta_{13})^{(1)}(\beta_{13})^{(1)})(G^{(1)} , T^{(1)}, G^{(2)} , T^{(2)})
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{13}^{(1)}(1) and (b_{13}^{(1)} depending also on t can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{13}^{(1)}(1)e^{(\beta_{13})^{(1)} t}) and \((Q_{13}^{(1)}(1)e^{(\beta_{13})^{(1)} t})\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_{13}^{(1)}(1) and (b_{13}^{(1)}(1), i = 13, 14, 15 depend only on \( T_{14} \) and respectively on \( G \) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From 19 to 24 it results

\[
G_i(t) \geq G_i^0 e^{-\int_0^t ((a_i^{(1)}(1) - a_i^{(1)}(1)(T_{14}^{(1)}(s_{13})) |G_{13}^{(1)} - G_{13}^{(2)}| d s_{13})} \geq 0
\]

\[
T_i(t) \geq T_i^0 e^{-(b_i^{(1)}(1))} > 0 \quad \text{for } t > 0
\]

**Definition of \((\overline{M}_{13})^{(1)}, (\overline{M}_{13})^{(1)}, and (\overline{M}_{13})^{(1)}\):**

**Remark 3:** If \( G_{13} \) is bounded, the same property have also \( G_{14} \) and \( G_{15} \). Indeed if

\[
G_{13} < (\overline{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq (\overline{M}_{13})^{(1)}(1) - (a_{14}^{(1)}) G_{14} \text{ and by integrating}
\]

\[
G_{14} \leq (\overline{M}_{13})^{(1)}(1) \leq G_{14}^0 + 2(a_{14}^{(1)}(1)(\overline{M}_{13})^{(1)}(1)/(a_{14}^{(1)}(1))
\]

In the same way, one can obtain

\[
G_{15} \leq (\overline{M}_{13})^{(1)}(1) \leq G_{15}^0 + 2(a_{15}^{(1)}(1)(\overline{M}_{13})^{(1)}(1)/(a_{15}^{(1)}(1))
\]

www.ijsrp.org
If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13}, G_{15} \) and \( G_{13}, G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} ((b_i)_{1}^{(1)} (G(t), t)) = (b_{14})_{1}^{(1)} \) then \( T_{14} \to \infty \).

**Definition of** \((m)_{1}^{(1)}\) and \( \varepsilon_1 \):

Indeed let \( t_1 \) be so that for \( t > t_1 \)

\[
(b_{14})_{1}^{(1)} - (b_i)_{1}^{(1)} (G(t), t) < \varepsilon_1, T_{13} (t) > (m)_{1}^{(1)}
\]

Then \( \frac{dT_{14}}{dt} \geq (a_{14})_{1}^{(1)} (m)_{1}^{(1)} - \varepsilon_1 T_{14} \) which leads to

\[
T_{14} \geq \left( \frac{(a_{14})_{1}^{(1)} (m)_{1}^{(1)}}{\varepsilon_1} \right) (1 - e^{-\varepsilon_1 t}) + T_0 e^{-\varepsilon_1 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_1 t} = \frac{1}{2} \) it results

\[
T_{14} \geq \left( \frac{(a_{14})_{1}^{(1)} (m)_{1}^{(1)}}{2} \right) \cdot t = \log \frac{2}{\varepsilon_1}
\]

By taking now \( \varepsilon_1 \) sufficiently small one sees that \( T_{14} \) is unbounded.

The same property holds for \( T_{15} \) if \( \lim_{t \to \infty} (b_{15})_{1}^{(1)} (G(t), t) = (b_{15})_{1}^{(1)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_i)_{1}^{(2)}}{(M_{16})_{1}^{(2)}} \), \( \frac{(b_i)_{1}^{(2)}}{(M_{16})_{1}^{(2)}} < 1 \) and to choose

\[
(\hat{P}_{16})_{1}^{(2)}, (\hat{Q}_{16})_{1}^{(2)} \text{ large to have}
\]

\[
\frac{(a_i)_{1}^{(2)}}{(M_{16})_{1}^{(2)}} \left[ (\hat{P}_{16})_{1}^{(2)} + ((\hat{P}_{16})_{1}^{(2)} + a_j^0) e^{-\frac{(Q_{16})_{1}^{(2)} + r_j^0}{r_j}} \right] \leq (\hat{P}_{16})_{1}^{(2)}
\]

\[
\frac{(b_i)_{1}^{(2)}}{(M_{16})_{1}^{(2)}} \left[ ((\hat{Q}_{16})_{1}^{(2)} + r_j^0) e^{-\frac{(Q_{16})_{1}^{(2)} + r_j^0}{r_j}} + (\hat{Q}_{16})_{1}^{(2)} \right] \leq (\hat{Q}_{16})_{1}^{(2)}
\]

In order that the operator \( A^{(2)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying

The operator \( A^{(2)} \) is a contraction with respect to the metric

\[
d \left( ((G_{13})_{1}^{(1)}, (T_{13})_{1}^{(1)}), ((G_{19})_{1}^{(2)}, (T_{19})_{1}^{(2)})) \right) =
\]

\[
\sup \max \left\{ G_i^{(1)} (t) - G_i^{(2)} (t) e^{-(A_{16})_{1}^{(2)} t}, \max \left\{ T_i^{(1)} (t) - T_i^{(2)} (t) e^{-(R_{16})_{1}^{(2)} t} \right\} \right\}
\]

Indeed if we denote

**Definition of** \( \tilde{G}_{19}, \tilde{T}_{19} : \ (\tilde{G}_{19}, \tilde{T}_{19}) = A^{(2)} (G_{19}, T_{19}) \)

It results

\[
\tilde{G}_{19}^{(1)} - \tilde{G}_{19}^{(2)} \leq \int_0^1 (a_{16})_{1}^{(2)} |G_{17}^{(1)} - G_{17}^{(2)} e^{-(R_{16})_{1}^{(2)} t_{16}} e^{(R_{16})_{1}^{(2)} t_{16}}| ds_{16} +
\]

\[
\int_0^1 ((a_{16})_{1}^{(2)} |G_{18}^{(1)} - G_{18}^{(2)} e^{-(R_{16})_{1}^{(2)} t_{16}} e^{(R_{16})_{1}^{(2)} t_{16}}| +
\]

www.ijsrp.org
\((a_{16})''(T_{17}^{(1)}, s(16))G_{16}^{(1)} - G_{16}^{(2)}e^{-\gamma_{16}^{(2)}s(16)}e^{\gamma_{16}^{(2)}s(16)} +
\)
\(G_{16}^{(2)}((a_{16})''(T_{17}^{(1)}, s(16)) - (a_{16})''(T_{17}^{(2)}, s(16)))e^{-\gamma_{16}^{(2)}s(16)}e^{\gamma_{16}^{(2)}s(16)}ds(16)
\)

Where \(s(16)\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| \left[ (G_{19})^{(1)} - (G_{19})^{(2)} \right] e^{-\gamma_{16}^{(2)}t} \leq \frac{1}{(M_{16})^{(2)}} \left( (a_{16})^{(2)} + (a_1')^{(2)} + (M_{16})^{(2)} + \left( \bar{M}_{16}^{(2)} \right)^{2} \right) \right| d\left( \left( (G_{19})^{(1)}, (T_{19})^{(1)}, (G_{19})^{(2)}, (T_{19})^{(2)} \right) \right)
\]

And analogous inequalities for \(G_t\) and \(T_t\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{16})''(2)\) and \((b_{16})''(2)\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\bar{P}_{16})^{(2)} e^{\gamma_{16}^{(2)}t}\) and \((\bar{Q}_{16})^{(2)} e^{\gamma_{16}^{(2)}t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_1')^{(2)}\) and \((b_1')^{(2)}, i = 16, 17, 18\) depend only on \(T_{17}\) and respectively on \((G_{19})(\text{on not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_t(0) = 0\) and \(T_t(t) = 0\)

From 19 to 24 it results

\[
G_t(t) \geq G_t^{(1)} e^{-\int_{0}^{t}[(a_1')^{(2)} - (a_1')^{(2)}(T_{17}(s(16), s(16)))ds(16)]} \geq 0
\]

\[
T_t(t) \geq T_t^{(1)} e^{-\bar{t}^{(2)}t} > 0 \quad \text{for } t > 0
\]

**Definition of** \((\bar{M}_{16})^{(2)}\), \((\bar{M}_{16})^{(2)}\), and \((\bar{M}_{16})^{(2)}\) :

**Remark 3:** if \(G_{16}\) is bounded, the same property have also \(G_{17}\) and \(G_{18}\) - indeed if

\[
G_{16} < (\bar{M}_{16})^{(2)} \text{ it follows } \frac{dG_{17}}{dt} \leq \left( (\bar{M}_{16})^{(2)} \right)_{1} - (a_1')^{(2)}G_{17} \text{ and by integrating}
\]

\[
G_{17} \leq \left( (\bar{M}_{16})^{(2)} \right)_{2} = G_{17}^{(1)} + 2(a_1')^{(2)}(\bar{M}_{16})^{(2)}_{1}/(a_1')^{(2)}
\]

In the same way, one can obtain

\[
G_{18} \leq \left( (\bar{M}_{16})^{(2)} \right)_{3} = G_{18}^{(1)} + 2(a_1')^{(2)}(\bar{M}_{16})^{(2)}_{2}/(a_1')^{(2)}
\]

If \(G_{17}\) or \(G_{18}\) is bounded, the same property follows for \(G_{16}\), \(G_{18}\), and \(G_{16}\), \(G_{17}\) respectively.

**Remark 4:** If \(G_{16}\) is bounded, from below, the same property holds for \(G_{17}\) and \(G_{18}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{17}\) is bounded from below.

**Remark 5:** If \(T_{16}\) is bounded from below and \(\lim_{t \to \infty} ((b_1')^{(2)} ((G_{19})(t), t)) = (b_1')^{(2)}\) then \(T_{17} \to \infty\).

**Definition of** \((m)^{(2)}\) and \(\varepsilon_2\) :

Indeed let \(t_2\) be so that for \(t > t_2\)

\[
(b_1')^{(2)} - (b_1')^{(2)} ((G_{19})(t), t) < \varepsilon_2, T_{16} (t) > (m)^{(2)}
\]

Then \(\frac{dT_{17}}{dt} \geq (a_1')^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}\) which leads to
It is now sufficient to take \( \left( \frac{a_j}{M_20}\right)^{(3)} \) and \( \left( \frac{b_j}{M_20}\right)^{(3)} < 1 \) and to choose

\[
\left( \tilde{P}_{20}\right)^{(3)} \quad \text{and} \quad \left( \tilde{Q}_{20}\right)^{(3)}
\]

large to have

\[
\left( \frac{a_j}{M_20}\right)^{(3)} \left[ \tilde{P}_{20}^{(3)} + \left( \tilde{P}_{20}^{(3)} + G_j^0 e^{\left( \frac{\tilde{P}_{20}^{(3)} + 0}{\tilde{t}_j}\right)} \right) \right] \leq \left( \tilde{P}_{20}\right)^{(3)}
\]

\[
\left( \frac{b_j}{M_20}\right)^{(3)} \left[ \left( \tilde{Q}_{20}^{(3)} + \tilde{t}_j^0 e^{\left( \frac{\tilde{Q}_{20}^{(3)} + 0}{\tilde{t}_j}\right)} + \left( \tilde{Q}_{20}\right)^{(3)} \right) \right] \leq \left( \tilde{Q}_{20}\right)^{(3)}
\]

In order that the operator \( \mathcal{A}(3) \) transforms the space of sextuples of functions \( G_i, T_i \) into itself

The operator \( \mathcal{A}(3) \) is a contraction with respect to the metric

\[
d \left( \left( G_{23}^{(1)}, (T_{23}^{(1)}), \left( G_{23}^{(2)}, (T_{23}^{(2)}) \right) \right) = sup_{i \in \mathbb{R}^+} \left[ \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-\left( \frac{\tilde{a}_{20}^{(3)}}{3}\right) t} \right] \times max_{i \in \mathbb{R}^+} \left[ \left| \tilde{t}_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left( \tilde{b}_{20}^{(3)}\right)} \right]
\]

Indeed if we denote

**Definition of \( \tilde{G}_{23}, \tilde{T}_{23} : \left( \left( G_{23}^{(1)}, (T_{23}^{(1)}), \left( G_{23}^{(2)}, (T_{23}^{(2)}) \right) \right) = \mathcal{A}(3) \left( \left( G_{23}^{(1)}, (T_{23}^{(1)}), \left( G_{23}^{(2)}, (T_{23}^{(2)}) \right) \right) \right) \)**

It results

\[
\tilde{G}_{20}^{(1)} - \tilde{G}_{20}^{(2)} \leq \int_{0}^{t} \left( \tilde{a}_{20}^{(3)} \right) \left( G_{21}^{(1)} - G_{21}^{(2)} \right) e^{-\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} ds_{20} + 
\]

\[
\int_{0}^{t} \left( \tilde{a}_{20}^{(3)} \right) \left( G_{20}^{(1)} - G_{20}^{(2)} \right) e^{-\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} ds_{20} + 
\]

\[
\left( \tilde{a}_{20}^{(3)} \right) \left( T_{21}^{(1)} , s_{20} \right) \left( G_{20}^{(1)} - G_{20}^{(2)} \right) e^{-\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} ds_{20} + 
\]

\[
\tilde{G}_{20}^{(2)} \left( \tilde{a}_{20}^{(3)} \right) \left( G_{20}^{(1)} , s_{20} \right) - \left( \tilde{a}_{20}^{(3)} \right) \left( T_{21}^{(2)} , s_{20} \right) e^{-\left( \tilde{a}_{20}^{(3)}\right) t} e^{\left( \tilde{a}_{20}^{(3)}\right) t} ds_{20}
\]

Where \( s_{20} \) represents integrand that is integrated over the interval \([0,t]\)

From the hypotheses it follows

\[
\left[ G_{20}^{(1)} - G_{20}^{(2)} \right] e^{-\left( \tilde{a}_{20}^{(3)}\right) t} \leq \frac{1}{\left( \tilde{a}_{20}^{(3)}\right)} \left( \tilde{a}_{20}^{(3)} + \left( \tilde{a}_{20}^{(3)} \right) + \left( \tilde{a}_{20}^{(3)} + \left( \tilde{a}_{20}^{(3)} \right) \right) \right) \right) \int_{0}^{t} \left( (G_{23}^{(1)} , (T_{23}^{(1)}), (G_{23}^{(2)} , (T_{23}^{(2)})) \right)
\]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( \left( \tilde{a}_{20}^{(3)}\right) \) and \( \left( \tilde{b}_{20}^{(3)}\right) \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition
necessary to prove the uniqueness of the solution bounded by \( (\bar{P}_{20})^{(3)} e^{(\mathcal{R}_{20})^{(3)} t} \) and \( (\bar{Q}_{20})^{(3)} e^{(\mathcal{R}_{20})^{(3)} t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{(3)}) \) and \( (b_{i}^{(3)}) \), \( i = 20, 21, 22 \) depend only on \( T_{21} \) and respectively on \( (G_{23})(and not on t) \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \)

From 19 to 24 it results

\[
G_i (t) \geq G^0_i e \left[ - \int_0^t (a_i^{(3)} - a_i^{(3)}(T_{21}(t), t)) dt \right] \geq 0
\]

\[
T_i (t) \geq T^0_i e \left[ -(b_i^{(3)}) t \right] > 0 \quad \text{for} \ t > 0
\]

**Definition of** \( (\bar{M}_{20})^{(3)} \) : 1, \( (\bar{M}_{20})^{(3)} \) : 2 and \( (\bar{M}_{20})^{(3)} \) : 3.

**Remark 3:** If \( G_{20} \) is bounded, the same property have also \( G_{21} \) and \( G_{22} \). Indeed if \( G_{20} < (\bar{M}_{20})^{(3)} \) it follows \( dG_{21} / dt \leq (\bar{M}_{20})^{(3)} - (a_{21})^{(3)}G_{21} \) and by integrating

\[
G_{21} \leq (\bar{M}_{20})^{(3)}_1 + 2(a_{21})^{(3)}(\bar{M}_{20})^{(3)}_1 / (a_{21})^{(3)}
\]

In the same way, one can obtain

\[
G_{22} \leq (\bar{M}_{20})^{(3)}_3 \leq G^0_{22} + 2(a_{22})^{(3)}((\bar{M}_{20})^{(3)}_2) / (a_{22})^{(3)}
\]

If \( G_{21} \) or \( G_{22} \) is bounded, the same property follows for \( G_{20} \), \( G_{22} \) and \( G_{20} \), \( G_{21} \) respectively.

**Remark 4:** If \( G_{20} \) is bounded, from below, the same property holds for \( G_{21} \) and \( G_{22} \). The proof is analogous to the preceding one. An analogous property is true if \( G_{21} \) is bounded from below.

**Remark 5:** If \( T_{20} \) is bounded from below and \( \lim_{t \to \infty} (b_{21}^{(3)}((G_{23})(t), t)) = (b_{21})^{(3)} \) then \( T_{21} \to \infty \).

