THE GENERAL THEORY OF WAVE FUNCTIONS, PROBABILITY DISTRIBUTIONS, BREMSSTRAHLUNG, ELECTRON DEFLECTION (BY AN ELECTRIC FIELD OR BY NUCLEUS) VIRTUAL PHOTON CREATIONS, LAMB SHIFT, GAMMA RAY PHOTONS ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS DECELERATION OF ACCELERATION FROM THE MOVING OBSERVER’S FRAME OF REFERENCE, INCREASE IN RELATIVISTIC MASS PAIR PRODUCTION OF THE PARTICLES COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS

1 DR K N PRASANNA KUMAR, 2 PROF B S KIRANAGI AND 3 PROF C S BAGEWADI

ABSTRACT: Model bestows its priority and preferential attention in the construction of a terra firma for wave function, probability wave distribution, Bremsstrahlung, Gamma ray Photons so that experimental Physicists could use it for prediction and prognostication purposes. In essence the continuation of the first model, the Model highlights and showcases the most important systemic properties. Relativistic study of the parameters is another dernier resort used for inclusion of relativistic effects on the parameters, which makes model to give conducive response, fortuitous and felicitous reciprocation towards relativistic calculations at the subatomic scales at Planck's length.

INTRODUCTION:

Imperative compatibilities and structural variabilities, character constitution exists between wave function and several other variables that are related to Quantum computation. There is an absolute and dire necessity of a model for optimum development of a system of those parameters and diffuse solidarity abstraction. We try to consummate the need with a solidarity abstraction of some of the most important and interesting variables in this model. Model delineated and disseminated in the following comprises of following variables:

1. Wave Functions
2. Probability Distributions
3. Bremsstrahlung
4. Electron Deflection (By an electric field or by nucleus)
5. Virtual Photon Creations
6. Lamb Shift
7. Gamma ray Photons
8. Energy produced due to electrons and positrons
9. Deceleration of acceleration from the moving observer’s frame of reference
10. Increase in relativistic mass
11. Pair production of the particles
12. Collisions of photons with atomic nucleus

PROBABILITY DISTRIBUTION AND WAVE FUNCTIONS: MODULE NUMBERED ONE

NOTATION:

$G_{13}$: CATEGORY ONE OF WAVE FUNCTIONS

$G_{14}$: CATEGORY TWO OF WAVE FUNCTIONS
\[ G_{15} : \text{CATEGORY THREE OF WAVE FUNCTIONS} \]
\[ T_{13} : \text{CATEGORY ONE OF PROBABILITY DISTRIBUTIONS} \]
\[ T_{14} : \text{CATEGORY TWO OF PROBABILITY DISTRIBUTIONS} \]
\[ T_{15} : \text{CATEGORY THREE OF PROBABILITY DISTRIBUTIONS} \]

**VIRTUAL PHOTON CREATIONS AND LAMB SHIFT MODULE NUMBERED TWO:**

\[ G_{16} : \text{CATEGORY ONE OF VIRTUAL PHOTON CREATIONS} \]
\[ G_{17} : \text{CATEGORY TWO OF VIRTUAL PHOTON CREATIONS} \]
\[ G_{18} : \text{CATEGORY THREE OF VIRTUAL PHOTON CREATIONS} \]
\[ T_{16} : \text{CATEGORY ONE OF LAMB SHIFT} \]
\[ T_{17} : \text{CATEGORY TWO OF LAMB SHIFT} \]
\[ T_{18} : \text{CATEGORY THREE OF LAMB SHIFT} \]

**ELECTRON DEFLECTION (BY MAGNETIC OR ELECTRIC FIELDS) AND BREMSSTRAHLUNG: MODULE NUMBERED THREE:**

\[ G_{20} : \text{CATEGORY ONE OF ELECTRON DEFLECTION} \]
\[ G_{21} : \text{CATEGORY TWO OF ELECTRON DEFLECTION} \]
\[ G_{22} : \text{CATEGORY THREE OF ELECTRON DEFLECTION} \]
\[ T_{20} : \text{CATEGORY ONE OF BREMSSTRAHLUNG} \]
\[ T_{21} : \text{CATEGORY TWO OF BREMSSTRAHLUNG} \]
\[ T_{22} : \text{CATEGORY THREE OF BREMSSTRAHLUNG} \]

**GAMMA RAY PHOTONS AND ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS COLLISION: MODULE NUMBERED FOUR:**

\[ G_{24} : \text{CATEGORY ONE OF ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS} \]
\[ G_{25} : \text{CATEGORY TWO OF ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS} \]
\[ G_{26} : \text{CATEGORY THREE OF ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS} \]
\( T_{24} \) : CATEGORY ONE OF GAMMA RAY PHOTONS
\( T_{25} \) : CATEGORY TWO OF GAMMA RAY PHOTONS
\( T_{26} \) : CATEGORY THREE OF GAMMA RAY PHOTONS

DECELERATION OF PARTICLE ACCELERATION FROM MOVING OBSERVER’S FRAME OF REFERENCE AND INCREASE IN RELATIVISTIC MASS: MODULE NUMBERED FIVE.

============================================================================= 

\( G_{28} \) : CATEGORY ONE OF INCREASE IN RELATIVISTIC MASS
\( G_{29} \) : CATEGORY TWO OF INCREASE IN RELATIVISTIC MASS
\( G_{30} \) : CATEGORY THREE OF INCREASE IN RELATIVISTIC MASS

\( T_{28} \) : CATEGORY ONE OF RELATIVISTIC EFFECTS FROM A MOVING OBSERVER’S FRAME OF REFERENCE
\( T_{29} \) : CATEGORY TWO OF RELATIVISTIC EFFECTS FROM A MOVING OBSERVER’S POINT OF VIEW
\( T_{30} \) : CATEGORY THREE OF RELATIVISTIC EFFECTS

PAIR PRODUCTION AND COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS: MODULE NUMBERED SIX.

============================================================================= 

\( G_{32} \) : CATEGORY ONE OF COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS
\( G_{33} \) : CATEGORY TWO OF COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS
\( G_{34} \) : CATEGORY THREE OF COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS

\( T_{32} \) : CATEGORY ONE OF PAIRS PRODUCTION
\( T_{33} \) : CATEGORY TWO OF PAIRS PRODUCTION
\( T_{34} \) : CATEGORY THREE OF PAIRS PRODUCTION

============================================================================= 

\( (a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} \), \( (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)} \), \( (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)} \), \( (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)} \)

are Accentuation coefficients

\( (a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)} \), \( (a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}, (b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)} \), \( (a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}, (b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)} \), \( (a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)}, (b_{29})^{(5)} \), \( (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)} \), \( (b_{28})^{(5)} \)

are Dissipation coefficients

www.ijsrp.org
GOVERNING EQUATIONS OF THE HOLISTIC SYSTEM:

(1) WAVE FUNCTIONS
(2) PROBABILITY DISTRIBUTIONS
(3) BREMSSTRAHLUNG
(4) ELECTRON DEFLECTION (BY AN ELECTRIC FIELD OR BY NUCLEUS)
(5) VIRTUAL PHOTON CREATIONS
(6) LAMB SHIFT
(7) GAMMA RAY PHOTONS
(8) ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS
(9) DECELERATION OF ACCELERATION FROM THE MOVING OBSERVER’S FRAME OF REFERENCE
(10) INCREASE IN RELATIVISTIC MASS
(11) PAIR PRODUCTION OF THE PARTICLES
(12) COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS

The differential system of this model is now (Module numbered one)

\[
\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[ (a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14}, t) \right]G_{13}
\]

\[
\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[ (a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14}, t) \right]G_{14}
\]

\[
\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[ (a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14}, t) \right]G_{15}
\]

\[
\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[ (b'_{13})^{(1)} - (b''_{13})^{(1)}(G, t) \right]T_{13}
\]

\[
\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[ (b'_{14})^{(1)} - (b''_{14})^{(1)}(G, t) \right]T_{14}
\]

\[
\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[ (b'_{15})^{(1)} - (b''_{15})^{(1)}(G, t) \right]T_{15}
\]

\[+(a''_{13})^{(1)}(T_{14}, t) = \text{First augmentation factor} \]

\[-(b''_{13})^{(1)}(G, t) = \text{First detritions factor} \]

The differential system of this model is now (Module numbered two)

\[
\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[ (a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17}, t) \right]G_{16}
\]

\[
\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[ (a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17}, t) \right]G_{17}
\]

\[
\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[ (a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17}, t) \right]G_{18}
\]

\[
\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[ (b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19}, t) \right]T_{16}
\]

\[
\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[ (b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19}, t) \right]T_{17}
\]
\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19}, t)]T_{18}
\]

\[+(a_{16})^{(2)}(T_{17}, t) = \text{First augmentation factor} \]

\[-(b_{16})^{(2)}((G_{19}), t) = \text{First detritions factor} \]

The differential system of this model is now (Module numbered three)

\[
\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - [(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)]G_{20}
\]

\[
\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - [(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21}, t)]G_{21}
\]

\[
\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - [(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21}, t)]G_{22}
\]

\[
\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23}, t)]T_{20}
\]

\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23}, t)]T_{21}
\]

\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23}, t)]T_{22}
\]

\[+(a_{20})^{(3)}(T_{21}, t) = \text{First augmentation factor} \]

\[-(b_{20})^{(3)}(G_{23}, t) = \text{First detritions factor} \]

The differential system of this model is now (Module numbered four)

\[
\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - [(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)]G_{24}
\]

\[
\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - [(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25}, t)]G_{25}
\]

\[
\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - [(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25}, t)]G_{26}
\]