**Definition of** \( (m)^{(3)} \) and \( \varepsilon_3 \):

Indeed let \( t_3 \) be so that for \( t > t_3 \)

\[
(b_{21})^{(3)} - (b_{1}^{(3)}((G_{23})(t), t)) < \varepsilon_3, T_{20} (t) > (m)^{(3)}
\]

Then \( dT_{21} / dt \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21} \) which leads to

\[
T_{21} \geq \frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3} (1 - e^{-\varepsilon_3 t}) + T^0_{21} e^{-\varepsilon_3 t} \quad \text{If we take} \ t \ \text{such that} \ e^{-\varepsilon_3 t} = \frac{1}{2} \ \text{it results}
\]

\[
T_{21} \geq \frac{(a_{21})^{(3)}(m)^{(3)}}{2} \quad t = log \frac{2}{\varepsilon_3} \quad \text{By taking now} \ \varepsilon_3 \ \text{sufficiently small one sees that} \ T_{21} \ \text{is unbounded}.
\]

The same property holds for \( T_{22} \) if \( \lim_{t \to \infty} (b_{22}^{(3)}((G_{23})(t), t)) = (b_{22})^{(3)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_{1})^{(3)}}{(\bar{M}_{24})^{(3)}} \) and \( \frac{(b_{1})^{(3)}}{(\bar{M}_{24})^{(3)}} \) less than 1 and to choose

\[
(\bar{P}_{24})^{(4)} \) and \( (\bar{Q}_{24})^{(4)} \) large to have

www.ijsrp.org
\[
\left( \frac{(a_i)(4)}{(M_{24})(4)} \right) \left( \frac{(b_i)(4)}{(M_{24})(4)} \right) \left[ (\bar{P}_{24})^{(4)} + \left( (\bar{P}_{24})^{(4)} + \bar{G}_0 \right) e^{-\left( \frac{(P_{24})(4) + \bar{G}_0}{T_i} \right)} \right] \leq (\bar{P}_{24})^{(4)}
\]

\[
\left( \frac{(b_i)(4)}{(M_{24})(4)} \right) \left[ \left( (\bar{Q}_{24})^{(4)} + T_i^{(4)} \right) e^{\left( \frac{(Q_{24})(4) + T_i^{(4)}}{T_i} \right)} \right] + (\bar{Q}_{24})^{(4)} \leq (\bar{Q}_{24})^{(4)}
\]

In order that the operator \( \mathcal{A}^{(4)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying IN to itself.

The operator \( \mathcal{A}^{(4)} \) is a contraction with respect to the metric
\[
d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right) = \sup_{i \in \mathbb{R}^n} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-((\bar{G}_{24})^{(4)}t)} \max_{i \in \mathbb{R}^n} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-((\bar{Q}_{24})^{(4)}t)}
\]

Indeed if we denote
\[
\text{Definition of} \quad (G_{27}), (T_{27}): \quad (G_{27}), (T_{27}) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))
\]

It results
\[
\left| \bar{a}_{24}^{(1)} - \bar{a}_{24}^{(2)} \right| \leq \int_0^t (a_{24})^{(4)} |G_{25}^{(1)} - G_{25}^{(2)}| e^{-((\bar{G}_{24})^{(4)}s_{(24)})} e^{((\bar{Q}_{24})^{(4)}s_{(24)})} ds_{(24)} + \\
\int_0^t (a_{24})^{(4)} |G_{24}^{(1)} - G_{24}^{(2)}| e^{-((\bar{G}_{24})^{(4)}s_{(24)})} e^{((\bar{Q}_{24})^{(4)}s_{(24)})} + \\
(a_{24})^{(4)} |T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-((\bar{G}_{24})^{(4)}s_{(24)})} e^{((\bar{Q}_{24})^{(4)}s_{(24)})} + \\
G_{24}^{(2)} |(a_{24})^{(4)} (T_{25}^{(1)}, s_{(24)}) - (a_{24})^{(4)} (T_{25}^{(2)}, s_{(24)}) | e^{-((\bar{G}_{24})^{(4)}s_{(24)})} e^{((\bar{Q}_{24})^{(4)}s_{(24)})} ds_{(24)}
\]

Where \( s_{(24)} \) represents integrand that is integrated over the interval \([0, t] \)

From the hypotheses it follows
\[
\left| (G_{27})^{(1)} - (G_{27})^{(2)} \right| e^{-((\bar{Q}_{24})^{(4)}t)} \leq \\
\frac{1}{(M_{24})(4)} \left[ (a_{24})^{(4)} + (a_{24})^{(4)} + (\bar{A}_{24})^{(4)} + (\bar{P}_{24})^{(4)} (\bar{Q}_{24})^{(4)} \right] d \left( ((G_{27})^{(1)}, (T_{27})^{(1)}), ((G_{27})^{(2)}, (T_{27})^{(2)}) \right)
\]

And analogous inequalities for \( G_i, T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( (a_{24})^{(4)} \) and \( (b_{24})^{(4)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\bar{P}_{24})^{(4)} e^{(\bar{Q}_{24})^{(4)}t} \) and \((\bar{Q}_{24})^{(4)} e^{(\bar{Q}_{24})^{(4)}t}\) respectively on \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i^{(4)} \) and \( (b_i^{(4)}, i = 24, 25, 26 \) depend only on \( T_{25} \) and respectively on \( (G_{27})(and not on \( t \) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

www.ijsrp.org
From 19 to 24 it results

\[ G_i(t) \geq G_i^0 e^{-\int_0^t (a_i')(a_i') e^{-\int_0^s (a_i')(a_i') ds} ds} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{-\int_0^t (b_i)(b_i') ds} > 0 \text{ for } t > 0 \]

Definition of \((\tilde{M}_{24})^{(4)}_1, (\tilde{M}_{24})^{(4)}_2, \text{ and } (\tilde{M}_{24})^{(4)}_3:\)

Remark 3: if \(G_{24}\) is bounded, the same property have also \(G_{25}\) and \(G_{26}\) indeed if \(G_{24} < (\tilde{M}_{24})^{(4)}\) it follows \(\frac{dG_{25}}{dt} \leq (\tilde{M}_{24})^{(4)}_1 - (a_{25})^{(4)} G_{25}\) and by integrating

\[ G_{25} \leq (\tilde{M}_{24})^{(4)}_2 = G_{25}^0 + 2(a_{25})^{(4)}(\tilde{M}_{24})^{(4)}_1/(a_{25})^{(4)} \]

In the same way, one can obtain

\[ G_{26} \leq (\tilde{M}_{24})^{(4)}_3 = G_{26}^0 + 2(a_{26})^{(4)}(\tilde{M}_{24})^{(4)}_2/(a_{26})^{(4)} \]

If \(G_{25}\) or \(G_{26}\) is bounded, the same property follows for \(G_{24}, G_{26}\) and \(G_{24}, G_{25}\) respectively.

Remark 4: if \(G_{24}\) is bounded, from below, the same property holds for \(G_{25}\) and \(G_{26}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{25}\) is bounded from below.

Remark 5: if \(T_{24}\) is bounded from below and \(\lim_{t \to \infty} ((b_{25})''(G_{27})(t), t)) = (b_{25})^{(4)}(t) \) then \(T_{25} \to \infty\).

Definition of \((m)^{(4)}\) and \(\varepsilon_4:\)

Indeed let \(t_4\) be so that for \(t > t_4\)

\[ (b_{25})^{(4)} - (b_{25})^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)} \]

Then \(\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}\) which leads to

\[ T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4} (1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}, \text{ if we take } t \text{ such that } e^{-\varepsilon_4 t} = \frac{1}{2} \text{ it results} \]

\[ T_{25} \geq \frac{(a_{25})^{(4)}(m)^{(4)}}{2}, t = \log \frac{2}{\varepsilon_4} \text{ By taking now } \varepsilon_4 \text{ sufficiently small one sees that } T_{25} \text{ is unbounded.} \]

The same property holds for \(T_{26}\) if \(\lim_{t \to \infty} (b_{26})''(G_{27})(t), t) = (b_{26})^{(4)}(t) \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \(G_{29}, G_{30}, T_{28}, T_{29}, T_{30} \)

It is now sufficient to take \(\frac{(a_i)^{(5)}}{(M_{28})^{(5)}} < 1\) and to choose

\( \tilde{P}_{28}^{(5)} \) and \( \tilde{Q}_{28}^{(5)} \) large to have

www.ijsrp.org
\[
\begin{align*}
\left(\frac{a_i}{M_{28}^{(5)}}\right)^5 \left[ (P_{28})^{(5)} + (\tilde{P}_{28})^{(5)} + G_i^{(0)} e^{-\left(\frac{(P_{28})^{(5)} + e^{G_i^{(0)}}}{\tau_j^{(i)}}\right)} \right] & \leq \left(\tilde{P}_{28}\right)^{(5)} \\
\left(\frac{b_i}{M_{28}^{(5)}}\right)^5 \left[ (\tilde{Q}_{28})^{(5)} + t_i^{(0)} e^{-\left(\frac{(Q_{28})^{(5)} + e^{Q_i^{(0)}}}{\tau_j^{(i)}}\right)} + (\tilde{Q}_{28})^{(5)} \right] & \leq \left(\tilde{Q}_{28}\right)^{(5)}
\end{align*}
\]

In order that the operator \(A^{(5)}\) transforms the space of sextuples of functions \(G_i, T_i\) into itself

The operator \(A^{(5)}\) is a contraction with respect to the metric

\[
d \left(\left(\frac{G_{31}}{(T_{31})^{(5)}}, (G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) =
\]

\[
sup_{t \in \mathbb{R}^+} \left[ G_i^{(1)}(t) - G_i^{(2)}(t) \right] e^{-\left(\tilde{Q}_{28}\right)^{(5)}t} \max_{t \in \mathbb{R}^+} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}t}
\]

Indeed, if we denote

**Definition of \((\tilde{G}_{31}), (\tilde{T}_{31})\):** \(\left(\tilde{G}_{31}, (\tilde{T}_{31})\right) = A^{(5)}(\left(G_{31}, (T_{31})\right))\)

It results

\[
\left| G_{28}^{(1)} - G_i^{(2)} \right| \leq \int_0^t (a_{28}^{(5)}) \left| G_{29}^{(1)} - G_{29}^{(2)} \right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} e^{\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} ds_{(28)} +
\]

\[
\int_0^t (a_{28}^{(5)} \left| G_{28}^{(1)} - G_{28}^{(2)} \right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} e^{-\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} +
\]

\[
\left(\tilde{a}_{28}^{(5)} \left| T_{29}^{(1)}(s_{(28)})\right| G_{28}^{(1)} - G_{28}^{(2)} \right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} e^{\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} +
\]

\[
\left(\tilde{a}_{28}^{(5)} \left| T_{29}^{(2)}(s_{(28)})\right| - (a_{28}^{(5)} \left| T_{29}^{(2)}(s_{(28)})\right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} e^{\left(\tilde{Q}_{28}\right)^{(5)}s_{(28)}} ds_{(28)}
\]

Where \(s_{(28)}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| (G_{31})^{(1)}(1) - (G_{31})^{(2)} \right| e^{-\left(\tilde{Q}_{28}\right)^{(5)}t} \leq {1 \over \left(\tilde{a}_{28}^{(5)}\right)^5} \left(\tilde{a}_{28}^{(5)} \left(\tilde{a}_{28}^{(5)} + \left(\tilde{A}_{28}^{(5)} + \left(\tilde{P}_{28}^{(5)} \left(\tilde{Q}_{28}^{(5)} + \left(\tilde{P}_{28}^{(5)} \left(\tilde{Q}_{28}^{(5)} \right) \right) d \left(\left(\tilde{G}_{31}, (\tilde{T}_{31})\right) \right) \right) \right) \right) \right)
\]

And analogous inequalities for \(G_i, T_i\). Taking into account the hypothesis (35,35,36) the result follows

**Remark 1:** The fact that we supposed \(a_{28}^{(5)}\) and \(b_{28}^{(5)}\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\tilde{P}_{28})^{(5)} e^{(\tilde{Q}_{28})^{(5)} t}\) and \((\tilde{Q}_{28})^{(5)} e^{(\tilde{Q}_{28})^{(5)} t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_{i}^{(5)})\) and \((b_{i}^{(5)}), i = 28, 29, 30\) depend only on \(T_{29}\) and respectively on \(G_{31}\) and **not on** \(t\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)
From GLOBAL EQUATIONS it results

\[ G_i(t) \geq G_i^0 e^{\left[-\int_{t_0}^{t} (a_i'(s)) (G_{28}(s)) ds\right]} \leq 0 \]

\[ T_i(t) \geq T_i^0 e^{-(b_i'(s))} > 0 \] for \( t > 0 \)

**Definition of** \( (\mathcal{M}_{28})^5 \) \( \left( (\mathcal{M}_{28})^5 \right)_1 \) \( (\mathcal{M}_{28})^5 \) : 

**Remark 3:** If \( G_{28} \) is bounded, the same property have also \( G_{29} \) and \( G_{30} \). Indeed if

\[ G_{28} < (\mathcal{M}_{28})^5 \] it follows \( \frac{dG_{29}}{dt} \leq (\mathcal{M}_{28})^5 - (a_{29}'(s)G_{29}) \) and by integrating

\[ G_{29} \leq (\mathcal{M}_{28})^5 \]

In the same way, one can obtain

\[ G_{30} \leq (\mathcal{M}_{28})^5 \]

If \( G_{29} \) or \( G_{30} \) is bounded, the same property follows for \( G_{28} \), \( G_{29} \) and \( G_{28} \), \( G_{29} \) respectively.

**Remark 4:** If \( G_{28} \) is bounded, from below, the same property holds for \( G_{29} \) and \( G_{30} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{29} \) is bounded from below.

**Remark 5:** If \( T_{28} \) is bounded from below and \( \lim_{t \to \infty} \left( (b_{29}'(s)) (G_{31}(s), t) \right) = (b_{29}'(s)) \) then \( T_{29} \to \infty \).

**Definition of** \( (m)^5 \) \( \varepsilon_5 \):

Indeed let \( t_5 \) be so that for \( t > t_5 \)

\[ (b_{29}'(s)) - (b_{29}'(s))(G_{31}(s), t) < \varepsilon_5, T_{29} (t) > (m)^5 \]

Then \( \frac{dT_{29}}{dt} \geq (a_{29}'(s))(m)^5 - \varepsilon_5 T_{29} \), which leads to

\[ T_{29} \geq \left( \frac{(a_{29}'(s))(m)^5}{\varepsilon_5} \right) \left( 1 - e^{-\varepsilon_5 t} \right) + T_{29}^0 e^{-\varepsilon_5 t} \]

If we take \( t \) such that \( e^{-\varepsilon_5 t} = \frac{1}{2} \) it results

\[ T_{29} \geq \left( \frac{(a_{29}'(s))(m)^5}{2} \right), \quad t = \log \frac{2}{\varepsilon_5} \]

By taking now \( \varepsilon_5 \) sufficiently small one sees that \( T_{29} \) is unbounded.

The same property holds for \( T_{30} \) if \( \lim_{t \to \infty} (b_{30}'(s)) (G_{31}(s), t) = (b_{30}'(s)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \frac{a_{32}(s)}{G_{32}^{(6)}} < 1 \) and to choose

\( (\bar{p}_{32})^{(6)} \) and \( (\bar{q}_{32})^{(6)} \) large to have
\[
\left. \frac{(a_i)^{(6)}}{(M_{32})^{(6)}} \right| \left( \mathcal{P}_{32} \right)^{(6)} + \left( \mathcal{P}_{32} \right)^{(6)} + G_i^{(6)} e^{-\frac{(\mathcal{Q}_{32})^{(6)} t}{7_i}} \right| \leq \left( \bar{\mathcal{P}}_{32} \right)^{(6)}
\]

\[
\left. \frac{(b_i)^{(6)}}{(M_{32})^{(6)}} \right| \left( \left( \mathcal{Q}_{32} \right)^{(6)} + T_{j}^{(6)} e^{-\frac{(\mathcal{Q}_{32})^{(6)} t}{7_j}} \right) + \left( \bar{\mathcal{Q}}_{32} \right)^{(6)} \right| \leq \left( \bar{\mathcal{Q}}_{32} \right)^{(6)}
\]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself.

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

\[
d \left( \left( G_{35}^{(1)}, T_{35}^{(1)} \right), \left( G_{35}^{(2)}, T_{35}^{(2)} \right) \right) =
\sup_{t \in \mathbb{R}} \left| G_i^{(1)}(t) - G_i^{(2)}(t) \right| e^{-((\mathcal{Q}_{32})^{(6)} t)} \leq \sup_{t \in \mathbb{R}} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \right| e^{-((\mathcal{Q}_{32})^{(6)} t)}
\]

Indeed if we denote

**Definition of** \((G_{35}), (T_{35}) : \left( (G_{35}), (T_{35}) \right) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))\)

It results

\[
\left| G_{32}^{(1)} - G_{32}^{(2)} \right| \leq \int_{t_0}^{t} \left| \left( a_{32}^{(6)} \right)^{(6)} G_{33}^{(1)}(t) - G_{33}^{(2)}(t) e^{-((\mathcal{Q}_{32})^{(6)} s_{32})} e^{((\mathcal{Q}_{32})^{(6)} s_{32})} ds_{32} \right| +
\int_{t_0}^{t} \left| \left( a_{32}^{(6)} \right)^{(6)} G_{32}^{(1)}(t) - G_{32}^{(2)}(t) e^{-((\mathcal{Q}_{32})^{(6)} s_{32})} e^{((\mathcal{Q}_{32})^{(6)} s_{32})} +
\left( a_{32}^{(6)} \right)^{(6)} T_{33}^{(1)}(s_{32}) - \left( a_{32}^{(6)} \right)^{(6)} T_{33}^{(2)}(s_{32}) \right| \leq \int_{t_0}^{t} \left| \left( a_{32}^{(6)} \right)^{(6)} G_{32}^{(1)}(t) - G_{32}^{(2)}(t) e^{-((\mathcal{Q}_{32})^{(6)} s_{32})} e^{((\mathcal{Q}_{32})^{(6)} s_{32})} ds_{32} \right|
\]

Where \( s_{32} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[
\left| (G_{35})^{(1)} - (G_{35})^{(2)} \right| e^{-((\mathcal{Q}_{32})^{(6)} t)} \leq \frac{1}{(M_{32})^{(6)}} \left| (a_{32}^{(6)})^{(6)} + (a_{32}^{(6)})^{(6)} + (\mathcal{A}_{32}^{(6)})^{(6)} + (\mathcal{P}_{32}^{(6)})^{(6)} (\mathcal{K}_{32}^{(6)})^{(6)} \right| d \left( \left( (G_{35})^{(1)}, (T_{35})^{(1)} \right), \left( (G_{35})^{(2)}, (T_{35})^{(2)} \right) \right)
\]

And analogous inequalities for \( G_i, T_i \). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{32})^{(6)}\) and \((b_{32})^{(6)}\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\mathcal{P}_{32})^{(6)} e^{((\mathcal{Q}_{32})^{(6)} t)}\) and \((\bar{\mathcal{Q}}_{32})^{(6)} e^{((\mathcal{Q}_{32})^{(6)} t)}\) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \((a_i)^{(6)}\) and \((b_i)^{(6)}\), \(i = 32, 33, 34\) depend only on \( T_{33} \) and respectively on \((G_{35})\) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)
From 69 to 32 it results
\[ G_i(t) \geq G_i^0 e^{\int_0^t (a_i''(s) - a_i''(t)) ds} \geq 0 \]
\[ T_i(t) \geq T_i^0 e^{(-b_i''(t)) t} > 0 \text{ for } t > 0 \]

**Definition of** \((\mathcal{M}_{32}(6))_1, (\mathcal{M}_{32}(6))_2, \text{ and } (\mathcal{M}_{32}(6))_3 : \)

**Remark 3:** If \(G_{32}\) is bounded, the same property have also \(G_{33}\) and \(G_{34}\). Indeed if
\[ G_{32} < (\mathcal{M}_{32}(6)) \text{ it follows } \frac{dG_{33}}{dt} \leq (\mathcal{M}_{32}(6))_1 - (a'_{33}) G_{33} \text{ and by integrating } 
G_{33} \leq (\mathcal{M}_{32}(6))_2 = G_{33}^0 + 2(a_{33}) (\mathcal{M}_{32}(6))_1 / (a'_{33}) (6) 
In the same way, one can obtain
\[ G_{34} \leq (\mathcal{M}_{32}(6))_3 = G_{34}^0 + 2(a_{34}) (\mathcal{M}_{32}(6))_2 / (a'_{34}) (6) \]

If \(G_{33}\) or \(G_{34}\) is bounded, the same property follows for \(G_{32}, G_{34}\) and \(G_{32}, G_{33}\) respectively.

**Remark 4:** If \(T_{32}\) is bounded from below and \(\lim_{t \to \infty} ((b''_{33}) (G_{35}(t), t)) = (b_{33}'(6)) \text{ then } T_{33} \to \infty. \)

**Definition of** \((m)(6)\) and \(\epsilon_6 : \)

Indeed let \(t_6\) be so that for \(t \geq t_6 \)
\[(b_{33})'(6) - (b_{33}'(6)) ((G_{35}(t), t)) < \epsilon_6, T_{32}(t) > (m)(6) \]

Then \(\frac{dT_{33}}{dt} \geq (a_{33}) (6)(m)(6) - \epsilon_6 T_{33}\) which leads to
\[ T_{33} \geq \left( (a_{33}) (6)(m)(6) \right) \frac{1}{\epsilon_6} \left( 1 - e^{-\epsilon_6 t} \right) + T_{33}^0 e^{-\epsilon_6 t} \] If we take \(t\) such that \(e^{-\epsilon_6 t} = \frac{1}{2}\) it results
\[ T_{33} \geq \left( \frac{(a_{33}) (6)(m)(6)}{2} \right) \] \( t = \log \frac{2}{\epsilon_6} \) By taking now \(\epsilon_6\) sufficiently small one sees that \(T_{33}\) is unbounded.

The same property holds for \(T_{34}\) if \(\lim_{t \to \infty} (b''_{34})(6) ((G_{35}(t), t), t) = (b_{34}'(6)) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1)(1), (\sigma_2)(1), (\tau_1)(1), (\tau_2)(1) : \)

(a) \(\sigma_1)(1), (\sigma_2)(1), (\tau_1)(1), (\tau_2)(1)\) for four constants satisfying
\[-(\sigma_2)(1) \leq -(a_{13}) (1) + (a_{14}) (1) - (a_{13}) (1)(T_{14}, t) + (a_{14}) (1)(T_{14}, t) \leq -(\sigma_1)(1) \]
\[-(r_2)^{(1)} \leq -(b_{13})^{(1)} + (b_{14})^{(1)} - (b_{13})^{(1)}(G, t) - (b_{14})^{(1)}(G, t) \leq -(r_1)^{(1)}\]

**Definition of** \((v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}\):

(b) By \((v_1)^{(1)} > 0, (v_2)^{(1)} < 0\) and respectively \((u_1)^{(1)} > 0, (u_2)^{(1)} < 0\) the roots of the equations

\[a_{14}^{(1)}(v^{(1)})^2 + (a_1)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0\]

and \(b_{14}^{(1)}(u^{(1)})^2 + (r_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0\)

**Definition of** \((\tilde{v}_1)^{(1)}, (\tilde{v}_2)^{(1)}, (\tilde{u}_1)^{(1)}, (\tilde{u}_2)^{(1)}\):

By \((\tilde{v}_1)^{(1)} > 0, (\tilde{v}_2)^{(1)} < 0\) and respectively \((\tilde{u}_1)^{(1)} > 0, (\tilde{u}_2)^{(1)} < 0\) the roots of the equations

\[a_{14}^{(1)}(v^{(1)})^2 + (\tilde{a}_2)^{(1)}v^{(1)} - (a_{13})^{(1)} = 0\]

and \(b_{14}^{(1)}(u^{(1)})^2 + (\tilde{r}_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0\)

**Definition of** \((m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}\):

(c) If we define \((m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}\) by

\[(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}\] if \((v_0)^{(1)} < (v_1)^{(1)}\)

\[(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\tilde{v}_1)^{(1)}\] if \((v_1)^{(1)} < (v_0)^{(1)} < (\tilde{v}_1)^{(1)}\),

and \([v_0)^{(1)} = \frac{G_{13}}{G_{14}}\]

\[(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}\] if \((v_1)^{(1)} < (v_0)^{(1)}\)

and analogously

\[(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}\] if \((u_0)^{(1)} < (u_1)^{(1)}\)

\[(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\tilde{u}_1)^{(1)}\] if \((u_1)^{(1)} < (u_0)^{(1)} < (\tilde{u}_1)^{(1)}\),

and \([u_0)^{(1)} = \frac{T_{13}}{T_{14}}\]

\[(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}\] if \((\tilde{u}_1)^{(1)} < (u_0)^{(1)}\)

are defined respectively.

Then the solution satisfies the inequalities

\[G_{13}^{(0)}e^{(s_1)^{(1)}-(p_{13})^{(1)}t} \leq G_{13}(t) \leq G_{13}^{(0)}e^{(s_1)^{(1)}t}\]

where \((p_i)^{(1)}\) is defined

\[
\frac{1}{(m_1)^{(1)}}G_{13}^{(0)}e^{(s_1)^{(1)}-(p_{13})^{(1)}t} \leq G_{13}(t) \leq \frac{1}{(m_2)^{(1)}}G_{13}^{(0)}e^{(s_1)^{(1)}t}
\]

\[
\frac{1}{(m_1)^{(1)}}G_{13}^{(0)}e^{(s_1)^{(1)}-(p_{13})^{(1)}t} e^{((s_1)^{(1)}-(p_{13})^{(1)}t} - e^{((s_2)^{(1)}-(p_{13})^{(1)}t} + G_{15}^{(0)}e^{-(s_2)^{(1)}t} \leq G_{15}(t) \leq G_{15}^{(0)}e^{-(s_2)^{(1)}t}
\]

\[
\frac{1}{(m_1)^{(1)}}T_{13}^{(0)}e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(m_2)^{(1)}}T_{13}^{(0)}e^{(R_1)^{(1)}t}
\]

\[
\frac{1}{(m_1)^{(1)}}T_{13}^{(0)}e^{(R_1)^{(1)}t} e^{((R_1)^{(1)}-(p_{13})^{(1)}t} + T_{15}^{(0)}e^{-(b_{13})^{(1)}t} \leq T_{15}(t) \leq T_{15}^{(0)}e^{-(b_{13})^{(1)}t}
\]

www.ijsrp.org
\[
\left(\frac{(a_{13})^{(1)}}{(\mu_{2})^{(1)}}\right)^{\frac{\tau_{2}}{T}} \left[e^{((R_{1})^{(1)}+(r_{13})^{(1)}))t} - e^{-(R_{2})^{(1)}t}\right] + \tau_{1}^{0} e^{-(R_{2})^{(1)}t}
\]

**Definition of \((S_{1})^{(1)}, (S_{2})^{(1)}, (R_{1})^{(1)}, (R_{2})^{(1)}\):**

Where \((S_{1})^{(1)} = (a_{13})^{(1)}(m_{2})^{(1)} - (a'_{13})^{(1)}\)
\((S_{2})^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}\)
\((R_{1})^{(1)} = (b_{13})^{(1)}(\mu_{2})^{(1)} - (b'_{13})^{(1)}\)
\((R_{2})^{(1)} = (b_{15})^{(1)} - (r_{15})^{(1)}\)

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_{1})^{(2)}, (\sigma_{2})^{(2)}, (\tau_{1})^{(2)}, (\tau_{2})^{(2)}\):**

(d) \(\sigma_{1})^{(2)}, (\sigma_{2})^{(2)}, (\tau_{1})^{(2)}, (\tau_{2})^{(2)}\) four constants satisfying

\[-(\sigma_{2})^{(2)} \leq -(a'_{16})^{(2)} + (a_{17})^{(2)} - (a_{16})^{(2)}(t_{17})t + (\mu_{17})^{(2)}(t_{17})t \leq -(\sigma_{1})^{(2)}\]
\[-(\tau_{2})^{(2)} \leq -(b'_{16})^{(2)} + (b_{17})^{(2)} - (b_{16})^{(2)}((G_{16})t - (b_{17})^{(2)}((G_{16})t \leq -(\tau_{1})^{(2)}\]

**Definition of \((v_{1})^{(2)}, (v_{2})^{(2)}, (u_{1})^{(2)}, (u_{2})^{(2)}\):**

By \((v_{1})^{(2)} > 0, (v_{2})^{(2)} < 0\) and respectively \((u_{1})^{(2)} > 0, (u_{2})^{(2)} < 0\) the roots

(e) of the equations \((a_{17})^{(2)}(v^{(2)})^{2} + (\sigma_{1})^{(2)}v^{(2)} - (a_{16})^{(2)} = 0\)

and \((b_{14})^{(2)}(u^{(2)})^{2} + (\tau_{1})^{(2)}u^{(2)} - (b_{16})^{(2)} = 0\)

**Definition of \((\bar{v}_{1})^{(2)}, (\bar{v}_{2})^{(2)}, (\bar{u}_{1})^{(2)}, (\bar{u}_{2})^{(2)}\):**

By \((\bar{v}_{1})^{(2)} > 0, (\bar{v}_{2})^{(2)} < 0\) and respectively \((\bar{u}_{1})^{(2)} > 0, (\bar{u}_{2})^{(2)} < 0\) the roots

of the equations \((a_{17})^{(2)}(v^{(2)})^{2} + (\sigma_{2})^{(2)}v^{(2)} - (a_{16})^{(2)} = 0\)

and \((b_{17})^{(2)}(u^{(2)})^{2} + (\tau_{2})^{(2)}u^{(2)} - (b_{16})^{(2)} = 0\)

**Definition of \((m_{1})^{(2)}, (m_{2})^{(2)}, (\mu_{1})^{(2)}, (\mu_{2})^{(2)}\):**

(f) If we define \((m_{1})^{(2)}, (m_{2})^{(2)}, (\mu_{1})^{(2)}, (\mu_{2})^{(2)}\) by

\((m_{2})^{(2)} = (v_{0})^{(2)}, (m_{1})^{(2)} = (v_{1})^{(2)}\) if \((v_{0})^{(2)} < (v_{1})^{(2)}\)
\((m_{2})^{(2)} = (v_{1})^{(2)}, (m_{1})^{(2)} = (\bar{v}_{1})^{(2)}\) if \((v_{1})^{(2)} < (v_{0})^{(2)}\)

and \((v_{0})^{(2)} = \frac{v_{16}}{v_{17}}\)

\((m_{2})^{(2)} = (v_{2})^{(2)}, (m_{1})^{(2)} = (v_{0})^{(2)}\) if \((v_{2})^{(2)} < (v_{0})^{(2)}\)

and analogously

\((\mu_{2})^{(2)} = (u_{0})^{(2)}, (\mu_{1})^{(2)} = (u_{1})^{(2)}\) if \((u_{0})^{(2)} < (u_{1})^{(2)}\)
\[
(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_4)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},
\]

and \( (u_0)^{(2)} = \frac{T_0}{T_0} \) 

( \mu_2)^{(2)} = (u_1)^{(2)}, (\mu_4)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)} 

Then the solution satisfies the inequalities

\[
G_0^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16} (t) \leq G_0^0 e^{(S_1)^{(2)}t}
\]

\((p_i)^{(2)}\) is defined

\[
\frac{1}{(m_1)^{(2)}} G_0^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17} (t) \leq \frac{1}{(m_2)^{(2)}} G_0^0 e^{(S_1)^{(2)}t}
\]

\[
\frac{(a_{18})^{(2)}G_0^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})^2} \left[ e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{(-S_2)^{(2)}t} \right] + G_{18}^0 e^{-S_2^{(2)}t} \leq G_{18} (t) \leq
\]

\[
T_0^0 e^{(R_{18})^{(2)}t} \leq T_{16} e^{(R_1)^{(2)}t + (r_{16})^{(2)}t}
\]

\[
\frac{1}{(m_2)^{(2)}} T_0 e^{(R_{16})^{(2)}t} \leq T_{16} e^{(R_1)^{(2)}t + (r_{16})^{(2)}t}
\]

\[
\frac{b_{18}}{(m_1)^{(2)}((R_{18})^{(2)} - (b_{18})^{(2)})^2} \left[ e^{(R_{18})^{(2)}t} - e^{-(b_{18})^{(2)}t} \right] + T_0 e^{-(b_{18})^{(2)}t} \leq T_{18} (t) \leq
\]

\[
\frac{(a_{18})^{(2)}T_0}{(m_2)^{(2)}((R_{18})^{(2)} + (r_{18})^{(2)})^2} \left[ e^{(R_{18})^{(2)}t + (r_{18})^{(2)}t} - e^{-(R_{18})^{(2)}t} \right] + T_0 e^{-(R_{18})^{(2)}t}
\]

**Definition of \((S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}\):**

- \((S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (b_{16})^{(2)}\)
- \((S_2)^{(2)} = (a_{10})^{(2)} - (b_{10})^{(2)}\)
- \((S_2)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16})^{(2)}\)
- \((R_2)^{(2)} = (b_{18})^{(2)} - (r_{18})^{(2)}\)

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\):**

(a) \(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[
-(\sigma_2)^{(3)} \leq -(a_{20}^{(3)} + (a_{21}^{(3)} - (a_{20}^{(3)}(T_{21}, t) + (a_{21}^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}
\]

\[
-(\tau_2)^{(3)} \leq -(b_{20}^{(3)} + (b_{21}^{(3)} - (b_{20}^{(3)}(G, t) - (b_{21}^{(3)}(G_{23}, t) \leq -(\tau_1)^{(3)}
\]

**Definition of \((v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}\):**

(b) By \((v_1)^{(3)} > 0, (v_2)^{(3)} < 0\) and respectively \((u_1)^{(3)} > 0, (u_2)^{(3)} < 0\) the roots of the equations

\[
(a_{21}^{(3)}(v)^{(3)} + (\sigma_1)^{(3)}v - (a_{20}^{(3)} = 0
\]

www.ijsrp.org
and \((b_{21}^{(3)}(u^{(3)})^2 + (\tau_1^{(3)}u^{(3)} - (b_{20}^{(3)}) = 0\) and

By \((\bar{v}_1^{(3)}) > 0, (\bar{v}_2^{(3)}) < 0\) and respectively \((\bar{u}_1^{(3)}) > 0, (\bar{u}_2^{(3)}) < 0\) the roots of the equations \(a_{21}^{(3)}(v^{(3)})^2 + (\sigma_2^{(3)}v^{(3)} - (a_{20}^{(3)}) = 0\)

and \((b_{21}^{(3)}(u^{(3)})^2 + (\tau_2^{(3)}u^{(3)} - (b_{20}^{(3)}) = 0\)

Definition of \((m_1^{(3)}), (m_2^{(3)}), (\mu_1^{(3)}), (\mu_2^{(3)})\):

(c) If we define \((m_1^{(3)}), (m_2^{(3)}), (\mu_1^{(3)}), (\mu_2^{(3)})\) by

\[(m_2^{(3)}) = (v_0^{(3)}, m_1^{(3)}) = (v_1^{(3)}), \text{ if } (v_0^{(3)}) < (v_1^{(3)})\]

\[(m_2^{(3)}) = (v_1^{(3)}, m_1^{(3)}) = (\bar{v}_1^{(3)}), \text{ if } (v_0^{(3)}) < (\bar{v}_1^{(3)})\]

and

\[m_2^{(3)} = (v_0^{(3)}, m_1^{(3)}) = (v_0^{(3)}), \text{ if } (v_0^{(3)}) < (v_0^{(3)})\]

and analogously

\[(\mu_2^{(3)}) = (u_0^{(3)}, \mu_1^{(3)}) = (u_1^{(3)}), \text{ if } (u_0^{(3)}) < (u_1^{(3)})\]

\[(\mu_2^{(3)}) = (u_1^{(3)}, \mu_1^{(3)}) = (\bar{u}_1^{(3)}), \text{ if } (u_0^{(3)}) < (\bar{u}_1^{(3)})\]

Then the solution satisfies the inequalities

\[G_0^{(20)}e^{((s_1^{(3)})-(p_20^{(3)})t)\leq G_20^{(20)}e^{(s_1^{(3)}t)\phantom{1}}\]

\[(p_{10})^{(3)}\text{ is defined}\]

\[
\frac{1}{(m_1^{(3)})(m_2^{(3)}(s_1^{(3)}-(p_20^{(3)})(s_1^{(3)}t)\leq G_21^{(21)}t \leq \frac{1}{(m_2^{(3)})(m_2^{(3)}(s_1^{(3)}-(p_20^{(3)})(s_1^{(3)}t)\leq G_22^{(22)}t \leq G_22^{(22)}e^{-(s_1^{(3)}t)\}
\]

\[T_0^{(20)}e^{(R_1^{(3)})t} \leq T_0^{(20)}e^{(R_1^{(3)}+(R_20^{(3)})t} \]

\[
\frac{1}{(\mu_1^{(3)})(\mu_2^{(3)})(R_1^{(3)}+(R_20^{(3)})t} \leq T_0^{(20)}e^{((R_1^{(3)}+(R_20^{(3)})t} \phantom{1}\]

\[\frac{1}{(\mu_1^{(3)})(R_1^{(3)}+(R_20^{(3)})t} \leq T_0^{(20)}e^{((R_1^{(3)}+(R_20^{(3)})t} \phantom{1} \]