\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}(G_{27}, t)]T_{24}
\]

\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}(G_{27}, t)]T_{25}
\]

\[
\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}(G_{27}, t)]T_{26}
\]

\[+(a_{24})^{(4)}(T_{25}, t) = \text{First augmentation factor} \]

\[-(b_{24})^{(4)}(G_{27}, t) = \text{First detritions factor} \]

The differential system of this model is now (Module numbered five)
\[
d\frac{G_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})(T_{29}, t)\right]G_{28}
\]
\[
d\frac{G_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})(T_{29}, t)\right]G_{29}
\]
\[
d\frac{G_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})(T_{29}, t)\right]G_{30}
\]
\[
d\frac{T_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})(G_{31}, t)\right]T_{28}
\]
\[
d\frac{T_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})(G_{31}, t)\right]T_{29}
\]
\[
d\frac{T_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})(G_{31}, t)\right]T_{30}
\]
\[
+ (a''_{28})(T_{29}, t) = \text{First augmentation factor}
\]
\[
- (b''_{28})(G_{31}, t) = \text{First detriments factor}
\]

The differential system of this model is now (Module numbered six)

\[
d\frac{G_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})(T_{33}, t)\right]G_{32}
\]
\[
d\frac{G_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})(T_{33}, t)\right]G_{33}
\]
\[
d\frac{G_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})(T_{33}, t)\right]G_{34}
\]
\[
d\frac{T_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})(G_{35}, t)\right]T_{32}
\]
\[
d\frac{T_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})(G_{35}, t)\right]T_{33}
\]
\[
d\frac{T_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})(G_{35}, t)\right]T_{34}
\]
\[
+ (a''_{32})(T_{33}, t) = \text{First augmentation factor}
\]
\[
- (b''_{32})(G_{35}, t) = \text{First detriments factor}
\]

**HOLISTIC CONCATENATED SYSTEM EQUATIONS HENCEFORTH REFERRED TO AS “GLOBAL EQUATIONS”**

**GOVERNING EQUATIONS OF THE HOLISTIC SYSTEM:**

**WAVE FUNCTIONS**
(2) PROBABILITY DISTRIBUTIONS
(3) BREMSSTRAHLUNG
(4) ELECTRON DEFLECTION (BY AN ELECTRIC FIELD OR BY NUCLEUS)
(5) VIRTUAL PHOTON CREATIONS
(6) LAMB SHIFT
(7) GAMMA RAY PHOTONS
(8) ENERGY PRODUCED DUE TO ELECTRONS AND POSITRONS
(9) DECELERATION OF ACCELERATION FROM THE MOVING OBSERVER’S FRAME OF REFERENCE
(10) INCREASE IN RELATIVISTIC MASS
(11) PAIR PRODUCTION OF THE PARTICLES
(12) COLLISIONS OF PHOTONS WITH ATOMIC NUCLEUS
\[ \frac{dT_{14}}{dt} = (b_{14})^{(1)} T_{13} - \left[ (b_{14}^{(1)}) (G(t)) \right] T_{14} \]

\[ \frac{dT_{15}}{dt} = (b_{15})^{(1)} T_{14} - \left[ (b_{15}^{(1)}) (G(t)) \right] T_{15} \]

Where \(- (b_{13})^{(1)} (G(t))\), \(- (b_{14})^{(1)} (G(t))\), \(- (b_{15})^{(1)} (G(t))\) are first detritus coefficients for category 1, 2 and 3

\(- (b_{16})^{(2)} (G(t))\), \(- (b_{17})^{(2)} (G(t))\), \(- (b_{18})^{(2)} (G(t))\) are second detritus coefficients for category 1, 2 and 3

\(- (b_{19})^{(3)} (G(t))\), \(- (b_{20})^{(3)} (G(t))\), \(- (b_{21})^{(3)} (G(t))\) are third detritus coefficients for category 1, 2 and 3

\(- (b_{22})^{(4)} (G(t))\), \(- (b_{23})^{(4)} (G(t))\), \(- (b_{24})^{(4)} (G(t))\) are fourth detritus coefficients for category 1, 2 and 3

\(- (b_{25})^{(5)} (G(t))\), \(- (b_{26})^{(5)} (G(t))\), \(- (b_{27})^{(5)} (G(t))\) are fifth detritus coefficients for category 1, 2 and 3

\(- (b_{28})^{(6)} (G(t))\), \(- (b_{29})^{(6)} (G(t))\), \(- (b_{30})^{(6)} (G(t))\) are sixth detritus coefficients for category 1, 2 and 3

\[ \frac{dG_{16}}{dt} = (a_{16})^{(2)} G_{17} - \left[ (a_{16}^{(2)}) \right] G_{16} \]

\[ \frac{dG_{17}}{dt} = (a_{17})^{(2)} G_{16} - \left[ (a_{17}^{(2)}) \right] G_{17} \]

\[ \frac{dG_{18}}{dt} = (a_{18})^{(2)} G_{17} - \left[ (a_{18}^{(2)}) \right] G_{18} \]

Where \(+ (a_{13})^{(1)} (T_{17}, t)\), \(+ (a_{14})^{(2)} (T_{17}, t)\), \(+ (a_{15})^{(2)} (T_{17}, t)\) are first augmentation coefficients for category 1, 2 and 3

\(+ (a_{16})^{(1)} (T_{17}, t)\), \(+ (a_{17})^{(1)} (T_{17}, t)\), \(+ (a_{18})^{(1)} (T_{17}, t)\) are second augmentation coefficient for category 1, 2 and 3

\(+ (a_{20})^{(3)} (T_{21}, t)\), \(+ (a_{21})^{(3)} (T_{21}, t)\), \(+ (a_{22})^{(3)} (T_{21}, t)\) are third augmentation coefficients for category 1, 2 and 3

\(+ (a_{23})^{(4)} (T_{25}, t)\), \(+ (a_{24})^{(4)} (T_{25}, t)\), \(+ (a_{25})^{(4)} (T_{25}, t)\) are fourth augmentation coefficients for category 1, 2 and 3

\(+ (a_{26})^{(5)} (T_{29}, t)\), \(+ (a_{27})^{(5)} (T_{29}, t)\), \(+ (a_{28})^{(5)} (T_{29}, t)\) are fifth augmentation coefficients for category 1, 2 and 3

\(+ (a_{31})^{(6)} (T_{33}, t)\), \(+ (a_{32})^{(6)} (T_{33}, t)\), \(+ (a_{33})^{(6)} (T_{33}, t)\) are sixth augmentation coefficients for category 1, 2 and 3

\[ \frac{dT_{16}}{dt} = (b_{16})^{(2)} T_{17} - \left[ (b_{16}^{(2)}) \right] T_{16} \]

\[ \frac{dT_{17}}{dt} = (b_{17})^{(2)} T_{16} - \left[ (b_{17}^{(2)}) \right] T_{17} \]

www.ijsrp.org
\[
\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \begin{pmatrix}
(b_{18})^{(2)} - (b_{18})^{(2)}(G_{19}, t) - (b_{18})^{(1,1)}(G, t) - (b_{18})^{(3,3,3)}(G_{23}, t) \\
-(b_{26})^{(4,4,4,4)}(G_{27}, t) - (b_{30})^{(5,5,5,5,5)}(G_{31}, t) - (b_{34})^{(6,6,6,6,6)}(G_{35}, t)
\end{pmatrix} T_{18}
\]

where \(- (b_{18})^{(2)}(G, t), -(b_{18})^{(1,1)}(G, t), -(b_{18})^{(3,3,3)}(G_{23}, t)\) are first detrition coefficients for category 1, 2 and 3

\[- (b_{26})^{(4,4,4,4)}(G_{27}, t), -(b_{30})^{(5,5,5,5,5)}(G_{31}, t), -(b_{34})^{(6,6,6,6,6)}(G_{35}, t)\] are second detrition coefficients for category 1, 2 and 3

\[- (b_{26})^{(1,1,1,1,1)}(G_{23}, t), -(b_{26})^{(1,1,1,1,1)}(G_{23}, t), -(b_{26})^{(3,3,3,3,3)}(G_{23}, t)\] are third detrition coefficients for category 1, 2 and 3

\[- (b_{26})^{(4,4,4,4,4)}(G_{27}, t), -(b_{26})^{(4,4,4,4,4)}(G_{27}, t), -(b_{26})^{(4,4,4,4,4)}(G_{27}, t)\] are fourth detrition coefficients for category 1, 2 and 3

\[- (b_{26})^{(5,5,5,5,5)}(G_{31}, t), -(b_{26})^{(5,5,5,5,5)}(G_{31}, t), -(b_{26})^{(5,5,5,5,5)}(G_{31}, t)\] are fifth detrition coefficients for category 1, 2 and 3

\[- (b_{26})^{(6,6,6,6,6,6,6)}(G_{35}, t), -(b_{26})^{(6,6,6,6,6,6,6)}(G_{35}, t)\] are sixth detrition coefficients for category 1, 2 and 3

\[
\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \begin{pmatrix}
(a_{20})^{(3)} + (a_{20})^{(3)}(T_{21}, t) + (a_{19})^{(2,2,2)}(T_{17}, t) + (a_{15})^{(1,1,1)}(T_{14}, t) \\
+(a_{24})^{(4,4,4,4,4)}(T_{25}, t) + (a_{20})^{(5,5,5,5,5,5)}(T_{29}, t) + (a_{34})^{(6,6,6,6,6,6,6)}(T_{33}, t)
\end{pmatrix} G_{20}
\]

\[
\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{22} - \begin{pmatrix}
(a_{21})^{(3)} + (a_{21})^{(3)}(T_{21}, t) + (a_{17})^{(2,2,2)}(T_{17}, t) + (a_{14})^{(1,1,1)}(T_{14}, t) \\
+(a_{25})^{(4,4,4,4,4,4)}(T_{25}, t) + (a_{20})^{(5,5,5,5,5,5,5)}(T_{29}, t) + (a_{34})^{(6,6,6,6,6,6,6,6)}(T_{33}, t)
\end{pmatrix} G_{21}
\]

\[
\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \begin{pmatrix}
(a_{22})^{(3)} + (a_{22})^{(3)}(T_{21}, t) + (a_{19})^{(2,2,2)}(T_{17}, t) + (a_{15})^{(1,1,1)}(T_{14}, t) \\
+(a_{26})^{(4,4,4,4,4,4,4)}(T_{25}, t) + (a_{20})^{(5,5,5,5,5,5,5,5)}(T_{29}, t) + (a_{34})^{(6,6,6,6,6,6,6,6,6)}(T_{33}, t)
\end{pmatrix} G_{22}
\]

\[
\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \begin{pmatrix}
(b_{20})^{(3)} - (b_{20})^{(3)}(G_{23}, t) - (b_{16})^{(2,2,2)}(G_{19}, t) - (b_{13})^{(1,1,1)}(G, t) \\
-(b_{24})^{(4,4,4,4,4,4)}(G_{27}, t) - (b_{30})^{(5,5,5,5,5,5,5)}(G_{31}, t) - (b_{34})^{(6,6,6,6,6,6,6,6)}(G_{35}, t)
\end{pmatrix} T_{20}
\]

\[
\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \begin{pmatrix}
(b_{21})^{(3)} - (b_{21})^{(3)}(G_{23}, t) - (b_{17})^{(2,2,2)}(G_{19}, t) - (b_{14})^{(1,1,1)}(G, t) \\
-(b_{25})^{(4,4,4,4,4,4,4)}(G_{27}, t) - (b_{29})^{(5,5,5,5,5,5,5,5)}(G_{31}, t) - (b_{33})^{(6,6,6,6,6,6,6,6,6)}(G_{35}, t)
\end{pmatrix} T_{21}
\]
\[
\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \begin{bmatrix}
(b_{22})^{(3)} & -(b_{22})^{(3)}(G_{23}, t) & -(b_{22})^{(2,2,2)}(G_{19}, t) & -(b_{22})^{(1,1,1)}(G_{t}) \\
-(b_{20})^{(4,4,4,4,4)} & (G_{27}, t) & -(b_{20})^{(5,5,5,5,5,5)} & -(b_{20})^{(6,6,6,6,6,6)}(G_{35}, t) \\
-(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) \\
-(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) & -(b_{20})^{(4,4,4,4,4)}(G_{27}, t) \\
\end{bmatrix} T_{22}
\]

These are first detritus coefficients for category 1, 2 and 3.

\[
\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix}
(b_{24})^{(4)} & -(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(5,5,5)}(G_{31}, t) & -(b_{24})^{(6,6,6)}(G_{35}, t) \\
-(b_{20})^{(4)} & (G_{27}, t) & -(b_{20})^{(5,5,5)} & -(b_{20})^{(6,6,6)}(G_{35}, t) \\
-(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) \\
-(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) & -(b_{20})^{(4)}(G_{27}, t) \\
\end{bmatrix} T_{24}
\]

These are second detritus coefficients for category 1, 2 and 3.

\[
\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix}
(b_{25})^{(4)} & -(b_{25})^{(4)}(G_{27}, t) & -(b_{25})^{(5,5,5)}(G_{31}, t) & -(b_{25})^{(6,6,6)}(G_{35}, t) \\
-(b_{21})^{(4)} & (G_{27}, t) & -(b_{21})^{(5,5,5)} & -(b_{21})^{(6,6,6)}(G_{35}, t) \\
-(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) \\
-(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) & -(b_{21})^{(4)}(G_{27}, t) \\
\end{bmatrix} T_{25}
\]

These are third detritus coefficients for category 1, 2 and 3.

\[
\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix}
(b_{26})^{(4)} & -(b_{26})^{(4)}(G_{27}, t) & -(b_{26})^{(5,5,5)}(G_{31}, t) & -(b_{26})^{(6,6,6)}(G_{35}, t) \\
-(b_{22})^{(4)} & (G_{27}, t) & -(b_{22})^{(5,5,5)} & -(b_{22})^{(6,6,6)}(G_{35}, t) \\
-(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) \\
-(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) & -(b_{22})^{(4)}(G_{27}, t) \\
\end{bmatrix} T_{26}
\]

These are fourth detritus coefficients for category 1, 2 and 3.

\[
\frac{dT_{27}}{dt} = (b_{27})^{(4)}T_{26} - \begin{bmatrix}
(b_{27})^{(4)} & -(b_{27})^{(4)}(G_{27}, t) & -(b_{27})^{(5,5,5)}(G_{31}, t) & -(b_{27})^{(6,6,6)}(G_{35}, t) \\
-(b_{23})^{(4)} & (G_{27}, t) & -(b_{23})^{(5,5,5)} & -(b_{23})^{(6,6,6)}(G_{35}, t) \\
-(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) \\
-(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) & -(b_{23})^{(4)}(G_{27}, t) \\
\end{bmatrix} T_{27}
\]

These are fifth detritus coefficients for category 1, 2 and 3.

\[
\frac{dT_{28}}{dt} = (b_{28})^{(4)}T_{27} - \begin{bmatrix}
(b_{28})^{(4)} & -(b_{28})^{(4)}(G_{27}, t) & -(b_{28})^{(5,5,5)}(G_{31}, t) & -(b_{28})^{(6,6,6)}(G_{35}, t) \\
-(b_{24})^{(4)} & (G_{27}, t) & -(b_{24})^{(5,5,5)} & -(b_{24})^{(6,6,6)}(G_{35}, t) \\
-(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) \\
-(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) & -(b_{24})^{(4)}(G_{27}, t) \\
\end{bmatrix} T_{28}
\]

These are sixth detritus coefficients for category 1, 2 and 3.

ISSN 2250-3153

www.ijsrp.org
\[
d\frac{T_{26}}{dt} = (b_{26})^{(4)} T_{25} - \begin{bmatrix}
(b_{26}''')^{(4)} \frac{d}{dt} G_{27} - b_{26}'')^{(4)} (G_{27}, t) - (b_{30}'')^{(5,5)} (G_{31} , t) - (b_{34}'')^{(6,6)} (G_{35} , t)
- (b_{13}''')^{(1,1,1,1)} (G, t) - (b_{16}''')^{(2,2,2,2)} (G_{19} , t) - (b_{22}''')^{(3,3,3,3)} (G_{23} , t)
\end{bmatrix} T_{26}
\]

Where \(- (b_{26}'')^{(4)} (G_{27}, t) - (b_{26}''')^{(4)} (G_{27}, t) - (b_{30}'')^{(4)} (G_{27}, t)\) are first detritus coefficients for category 1, 2 and 3

\(- (b_{26})^{(5,5)} (G_{31} , t) - (b_{26})^{(6,6)} (G_{35} , t)\) are second detritus coefficients for category 1, 2 and 3

\(- (b_{26})^{(5,5)} (G_{31} , t) - (b_{26})^{(6,6)} (G_{35} , t)\) are third detritus coefficients for category 1, 2 and 3

\(- (b_{26})^{(5,5)} (G_{31} , t) - (b_{26})^{(6,6)} (G_{35} , t)\) are fourth detritus coefficients for category 1, 2 and 3

\(- (b_{26})^{(5,5)} (G_{31} , t) - (b_{26})^{(6,6)} (G_{35} , t)\) are fifth detritus coefficients for category 1, 2 and 3

\(- (b_{26})^{(5,5)} (G_{31} , t) - (b_{26})^{(6,6)} (G_{35} , t)\) are sixth detritus coefficients for category 1, 2 and 3

\[
d\frac{G_{28}}{dt} = (a_{28})^{(5)} G_{29} - \begin{bmatrix}
(a_{28}''')^{(5)} + (a_{28}'')^{(4,4)} (T_{29}, t) + (a_{28}'')^{(4,4)} (T_{25} , t) + (a_{28}')^{(6,6,6)} (T_{33} , t)
+ (a_{13}''')^{(1,1,1,1)} (T_{14} , t) + (a_{16}''')^{(2,2,2,2,2)} (T_{17} , t) + (a_{20}''')^{(3,3,3,3,3)} (T_{21} , t)
\end{bmatrix} G_{28}
\]

\[
d\frac{G_{29}}{dt} = (a_{29})^{(5)} G_{29} - \begin{bmatrix}
(a_{29}''')^{(5)} + (a_{29}'')^{(4,4)} (T_{29}, t) + (a_{29}'')^{(4,4)} (T_{25} , t) + (a_{29}')^{(6,6,6)} (T_{33} , t)
+ (a_{14}''')^{(1,1,1,1)} (T_{14} , t) + (a_{17}''')^{(2,2,2,2,2)} (T_{17} , t) + (a_{21}''')^{(3,3,3,3,3)} (T_{21} , t)
\end{bmatrix} G_{29}
\]

\[
d\frac{G_{30}}{dt} = (a_{30})^{(5)} G_{29} - \begin{bmatrix}
(a_{30}''')^{(5)} + (a_{30}'')^{(4,4)} (T_{29}, t) + (a_{30}'')^{(4,4)} (T_{25} , t) + (a_{30}')^{(6,6,6)} (T_{33} , t)
+ (a_{15}''')^{(1,1,1,1)} (T_{14} , t) + (a_{18}''')^{(2,2,2,2,2)} (T_{17} , t) + (a_{22}''')^{(3,3,3,3,3)} (T_{21} , t)
\end{bmatrix} G_{30}
\]