Definition of \((S_1^{(3)}), (S_2^{(3)}), (R_1^{(3)}), (R_2^{(3)})\):

Where \((S_1^{(3)}) = (a_{20}^{(3)}(m_2^{(3)}) - (a_{20}^{(3)})\)

\[(S_2^{(3)}) = (a_{22}^{(3)} - (p_{22}^{(3)})\]

www.ijsrp.org
\[(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{20})^{(3)}\]
\[(R_2)^{(3)} = (b_{22}')(3) - (r_{22})^{(3)}\]

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}:\)

\[d) \quad (\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}\] four constants satisfying

\[-(\sigma_2)^{(4)} \leq -(a_{24})^{(4)} + (a_{25})^{(4)} - (a_{24})^{(4)}(T_{25}, t) + (a_{25})^{(4)}(T_{25}, t) \leq - (\sigma_1)^{(4)}\]
\[-(\tau_2)^{(4)} \leq -(b_{24})^{(4)} + (b_{25})^{(4)} - (b_{24})^{(4)}(G_{27}, t) - (b_{25})^{(4)}((G_{27}, t) \leq - (\tau_1)^{(4)}\]

**Definition of** \((v_1)^{(4)}, (v_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, (v)^{(4)}, u^{(4)}:\)

(e) By \((v_1)^{(4)} > 0, (v_2)^{(4)} < 0\) and respectively \((u_1)^{(4)} > 0, (u_2)^{(4)} < 0\) the roots of the equations \((a_{25})^{(4)}(v)^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0\)
and \((b_{25})^{(4)}(u)^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0\)

**Definition of** \((\bar{v}_1)^{(4)}, (\bar{v}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}:\)

By \((\bar{v}_1)^{(4)} > 0, (\bar{v}_2)^{(4)} < 0\) and respectively \((\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0\) the roots of the equations \((a_{25})^{(4)}(v)^{(4)})^2 + (\sigma_1)^{(4)}v^{(4)} - (a_{24})^{(4)} = 0\)
and \((b_{25})^{(4)}(u)^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0\)

**Definition of** \((m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (v_0)^{(4)}:\)

(f) If we define \((m_2)^{(4)} = (v_0)^{(4)}, (m_1)^{(4)} = (v_1)^{(4)}, \) if \((v_0)^{(4)} < (v_1)^{(4)}\)

\[(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (\bar{v}_1)^{(4)}, \) if \((v_0)^{(4)} < (v_0)^{(4)} < (\bar{v}_1)^{(4)},\]

and \[(v_0)^{(4)} = \frac{v_{24}}{a_{25}}\]

\[(m_2)^{(4)} = (v_1)^{(4)}, (m_1)^{(4)} = (v_0)^{(4)}, \) if \((v_0)^{(4)} < (v_0)^{(4)}\)

and analogously

\[(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \) if \((u_0)^{(4)} < (u_1)^{(4)}\)

\[(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \) if \((u_0)^{(4)} < (u_0)^{(4)} < (u_1)^{(4)},\]

and \[(u_0)^{(4)} = \frac{u_{12}}{\tau_{25}}\]

\[(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \) if \((u_1)^{(4)} < (u_0)^{(4)}\) where \((u_1)^{(4)}, (\bar{u}_1)^{(4)}\)
are defined by 59 and 64 respectively

Then the solution satisfies the inequalities
\[ G_{24}^0 e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{24} e^{(S_1)^{(4)}t} \]

where \((p_{24})^{(4)}\) is defined

\[
\frac{1}{(m_1)^{(4)}} G_{24}^{0} e^{((S_1)^{(4)} - (p_{24})^{(4)})t} \leq G_{25} (t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^{0} e^{(S_1)^{(4)}t} \\
= \left[ \frac{e^{((S_1)^{(4)} - (p_{24})^{(4)})t} - e^{(S_2)^{(4)}t}}{e^{(S_1)^{(4)}t} - e^{(p_{24})^{(4)}t}} \right] + G_{26}^0 e^{- (S_2)^{(4)}t} \leq G_{26} (t) \leq \\
\frac{1}{(m_2)^{(4)}} G_{24}^{0} e^{(S_1)^{(4)}t} \left[ e^{(R_1)^{(4)}t} - e^{-(R_2)^{(4)}t} \right] + T_{26}^0 e^{- (R_2)^{(4)}t} \\
= T_{24}^0 e^{((R_1)^{(4)} + (R_2)^{(4)})t} \]

**Definition of \((S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}):**

Where \((S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24}')^{(4)}\)

\[(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}\]

\[(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24}')^{(4)}\]

\[(R_2)^{(4)} = (b_{26})^{(4)} - (\tau_{26})^{(4)}\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}):**

\[(g) \quad (\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)} \quad \text{four constants satisfying} \]

\[-(\sigma_1)^{(5)} \leq -(a_{28}')^{(5)} + (a_{29})^{(5)} - (a_{28}')^{(5)}(T_{29}, t) + (a_{29})^{(5)}(T_{29}, t) \leq -(\sigma_1)^{(5)} \]

\[-(\tau_2)^{(5)} \leq -(b_{28}')^{(5)} + (b_{29})^{(5)} - (b_{28}')^{(5)}(G_{31}, t) - (b_{29})^{(5)}(G_{31}, t) \leq -(\tau_1)^{(5)} \]

**Definition of \((v_1)^{(5)}, (v_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, v^{(5)}, u^{(5)}):**

\[(h) \quad \text{By} \quad (v_1)^{(5)} > 0, (v_2)^{(5)} < 0 \quad \text{and respectively} \quad (u_1)^{(5)} > 0, (u_2)^{(5)} < 0 \quad \text{the roots of} \quad \text{the equations} \]

\[(a_{20})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{20})^{(5)} = 0 \]

\[(b_{20})^{(5)}(u^{(5)})^2 + (\tau_1)^{(5)}u^{(5)} - (b_{20})^{(5)} = 0 \quad \text{and} \]

**Definition of \((\bar{v}_1)^{(5)}, (\bar{v}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}):**

\[\text{By} \quad (\bar{v}_1)^{(5)} > 0, (\bar{v}_2)^{(5)} < 0 \quad \text{and respectively} \quad (\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0 \quad \text{the roots of} \quad \text{the equations} \]

\[(a_{20})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{20})^{(5)} = 0 \]

www.ijsrp.org
and \((b_{28})^5 (u^5)^2 + (r_{2})^5 u^5 - (b_{28})^5 = 0\)

**Definition of** \((m_1)^5, (m_2)^5, (\mu_1)^5, (\mu_2)^5, (v_0)^5\):

(i) If we define \((m_1)^5, (m_2)^5, (\mu_1)^5, (\mu_2)^5\) by

\[(m_2)^5 = (v_0)^5, (m_1)^5 = (v_1)^5, \text{ if } (v_0)^5 < (v_1)^5\]

\[(m_2)^5 = (v_1)^5, (m_1)^5 = (\bar{v}_1)^5, \text{ if } (v_0)^5 < (\bar{v}_1)^5\]

and

\[(v_0)^5 = \frac{g_{28}^5}{g_{29}^5}\]

\[(m_2)^5 = (v_1)^5, (m_1)^5 = (v_0)^5, \text{ if } (\bar{v}_1)^5 < (v_0)^5\]

and analogously

\[(\mu_2)^5 = (u_0)^5, (\mu_1)^5 = (u_1)^5, \text{ if } (u_0)^5 < (u_1)^5\]

\[(\mu_2)^5 = (u_1)^5, (\mu_1)^5 = (\bar{u}_1)^5, \text{ if } (u_0)^5 < (\bar{u}_1)^5\]

and

\[u_0)^5 = \frac{g_{28}^5}{g_{29}^5}\]

\[(\mu_2)^5 = (u_1)^5, (\mu_1)^5 = (u_0)^5, \text{ if } (\bar{u}_1)^5 < (u_0)^5\]

where \((u_1)^5, (\bar{u}_1)^5\) are defined respectively.

Then the solution satisfies the inequalities

\[G_{20}^0 e^{((s_1)^5-(p_{28})^5) t} \leq G_{20}^0 e^{(s_1)^5 t} \leq G_{20}^0 e^{(s_1)^5 t}\]

where \((p_1)^5\) is defined.

\[\frac{1}{(m_2)^5} G_{20}^0 e^{((s_1)^5-(p_{28})^5) t} \leq G_{20}^0 e^{(s_1)^5 t} \leq \frac{1}{(m_2)^5} G_{20}^0 e^{(s_1)^5 t}\]

\[\frac{g_{28}^5 G_{10}^0 e^{(s_1)^5-(p_{28})^5) t}}{(m_1)^5 G_{20}^0 e^{(s_1)^5-(p_{28})^5) t}} e^{((s_1)^5-(p_{28})^5) t} - e^{-(s_2)^5 t]\]

\[\leq G_{20}^0 e^{(s_1)^5 t}\]

\[\leq G_{20}^0 e^{(s_1)^5 t}\]

\[\frac{G_{0}^0 e^{(s_1)^5} t}{T_{28}^0} \leq T_{28}^0 e^{((r_1)^5+(p_{28})^5) t}\]

\[\frac{1}{(\mu_1)^5} T_{28}^0 e^{((r_1)^5+(p_{28})^5) t} \leq T_{28}^0 e^{((r_1)^5+(p_{28})^5) t}\]

\[\frac{g_{28}^5 p_{30}^5}{(\mu_1)^5 (R_{1})^5 (b_{30})^5} e^{(r_1)^5} t - e^{-(b_{30})^5 t}\]

\[\leq T_{30}^0 e^{-(b_{30})^5 t}\]

**Definition of** \((s_1)^5, (s_2)^5, (R_1)^5, (R_2)^5\):

Where \((s_1)^5 = (a_{28})^5 (m_2)^5 - (a_{28})^5\)

\[(s_2)^5 = (a_{28})^5 (m_2)^5 - (a_{28})^5\]

\[(R_1)^5 = (b_{28})^5 (\mu_2)^5 - (b_{28})^5\]

\[(R_2)^5 = (b_{28})^5 (\mu_2)^5 - (b_{28})^5\]

www.ijsrp.org
Behavior of the solutions

If we denote and define

**Definition of** \((\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}:\)

\((j)\) \((\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}\) four constants satisfying

\[-(\sigma_2)^{(6)} \leq -(a_{32}^{(6)})^2 + (a_{33}^{(6)})^2 (T_{33}, t) + (a_{33}^{(6)})^2 T_{33}, t \leq -(\sigma_1)^{(6)}\]

\[-(\tau_2)^{(6)} \leq -(b_{32}^{(6)})^2 + (b_{33}^{(6)})^2 (G_{33}, t) - (b_{33}^{(6)})^2 (G_{33}, t) \leq -(\tau_1)^{(6)}\]

**Definition of** \((v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}:\)

\((k)\) By \((v_1)^{(6)} > 0, (v_2)^{(6)} < 0\) and respectively \((u_1)^{(6)} > 0, (u_2)^{(6)} < 0\) the roots of the equations

\((a_{32}^{(6)}(v^{(6)})^2 + (a_{33}^{(6)})^2 v^{(6)} - (a_{32}^{(6)})^2 = 0\)

and \((b_{32}^{(6)}(u^{(6)})^2 + (b_{33}^{(6)})^2 u^{(6)} - (b_{32}^{(6)})^2 = 0\)

**Definition of** \((\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}:\)

By \((\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0\) and respectively \((\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0\) the roots of the equations

\((a_{33}^{(6)}(v^{(6)})^2 + (a_{32}^{(6)})^2 v^{(6)} - (a_{33}^{(6)})^2 = 0\)

and \((b_{33}^{(6)}(u^{(6)})^2 + (b_{32}^{(6)})^2 u^{(6)} - (b_{33}^{(6)})^2 = 0\)

**Definition of** \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}:\)

\((l)\) If we define \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}\) by

\[(m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{if} \ (v_0)^{(6)} < (v_1)^{(6)}\]

\[(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (\bar{v}_0)^{(6)}, \text{if} \ (v_0)^{(6)} < (\bar{v}_1)^{(6)}, \text{and} \]

\[(V_0)^{(6)} = \frac{\sigma_2^{(6)}}{\sigma_3^{(6)}}\]

\[(m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{if} \ (\bar{v}_1)^{(6)} < (v_0)^{(6)}\]

and analogously

\[(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{if} \ (u_0)^{(6)} < (u_1)^{(6)}\]

\[(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_0)^{(6)}, \text{if} \ (u_0)^{(6)} < (\bar{u}_1)^{(6)}, \text{and} \]

\[(U_0)^{(6)} = \frac{\sigma_2^{(6)}}{\sigma_3^{(6)}}\]

\[(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{if} \ (\bar{u}_1)^{(6)} < (u_0)^{(6)}\]

where \((u_1)^{(6)}, (\bar{u}_1)^{(6)}\) are defined respectively

Then the solution satisfies the inequalities

\[G_0^{(6)} e^{(S_1)^{(6)} - (P_{32})^{(6)}} t \leq G_0^{(6)} e^{(S_1)^{(6)}}, \text{where} \ (P_i)^{(6)} \text{is defined} \]

\[\frac{1}{(m_1)^{(6)}} \frac{G_0^{(6)} e^{(S_1)^{(6)} - (P_{32})^{(6)}} t}{(m_2)^{(6)}} \leq G_0^{(6)} e^{(S_1)^{(6)}} \text{and} \]

\[\frac{1}{(m_2)^{(6)}} \frac{G_0^{(6)} e^{(S_1)^{(6)} - (P_{32})^{(6)}} t}{(m_1)^{(6)}} \leq G_0^{(6)} e^{(S_1)^{(6)}}\]
\[
\left(\frac{a_{32}}{m_1}\right)^6 \leq \left(\frac{a_{32} - p_{32}}{m_2}\right)^6 \left(e^{(S_1(t) - p_{32})t} - e^{-(S_2(t))t}\right) + G_{34}^0 e^{-(S_2(t))t} \leq G_{34}(t) \leq \left(\frac{a_{32}}{m_1}\right)^6 \end{equation}
\]

\[
\begin{align*}
T_{32}^0 e^{(R_1(t))t} & \leq T_{32}^0 e^{((R_1) + (R_2))t} \\
\frac{1}{\mu_1^6} T_{32}^0 e^{(R_1(t))t} & \leq \frac{1}{\mu_2^6} T_{32}^0 e^{((R_1) + (R_2))t} \\
\frac{a_{34}^6}{(R_1(t)) + (R_2(t))} & \leq \frac{a_{34}^6}{(R_1(t)) + (R_2(t))} \left(e^{(R_1(t))t} - e^{-(R_2(t))t}\right) + T_{34}^0 e^{-(R_2(t))t} \leq T_{34}(t) \leq \end{align*}
\]

**Definition of** \( (S_1(t), S_2(t), R_1(t), R_2(t)) \):

Where \( (S_1(t)) = (a_{32} - m_2)(a_{32} - p_{34}) \)

\( (S_2(t)) = (a_{34} - p_{34}) \)

\( (R_1(t)) = (b_{32} - \mu_2)(a_{34} - p_{34}) \)

\( (R_2(t)) = (b_{34} - \mu_2)(a_{34} - p_{34}) \)

**Proof:** From GLOBAL EQUATIONS we obtain

\[
\frac{dx^{(1)}}{dt} = (a_{13}^{(1)} - (a_{14}^{(1)} + (a_{13}^{(1)})(T_{14}, t)) - (a_{14}^{(1)})(T_{14}, t)v^{(1)} - (a_{14}^{(1)}v^{(1)})
\]

**Definition of** \( v^{(1)} \)

\[
\begin{align*}
\text{It follows} \quad -\left( (a_{14}^{(1)})v^{(1)} \right)^2 + (\sigma_2^{(1)})v^{(1)} - (a_{13}^{(1)}) \leq \frac{dx^{(1)}}{dt} & \leq -\left( (a_{14}^{(1)})v^{(1)} \right)^2 + (\sigma_1^{(1)})v^{(1)} - (a_{13}^{(1)}) \\
\end{align*}
\]

From which one obtains

**Definition of** \( (\bar{v}_1)^{(1)}, (v_0)^{(1)} \):

\[
\begin{align*}
(a) & \text{ For } 0 < \left(\frac{v_0}{\bar{v}_1}\right) = \frac{C_{14}}{C_{13}} < (v_1)^{(1)} < (\bar{v}_1)^{(1)} \\
\text{it follows} \quad (v_0)^{(1)} & \leq v^{(1)}(t) \leq (v_1)^{(1)} \\
\text{In the same manner} \quad \text{we get} \\
\text{In the same manner} \\
\end{align*}
\]

www.ijsrp.org
From which we deduce $(v_0)^{(1)} \leq v^{(1)}(t) \leq (\bar{v}_1)^{(1)}$

(b) If $0 < (v_2)^{(1)} < (v_0)^{(1)} = \frac{g_1^b}{g_1^d} < (\bar{v}_1)^{(1)}$ we find like in the previous case,

$$(v_1)^{(1)} \leq \frac{(v_1)^{(1)} + (c_1)^{(1)}(v_2)^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1}{1 + c_1^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1} \leq v^{(1)}(t) \leq \frac{(v_1)^{(1)} + (c_1)^{(1)}(v_2)^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1}{1 + c_1^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1} \leq (\bar{v}_1)^{(1)}$$

(c) If $0 < (v_1)^{(1)} \leq (\bar{v}_2)^{(1)} \leq \frac{g_1^b}{g_1^d}$, we obtain

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq \frac{(v_1)^{(1)} + (c_1)^{(1)}(v_2)^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1}{1 + c_1^{(1)} - (a_1)^{(1)}(1) \cdot (v_1)^{(1)} - (v_2)^{(1)} + 1} \leq (v_0)^{(1)}$$

And so with the notation of the first part of condition (c), we have

**Definition of** $v^{(1)}(t)$ :-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad v^{(1)}(t) = \frac{g_1^b(t)}{g_1^d(t)}$$

In a completely analogous way, we obtain

**Definition of** $u^{(1)}(t)$ :-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad u^{(1)}(t) = \frac{T_1^b(t)}{T_1^d(t)}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If $(a_1)^{(1)} = (a_1)^{(1)}$, then $(\sigma_1)^{(1)} = (\sigma_1)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $g_13(t) = (v_0)^{(1)}g_14(t)$ this also defines $(v_0)^{(1)}$ for the special case

Analogously if $(b_1)^{(1)} = (b_1)^{(1)}$, then $(\tau_1)^{(1)} = (\tau_1)^{(1)}$ and then

$$(u_1)^{(1)} = (\bar{u}_1)^{(1)}$$

if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_13(t) = (u_0)^{(1)}T_14(t)$.

This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

we obtain

$$\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - ((a_{16})^{(2)}(a_{17})^{(2)} + (a_{17})^{(2)}(T_17,t) - (a_{17})^{(2)}(T_17,t)v^{(2)} - (a_{17})^{(2)}v^{(2)}$$

**Definition of** $v^{(2)}$ :-

$$v^{(2)} = \frac{G_{16}}{G_{17}}$$

It follows
\[-\left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16})^{(2)} \right) \leq \frac{dv^{(2)}}{dt} \leq -\left( (a_{17})^{(2)}(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16})^{(2)} \right)\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(2)}, (v_0)^{(2)} \):\)

\[(d) \text{ For } 0 < (v_0)^{(2)} = \frac{G_0}{G_1} < (\bar{v}_1)^{(2)} < \frac{G_6}{G_7}, \text{ we find like in the previous case,} \]

\[v^{(2)}(t) \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \quad \frac{(C)^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}\]

it follows \((v_0)^{(2)} \leq v^{(2)}(t) \leq (v_1)^{(2)}\)

In the same manner, we get

\[v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \quad \frac{(\bar{C})^{(2)} = \frac{(v_1)^{(2)} - (v_0)^{(2)}}{(v_0)^{(2)} - (v_2)^{(2)}}\]

From which we deduce \((v_0)^{(2)} \leq v^{(2)}(t) \leq (\bar{v}_1)^{(2)}\)

\[(e) \text{ If } 0 < (v_1)^{(2)} < (v_0)^{(2)} = \frac{G_6}{G_7} < (\bar{v}_1)^{(2)} \text{ we find like in the previous case,} \]

\[(v_1)^{(2)} \leq \frac{(v_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \quad \frac{(v_1)^{(2)} = (v_0)^{(2)} = \frac{G_6}{G_7}, }{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \end{array}\]

\[(f) \text{ If } 0 < (v_2)^{(2)} \leq (\bar{v}_2)^{(2)} < (v_0)^{(2)} = \frac{G_6}{G_7}, \text{ we obtain} \]

\[(v_1)^{(2)} \leq v^{(2)}(t) \leq \frac{(\bar{v}_1)^{(2)} + (C)^{(2)}(v_2)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \quad \frac{(v_1)^{(2)} = (v_0)^{(2)} = \frac{G_6}{G_7}, }{1 + (C)^{(2)}e^{-[a_{17}]^{(2)}[v_1^{(2)} - (v_0)^{(2)}]}}, \end{array}\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(2)}(t) \):\)

\[(m_2)^{(2)} \leq v^{(2)}(t) \leq (m_1)^{(2)} \end{array}, \quad \frac{(v_1)^{(2)} = \frac{G_16(t)}{G_17(t)}\]

In a completely analogous way, we obtain

**Definition of** \(u^{(2)}(t) \):\)

\[(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)} \end{array}, \quad \frac{(u_1)^{(2)} = \frac{T_16(t)}{T_17(t)}\]

**Particular case:**

If \((a_{16})^{(2)} = (a_{17})^{(2)} = (\sigma_1)^{(2)} = (\sigma_2)^{(2)}\) and in this case \((v_1)^{(2)} = (\bar{v}_1)^{(2)}\) if in addition \((v_0)^{(2)} = (v_1)^{(2)}\) then \(v^{(2)}(t) = (v_0)^{(2)}\) and as a consequence \(G_16(t) = (v_0)^{(2)}G_17(t)\)

Analogously if \((b_{16})^{(2)} = (b_{17})^{(2)} = (\tau_1)^{(2)} = (\tau_2)^{(2)}\) and then

\[(u_1)^{(2)} = (\bar{u}_1)^{(2)}\) if in addition \((u_0)^{(2)} = (u_1)^{(2)}\) then \(T_16(t) = (u_0)^{(2)}T_17(t)\) This is an important
consequence of the relation between \( (v_1)^{(2)} \) and \( (\tilde{v}_1)^{(2)} \)

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - (a_{20}')^{(3)} - (a_{21})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}, t) - (a_{21}^{''})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)}
\]

**Definition of** \( v^{(3)} \) :.

\[
v^{(3)} = \frac{g_{20}}{g_{21}}
\]

It follows

\[
-(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)} \leq \frac{dv^{(3)}}{dt} \leq -(a_{21})^{(3)}(v^{(3)})^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}
\]

From which one obtains

(a) For \( 0 < (v_0)^{(3)} = \frac{g_0}{g_{21}} < (v_1)^{(3)} < (\tilde{v}_1)^{(3)} \)

\[
v^{(3)}(t) \geq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}}{1 + (C)^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_0)^{(3)}]t}} \leq (C)^{(3)} = \frac{(v_1)^{(3)} - (v_0)^{(3)}}{(v_0)^{(3)} - (v_2)^{(3)}}
\]

it follows \( (v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)} \)

In the same manner, we get

\[
v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (\tilde{C})^{(3)}[v_2)^{(3)}]}{1 + (\tilde{C})^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq \frac{(v_1)^{(3)} + (\tilde{C})^{(3)}}{1 + (\tilde{C})^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq (\tilde{v}_1)^{(3)}
\]

**Definition of** \( (\tilde{v}_1)^{(3)} \) :.

From which we deduce \( (v_0)^{(3)} \leq v^{(3)}(t) \leq (\tilde{v}_1)^{(3)} \)

(b) If \( 0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{g_0}{g_{21}} < (\tilde{v}_1)^{(3)} \) we find like in the previous case,

\[
(v_1)^{(3)} \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}}{1 + (C)^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_0)^{(3)}]t}} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (\tilde{C})^{(3)}(v_2)^{(3)}]}{1 + (\tilde{C})^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq (\tilde{v}_1)^{(3)}
\]

(c) If \( 0 < (v_1)^{(3)} \leq (\tilde{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{g_0}{g_{21}}, \) we obtain

\[
(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(v_1)^{(3)} + (C)^{(3)}(v_2)^{(3)}]}{1 + (C)^{(3)}e^{-(a_{21})^{(3)}[(v_1)^{(3)} - (v_2)^{(3)}]t}} \leq (v_0)^{(3)}
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(3)}(t) \) :
In a completely analogous way, we obtain

**Definition of** \( u^{(3)}(t) \) :

\[
(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}, \quad v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case :**

If \((a_{20}^{(3)}) = (a_{21}^{(3)})\), then \((\sigma_1)^{(3)} = (\sigma_2)^{(3)}\) and in this case \((v_1)^{(3)} = (\bar{v}_1)^{(3)}\) if in addition \((v_0)^{(3)} = (v_1)^{(3)}\) then \(v^{(3)}(t) = (v_0)^{(3)}\) and as a consequence \(G_{20}(t) = (v_0)^{(3)}G_{21}(t)\)

Analogously if \((b_{20}^{(3)}) = (b_{21}^{(3)})\), then \((\tau_1)^{(3)} = (\tau_2)^{(3)}\) and then \((u_1)^{(3)} = (\bar{u}_1)^{(3)}\) if in addition \((u_0)^{(3)} = (u_1)^{(3)}\) then \(T_{20}(t) = (u_0)^{(3)}T_{21}(t)\) This is an important consequence of the relation between \((v_1)^{(3)}\) and \((\bar{v}_1)^{(3)}\)

**From GLOBAL EQUATIONS we obtain**

\[
\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a_{24}')^{(4)} - (a_{25})^{(4)} + (a_{25}')^{(4)}(T_{25}, t)\right) - (a_{25}''(4)(T_{25}, t)v^{(4)} - (a_{25}''(4)v^{(4)}
\]

**Definition of** \( v^{(4)} \) :

\[
v^{(4)} = \frac{G_{24}}{G_{25}}
\]

**It follows**

\[
-(a_{25}^{(4)}v^{(4)})^2 + (\sigma_2^{(4)}v^{(4)} - (a_{24}^{(4)})) \leq \frac{dv^{(4)}}{dt} \leq -(a_{25}^{(4)}v^{(4)})^2 + (\sigma_4^{(4)}v^{(4)} - (a_{24}^{(4)}))
\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(4)}\), \((v_0)^{(4)}\) :

**For** 0 < \((v_0)^{(4)} = \frac{G_{24}}{G_{25}} < (v_1)^{(4)} < (\bar{v}_1)^{(4)}\)

\[
v^{(4)}(t) \geq \frac{(v_1)^{(4)} + (C)^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)}(v_1)^{(4)} - (v_0)^{(4)}))t}}{4 + (C)^{(4)}e^{-(a_{25}^{(4)}(v_1)^{(4)} - (v_0)^{(4)))}}
\]

it follows \((v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)}\)

**In the same manner**, we get

\[
v^{(4)}(t) \leq \frac{(\bar{v}_1)^{(4)} + (\bar{C})^{(4)}(v_2)^{(4)}e^{-(a_{25}^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)}))t}}{4 + (\bar{C})^{(4)}e^{-(a_{25}^{(4)}(\bar{v}_1)^{(4)} - (\bar{v}_2)^{(4)))}}
\]

From which we deduce \((v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)}\)

**If** 0 < \((v_1)^{(4)} < (v_0)^{(4)} = \frac{G_{24}}{G_{25}} < (\bar{v}_1)^{(4)}\) we find like in the previous case,
\[ (v_1)^{(4)} \leq \frac{(v_3)^{(4)} + (v_4)^{(4)} e^{-(\alpha_{25})[(v_1)^{(4)} - (v_2)^{(4)}] t}}{1 + (\alpha_{25}) e^{-(\alpha_{25})[(v_1)^{(4)} - (v_2)^{(4)}] t}} \leq v^{(4)}(t) \leq \frac{(\tilde{v}_1)^{(4)} + (\tilde{v}_2)^{(4)} e^{-(\alpha_{25})[(v_3)^{(4)} - (v_2)^{(4)}] t}}{1 + (\alpha_{25}) e^{-(\alpha_{25})[(v_3)^{(4)} - (v_2)^{(4)}] t}} \leq (v_0)^{(4)} \]

(f) If \( 0 < (v_1)^{(4)} \leq (\tilde{v}_1)^{(4)} \leq (v_0)^{(4)} = \frac{G'_2}{G_2} \), we obtain

\[ (v_1)^{(4)} \leq v^{(4)}(t) \leq \frac{(v_3)^{(4)} + (v_4)^{(4)} e^{-(\alpha_{25})[(v_1)^{(4)} - (v_2)^{(4)}] t}}{1 + (\alpha_{25}) e^{-(\alpha_{25})[(v_1)^{(4)} - (v_2)^{(4)}] t}} \leq (v_0)^{(4)} \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( v^{(4)}(t) \):

\[ (m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad v^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(4)}(t) \):

\[ (\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \( (a''_{24})^{(4)} = (a''_{25})^{(4)} \), then \( (\sigma_1)^{(4)} = (\sigma_2)^{(4)} \) and in this case \( (v_1)^{(4)} = (\tilde{v}_1)^{(4)} \) if in addition \( (v_0)^{(4)} = (v_1)^{(4)} \) then \( v^{(4)}(t) = (v_0)^{(4)} \) and as a consequence \( G_{24}(t) = (v_0)^{(4)} G_{25}(t) \) this also defines \( (v_0)^{(4)} \) for the special case.

Analogously if \( (b''_{24})^{(4)} = (b''_{25})^{(4)} \), then \( (\tau_1)^{(4)} = (\tau_2)^{(4)} \) and then \( (\mu_1)^{(4)} = (\tilde{\mu}_1)^{(4)} \) if in addition \( (u_0)^{(4)} = (u_1)^{(4)} \) then \( T_{24}(t) = (u_0)^{(4)} T_{25}(t) \) This is an important consequence of the relation between \( (v_1)^{(4)} \) and \( (\tilde{v}_1)^{(4)} \), and definition of \( (u_0)^{(4)} \).

From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - (a'_{28})^{(5)} - (a''_{28})^{(5)}(T_{28}, t) - (a_{29})^{(5)}(T_{29}, t) v^{(5)} - (a_{20})^{(5)} v^{(5)} \]

**Definition of** \( v^{(5)} \):

\[ v^{(5)} = \frac{G_{20}}{G_{28}} \]

It follows

\[ -(a_{29})^{(5)} v^{(5)} - (a_{28})^{(5)} \leq \frac{dv^{(5)}}{dt} \leq -(a_{29})^{(5)} v^{(5)} + (a_{28})^{(5)} v^{(5)} - (a_{20})^{(5)} v^{(5)} \]

From which one obtains

**Definition of** \( (\tilde{v}_1)^{(5)}, (v_0)^{(5)} \):

\[ (g) \text{ For } 0 < (v_0)^{(5)} = \frac{G'_2}{G_{25}} < (v_1)^{(5)} < (\tilde{v}_1)^{(5)} \]
\[ \nu^{(5)}(t) = \frac{\nu_1^{(5)} + (\nu_2^{(5)})_e^{-[(a_{29})^{(5)}(v_1^{(5)})-(\nu_0^{(5)})]} \left[ e \right]}{5 + (C^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]}],} \quad \left( C^{(5)} = \frac{\nu_1^{(5)} - (v_0^{(5)})}{(v_0^{(5)}) - (v_2^{(5)})} \right) \]

It follows \( (\nu_0^{(5)}) \leq \nu^{(5)}(t) \leq (\nu_1^{(5)}) \)

In the same manner, we get

\[ \nu^{(5)}(t) \leq \left( \frac{(v_1^{(5)}) + (\nu_2^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]} \left[ e \right]}{5 + (C^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]}],} \right) \]

From which we deduce \( (\nu_0^{(5)}) \leq \nu^{(5)}(t) \leq (\nu_1^{(5)}) \)

(h) If \( 0 < (\nu_1^{(5)}) < (\nu_0^{(5)}) = \frac{\nu_0}{\nu_2^{(5)}} < (\nu_2^{(5)}) \) we find like in the previous case,

\[ (\nu_1^{(5)}) \leq \left( \frac{(v_1^{(5)}) + (\nu_2^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]} \left[ e \right]}{5 + (C^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]}],} \right) \leq (\nu_0^{(5)}) \]

(i) If \( 0 < (\nu_1^{(5)}) \leq (\nu_2^{(5)}) \leq \left( \frac{(v_0^{(5)}) = \frac{\nu_0}{\nu_2^{(5)}}}, \right) \)

we obtain

\[ (\nu_0^{(5)}) \leq \nu^{(5)}(t) \leq \left( \frac{(v_1^{(5)}) + (\nu_2^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]} \left[ e \right]}{5 + (C^{(5)})_e^{-[(a_{29})(v_1^{(5)})-(\nu_0^{(5)})]}],} \right) \leq (\nu_0^{(5)}) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \( \nu^{(5)}(t) := \)

\[ (m_2^{(5)}) \leq \nu^{(5)}(t) \leq \left( m_1^{(5)} \right), \quad \nu^{(5)}(t) = \frac{G_{29}(t)}{G_{29}(t)} \]

In a completely analogous way, we obtain

**Definition of** \( u^{(5)}(t) := \)

\[ (\mu_2^{(5)}) \leq u^{(5)}(t) \leq \left( \mu_1^{(5)} \right), \quad u^{(5)}(t) = \frac{T_{29}(t)}{T_{29}(t)} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( (a_{29}^{(5)}) = (a_{29}^{(5)}) \), then \( (\sigma_1^{(5)}) = (\sigma_2^{(5)}) \) and in this case \( (\nu_1^{(5)}) = (\nu_3^{(5)}) \) if in addition \( (\nu_0^{(5)}) = (\nu_3^{(5)}) \) then \( \nu^{(5)}(t) = (\nu_0^{(5)}) \) as a consequence \( G_{29}(t) = (\nu_0^{(5)}) \) \( G_{29}(t) \) this also defines \( (\nu_0^{(5)}) \) for the special case.