Where \(+ (a_{28}'')^{(4,4)} (T_{29}, t) + (a_{28})^{(4,4)} (T_{25} , t) + (a_{28})^{(6,6,6)} (T_{33} , t)\) are first augmentation coefficients for category 1, 2 and 3

And \(+ (a_{28}')^{(6,6,6)} (T_{25} , t) + (a_{28}')^{(6,6,6)} (T_{33} , t) + (a_{13}'')^{(6,6,6)} (T_{23} , t)\) are second augmentation coefficient for category 1, 2 and 3

\(+ (a_{28}')^{(6,6,6)} (T_{25} , t) + (a_{28}')^{(6,6,6)} (T_{33} , t) + (a_{13}'')^{(6,6,6)} (T_{23} , t)\) are third augmentation coefficient for category 1, 2 and 3

\(+ (a_{13}')^{(6,6,6)} (T_{23} , t) + (a_{13}')^{(6,6,6)} (T_{23} , t) + (a_{17}')^{(6,6,6)} (T_{23} , t)\) are fourth augmentation coefficients for category 1, 2, and 3

\(+ (a_{28}')^{(6,6,6)} (T_{25} , t) + (a_{28}')^{(6,6,6)} (T_{33} , t) + (a_{13}'')^{(6,6,6)} (T_{23} , t)\) are fifth augmentation coefficients for category 1, 2, and 3

\(+ (a_{28}')^{(6,6,6)} (T_{25} , t) + (a_{28}')^{(6,6,6)} (T_{33} , t) + (a_{13}'')^{(6,6,6)} (T_{23} , t)\) are sixth augmentation coefficients for category 1, 2, and 3

\[
d\frac{T_{28}}{dt} = (b_{28})^{(5)} T_{28} - \begin{bmatrix}
(b_{28}''')^{(5)} - (b_{28})^{(5)} (G_{31} , t) - (b_{28})^{(4,4)} (G_{27} , t) - (b_{13})^{(6,6,6)} (G_{31} , t)
- (b_{13}'')^{(1,1,1,1)} (G, t) - (b_{16}'')^{(2,2,2,2,2)} (G_{19} , t) - (b_{22}'')^{(3,3,3,3,3)} (G_{23} , t)
\end{bmatrix} T_{28}
\]

\[
d\frac{T_{29}}{dt} = (b_{29})^{(5)} T_{29} - \begin{bmatrix}
(b_{29}''')^{(5)} - (b_{29})^{(5)} (G_{31} , t) - (b_{29})^{(4,4)} (G_{27} , t) - (b_{29})^{(6,6,6)} (G_{35} , t)
- (b_{14}'')^{(1,1,1,1)} (G, t) - (b_{17}'')^{(2,2,2,2,2)} (G_{19} , t) - (b_{21}'')^{(3,3,3,3,3)} (G_{23} , t)
\end{bmatrix} T_{29}
\]

\[
d\frac{T_{30}}{dt} = (b_{30})^{(5)} T_{29} - \begin{bmatrix}
(b_{30}''')^{(5)} - (b_{30})^{(5)} (G_{31} , t) - (b_{30})^{(4,4)} (G_{27} , t) - (b_{30})^{(6,6,6)} (G_{35} , t)
- (b_{15}'')^{(1,1,1,1)} (G, t) - (b_{18}'')^{(2,2,2,2,2)} (G_{19} , t) - (b_{22}'')^{(3,3,3,3,3)} (G_{23} , t)
\end{bmatrix} T_{30}
\]
where \(-b_{23}(G_{35}, t)\), \(-(b_{24})^5(G_{31}, t)\), \(-(b_{25})^5(G_{32}, t)\) are first detritus coefficients for category 1, 2 and 3
\(-b_{26}(G_{27}, t)\), \(-b_{27}(G_{27}, t)\), \(-b_{28}(G_{27}, t)\) are second detritus coefficients for category 1, 2 and 3
\(-b_{29}(G_{30}, t)\), \(-b_{30}(G_{30}, t)\), \(-b_{31}(G_{30}, t)\) are third detritus coefficients for category 1, 2 and 3
\(-b_{32}(G_{31}, t)\), \(-b_{33}(G_{31}, t)\), \(-b_{34}(G_{31}, t)\) are fourth detritus coefficients for category 1, 2, and 3
\(-b_{35}(G_{32}, t)\), \(-b_{36}(G_{32}, t)\), \(-b_{37}(G_{32}, t)\) are fifth detritus coefficients for category 1, 2, and 3
\(-b_{38}(G_{32}, t)\), \(-b_{39}(G_{32}, t)\), \(-b_{40}(G_{32}, t)\) are sixth detritus coefficients for category 1, 2, and 3

\[
\frac{dG_{32}}{dt} = (a_{32})^6G_{33} - \left[ (a_{32})^6 + (a_{33})^6(T_{33}, t) + (a_{30})^6(T_{33}, t) + (a_{24})^6(T_{25}, t) + (a_{13})^6(T_{14}, t) + (a_{16})^6(T_{14}, t) + (a_{20})^6(T_{21}, t) \right] G_{32}
\]

\[
\frac{dG_{33}}{dt} = (a_{33})^6G_{32} - \left[ (a_{33})^6 + (a_{30})^6(T_{33}, t) + (a_{25})^6(T_{25}, t) + (a_{14})^6(T_{14}, t) + (a_{17})^6(T_{14}, t) + (a_{21})^6(T_{21}, t) \right] G_{33}
\]

\[
\frac{dG_{34}}{dt} = (a_{34})^6G_{33} - \left[ (a_{34})^6 + (a_{30})^6(T_{33}, t) + (a_{25})^6(T_{25}, t) + (a_{15})^6(T_{14}, t) + (a_{18})^6(T_{14}, t) + (a_{22})^6(T_{21}, t) \right] G_{34}
\]

\[
+ (a_{32})^6(T_{32}, t) + (a_{33})^6(T_{33}, t) + (a_{30})^6(T_{30}, t) \text{ are first augmentation coefficients for category 1, 2 and 3}
\]

\[
+ (a_{25})^6(T_{25}, t) + (a_{26})^6(T_{26}, t) \text{ are second augmentation coefficients for category 1, 2 and 3}
\]

\[
+ (a_{14})^6(T_{14}, t) + (a_{18})^6(T_{18}, t) \text{ are third augmentation coefficients for category 1, 2 and 3}
\]

\[
+ (a_{21})^6(T_{21}, t) \text{ are fourth augmentation coefficients}
\]

\[
+ (a_{22})^6(T_{22}, t) \text{ are fifth augmentation coefficients}
\]

\[
+ (a_{23})^6(T_{23}, t) + (a_{24})^6(T_{24}, t) \text{ sixth augmentation coefficients}
\]

\[
\frac{dT_{32}}{dt} = (b_{32})^6T_{33} - \left[ (b_{32})^6 - (b_{33})^6(G_{35}, t) - (b_{29})^6(G_{31}, t) - (b_{24})^6(G_{27}, t) - (b_{13})^6(G_{11}, t) - (b_{16})^6(G_{11}, t) - (b_{20})^6(G_{23}, t) \right] T_{32}
\]

\[
\frac{dT_{33}}{dt} = (b_{33})^6T_{32} - \left[ (b_{33})^6 - (b_{34})^6(G_{35}, t) - (b_{29})^6(G_{31}, t) - (b_{25})^6(G_{27}, t) - (b_{14})^6(G_{11}, t) - (b_{17})^6(G_{11}, t) - (b_{22})^6(G_{23}, t) \right] T_{33}
\]

\[
\frac{dT_{34}}{dt} = (b_{34})^6T_{33} - \left[ (b_{34})^6 - (b_{35})^6(G_{35}, t) - (b_{30})^6(G_{31}, t) - (b_{26})^6(G_{27}, t) - (b_{15})^6(G_{11}, t) - (b_{18})^6(G_{11}, t) - (b_{23})^6(G_{23}, t) \right] T_{34}
\]

\[
- (b_{35})^6(G_{35}, t) - (b_{36})^6(G_{36}, t) - (b_{37})^6(G_{37}, t) \text{ are first detritus coefficients for category 1, 2 and 3}
\]

\[
- (b_{38})^6(G_{38}, t) - (b_{39})^6(G_{39}, t) - (b_{40})^6(G_{40}, t) \text{ are second detritus coefficients for category 1, 2 and 3}
\]

\[
- (b_{41})^6(G_{41}, t) - (b_{42})^6(G_{42}, t) - (b_{43})^6(G_{43}, t) \text{ are third detritus coefficients for category 1, 2 and 3}
\]

www.ijsrp.org
Where we suppose

(A) \((a_i^{(1)}, (a_i^{(1)}, (a_i^{(1)}, (b_i^{(1)}, (b_i^{(1)}, (b_i^{(1)} > 0, \)

\[i, j = 13, 14, 15\]

(B) The functions \((a_i^{(1)}, (b_i^{(1)} are positive continuous increasing and bounded.