Analogously if \( (b_{29}^{(5)}) = (b_{29}^{(5)}) \), then \( (\tau_1^{(5)}) = (\tau_2^{(5)}) \) and then \( (\nu_1^{(5)}) = (\nu_3^{(5)}) \) if in addition \( (\nu_0^{(5)}) = (\nu_3^{(5)}) \) then \( T_{29}(t) = (\nu_0^{(5)}) \) \( T_{29}(t) \) This is an important consequence of the relation between \( (\nu_1^{(5)}) \) and \( (\nu_3^{(5)}) \), and definition of \( (\nu_0^{(5)}) \).

we obtain
\[
\frac{dv^6(t)}{dt} = (a_{32}^6)^{(6)} - \left((a_{32}^6)^{(6)} - (a_{32}^6)^{(6)} + (a_{32}^6)^{(6)}(T_{33}, t) - (a_{32}^6)^{(6)}(T_{33}, t)v^6(t) - (a_{32}^6)^{(6)}v^6(t)\right)
\]

\text{Definition of } v^6(t) : \quad v^6(t) = \frac{M_2}{M_3}

It follows

\[- \left((a_{32}^6)^{(6)}v^6(t)^2 + (\sigma_2^6)\right) v^6(t) - (a_{32}^6)^{(6)} \leq \frac{dv^6(t)}{dt} \leq - \left((a_{32}^6)^{(6)}v^6(t)^2 + (\sigma_1^6)\right) v^6(t) - (a_{32}^6)^{(6)}\]

From which one obtains

\text{Definition of } (\bar{v}_1)^{(6)}, (v_0)^{(6)} : 

\[(j) \quad \text{For } 0 < (v_0)^{(6)} = \frac{\theta_2}{\theta_3} < (v_1)^{(6)} < (\bar{v}_1)^{(6)} \]

\[v^6(t) \geq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \quad \bar{v}^6(t) \geq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \]

it follows \((v_0)^{(6)} \leq v^6(t) \leq (v_1)^{(6)}\)

In the same manner, we get

\[v^6(t) \leq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \quad \bar{v}^6(t) \leq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \]

From which we deduce \((v_0)^{(6)} \leq v^6(t) \leq (v_1)^{(6)}\)

\[(k) \quad \text{If } 0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{\theta_2}{\theta_3} < (\bar{v}_1)^{(6)} \text{ we find like in the previous case,} \]

\[v^6(t) \leq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \quad \bar{v}^6(t) \leq \frac{(v_1)^{(6)}(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_0)^{(6)}]}{1+(C(t)v_2)^{(6)}e^{-[a_{33}^6]((v_1)^{(6)}-v_2)^{(6)}]}}, \]

\[
\text{And so with the notation of the first part of condition (c), we have}
\]

\text{Definition of } v^{(6)}(t) : 

\[m_2^{(6)} \leq v^{(6)}(t) \leq m_1^{(6)} \quad v^{(6)}(t) = \frac{M_2}{M_3}
\]

In a completely analogous way, we obtain

\text{Definition of } u^{(6)}(t) : 

\[\mu_2^{(6)} \leq u^{(6)}(t) \leq \mu_1^{(6)} \quad u^{(6)}(t) = \frac{M_2}{M_3}
\]

www.ijsrp.org
Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_{12}''')^{(6)} = (a_{22}''')^{(6)}\), then \((σ_1)^{(6)} = (σ_2)^{(6)}\) and in this case \((v_1)^{(6)} = (v_2)^{(6)}\) if in addition \((v_0)^{(6)} = (v_1)^{(6)}\) then \(v^{(6)}(t) = (v_0)^{(6)}\) and as a consequence \(G_{32}(t) = (v_0)^{(6)}G_{33}(t)\) this also defines \((v_0)^{(6)}\) for the special case.

Analogously if \((b_{12}''')^{(6)} = (b_{22}''')^{(6)}\), then \((τ_1)^{(6)} = (τ_2)^{(6)}\) and then \((u_1)^{(6)} = (u_2)^{(6)}\) if in addition \((u_0)^{(6)} = (u_1)^{(6)}\) then \(T_{32}(t) = (u_0)^{(6)}T_{33}(t)\) This is an important consequence of the relation between \((v_1)^{(6)}\) and \((v_2)^{(6)}\), and definition of \((u_0)^{(6)}\).

We can prove the following

**Theorem 3:** If \((a_i^{(1)})\) and \((b_i^{(1)})\) are independent on \(t\) , and the conditions

\[
(a_{13}'^{(1)}(a_{14}'^{(1)} - (a_{13}'^{(1)}(a_{14}'))^{(1)}) < 0 \\
(a_{13}'^{(1)}(a_{14}'^{(1)} - (a_{13}'^{(1)}(a_{14}'))^{(1)} + (a_{13}'^{(1)}(p_{13}'))^{(1)} + (a_{14}'^{(1)}(p_{14}'))^{(1)} + (p_{13}')^{(1)}(p_{14}')^{(1)}) > 0 \\
(b_{13}'^{(1)}(b_{14}'^{(1)} - (b_{13}'^{(1)}(b_{14}'))^{(1)}) > 0 , \\
(b_{13}'^{(1)}(b_{14}'^{(1)} - (b_{13}'^{(1)}(b_{14}'))^{(1)} - (b_{13}'^{(1)}(r_{14}'))^{(1)} - (b_{14}'^{(1)}(r_{14}'))^{(1)} + (r_{13}'^{(1)}(r_{14}'))^{(1)} < 0 \]

with \((p_{13})^{(1)}, (r_{14})^{(1)}\) as defined, then the system

If \((a''^{(2)})\) and \((b''^{(2)})\) are independent on \(t\) , and the conditions

\[
(a_{16}'^{(2)}(a_{17}'^{(2)} - (a_{16}'^{(2)}(a_{17}'))^{(2)}) < 0 \\
(a_{16}'^{(2)}(a_{17}'^{(2)} - (a_{16}'^{(2)}(a_{17}'))^{(2)} + (a_{16}'^{(2)}(p_{16}'))^{(2)} + (a_{17}'^{(2)}(p_{17}'))^{(2)} + (p_{16}')^{(2)}(p_{17}')^{(2)}) > 0 \\
(b_{16}'^{(2)}(b_{17}'^{(2)} - (b_{16}'^{(2)}(b_{17}'))^{(2)}) > 0 , \\
(b_{16}'^{(2)}(b_{17}'^{(2)} - (b_{16}'^{(2)}(b_{17}'))^{(2)} - (b_{16}'^{(2)}(r_{17}'))^{(2)} - (b_{17}'^{(2)}(r_{17}'))^{(2)} + (r_{16}'^{(2)}(r_{17}'))^{(2)} < 0 \]

with \((p_{16})^{(2)}, (r_{17})^{(2)}\) as defined are satisfied , then the system

If \((a''^{(3)})\) and \((b''^{(3)})\) are independent on \(t\) , and the conditions

\[
(a_{20}'^{(3)}(a_{21}'^{(3)} - (a_{20}'^{(3)}(a_{21}'))^{(3)}) < 0 \\
(a_{20}'^{(3)}(a_{21}'^{(3)} - (a_{20}'^{(3)}(a_{21}'))^{(3)} + (a_{20}'^{(3)}(p_{20}'))^{(3)} + (a_{21}'^{(3)}(p_{21}'))^{(3)} + (p_{20}')^{(3)}(p_{21}')^{(3)}) > 0 \\
(b_{20}'^{(3)}(b_{21}'^{(3)} - (b_{20}'^{(3)}(b_{21}'))^{(3)}) > 0 , \\
(b_{20}'^{(3)}(b_{21}'^{(3)} - (b_{20}'^{(3)}(b_{21}'))^{(3)} - (b_{20}'^{(3)}(r_{21}'))^{(3)} - (b_{21}'^{(3)}(r_{21}'))^{(3)} + (r_{20}')^{(3)}(r_{21}')^{(3)} < 0 \]

with \((p_{20})^{(3)}, (r_{21})^{(3)}\) as defined are satisfied , then the system

If \((a''^{(4)})\) and \((b''^{(4)})\) are independent on \(t\) , and the conditions

\[
(a_{24}'^{(4)}(a_{25}'^{(4)} - (a_{24}'^{(4)}(a_{25}'))^{(4)}) < 0 \\
(a_{24}'^{(4)}(a_{25}'^{(4)} - (a_{24}'^{(4)}(a_{25}'))^{(4)} + (a_{24}'^{(4)}(p_{24}'))^{(4)} + (a_{25}'^{(4)}(p_{25}'))^{(4)} + (p_{24}')^{(4)}(p_{25}')^{(4)}) > 0 \]

www.ijsrp.org
\[(b_{24}^*(4)b_{25}^*(4) - (b_{24}^*(4)b_{25}^*(4)) > 0,\]
\[(b_{24}^*(4)b_{25}^*(4) - (b_{24}^*(4)b_{25}^*(4)) - (b_{24}^*(4)r_{25}^*(4)) - (b_{25}^*(4)r_{25}^*(4)) + (r_{24}^*(4)r_{25}^*(4)) < 0\]

with \((p_{24}^*(4), r_{25}^*(4))\) as defined are satisfied, then the system

if \((a_i^*(5) \text{ and } h_i^*(5))\) are independent on \(t\), and the conditions

\[(a_{28}^*(5)a_{29}^*(5) - (a_{28}^*(5)a_{29}^*(5)) < 0\]
\[(a_{28}^*(5)a_{29}^*(5) - (a_{28}^*(5)a_{29}^*(5)) + (a_{28}^*(5)p_{28}^*(5) + (a_{29}^*(5)p_{28}^*(5)) + (p_{28}^*(5)p_{29}^*(5)) > 0\]
\[(b_{28}^*(5)b_{29}^*(5) - (b_{28}^*(5)b_{29}^*(5)) > 0,\]
\[(b_{28}^*(5)b_{29}^*(5) - (b_{28}^*(5)b_{29}^*(5)) - (b_{28}^*(5)r_{29}^*(5)) - (b_{29}^*(5)r_{29}^*(5)) + (r_{28}^*(5)r_{29}^*(5)) < 0\]

with \((p_{28}^*(5), r_{29}^*(5))\) as defined satisfied, then the system

if \((a_i^*(6) \text{ and } h_i^*(6))\) are independent on \(t\), and the conditions

\[(a_{32}^*(6)a_{33}^*(6) - (a_{32}^*(6)a_{33}^*(6)) < 0\]
\[(a_{32}^*(6)a_{33}^*(6) - (a_{32}^*(6)a_{33}^*(6)) + (a_{32}^*(6)p_{32}^*(6) + (a_{33}^*(6)p_{32}^*(6)) + (p_{32}^*(6)p_{33}^*(6)) > 0\]
\[(b_{32}^*(6)b_{33}^*(6) - (b_{32}^*(6)b_{33}^*(6)) > 0,\]
\[(b_{32}^*(6)b_{33}^*(6) - (b_{32}^*(6)b_{33}^*(6)) - (b_{32}^*(6)r_{33}^*(6)) - (b_{33}^*(6)r_{33}^*(6)) + (r_{32}^*(6)r_{33}^*(6)) < 0\]

with \((p_{32}^*(6), r_{33}^*(6))\) as defined are satisfied, then the system

\[
\begin{align*}
(a_{13}^{(1)})G_{14} - [(a_{13}^{(1)}) + (a_{13}^{(1)})(T_{14})]G_{13} &= 0 \\
(a_{14}^{(1)})G_{14} - [(a_{14}^{(1)}) + (a_{14}^{(1)})(T_{14})]G_{14} &= 0 \\
(a_{15}^{(1)})G_{14} - [(a_{15}^{(1)}) + (a_{15}^{(1)})(T_{14})]G_{15} &= 0 \\
(b_{13}^{(1)})T_{14} - [(b_{13}^{(1)}) - (b_{13}^{(1)})(G)]T_{13} &= 0 \\
(b_{14}^{(1)})T_{14} - [(b_{14}^{(1)}) - (b_{14}^{(1)})(G)]T_{14} &= 0 \\
(b_{15}^{(1)})T_{14} - [(b_{15}^{(1)}) - (b_{15}^{(1)})(G)]T_{15} &= 0
\end{align*}
\]

has a unique positive solution, which is an equilibrium solution for the system

\[
\begin{align*}
(a_{16}^{(2)})G_{17} - [(a_{16}^{(2)}) + (a_{16}^{(2)})(T_{17})]G_{16} &= 0 \\
(a_{17}^{(2)})G_{16} - [(a_{17}^{(2)}) + (a_{17}^{(2)})(T_{17})]G_{17} &= 0 \\ 
(a_{18}^{(2)})G_{17} - [(a_{18}^{(2)}) + (a_{18}^{(2)})(T_{17})]G_{18} &= 0 \\
(b_{16}^{(2)})T_{17} - [(b_{16}^{(2)}) - (b_{16}^{(2)})(G)]T_{16} &= 0 \\
(b_{17}^{(2)})T_{16} - [(b_{17}^{(2)}) - (b_{17}^{(2)})(G)]T_{17} &= 0 \\
(b_{18}^{(2)})T_{17} - [(b_{18}^{(2)}) - (b_{18}^{(2)})(G)]T_{18} &= 0
\end{align*}
\]
has a unique positive solution, which is an equilibrium solution for

\[(a_{20})^3]G_{21} - [(a'_{20})^3 + (a''_{20})^3]G_{20} = 0\]

\[(a_{21})^3]G_{20} - [(a'_{21})^3 + (a''_{21})^3]G_{21} = 0\]

\[(a_{22})^3]G_{21} - [(a'_{22})^3 + (a''_{22})^3]G_{22} = 0\]

\[(b_{20})^3]T_{21} - [(b'_{20})^3 - (b''_{20})^3]T_{20} = 0\]

\[(b_{21})^3]T_{20} - [(b'_{21})^3 - (b''_{21})^3]T_{21} = 0\]

\[(b_{22})^3]T_{21} - [(b'_{22})^3 - (b''_{22})^3]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution

\[(a_{24})^4]G_{25} - [(a'_{24})^4 + (a''_{24})^4]G_{24} = 0\]

\[(a_{25})^4]G_{24} - [(a'_{25})^4 + (a''_{25})^4]G_{25} = 0\]

\[(a_{26})^4]G_{25} - [(a'_{26})^4 + (a''_{26})^4]G_{26} = 0\]

\[(b_{24})^4]T_{25} - [(b'_{24})^4 - (b''_{24})^4]T_{24} = 0\]

\[(b_{25})^4]T_{24} - [(b'_{25})^4 - (b''_{25})^4]T_{25} = 0\]

\[(b_{26})^4]T_{25} - [(b'_{26})^4 - (b''_{26})^4]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^5]G_{29} - [(a'_{28})^5 + (a''_{28})^5]G_{28} = 0\]

\[(a_{29})^5]G_{28} - [(a'_{29})^5 + (a''_{29})^5]G_{29} = 0\]

\[(a_{30})^5]G_{29} - [(a'_{30})^5 + (a''_{30})^5]G_{30} = 0\]

\[(b_{28})^5]T_{29} - [(b'_{28})^5 - (b''_{28})^5]T_{28} = 0\]

\[(b_{29})^5]T_{28} - [(b'_{29})^5 - (b''_{29})^5]T_{29} = 0\]

\[(b_{30})^5]T_{29} - [(b'_{30})^5 - (b''_{30})^5]T_{30} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{32})^6]G_{33} - [(a'_{32})^6 + (a''_{32})^6]G_{32} = 0\]

\[(a_{33})^6]G_{32} - [(a'_{33})^6 + (a''_{33})^6]G_{33} = 0\]

\[(a_{34})^6]G_{33} - [(a'_{34})^6 + (a''_{34})^6]G_{34} = 0\]
Indeed the first two equations have a nontrivial solution $G_{13}, G_{14}$ if

$$F(T) = (a_{13}')^{(1)}(a_{14}')^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}')^{(1)}(a_{14}'')^{(1)}(T_{14}) + (a_{14}')^{(1)}(a_{13})^{(1)}(T_{14}) + (a_{13}')^{(1)}(T_{14})(a_{14})^{(1)}(T_{14}) = 0$$

(a) Indeed the first two equations have a nontrivial solution $G_{16}, G_{17}$ if

$$F(T_{19}) = (a_{16}')^{(2)}(a_{17}')^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16}')^{(2)}(a_{17}'')^{(2)}(T_{17}) + (a_{17}')^{(2)}(a_{16})^{(2)}(T_{17}) + (a_{16}')^{(2)}(T_{17})(a_{17})^{(2)}(T_{17}) = 0$$

(a) Indeed the first two equations have a nontrivial solution $G_{20}, G_{21}$ if

$$F(T_{23}) = (a_{20}')^{(3)}(a_{21}')^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20}')^{(3)}(a_{21}'')^{(3)}(T_{21}) + (a_{21}')^{(3)}(a_{20})^{(3)}(T_{21}) + (a_{20}')^{(3)}(T_{21})(a_{21})^{(3)}(T_{21}) = 0$$

(a) Indeed the first two equations have a nontrivial solution $G_{24}, G_{25}$ if

$$F(T_{27}) = (a_{24}')^{(4)}(a_{25}')^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24}')^{(4)}(a_{25}'')^{(4)}(T_{25}) + (a_{25}')^{(4)}(a_{24})^{(4)}(T_{25}) + (a_{24}')^{(4)}(T_{25})(a_{25})^{(4)}(T_{25}) = 0$$

(a) Indeed the first two equations have a nontrivial solution $G_{28}, G_{29}$ if

$$F(T_{31}) = (a_{28}')^{(5)}(a_{29}')^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28}')^{(5)}(a_{29}'')^{(5)}(T_{29}) + (a_{29}')^{(5)}(a_{28})^{(5)}(T_{29}) + (a_{28}')^{(5)}(T_{29})(a_{29})^{(5)}(T_{29}) = 0$$

(a) Indeed the first two equations have a nontrivial solution $G_{32}, G_{33}$ if

$$F(T_{35}) = (a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32}')^{(6)}(a_{33}'')^{(6)}(T_{33}) + (a_{33}')^{(6)}(a_{32})^{(6)}(T_{33}) + (a_{32}')^{(6)}(T_{33})(a_{33})^{(6)}(T_{33}) = 0$$

www.ijsrp.org
Definition and uniqueness of $T'_{14}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(1)}(T'_{14})$ being increasing, it follows that there exists a unique $T'_{14}$ for which $f(T'_{14}) = 0$. With this value, we obtain from the three first equations

\[ G_{13} = \frac{\frac{a_{13}(1)}{G_{14}}}{[\frac{a_{13}(1)}{G_{14}}]} \quad , \quad G_{15} = \frac{\frac{a_{15}(1)}{G_{14}}}{[\frac{a_{15}(1)}{G_{14}}]} \]

Definition and uniqueness of $T'_{17}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(2)}(T'_{17})$ being increasing, it follows that there exists a unique $T'_{17}$ for which $f(T'_{17}) = 0$. With this value, we obtain from the three first equations

\[ G_{16} = \frac{\frac{a_{16}(2)}{G_{17}}}{[\frac{a_{16}(2)}{G_{17}}]} \quad , \quad G_{18} = \frac{\frac{a_{18}(2)}{G_{17}}}{[\frac{a_{18}(2)}{G_{17}}]} \]

Definition and uniqueness of $T'_{21}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique $T'_{21}$ for which $f(T'_{21}) = 0$. With this value, we obtain from the three first equations

\[ G_{20} = \frac{\frac{a_{20}(3)}{G_{21}}}{[\frac{a_{20}(3)}{G_{21}}]} \quad , \quad G_{22} = \frac{\frac{a_{22}(3)}{G_{21}}}{[\frac{a_{22}(3)}{G_{21}}]} \]