**Definition of** \((p_i^{(1)}, (r_i^{(1)}:\)

\[(a_i^{(1)}(T_{14}, t) \leq (p_i^{(1)} \leq (A_{13}^{(1)}\]

\[(b_i^{(1)}(G, t) \leq (r_i^{(1)} \leq (B_{13}^{(1)})\]

\(\lim_{G \to -\infty} (a_i^{(1)}(T_{14}, t) = (p_i^{(1)}\]

\(\lim_{G \to -\infty} (b_i^{(1)}(G, t) = (r_i^{(1)})\]

**Definition of** \((A_{13}^{(1)}, (B_{13}^{(1)}:\)

Where \((A_{13}^{(1)}, (B_{13}^{(1)}, (p_i^{(1)}, (r_i^{(1)}) are positive constants and \(i = 13, 14, 15\)

They satisfy Lipschitz condition:

\[|a_i^{(1)}(T_{14}, t) - (A_{13}^{(1)} (T_{14}, t)| \leq (k_{13}^{(1)} |T_{14} - T_{14}| e^{-(\gamma_{13}^{(1)}) t}\]

\[|b_i^{(1)}(G', t) - (B_{13}^{(1)}(G, t)| \leq (k_{13}^{(1)} |G - G'| e^{-(\gamma_{13}^{(1)}) t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i^{(1)}(T_{14}, t) and (a_i^{(1)}(T_{14}, t') . (T_{14}, t) and (T_{14}, t') are points belonging to the interval \([k_{13}^{(1)}, (B_{13}^{(1)})]. It is to be noted that \((a_i^{(1)}(T_{14}, t) is uniformly continuous. In the eventuality of the fact, that if \((k_{13}^{(1)} = 1 then the function \((a_i^{(1)}(T_{14}, t), the first augmentation coefficient WOULD be absolutely continuous.

**Definition of** \((M_{13}^{(1)}, (k_{13}^{(1)}:\)

(D) \((M_{13}^{(1)}, (k_{13}^{(1)}, are positive constants

\[\frac{(a_i^{(1)}}{(M_{13}^{(1)}) , \frac{(b_i^{(1)}}{(k_{13}^{(1)}) < 1\]

**Definition of** \((\hat{M}_{13}^{(1)}, (\hat{k}_{13}^{(1)}:\)

(E) There exists two constants \((\hat{M}_{13}^{(1)} and (\hat{k}_{13}^{(1}) which together with \((M_{13}^{(1)}, (k_{13}^{(1)}, (A_{13}^{(1)}) and \((B_{13}^{(1)}) and the constants \((a_i^{(1)}, (a_i^{(1)}, (b_i^{(1)}, (b_i^{(1)}, (p_i^{(1)}, (r_i^{(1)}) , (i = 13, 14, 15,\]

satisfy the inequalities

www.ijsrp.org
\[
\frac{1}{(M_{13})^2}[(a)_{1}^{(3)} + (a')_{1}^{(3)} + (\bar{A}_{13})^{(3)} + (\bar{P}_{13})^{(3)}(\bar{k}_{13})^{(3)}] < 1
\]
\[
\frac{1}{(M_{13})^2}[(b)_{1}^{(3)} + (b')_{1}^{(3)} + (\bar{B}_{13})^{(3)} + (\bar{Q}_{13})^{(3)}(\bar{k}_{13})^{(3)}] < 1
\]

Where we suppose

(F) \((a_{ij})^{(2)}, (a'_{ij})^{(2)}, (a''_{ij})^{(2)}, (b_{ij})^{(2)}, (b'_{ij})^{(2)}, (b''_{ij})^{(2)} > 0, \quad i, j = 16, 17, 18\)

(G) The functions \((a''_{ij})^{(2)}, (b''_{ij})^{(2)}\) are positive continuous increasing and bounded.

**Definition of** \((p_{i})^{(2)}, (r_{i})^{(2)}:\)

\[(a_{i}^{(3)}(T_{17}, t) \leq (p_{i})^{(2)} \leq (\bar{A}_{16})^{(2)}\]
\[(b_{i}^{(3)}(G_{19}, t) \leq (r_{i})^{(2)} \leq (\bar{B}_{16})^{(2)}\]

(H) \(\lim_{T_{17} \to \infty}(a_{i}^{(3)}(T_{17}, t) = (p_{i})^{(2)}\]
\(\lim_{G_{19} \to \infty}(b_{i}^{(3)}(G_{19}, t) = (r_{i})^{(2)}\]

**Definition of** \((\bar{A}_{16})^{(2)}, (\bar{B}_{16})^{(2)}:\)

Where \((\bar{A}_{16})^{(2)}, (\bar{B}_{16})^{(2)}, (p_{i})^{(2)}, (r_{i})^{(2)}\) are positive constants and \(i = 16, 17, 18\)

They satisfy Lipschitz condition:

\[|a_{i}^{(3)}(T_{17}, t) - (a'_{i})^{(3)}(T_{17}, t)| \leq (\bar{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-\alpha_{16}(2)t}\]
\[|b_{i}^{(3)}(G_{19}, t) - (b'_{i})^{(3)}(G_{19}, t)| \leq (\bar{B}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-\alpha_{16}(2)t}\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_{i}^{(3)}(T_{17}, t)\) and\((a'_{i})^{(3)(T_{17}, t)\} and \((T_{17}, t)\) and \((T_{17}, t)\) are points belonging to the interval \([\bar{k}_{16})^{(2)}, (\bar{M}_{16})^{(3)}\]. It is to be noted that \((a_{i}^{(3)}(T_{17}, t)\) is uniformly continuous. In the eventuality of the fact, if \((\bar{M}_{16})^{(3)} = 1\) then the function \((a_{i}^{(3)}(T_{17}, t)\), the SECOND augmentation coefficient would be absolutely continuous.

**Definition of** \((\bar{M}_{16})^{(3)}, (\bar{k}_{16})^{(2)}:\)

(I) \((\bar{M}_{16})^{(3)}, (\bar{k}_{16})^{(2)}\) are positive constants

\[\frac{(\bar{a})^{(2)}_{16}}{(\bar{M}_{16})^{(3)}}, \frac{(\bar{b})^{(2)}_{16}}{(\bar{M}_{16})^{(3)}} < 1\]

**Definition of** \((\bar{P}_{13})^{(3)}, (\bar{Q}_{13})^{(3)}:\)

There exist two constants \((\bar{P}_{16})^{(2)}\) and \((\bar{Q}_{16})^{(2)}\) which together with \((\bar{M}_{16})^{(3)}\), \((\bar{k}_{16})^{(2)}\), \((\bar{A}_{16})^{(2)}\) and \((\bar{B}_{16})^{(2)}\) and the constants \((a_{i})^{(2)}, (a'_{i})^{(2)}, (b_{i})^{(2)}, (b'_{i})^{(2)}, (p_{i})^{(2)}, (r_{i})^{(2)}, t = 16, 17, 18,\)

satisfy the inequalities

\[\frac{1}{(M_{16})^{3}}[(a)_{i}^{(3)} + (a'_{i})^{(3)} + (\bar{A}_{16})^{(2)} + (\bar{P}_{13})^{(3)}(\bar{k}_{16})^{(2)}] < 1\]
\[\frac{1}{(M_{16})^{3}}[(b)_{i}^{(3)} + (b'_{i})^{(3)} + (\bar{B}_{16})^{(2)} + (\bar{Q}_{13})^{(3)}(\bar{k}_{16})^{(2)}] < 1\]

Where we suppose

(J) \((a_{ij})^{(3)}, (a'_{ij})^{(3)}, (a''_{ij})^{(3)}, (b_{ij})^{(3)}, (b'_{ij})^{(3)}, (b''_{ij})^{(3)} > 0, \quad i, j = 20, 21, 22\)

www.ijsrp.org
The functions \( (a_i^\prime\prime)^{(3)}, (b_i^\prime\prime)^{(3)} \) are positive continuous increasing and bounded.

**Definition of** \( (p_i)^{(3)}, (r_i)^{(3)} \):

\[
(a_i^\prime\prime)^{(3)}(T_{21}, t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}
\]

\[
(b_i^\prime\prime)^{(3)}(G_{23}, t) \leq (r_i)^{(3)} \leq (\hat{B}_{20})^{(3)}
\]

\[
\lim_{t \to \infty}(a_i^\prime\prime)^{(3)}(T_{21}, t) = (p_i)^{(3)}
\]

\[
\lim_{G \to \infty}(b_i^\prime\prime)^{(3)}(G_{23}, t) = (r_i)^{(3)}
\]

**Definition of** \( (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)} \):

Where \( (\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)} \) are positive constants and \( i = 20, 21, 22 \)

They satisfy Lipschitz condition:

\[
| (a_i^\prime\prime)^{(3)}(T_{21}, t) - (a_i^\prime\prime)^{(3)}(T_{21}, t) | \leq (k_{20})^{(3)}|T_{21} - T_{21}^\prime|e^{-(q_{20})^{(3)}t}
\]

\[
| (b_i^\prime\prime)^{(3)}(G_{23}, t) - (b_i^\prime\prime)^{(3)}(G_{23}, t) | \leq (k_{20})^{(3)}|G_{23} - G_{23}^\prime|e^{-(q_{20})^{(3)}t}
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_i^\prime\prime)^{(3)}(T_{21}, t) \)
and\( (a_i^\prime\prime)^{(3)}(T_{21}, t) \). \( (T_{21}, t) \) are points belonging to the interval \( [1, (M)_{20})^{(3)} \). It is to be noted that \( (a_i^\prime\prime)^{(3)}(T_{21}, t) \) is uniformly continuous. In the eventuality of the fact, that if \( (\hat{M}_{20})^{(3)} = 1 \) then the function \( (a_i^\prime\prime)^{(3)}(T_{21}, t) \), the THIRD augmentation coefficient, would be absolutely continuous.