Definition and uniqueness of $T'_{25}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique $T'_{25}$ for which $f(T'_{25}) = 0$. With this value, we obtain from the three first equations

\[ G_{24} = \frac{\frac{a_{24}(4)}{G_{25}}}{[\frac{a_{24}(4)}{G_{25}}]} \quad , \quad G_{26} = \frac{\frac{a_{26}(4)}{G_{25}}}{[\frac{a_{26}(4)}{G_{25}}]} \]

Definition and uniqueness of $T'_{29}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique $T'_{29}$ for which $f(T'_{29}) = 0$. With this value, we obtain from the three first equations

\[ G_{28} = \frac{\frac{a_{28}(5)}{G_{29}}}{[\frac{a_{28}(5)}{G_{29}}]} \quad , \quad G_{30} = \frac{\frac{a_{30}(5)}{G_{29}}}{[\frac{a_{30}(5)}{G_{29}}]} \]

Definition and uniqueness of $T'_{33}$:

After hypothesis $f(0) < 0$, $f(\infty) > 0$ and the functions $(a_i')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique $T'_{33}$ for which $f(T'_{33}) = 0$. With this value, we obtain from the three first equations

\[ G_{32} = \frac{\frac{a_{32}(6)}{G_{33}}}{[\frac{a_{32}(6)}{G_{33}}]} \quad , \quad G_{34} = \frac{\frac{a_{34}(6)}{G_{33}}}{[\frac{a_{34}(6)}{G_{33}}]} \]

(e) By the same argument, the equations 92,93 admit solutions $G_{13}, G_{14}$ if

\[ \varphi(G) = \left( b_{13}'(1)(b_{14}'(1) - (b_{13})^{(1)}(b_{14})^{(1)} - \left[ (b_{13})^{(1)}(b_{14})^{(1)}(G) + (b_{14})^{(1)}(b_{13})^{(1)}(G) \right] + (b_{13})^{(1)}(G) (b_{14})^{(1)}(G) = 0 \]

Where in $G(G_{13}, G_{14}, G_{15})$, $G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that $\varphi$ is a
decreasing function in $G_{14}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{14}$ such that $\phi(G') = 0$

(f) By the same argument, the equations 92, 93 admit solutions $G_{16}, G_{17}$ if

$$\phi(G_{19}) = (b_{16}(2)(b_{17}(2)) - (b_{16}(2)(b_{17}(2)) -$$

$$[b_{16}^2](b_{17}^2)(G_{19}) + (b_{17}^2)(b_{16}^2)(G_{19})] + (b_{16}^2)(b_{17}^2)(G_{19}) = 0$$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that $\phi$ is a decreasing function in $G_{17}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{14}$ such that $\phi((G_{19})') = 0$

(g) By the same argument, the concatenated equations admit solutions $G_{20}, G_{21}$ if

$$\phi(G_{23}) = (b_{20}^3)(b_{21}^3) - (b_{20}^3)(b_{21}^3) -$$

$$[b_{20}^3](b_{21}^3)(G_{23}) + (b_{21}^3)(b_{20}^3)(G_{23})] + (b_{20}^3)(b_{21}^3)(G_{23}) = 0$$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that $\phi$ is a decreasing function in $G_{21}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{21}$ such that $\phi((G_{23})') = 0$

(h) By the same argument, the equations of modules admit solutions $G_{24}, G_{25}$ if

$$\phi(G_{27}) = (b_{24}^4)(b_{25}^4) - (b_{24}^4)(b_{25}^4) -$$

$$[b_{24}^4](b_{25}^4)(G_{27}) + (b_{25}^4)(b_{24}^4)(G_{27})] + (b_{24}^4)(b_{25}^4)(G_{27}) = 0$$

Where in $G_{27}(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that $\phi$ is a decreasing function in $G_{25}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{25}$ such that $\phi((G_{27})') = 0$

(i) By the same argument, the equations (modules) admit solutions $G_{26}, G_{29}$ if

$$\phi(G_{31}) = (b_{28}^5)(b_{29}^5) - (b_{28}^5)(b_{29}^5) -$$

$$[b_{28}^5](b_{29}^5)(G_{31}) + (b_{29}^5)(b_{28}^5)(G_{31})] + (b_{28}^5)(b_{29}^5)(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that $\phi$ is a decreasing function in $G_{29}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{29}$ such that $\phi((G_{31})') = 0$

(j) By the same argument, the equations (modules) admit solutions $G_{32}, G_{33}$ if

$$\phi(G_{35}) = (b_{32}^6)(b_{33}^6) - (b_{32}^6)(b_{33}^6) -$$

$$[b_{32}^6](b_{33}^6)(G_{35}) + (b_{33}^6)(b_{32}^6)(G_{35})] + (b_{32}^6)(b_{33}^6)(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that $\phi$ is a decreasing function in $G_{33}$ taking into account the hypothesis $\phi(0) > 0$, $\phi(\infty) < 0$ it follows that there exists a unique $G'_{33}$ such that $\phi(G') = 0$
Finally we obtain the unique solution of 89 to 94

\[ G_{14}^* \text{ given by } \varphi(G^*) = 0, \quad T_{14}^* \text{ given by } f(T_{14}^*) = 0 \text{ and } \]

\[ G_{13}^* = \frac{(a_{13})^1G_{14}^*}{[e_{13}]^1+(a_{13})^2(T_{14}^*)^1}, \quad G_{15}^* = \frac{(a_{15})^1G_{14}^*}{[e_{13}]^1+(a_{15})^2(T_{14}^*)^1} \]

\[ T_{13}^* = \frac{(b_{13})^1T_{14}^*}{[b_{13}]^1-(b_{13})^2(G^*)^1}, \quad T_{15}^* = \frac{(b_{15})^1T_{14}^*}{[b_{15}]^1-(b_{15})^2(G^*)^1} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{17}^* \text{ given by } \varphi((G_{19})^*) = 0, \quad T_{17}^* \text{ given by } f(T_{17}^*) = 0 \text{ and } \]

\[ G_{16}^* = \frac{(a_{16})^2G_{17}^*}{[e_{16}]^2+(a_{16})^2(T_{17}^*)^2}, \quad G_{18}^* = \frac{(a_{18})^2G_{17}^*}{[e_{16}]^2+(a_{18})^2(T_{17}^*)^2} \]

\[ T_{16}^* = \frac{(b_{16})^2T_{17}^*}{[b_{16}]^2-(b_{16})^2((G_{19})^*)^2}, \quad T_{18}^* = \frac{(b_{18})^2T_{17}^*}{[b_{18}]^2-(b_{18})^2((G_{19})^*)^2} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{21}^* \text{ given by } \varphi((G_{23})^*) = 0, \quad T_{21}^* \text{ given by } f(T_{21}^*) = 0 \text{ and } \]

\[ G_{20}^* = \frac{(a_{20})^3G_{21}^*}{[e_{20}]^3+(a_{20})^3(T_{21}^*)^3}, \quad G_{22}^* = \frac{(a_{22})^3G_{21}^*}{[e_{20}]^3+(a_{22})^3(T_{21}^*)^3} \]

\[ T_{20}^* = \frac{(b_{20})^3T_{21}^*}{[b_{20}]^3-(b_{20})^3((G_{23})^*)^3}, \quad T_{22}^* = \frac{(b_{22})^3T_{21}^*}{[b_{22}]^3-(b_{22})^3((G_{23})^*)^3} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{25}^* \text{ given by } \varphi(G_{27}^*) = 0, \quad T_{25}^* \text{ given by } f(T_{25}^*) = 0 \text{ and } \]

\[ G_{24}^* = \frac{(a_{24})^4G_{25}^*}{[e_{24}]^4+(a_{24})^4(T_{25}^*)^4}, \quad G_{26}^* = \frac{(a_{26})^4G_{25}^*}{[e_{24}]^4+(a_{26})^4(T_{25}^*)^4} \]

\[ T_{24}^* = \frac{(b_{24})^4T_{25}^*}{[b_{24}]^4-(b_{24})^4((G_{27})^*)^4}, \quad T_{26}^* = \frac{(b_{26})^4T_{25}^*}{[b_{26}]^4-(b_{26})^4((G_{27})^*)^4} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{29}^* \text{ given by } \varphi((G_{31})^*) = 0, \quad T_{29}^* \text{ given by } f(T_{29}^*) = 0 \text{ and } \]

\[ G_{28}^* = \frac{(a_{28})^5G_{29}^*}{[e_{28}]^5+(a_{28})^5(T_{29}^*)^5}, \quad G_{30}^* = \frac{(a_{30})^5G_{29}^*}{[e_{28}]^5+(a_{30})^5(T_{29}^*)^5} \]

\[ T_{28}^* = \frac{(b_{28})^5T_{29}^*}{[b_{28}]^5-(b_{28})^5((G_{31})^*)^5}, \quad T_{30}^* = \frac{(b_{30})^5T_{29}^*}{[b_{28}]^5-(b_{28})^5((G_{31})^*)^5} \]

Obviously, these values represent an equilibrium solution
Finally we obtain the unique solution

\[ \begin{align*}
G'_{33} & = \frac{(a_{32})^6 G'_{33}}{[a_{32}]^6 + (a_{32})^6 G'_{33}]}, \quad G'_{34} = \frac{(a_{34})^6 G'_{33}}{[a_{34}]^6 + (a_{34})^6 G'_{33}]}, \\
T'_{33} & = \frac{(b_{32})^6 T'_{33}}{[b_{32}]^6 - (b_{32})^6 G'_{33}[G'_{33}]}, \quad T'_{34} = \frac{(b_{34})^6 T'_{33}}{[b_{34}]^6 - (b_{34})^6 (G'_{33})[G'_{33}]}. 
\end{align*} \]

Obviously, these values represent an equilibrium solution

**ASYMPTOTIC STABILITY ANALYSIS**

**Theorem 4:** If the conditions of the previous theorem are satisfied and if the functions \((a_i)^{(1)}\) and \((b_i)^{(1)}\) Belong to \(C^3(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

**Proof:** Denote

**Definition of \(G_i, T_i\):**

\[ G_i = G_i^* + \mathcal{G}_i, \quad T_i = T_i^* + \mathcal{T}_i \]

\[ \frac{\partial (a_{i6})^{(1)}}{\partial t_{i6}} (T_{i6}) = (q_{i6})^{(1)} + \frac{\partial (b_{i6})^{(1)}}{\partial q_{i6}} (G^*) = s_{ij} \]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{13}}{dt} = -((a_{13})^{(1)} + (p_{13})^{(1)}) \mathcal{G}_{13} + (a_{13})^{(1)} \mathcal{G}_{14} - (q_{13})^{(1)} G_{13} T_{14} \]

\[ \frac{dG_{14}}{dt} = -((a_{14})^{(1)} + (p_{14})^{(1)}) \mathcal{G}_{14} + (a_{14})^{(1)} \mathcal{G}_{13} - (q_{14})^{(1)} G_{14} T_{14} \]

\[ \frac{dG_{15}}{dt} = -((a_{15})^{(1)} + (p_{15})^{(1)}) \mathcal{G}_{15} + (a_{15})^{(1)} \mathcal{G}_{14} - (q_{15})^{(1)} G_{15} T_{14} \]

\[ \frac{dT_{13}}{dt} = -((b_{13})^{(1)} - (r_{13})^{(1)}) T_{13} + (b_{13})^{(1)} T_{14} + \sum_{j=13}^{15} (s_{13}(j) T_{13} \mathcal{G}_j) \]

\[ \frac{dT_{14}}{dt} = -((b_{14})^{(1)} - (r_{14})^{(1)}) T_{14} + (b_{14})^{(1)} T_{13} + \sum_{j=13}^{15} (s_{14}(j) T_{14} \mathcal{G}_j) \]

\[ \frac{dT_{15}}{dt} = -((b_{15})^{(1)} - (r_{15})^{(1)}) T_{15} + (b_{15})^{(1)} T_{14} + \sum_{j=13}^{15} (s_{15}(j) T_{15} \mathcal{G}_j) \]

If the conditions of the previous theorem are satisfied and if the functions \((a_i)^{(2)}\) and \((b_i)^{(2)}\) Belong to \(C^3(\mathbb{R}_+)\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of \(G_i, T_i\):**

\[ G_i = G_i^* + \mathcal{G}_i, \quad T_i = T_i^* + \mathcal{T}_i \]

\[ \frac{\partial (a_{i7})^{(2)}}{\partial t_{i7}} (T_{i7}) = (q_{i7})^{(2)} + \frac{\partial (b_{i7})^{(2)}}{\partial q_{i7}} (G_{17}) = s_{ij} \]

taking into account equations (global) and neglecting the terms of power 2, we obtain

\[ \frac{dG_{16}}{dt} = -((a_{16})^{(2)} + (p_{16})^{(2)}) \mathcal{G}_{16} + (a_{16})^{(2)} \mathcal{G}_{17} - (q_{16})^{(2)} G_{16} T_{17} \]
\[
\begin{align*}
\frac{dG_{17}}{dt} &= -\left((a_{17})^{(2)} + (p_{17})^{(2)}\right)G_{17} + (a_{17})^{(2)}G_{16} + (q_{17})^{(2)}G_{17}T_{17} \\
\frac{dG_{18}}{dt} &= -\left((a_{18})^{(2)} + (p_{18})^{(2)}\right)G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}T_{17} \\
\frac{dT_{16}}{dt} &= -\left((b_{16})^{(2)} - (r_{16})^{(2)}\right)T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} \left(s^{(16)}(j)T_{16}G_{j}\right) \\
\frac{dT_{17}}{dt} &= -\left((b_{17})^{(2)} - (r_{17})^{(2)}\right)T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} \left(s^{(17)}(j)T_{17}G_{j}\right) \\
\frac{dT_{18}}{dt} &= -\left((b_{18})^{(2)} - (r_{18})^{(2)}\right)T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} \left(s^{(18)}(j)T_{18}G_{j}\right)
\end{align*}
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(3)}) and (b_i^{(3)})\) Belong to \(C^{(3)}(\mathbb{R}+)\) then the above equilibrium point is asymptotically stabl

Denote

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (G_{21})^{(3)}}{\partial T_{21}} = (q_{21})^{(3)}, \quad \frac{\partial (b_{22})^{(3)}}{\partial G_j} = (G_{22})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\begin{align*}
\frac{dG_{20}}{dt} &= -\left((a_{20})^{(3)} + (p_{20})^{(3)}\right)G_{20} + (a_{20})^{(3)}G_{21} - (q_{20})^{(3)}G_{20}T_{21} \\
\frac{dG_{21}}{dt} &= -\left((a_{21})^{(3)} + (p_{21})^{(3)}\right)G_{21} + (a_{21})^{(3)}G_{20} - (q_{21})^{(3)}G_{21}T_{21} \\
\frac{dG_{22}}{dt} &= -\left((a_{22})^{(3)} + (p_{22})^{(3)}\right)G_{22} + (a_{22})^{(3)}G_{21} - (q_{22})^{(3)}G_{22}T_{21} \\
\frac{dT_{20}}{dt} &= -\left((b_{20})^{(3)} - (r_{20})^{(3)}\right)T_{20} + (b_{20})^{(3)}T_{21} + \sum_{j=20}^{22} \left(s^{(20)}(j)T_{20}G_j\right) \\
\frac{dT_{21}}{dt} &= -\left((b_{21})^{(3)} - (r_{21})^{(3)}\right)T_{21} + (b_{21})^{(3)}T_{20} + \sum_{j=20}^{22} \left(s^{(21)}(j)T_{21}G_j\right) \\
\frac{dT_{22}}{dt} &= -\left((b_{22})^{(3)} - (r_{22})^{(3)}\right)T_{22} + (b_{22})^{(3)}T_{21} + \sum_{j=20}^{22} \left(s^{(22)}(j)T_{22}G_j\right)
\end{align*}
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_i^{(4)}) and (b_i^{(4)})\) Belong to \(C^{(4)}(\mathbb{R}+)\) then the above equilibrium point is asymptotically stabl

Denote

**Definition of** \(G_i, T_i\) :-

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (G_{25})^{(4)}}{\partial T_{25}} = (q_{25})^{(4)}, \quad \frac{\partial (b_{27})^{(4)}}{\partial G_j} = (G_{27})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\begin{align*}
\frac{dG_{24}}{dt} &= -\left((a_{24})^{(4)} + (p_{24})^{(4)}\right)G_{24} + (a_{24})^{(4)}G_{25} - (q_{24})^{(4)}G_{24}T_{25}
\end{align*}
\]
\[
\frac{dG_{25}}{dt} = -\left((a_{25}^{r})^{(4)} + (p_{25})^{(4)}\right)G_{25} + (a_{25})^{(4)}G_{24} - (q_{25})^{(4)}G_{25}T_{25}
\]

\[
\frac{dG_{26}}{dt} = -\left((a_{26}^{r})^{(4)} + (p_{26})^{(4)}\right)G_{26} + (a_{26})^{(4)}G_{25} + (q_{26})^{(4)}G_{26}T_{25}
\]

\[
\frac{dT_{24}}{dt} = -\left((b_{24}^{r})^{(4)} - (r_{24})^{(4)}\right)T_{24} + (b_{24})^{(4)}T_{25} + \sum_{i=24}^{26}(s_{(24)(i)}T_{24}G_{i})
\]

\[
\frac{dT_{25}}{dt} = -\left((b_{25}^{r})^{(4)} - (r_{25})^{(4)}\right)T_{25} + (b_{25})^{(4)}T_{24} + \sum_{i=24}^{26}(s_{(25)(i)}T_{25}G_{i})
\]

\[
\frac{dT_{26}}{dt} = -\left((b_{26}^{r})^{(4)} - (r_{26})^{(4)}\right)T_{26} + (b_{26})^{(4)}T_{25} + \sum_{i=24}^{26}(s_{(26)(i)}T_{26}G_{i})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_{i}^{r})^{(5)}\) and \((b_{i}^{r})^{(5)}\) belong to \(C^{(5)}(\mathbb{R}^{+})\) then the above equilibrium point is asymptotically stable.