**Definition of** \( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)} \):

\( (K) \) \( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)}, \) are positive constants

\[
\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1
\]

There exists two constants \( (\hat{P}_{20})^{(3)} \) and \( (\hat{Q}_{20})^{(3)} \) which together with \( (\hat{M}_{20})^{(3)}, (\hat{K}_{20})^{(3)} \), \( (\hat{A}_{20})^{(3)} \), \( (\hat{B}_{20})^{(3)} \), and the constants \( (a_i)^{(3)}, (a_i)^{(3)}, (b_i)^{(3)}, (b_i)^{(3)}, (p_i)^{(3)}, (r_i)^{(3)} \), \( i = 20, 21, 22 \), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i)^{(3)}] + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)} (\hat{K}_{20})^{(3)}] < 1
\]

\[
\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i)^{(3)}] + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{K}_{20})^{(3)}] < 1
\]

Where we suppose

\[
(a_i)^{(4)}, (a_i)^{(4)}, (a_i)^{(4)}, (b_i)^{(4)}, (b_i)^{(4)}, (b_i)^{(4)}, (b_i)^{(4)}, (b_i)^{(4)} > 0, \quad i, j = 24, 25, 26
\]

**Definition of** \( (p_i)^{(4)}, (r_i)^{(4)} \):

\[
(a_i^\prime\prime)^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}
\]

\[
(b_i^\prime\prime)^{(4)}(G_{27}, t) \leq (r_i)^{(4)} \leq (b_i)^{(4)} \leq (\hat{B}_{24})^{(4)}
\]
\( (N) \quad \lim_{T_2 \to \infty} (a_i^{''})^{(4)}(T_{25}, t) = (p_i)^{(4)} \\
\lim_{G \to \infty} (b_{i'}^{''})^{(4)}(G_{27}, t) = (r_{i'})^{(4)} \\

**Definition of** \( (\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)} \): \\
Where \( (\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_{i'})^{(4)} \) are positive constants and \( i = 24, 25, 26 \) \\

They satisfy Lipschitz condition: \\
\[ |(a_i^{''})^{(4)}(T_{25}, t) - (a_i^{''})^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{R}_{24})^{(4)} t} \]
\[ |(b_{i'}^{''})^{(4)}(G_{27}, t) - (b_{i'}^{''})^{(4)}(G_{27}, t)| < (\hat{k}_{24})^{(4)} |(G_{27}) - (G_{27})'| e^{-(\hat{R}_{24})^{(4)} t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \( (a_i^{''})^{(4)}(T_{25}, t) \) and \( (b_{i'}^{''})^{(4)}(G_{27}, t) \) and \( T_{25}, t \) and \( T_{25}, t \) are points belonging to the interval \( [(\hat{k}_{24})^{(4)}, (\hat{R}_{24})^{(4)}] \). It is to be noted that \( (a_i^{''})^{(4)}(T_{25}, t) \), \( (b_{i'}^{''})^{(4)}(G_{27}, t) \) are uniformly continuous. In the eventuality of the fact, that if \( (\hat{R}_{24})^{(4)} = 4 \) then the function \( (a_i^{''})^{(4)}(T_{25}, t) \), the FOURTH augmentation coefficient **WOULD** be absolutely continuous.

**Definition of** \( (\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)} \): \\
\( (\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)} \), are positive constants \\
\[ \frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \cdot \frac{(b_{i'})^{(4)}}{(\hat{M}_{24})^{(4)}} < 1 \]

**Definition of** \( (\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)} \): \\
(Q) There exists two constants \( (\hat{P}_{24})^{(4)} \) and \( (\hat{Q}_{24})^{(4)} \) which together with \( (\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} \) and \( (\hat{B}_{24})^{(4)} \) and the constants \( (a_i)^{(4)}, (a_{i'})^{(4)}, (b_{i'})^{(4)}, (b_{i'})^{(4)}, (p_i)^{(4)}, (r_{i'})^{(4)} \), \( i = 24, 25, 26 \), satisfy the inequalities \\
\[ \frac{1}{(\hat{M}_{24})^{(4)}} \left[ (a_i)^{(4)} + (a_{i'})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{B}_{24})^{(4)} (\hat{k}_{24})^{(4)} \right] < 1 \]
\[ \frac{1}{(\hat{M}_{24})^{(4)}} \left[ (b_{i'})^{(4)} + (b_{i'})^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)} \right] < 1 \]

Where we suppose \\
\( (a_i)^{(5)}, (a_{i'})^{(5)}, (b_{i'})^{(5)}, (b_{i'})^{(5)}, (b_{i'})^{(5)} > 0, \quad i, j = 28, 29, 30 \) \\
(S) The functions \( (a_i)^{(5)}, (b_{i'})^{(5)} \) are positive continuous increasing and bounded. 

**Definition of** \( (p_i)^{(5)}, (r_{i'})^{(5)} \): \\
\( (a_i)^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{29})^{(5)} \)
\( (b_{i'})^{(5)}(G_{31}, t) \leq (r_{i'})^{(5)} \leq (b_{i'})^{(5)} \leq (\hat{B}_{28})^{(5)} \)

\( (T) \quad \lim_{T_2 \to \infty} (a_i^{''})^{(5)}(T_{29}, t) = (p_i)^{(5)} \\
\lim_{G \to \infty} (b_{i'}^{''})^{(5)}(G_{31}, t) = (r_{i'})^{(5)} \)

**Definition of** \( (\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)} \): \\

www.ijsrp.org
Where \((\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, (r_i)^{(5)}\) are positive constants and \(i = 28, 29, 30\)

They satisfy Lipschitz condition:

\[
|\left(a_i''\right)^{(5)}(T'_{29}, t) - (a_i''\right)^{(5)}(T'_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T'_{29} - T'_{29}|e^{-(\hat{B}_{28})^{(5)}}t \\
|\left(b_i''\right)^{(5)}(G_{31}), t) - (b_i''\right)^{(5)}(G_{31}), t)| \leq (\hat{k}_{28})^{(5)}|((G_{31}) - (G_{31})'|e^{-(\hat{B}_{28})^{(5)}}t \\
\]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i''\right)^{(5)}(T'_{29}, t)\) and \((a_i''\right)^{(5)}(T'_{29}, t)\) . \((T'_{29}, t)\) and \((T'_{29}, t)\) are points belonging to the interval \([\hat{k}_{28})^{(5)}, (\hat{B}_{28})^{(5)}\] . It is to be noted that \((a_i''\right)^{(5)}(T'_{29}, t)\) is uniformly continuous. In the eventuality of the fact, that if \((\hat{B}_{28})^{(5)} = 5\) then the function \((a_i''\right)^{(5)}(T'_{29}, t)\) , the FIFTH augmentation coefficient attributable would be absolutely continuous.

Definition of \((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}\) :

\[
(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}\) are positive constants
\[
(a_i)^{(5)}, (b_i)^{(5)}, (\hat{M}_{28})^{(5)}, (\hat{M}_{28})^{(5)} < 1
\]

Definition of \((\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}\) :

There exists two constants \((\hat{P}_{28})^{(5)}\) and \((\hat{Q}_{28})^{(5)}\) which together with \((\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}, (\hat{B}_{28})^{(5)}\) and \((\hat{B}_{28})^{(5)}\) and the constants \((a_i)^{(5)}, (b_i)^{(5)}, (\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}\) , \((\hat{k}_{28})^{(5)}, i = 28, 29, 30\), satisfy the inequalities

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[\left(a_i\right)^{(5)} + (a_i)^{(5)}] \leq (\hat{A}_{28})^{(5)} + (\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)} < 1
\]

\[
\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)} + (b_i)^{(5)}] \leq (\hat{B}_{28})^{(5)} + (\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)} < 1
\]

Where we suppose

\[
(a_i)^{(6)}, (a_i)^{(6)}, (a_i)^{(6)}, (b_i)^{(6)}, (b_i)^{(6)}, (b_i)^{(6)}, (b_i)^{(6)} > 0, \quad i, j = 32, 33, 34
\]

(W) The functions \((a_i)^{(6)}, (b_i)^{(6)}\) are positive continuous increasing and bounded.

Definition of \((p_j)^{(6)}, (r_i)^{(6)}\) :

\[
(a_i)^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (A_{32})^{(6)}
\]

\[
(b_i)^{(6)}(G_{33}, t) \leq (r_i)^{(6)} \leq (b_i)^{(6)} \leq (B_{32})^{(6)}
\]

(X) \[
\lim_{T_2 \to \infty} (a_i)^{(6)}(T_{33}, t) = (p_j)^{(6)}
\]

\[
\lim_{G \to \infty} (b_i)^{(6)}(G_{33}, t) = (r_i)^{(6)}
\]

Definition of \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}\) :

Where \((\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_j)^{(6)}, (r_i)^{(6)}\) are positive constants and \(i = 32, 33, 34\)

They satisfy Lipschitz condition:

\[
|\left(a_i''\right)^{(6)}(T'_{33}, t) - (a_i''\right)^{(6)}(T'_{33}, t)| \leq (\hat{k}_{32})^{(6)}|T'_{33} - T'_{33}|e^{-(\hat{B}_{32})^{(6)}}t
\]

www.ijsrp.org
\[ \left| (b_i''(G_{35})', t) - (b_i''(G_{35}), t) \right| < (\tilde{k}_{32})^{(6)} \left| (G_{35}) - (G_{35})' \right| e^{-(\tilde{R}_{32})^{(6)}t} \]

With the Lipschitz condition, we place a restriction on the behavior of functions \((a_i''(T_{33}', t)\) and \((a_i''(T_{33}, t))\). \((T_{33}, t)\) and \((T_{33}, t)\) are points belonging to the interval \([\tilde{k}_{32}^{(6)}, (\tilde{M}_{32})^{(6)}]\). It is to be noted that \((a_i''(T_{33}, t))\) is uniformly continuous. In the eventuality of the fact, that if \((\tilde{M}_{32})^{(6)} = 6\) then the function \((a_i''(T_{33}, t))\), the SIXTH augmentation coefficient would be absolutely continuous.