**Definition of** \(G_{i}, T_{i} :\)

\[G_{i} = G_{i}^{*} + G_{i}, \quad T_{i} = T_{i}^{*} + T_{i}\]

\[\frac{\partial (a_{29})^{(5)}}{\partial a_{29}}(T_{29}^{*}) = (q_{29})^{(5)}, \quad \frac{\partial (b_{29})^{(5)}}{\partial b_{29}}(G_{31}^{*}) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -\left((a_{28}^{r})^{(5)} + (p_{28})^{(5)}\right)G_{28} + (a_{28})^{(5)}G_{29} - (q_{28})^{(5)}G_{28}T_{29}
\]

\[
\frac{dG_{29}}{dt} = -\left((a_{29}^{r})^{(5)} + (p_{29})^{(5)}\right)G_{29} + (a_{29})^{(5)}G_{28} - (q_{29})^{(5)}G_{29}T_{29}
\]

\[
\frac{dG_{30}}{dt} = -\left((a_{30}^{r})^{(5)} + (p_{30})^{(5)}\right)G_{30} + (a_{30})^{(5)}G_{29} - (q_{30})^{(5)}G_{30}T_{29}
\]

\[
\frac{dT_{28}}{dt} = -\left((b_{28}^{r})^{(5)} - (r_{28})^{(5)}\right)T_{28} + (b_{28})^{(5)}T_{29} + \sum_{i=28}^{30}(s_{(28)(i)}T_{28}G_{i})
\]

\[
\frac{dT_{29}}{dt} = -\left((b_{29}^{r})^{(5)} - (r_{29})^{(5)}\right)T_{29} + (b_{29})^{(5)}T_{28} + \sum_{i=28}^{30}(s_{(29)(i)}T_{29}G_{i})
\]

\[
\frac{dT_{30}}{dt} = -\left((b_{30}^{r})^{(5)} - (r_{30})^{(5)}\right)T_{30} + (b_{30})^{(5)}T_{29} + \sum_{i=28}^{30}(s_{(30)(i)}T_{30}G_{i})
\]

If the conditions of the previous theorem are satisfied and if the functions \((a_{i}^{r})^{(6)}\) and \((b_{i}^{r})^{(6)}\) belong to \(C^{(6)}(\mathbb{R}^{+})\) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \(G_{i}, T_{i} :\)

\[G_{i} = G_{i}^{*} + G_{i}, \quad T_{i} = T_{i}^{*} + T_{i}\]

\[\frac{\partial (a_{33})^{(6)}}{\partial a_{33}}(T_{33}^{*}) = (q_{33})^{(6)}, \quad \frac{\partial (b_{35})^{(6)}}{\partial b_{35}}(G_{35}^{*}) = s_{ij}\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain
\[
\frac{d\mathcal{G}_{32}}{dt} = -(a'_{32})^{(6)} + (p_{32})^{(6)}\mathcal{G}_{32} + (a_{32})^{(6)}\mathcal{G}_{33} - (q_{32})^{(6)}G^*_{32} T_{33}
\]
\[
\frac{d\mathcal{G}_{33}}{dt} = -(a'_{33})^{(6)} + (p_{33})^{(6)}\mathcal{G}_{33} + (a_{33})^{(6)}\mathcal{G}_{32} - (q_{33})^{(6)}G^*_{33} T_{33}
\]
\[
\frac{d\mathcal{G}_{34}}{dt} = -(a'_{34})^{(6)} + (p_{34})^{(6)}\mathcal{G}_{34} + (a_{34})^{(6)}\mathcal{G}_{33} - (q_{34})^{(6)}G^*_{34} T_{33}
\]
\[
\frac{dT_{32}}{dt} = -(b'_{32})^{(6)} - (r_{32})^{(6)}T_{32} + (b_{32})^{(6)}T_{33} + \sum_{j=32}^{34}(s_{(32)(j)}T_{33}^2 \mathcal{G}_j)
\]
\[
\frac{dT_{33}}{dt} = -(b'_{33})^{(6)} - (r_{33})^{(6)}T_{33} + (b_{33})^{(6)}T_{32} + \sum_{j=32}^{34}(s_{(33)(j)}T_{33}^2 \mathcal{G}_j)
\]
\[
\frac{dT_{34}}{dt} = -(b'_{34})^{(6)} - (r_{34})^{(6)}T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34}(s_{(34)(j)}T_{33}^2 \mathcal{G}_j)
\]

The characteristic equation of this system is
\[
\begin{align*}
&\left(\lambda(1)^{(1)} + (b'_{13})^{(1)} - (r_{15})^{(1)}\right)\left(\lambda(1)^{(1)} + (a'_{15})^{(1)} + (p_{15})^{(1)}\right) \\
&\left(\left(\lambda(1)^{(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G^*_{13}\right) \\
&\left(\left(\lambda(1)^{(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\right)s_{(14)(14)}T_{14} + (b_{14})^{(1)}s_{(13)(14)}T_{13}\right) \\
&\left(\left(\lambda(1)^{(1)} + (b_{14})^{(1)} - (r_{13})^{(1)}\right)s_{(13)(13)}T_{13}\right) \\
&\left(\left(\lambda(1)^{2} + (a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda(1)^{1})\right) \\
&\left(\left(\lambda(1)^{2} + (b'_{13})^{(1)} + (b'_{14})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda(1)^{1})\right) \\
&\left(\left(\lambda(1)^{2} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(q_{15})^{(1)}G^*_{15}\right) \\
&\left(\left(\lambda(1)^{2} + (a'_{13})^{(1)} + (p_{13})^{(1)}\right)(a_{15})^{(1)}(q_{14})^{(1)}G^*_{14} + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G^*_{13}\right) \\
&\left(\left(\lambda(1)^{1} + (b_{13})^{(1)} - (r_{12})^{(1)}\right)s_{(14)(15)}T_{14} + (b_{14})^{(1)}s_{(13)(15)}T_{13}\right) = 0
\end{align*}
\]
\[
\left( (\lambda)^{(2)} \right)^2 + ( (a_{16})^{(2)} + (a_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)} ) (\lambda)^{(2)} \\
\left( (\lambda)^{(2)} \right)^2 + ( (b_{16}^{(2)})^{(2)} + (b_{17}^{(2)})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)} ) (\lambda)^{(2)} \\
+ \left( (\lambda)^{(2)} \right)^2 + ( (a_{16}^{(2)})^{(2)} + (a_{17}^{(2)})^{(2)} + (p_{16}^{(2)})^{(2)} + (p_{17}^{(2)})^{(2)} ) (q_{18})^{(2)} G_{18} \\
+ (\lambda)^{(2)} + (a_{16}^{(2)})^{(2)} + (p_{16}^{(2)})^{(2)} ( (a_{18})^{(2)} (q_{17})^{(2)} G_{17} + (a_{17})^{(2)} (a_{18})^{(2)} (q_{16})^{(2)} G_{16} ) \\
\left( (\lambda)^{(2)} + (b_{16}^{(2)})^{(2)} - (r_{16})^{(2)} ) S_{(17),(18)} T_{17}^2 + (b_{17})^{(2)} S_{(16),(18)} T_{16}^2 \right) = 0
\]

\[
+ (\lambda)^{(3)} + (b_{22})^{(3)} - (r_{22})^{(3)} ((\lambda)^{(3)} + (a_{22})^{(3)} + (p_{22})^{(3)} ) \\
\left[ ( (\lambda)^{(3)} + (a_{20}^{(3)})^{(3)} (q_{21})^{(3)} G_{21}^{(3)} + (a_{21})^{(3)} (q_{26})^{(3)} G_{20}^{(3)} ) \right] \\
\left( (\lambda)^{(3)} + (b_{20}^{(3)})^{(3)} - (r_{20})^{(3)} ) S_{(21),(21)} T_{21}^2 + (b_{21})^{(3)} S_{(20),(21)} T_{21}^2 \\
+ (\lambda)^{(3)} + (a_{21}^{(3)})^{(3)} + (p_{21})^{(3)} ) (q_{20})^{(3)} G_{20}^{(3)} + (a_{20})^{(3)} (q_{21})^{(3)} G_{21}^{(3)} \\
\left( (\lambda)^{(3)} + (b_{20}^{(3)})^{(3)} - (r_{20})^{(3)} ) S_{(21),(20)} T_{21}^2 + (b_{21})^{(3)} S_{(20),(21)} T_{21}^2 \\
\left( (\lambda)^{(3)} + (b_{20}^{(3)})^{(3)} - (r_{20})^{(3)} ) S_{(21),(20)} T_{21}^2 + (b_{20})^{(3)} S_{(20),(20)} T_{20}^2 \\
\left( (\lambda)^{(3)} )^{(3)} + (a_{21}^{(3)})^{(3)} + (p_{21})^{(3)} ) (q_{20})^{(3)} G_{20}^{(3)} \\
\left( (\lambda)^{(3)} )^{(3)} + (b_{21}^{(3)})^{(3)} - (r_{21})^{(3)} ) (\lambda)^{(3)} \\
+ (\lambda)^{(3)} + (a_{20}^{(3)})^{(3)} + (p_{20})^{(3)} ) ( (a_{22})^{(3)} (q_{21})^{(3)} G_{21}^{(3)} + (a_{21})^{(3)} (a_{22})^{(3)} (q_{20})^{(3)} G_{20}^{(3)} ) \\
\left( (\lambda)^{(3)} + (b_{20}^{(3)})^{(3)} - (r_{20})^{(3)} ) S_{(21),(22)} T_{21}^2 + (b_{21})^{(3)} S_{(20),(22)} T_{20}^2 \right) \right) = 0
\]

\[
+ (\lambda)^{(4)} + (b_{26}^{(4)})^{(4)} - (r_{26})^{(4)} ((\lambda)^{(4)} + (a_{26}^{(4)})^{(4)} + (p_{26})^{(4)} ) \\
\left[ ( (\lambda)^{(4)} + (a_{24}^{(4)})^{(4)} (q_{25})^{(4)} G_{25}^{(4)} + (a_{25})^{(4)} (q_{24})^{(4)} G_{24}^{(4)} ) \right] \\
\left( (\lambda)^{(4)} + (b_{24}^{(4)})^{(4)} - (r_{24})^{(4)} ) S_{(25),(25)} T_{25}^2 + (b_{25})^{(4)} S_{(24),(25)} T_{25}^2 \\
+ (\lambda)^{(4)} + (a_{25}^{(4)})^{(4)} + (p_{25})^{(4)} ) (q_{24})^{(4)} G_{24}^{(4)} + (a_{24})^{(4)} (q_{25})^{(4)} G_{25}^{(4)} \\
\left( (\lambda)^{(4)} + (b_{24}^{(4)})^{(4)} - (r_{24})^{(4)} ) S_{(25),(24)} T_{25}^2 + (b_{25})^{(4)} S_{(24),(24)} T_{24}^2 \\
\left( (\lambda)^{(4)} )^{(4)} + (a_{24}^{(4)})^{(4)} + (p_{24})^{(4)} ) (p_{25})^{(4)} ) (\lambda)^{(4)} \\
\right)
\]
\[
\left( (\lambda)^{(4)} \right)^2 + ( (b_{24}^{(4)})^2 + (b_{25}^{(4)})^2 - (r_{24}^{(4)}) + (r_{25}^{(4)}) ) (\lambda)^{(4)} \\
+ \left( (\lambda)^{(4)} \right)^2 + ( (a_{24}^{(4)})^2 + (a_{25}^{(4)})^2 + (p_{24}^{(4)}) + (p_{25}^{(4)}) ) (\lambda)^{(4)} ) (q_{26}^{(4)}G_{26} \\
+ (\lambda)^{(4)} + (a_{24}^{(4)}) + (p_{24}^{(4)}) ) ( (a_{26}^{(4)})(q_{25}^{(4)}G_{25} + (a_{25}^{(4)})(a_{26}^{(4)})(q_{24}^{(4)}G_{24} \\
\left( (\lambda)^{(4)} + (b_{24}^{(4)}) - (r_{24}^{(4)}) ) s_{(25),(26)}T_{25} + (b_{25}^{(4)}) s_{(24),(26)}T_{24} ) = 0 \\
+ \\
(\lambda)^{(5)} + (b_{30}^{(5)}) - (r_{30}^{(5)}) ) ( (\lambda)^{(5)} + (a_{30}^{(5)}) + (p_{30}^{(5)}) \\
\left[ (\lambda)^{(5)} + (a_{28}^{(5)}) + (p_{28}^{(5)}) (q_{29}^{(5)}G_{29} + (a_{29}^{(5)})(q_{28}^{(5)}G_{28} \\
(\lambda)^{(5)} + (b_{30}^{(5)} - (r_{30}^{(5)}) ) s_{(29),(30)}T_{30} + (b_{29}^{(5)}) s_{(28),(29)}T_{29} \\
+ \left( (\lambda)^{(5)} + (a_{28}^{(5)}) + (p_{28}^{(5)}) ( (a_{26}^{(5)})(q_{25}^{(5)}G_{25} + (a_{25}^{(5)})(a_{26}^{(5)})(q_{24}^{(5)}G_{24} \\
\left( (\lambda)^{(5)} + (b_{28}^{(5)} - (r_{28}^{(5)}) ) s_{(29),(30)}T_{30} + (b_{29}^{(5)}) s_{(28),(29)}T_{29} \\
\left[ (\lambda)^{(5)} + (a_{28}^{(5)}) + (p_{28}^{(5)}) ( (a_{24}^{(5)})(q_{25}^{(5)}G_{25} + (a_{24}^{(5)})(a_{25}^{(5)})(q_{24}^{(5)}G_{24} \\
\left( (\lambda)^{(5)} + (b_{28}^{(5)} - (r_{28}^{(5)}) ) s_{(29),(30)}T_{30} + (b_{29}^{(5)}) s_{(28),(29)}T_{29} ) = 0 \\
+ \\
(\lambda)^{(6)} + (b_{34}^{(6)}) - (r_{34}^{(6)}) ) ( (\lambda)^{(6)} + (a_{34}^{(6)}) + (p_{34}^{(6)}) \\
\left[ (\lambda)^{(6)} + (a_{32}^{(6)}) + (p_{32}^{(6)}) (q_{33}^{(6)}G_{33} + (a_{33}^{(6)})(q_{32}^{(6)}G_{32} \\
\left( (\lambda)^{(6)} + (b_{34}^{(6)}) - (r_{34}^{(6)}) ) s_{(33),(34)}T_{33} + (b_{33}^{(6)}) s_{(32),(33)}T_{33} \\
+ \left( (\lambda)^{(6)} + (a_{33}^{(6)}) + (p_{33}^{(6)}) (q_{32}^{(6)}G_{32} + (a_{32}^{(6)})(q_{33}^{(6)}G_{33} \\
\left( (\lambda)^{(6)} + (b_{32}^{(6)}) - (r_{32}^{(6)}) ) s_{(33),(32)}T_{33} + (b_{31}^{(6)}) s_{(31),(32)}T_{32} \\
\left( (\lambda)^{(6)} + (a_{32}^{(6)}) + (p_{32}^{(6)}) (q_{33}^{(6)}G_{33} + (a_{33}^{(6)})(q_{32}^{(6)}G_{32} \\
\left( (\lambda)^{(6)} + (b_{32}^{(6)}) + (b_{33}^{(6)} - (r_{32}^{(6)}) + (r_{33}^{(6)}) (\lambda)^{(6)} \\
\left[ (\lambda)^{(6)} + (b_{34}^{(6)}) + (b_{33}^{(6)} - (r_{32}^{(6)}) + (r_{33}^{(6)}) (\lambda)^{(6)} \\
www.ijsrp.org
\[
+ \left( (\lambda^6) + (a_{32}^6) + (a_{33}^6) + (p_{32}^6) + (p_{33}^6) \right) \left( q_{34}^6 \right) G_{34}^6 \\
+ \left( (\lambda^6) + (a_{32}^6) + (p_{32}^6) \right) \left( (a_{34}^6) (q_{33}^6) G_{33}^6 + (a_{33}^6) (a_{34}^6) (q_{32}^6) G_{32}^6 \right) \\
\left( ((\lambda^6) + (b_{32}^6) - (r_{32}^6)) s_{(33),(34)} T_{33}^* + (b_{33}^6) s_{(32),(34)} T_{32}^* \right) = 0
\]

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

**Acknowledgments:**

The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature's Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive

**REFERENCES**


3. A HAIMOVICI: “On the growth of a two species ecological system divided on age groups”. Tensor, Vol 37 (1982), Commemoration volume dedicated to Professor Akitsugu Kawaguchi on his 80th birthday


www.ijsrp.org


====================================================================================================
First Author: ¹Mr. K. N.Prasanna Kumar has three doctorates one each in Mathematics, Economics, Political Science. Thesis was based on Mathematical Modeling. He was recently awarded D.litt. for his work on ‘Mathematical Models in Political Science’--- Department of studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India Corresponding Author:drknpkumar@gmail.com

Second Author: ²Prof. B.S Kiranagi is the Former Chairman of the Department of Studies in Mathematics, Manasa Gangotri and present Professor Emeritus of UGC in the Department. Professor Kiranagi has guided over 25 students and he has received many encomiums and laurels for his contribution to Co homology Groups and Mathematical Sciences. Known for his prolific writing, and one of the senior most Professors of the country, he has over 150 publications to his credit. A prolific writer and a prodigious thinker, he has to his credit several books on Lie Groups, Co Homology Groups, and other mathematical application topics, and excellent publication history.-- UGC Emeritus Professor (Department of studies in Mathematics), Manasagangotri, University of Mysore, Karnataka, India

Third Author: ³Prof. C.S. Bagewadi is the present Chairman of Department of Mathematics and Department of Studies in Computer Science and has guided over 25 students. He has published articles in both national and international journals. Professor Bagewadi specializes in Differential Geometry and its wide-ranging ramifications. He has to his credit more than 159 research papers. Several Books on Differential Geometry, Differential Equations are coauthored by him--- Chairman, Department of studies in Mathematics and Computer science, Jnanasahyadri Kuvempu University, Shankarghatta, Shimoga district, Karnataka, India