**Definition of \((\tilde{M}_{32})^{(6)}, (\tilde{k}_{32})^{(6)}\):**

\((\tilde{M}_{32})^{(6)}, (\tilde{k}_{32})^{(6)}\), are positive constants

\[
\left( \frac{a_i^{(6)}}{\tilde{M}_{32}^{(6)}}, \frac{b_i^{(6)}}{\tilde{M}_{32}^{(6)}} \right) < 1
\]

**Definition of \((\tilde{P}_{32})^{(6)}, (\tilde{Q}_{32})^{(6)}\):**

There exists two constants \((\tilde{P}_{32})^{(6)}\) and \((\tilde{Q}_{32})^{(6)}\) which together with

\((\tilde{M}_{32})^{(6)}, (\tilde{k}_{32})^{(6)}, (\tilde{A}_{32})^{(6)}\) and \((\tilde{B}_{32})^{(6)}\) and the constants

\((a_i^{(6)}), (a_i^{(6)}), (b_i^{(6)}), (b_i^{(6)}), (p_i^{(6)}), (r_i^{(6)}), i = 32, 33, 34,\)

satisfy the inequalities

\[
\frac{1}{(\tilde{M}_{32})^{(6)}} \left[ (a_i^{(6)}) + (a_i')^{(6)} + (\tilde{A}_{32})^{(6)} + (\tilde{P}_{32})^{(6)} (\tilde{k}_{32})^{(6)} \right] < 1
\]

\[
\frac{1}{(\tilde{M}_{32})^{(6)}} \left[ (b_i^{(6)}) + (b_i')^{(6)} + (\tilde{B}_{32})^{(6)} + (\tilde{Q}_{32})^{(6)} (\tilde{k}_{32})^{(6)} \right] < 1
\]

**Theorem 1:** if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

**Definition of \(G_i(0), T_i(0)\):**

\(G_i(t) \leq (\tilde{P}_{13})^{(1)} e^{(\tilde{R}_{13})^{(1)}t}, \quad G_i(0) = G_i^0 > 0\)

\(T_i(t) \leq (\tilde{Q}_{13})^{(1)} e^{(\tilde{R}_{13})^{(1)}t}, \quad T_i(0) = T_i^0 > 0\)

**Definition of \(G_i(0), T_i(0)\):**

\(G_i(t) \leq (\tilde{P}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)}t}, \quad G_i(0) = G_i^0 > 0\)

\(T_i(t) \leq (\tilde{Q}_{16})^{(2)} e^{(\tilde{R}_{16})^{(2)}t}, \quad T_i(0) = T_i^0 > 0\)

**Definition of \(G_i(0), T_i(0)\):**

\(G_i(t) \leq (\tilde{P}_{20})^{(3)} e^{(\tilde{R}_{20})^{(3)}t}, \quad G_i(0) = G_i^0 > 0\)

\(T_i(t) \leq (\tilde{Q}_{20})^{(3)} e^{(\tilde{R}_{20})^{(3)}t}, \quad T_i(0) = T_i^0 > 0\)

**Definition of \(G_i(0), T_i(0)\):**

\[
\quad
\]

www.ijsrp.org
\[ G_i(t) \leq (\hat{P}_{24})^{(4)} e^{(R_{24})^{(4)}t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{24})^{(4)} e^{(R_{24})^{(4)}t}, \quad T_i(0) = T^0_i > 0 \]

**Definition of \( G_i(0), T_i(0) \):**

\[ G_i(t) \leq (\hat{P}_{28})^{(5)} e^{(R_{28})^{(5)}t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(R_{28})^{(5)}t}, \quad T_i(0) = T^0_i > 0 \]

**Definition of \( G_i(0), T_i(0) \):**

\[ G_i(t) \leq (\hat{P}_{32})^{(6)} e^{(R_{32})^{(6)}t}, \quad G_i(0) = G^0_i > 0 \]

\[ T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(R_{32})^{(6)}t}, \quad T_i(0) = T^0_i > 0 \]

**Proof:** Consider operator \( \mathcal{A}^{(1)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G_i^0 \leq (\hat{P}_{13})^{(1)}, T_i^0 \leq (\hat{Q}_{13})^{(1)}, \]

\[ 0 \leq G_i(t) - G^0_i \leq (\hat{P}_{13})^{(1)} e^{(R_{13})^{(1)}t} \]

\[ 0 \leq T_i(t) - T^0_i \leq (\hat{Q}_{13})^{(1)} e^{(R_{13})^{(1)}t} \]

By

\[ \bar{G}_{13}(t) = G^0_{13} + \int_0^t \left[ (a_{13})^{(1)} G_{14}(s_{13}) - (a'_{13})^{(1)} (T_{14}(s_{13}), s_{13}) \right] G_{13}(s_{13}) ds_{13} \]

\[ \bar{G}_{14}(t) = G^0_{14} + \int_0^t \left[ (a_{14})^{(1)} G_{13}(s_{13}) - (a'_{14})^{(1)} (T_{14}(s_{13}), s_{13}) \right] G_{14}(s_{13}) ds_{13} \]

\[ \bar{G}_{15}(t) = G^0_{15} + \int_0^t \left[ (a_{15})^{(1)} G_{14}(s_{13}) - (a'_{15})^{(1)} (T_{14}(s_{13}), s_{13}) \right] G_{15}(s_{13}) ds_{13} \]

\[ \bar{T}_{13}(t) = T^0_{13} + \int_0^t \left[ (b_{13})^{(1)} T_{14}(s_{13}) - (b'_{13})^{(1)} (G(s_{13}), s_{13}) \right] T_{13}(s_{13}) ds_{13} \]

\[ \bar{T}_{14}(t) = T^0_{14} + \int_0^t \left[ (b_{14})^{(1)} T_{13}(s_{13}) - (b'_{14})^{(1)} (G(s_{13}), s_{13}) \right] T_{14}(s_{13}) ds_{13} \]

\[ \bar{T}_{15}(t) = T^0_{15} + \int_0^t \left[ (b_{15})^{(1)} T_{14}(s_{13}) - (b'_{15})^{(1)} (G(s_{13}), s_{13}) \right] T_{15}(s_{13}) ds_{13} \]

Where \( s_{13} \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( \mathcal{A}^{(2)} \) defined on the space of sextuples of continuous functions \( G_i, T_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G^0_i, \quad T_i(0) = T^0_i, \quad G_i^0 \leq (\hat{P}_{16})^{(2)}, T_i^0 \leq (\hat{Q}_{16})^{(2)}, \]

www.ijsrp.org
\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{16})^{(2)} e^{(A_{16})^{(2)}} t \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{16})^{(2)} e^{(A_{16})^{(2)}} t \]

By

\[
\tilde{G}_{16}(t) = G_{16}^0 + \int_0^t \left( (a_{16})^{(2)} G_{17}(s(16)) - \left( (a'_{16})^{(2)} + a''_{16} \right)^{(2)} (T_{17}(s(16)), s(16)) \right) \text{d}s(16)
\]

\[
\tilde{G}_{17}(t) = G_{17}^0 + \int_0^t \left( (a_{17})^{(2)} G_{16}(s(16)) - \left( (a'_{17})^{(2)} + (a_{17})^{(2)} (T_{17}(s(16)), s(17)) \right) \right) \text{d}s(16)
\]

\[
\tilde{G}_{18}(t) = G_{18}^0 + \int_0^t \left( (a_{18})^{(2)} G_{17}(s(16)) - \left( (a'_{18})^{(2)} + (a_{18})^{(2)} (T_{17}(s(16)), s(16)) \right) \right) \text{d}s(16)
\]

\[
\tilde{T}_{16}(t) = T_{16}^0 + \int_0^t \left( (b_{16})^{(2)} T_{17}(s(16)) - \left( (b'_{16})^{(2)} - (b_{16})^{(2)} (G(s(16)), s(16)) \right) \right) \text{d}s(16)
\]

\[
\tilde{T}_{17}(t) = T_{17}^0 + \int_0^t \left( (b_{17})^{(2)} T_{16}(s(16)) - \left( (b'_{17})^{(2)} - (b_{17})^{(2)} (G(s(16)), s(16)) \right) \right) \text{d}s(16)
\]

\[
\tilde{T}_{18}(t) = T_{18}^0 + \int_0^t \left( (b_{18})^{(2)} T_{17}(s(16)) - \left( (b'_{18})^{(2)} - (b_{18})^{(2)} (G(s(16)), s(16)) \right) \right) \text{d}s(16)
\]

Where \( s(16) \) is the integrand that is integrated over an interval \((0, t)\)

**Proof:**

Consider operator \( A^{(3)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i(0) = (\tilde{P}_{20})^{(3)}, T_i(0) = (\tilde{Q}_{20})^{(3)} \]

\[ 0 \leq G_i(t) - G_i^0 \leq (\tilde{P}_{20})^{(3)} e^{(A_{20})^{(3)} t} \]

\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{20})^{(3)} e^{(A_{20})^{(3)} t} \]

By

\[
\tilde{G}_{20}(t) = G_{20}^0 + \int_0^t \left( (a_{20})^{(3)} G_{21}(s(20)) - \left( (a'_{20})^{(3)} + a''_{20} \right)^{(3)} (T_{21}(s(20)), s(20)) \right) \text{d}s(20)
\]

\[
\tilde{G}_{21}(t) = G_{21}^0 + \int_0^t \left( (a_{21})^{(2)} G_{20}(s(20)) - \left( (a'_{21})^{(2)} + a''_{21} \right)^{(2)} (T_{21}(s(20)), s(20)) \right) \text{d}s(20)
\]

\[
\tilde{G}_{22}(t) = G_{22}^0 + \int_0^t \left( (a_{22})^{(2)} G_{21}(s(20)) - \left( (a'_{22})^{(2)} + a''_{22} \right)^{(2)} (T_{21}(s(20)), s(20)) \right) \text{d}s(20)
\]

\[
\tilde{T}_{20}(t) = T_{20}^0 + \int_0^t \left( (b_{20})^{(3)} T_{21}(s(20)) - \left( (b'_{20})^{(3)} - (b_{20})^{(3)} (G(s(20)), s(20)) \right) \right) \text{d}s(20)
\]

\[
\tilde{T}_{21}(t) = T_{21}^0 + \int_0^t \left( (b_{21})^{(3)} T_{20}(s(20)) - \left( (b'_{21})^{(3)} - (b_{21})^{(3)} (G(s(20)), s(20)) \right) \right) \text{d}s(20)
\]

\[
\tilde{T}_{22}(t) = T_{22}^0 + \int_0^t \left( (b_{22})^{(3)} T_{21}(s(20)) - \left( (b'_{22})^{(3)} - (b_{22})^{(3)} (G(s(20)), s(20)) \right) \right) \text{d}s(20)
\]

Where \( s(20) \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( A^{(4)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+ \) which satisfy

\[ G_i(0) = G_i^0, T_i(0) = T_i^0, G_i(0) = (\tilde{P}_{24})^{(4)}, T_i(0) = (\tilde{Q}_{24})^{(4)}, \]

www.ijsrp.org
\[ 0 \leq G_i(t) - G_i^0 \leq (P_{24})^{(4)}e^{(\tilde{G}_{24})^{(4)}t} \]
\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{24})^{(4)}e^{(\tilde{\tilde{G}}_{24})^{(4)}t} \]

By
\[ \tilde{G}_{24}(t) = G_{24}^0 + \int_0^t \left[ (a_{24})^{(4)}G_{25}(s) - \left( (a_{24})^{(4)} + a_{24}''^{(4)} \right) \left( T_{25}(s), s \right) \right] ds \]
\[ \tilde{G}_{25}(t) = G_{25}^0 + \int_0^t \left[ (a_{25})^{(4)}G_{26}(s) - \left( (a_{25})^{(4)} + a_{25}''^{(4)} \right) \left( T_{25}(s), s \right) \right] ds \]
\[ \tilde{G}_{26}(t) = G_{26}^0 + \int_0^t \left[ (a_{26})^{(4)}G_{27}(s) - \left( (a_{26})^{(4)} + a_{26}''^{(4)} \right) \left( T_{25}(s), s \right) \right] ds \]
\[ \tilde{T}_{24}(t) = T_{24}^0 + \int_0^t \left[ (b_{24})^{(4)}T_{25}(s) - \left( (b_{24})^{(4)} + b_{24}''^{(4)} \right) \left( G(s), s \right) \right] ds \]
\[ \tilde{T}_{25}(t) = T_{25}^0 + \int_0^t \left[ (b_{25})^{(4)}T_{26}(s) - \left( (b_{25})^{(4)} + b_{25}''^{(4)} \right) \left( G(s), s \right) \right] ds \]
\[ \tilde{T}_{26}(t) = T_{26}^0 + \int_0^t \left[ (b_{26})^{(4)}T_{27}(s) - \left( (b_{26})^{(4)} + b_{26}''^{(4)} \right) \left( G(s), s \right) \right] ds \]

Where \( s \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(5)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\[ G_i(0) = G_i^0, \quad T_i(0) = T_i^0, \quad G_i^0 \leq (P_{28})^{(5)} \]
\[ 0 \leq G_i(t) - G_i^0 \leq (P_{28})^{(5)}e^{(\tilde{G}_{28})^{(5)}t} \]
\[ 0 \leq T_i(t) - T_i^0 \leq (\tilde{Q}_{28})^{(5)}e^{(\tilde{\tilde{G}}_{28})^{(5)}t} \]

By
\[ \tilde{G}_{28}(t) = G_{28}^0 + \int_0^t \left[ (a_{28})^{(5)}G_{29}(s) - \left( (a_{28})^{(5)} + a_{28}''^{(5)} \right) \left( T_{29}(s), s \right) \right] ds \]
\[ \tilde{G}_{29}(t) = G_{29}^0 + \int_0^t \left[ (a_{29})^{(5)}G_{30}(s) - \left( (a_{29})^{(5)} + a_{29}''^{(5)} \right) \left( T_{29}(s), s \right) \right] ds \]
\[ \tilde{G}_{30}(t) = G_{30}^0 + \int_0^t \left[ (a_{30})^{(5)}G_{31}(s) - \left( (a_{30})^{(5)} + a_{30}''^{(5)} \right) \left( T_{29}(s), s \right) \right] ds \]
\[ \tilde{T}_{28}(t) = T_{28}^0 + \int_0^t \left[ (b_{28})^{(5)}T_{29}(s) - \left( (b_{28})^{(5)} + b_{28}''^{(5)} \right) \left( G(s), s \right) \right] ds \]
\[ \tilde{T}_{29}(t) = T_{29}^0 + \int_0^t \left[ (b_{29})^{(5)}T_{30}(s) - \left( (b_{29})^{(5)} + b_{29}''^{(5)} \right) \left( G(s), s \right) \right] ds \]
\[ \tilde{T}_{30}(t) = T_{30}^0 + \int_0^t \left[ (b_{30})^{(5)}T_{31}(s) - \left( (b_{30})^{(5)} + b_{30}''^{(5)} \right) \left( G(s), s \right) \right] ds \]

Where \( s \) is the integrand that is integrated over an interval \((0, t)\)

Consider operator \( \mathcal{A}^{(6)} \) defined on the space of sextuples of continuous functions \( G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) which satisfy
\(G_i(0) = G_i^0, \ T_i(0) = T_i^0, \ G_i^0 \leq (\bar{P}_{32})^{(6)}, \ T_i^0 \leq (\bar{Q}_{32})^{(6)},\)

\[0 \leq G_i(t) - G_i^0 \leq (\bar{P}_{32})^{(6)} e^{(\bar{Q}_{32})^{(6)} t}\]

\[0 \leq T_i(t) - T_i^0 \leq (\bar{Q}_{32})^{(6)} e^{(\bar{Q}_{32})^{(6)} t}\]

By

\[\tilde{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32}^{(6)}) G_{33}(s_{32}) - (a_{32}^{(6)} + a_{32}^{(6)})(T_{33}(s_{32}), s_{32})) G_{32}(s_{32})\right] ds_{32}\]

\[\tilde{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33}^{(6)}) G_{32}(s_{32}) - (a_{33}^{(6)} + a_{33}^{(6)})(T_{33}(s_{32}), s_{32})) G_{33}(s_{32})\right] ds_{32}\]

\[\tilde{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34}^{(6)}) G_{33}(s_{32}) - (a_{34}^{(6)} + a_{34}^{(6)})(T_{33}(s_{32}), s_{32})) G_{34}(s_{32})\right] ds_{32}\]

\[\tilde{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32}^{(6)}) T_{33}(s_{32}) - (b_{32}^{(6)} + b_{32}^{(6)}) G(s_{32}) T_{32}(s_{32})\right] ds_{32}\]

\[\tilde{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33}^{(6)}) T_{32}(s_{32}) - (b_{33}^{(6)} + b_{33}^{(6)}) G(s_{32}) T_{33}(s_{32})\right] ds_{32}\]

\[\tilde{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34}^{(6)}) T_{33}(s_{32}) - (b_{34}^{(6)} + b_{34}^{(6)}) G(s_{32}) T_{34}(s_{32})\right] ds_{32}\]

Where \(s_{32}\) is the integrand that is integrated over an interval \((0, t)\)

(a) The operator \(\mathcal{A}^{(1)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13}^{(1)}) G_{14} + (\bar{P}_{13})^{(1)} e^{(\bar{Q}_{13})^{(1)} t} G_{13}\right] ds_{13} =
\]

\[1 + (a_{13}^{(1)}) t G_{14}^0 + \left(\frac{(a_{13}^{(1)}) (\bar{P}_{13})^{(1)}}{(\bar{Q}_{13})^{(1)}} e^{(\bar{Q}_{13})^{(1)} t} - 1\right)\]

From which it follows that

\[(G_{13}(t) - G_{13}^0) e^{-(\bar{Q}_{13})^{(1)} t} \leq \left(\frac{(a_{13}^{(1)}) (\bar{P}_{13})^{(1)}}{(\bar{Q}_{13})^{(1)}} e^{(\bar{Q}_{13})^{(1)} t} + (\bar{P}_{13})^{(1)}\right)\]

\((G_{13}^0)\) is as defined in the statement of theorem 1

Analogous inequalities hold also for \(G_{14}, G_{15}, T_{13}, T_{14}, T_{15}\)

(b) The operator \(\mathcal{A}^{(2)}\) maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

\[G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16}^{(2)}) G_{17} + (\bar{P}_{16})^{(6)} e^{(\bar{Q}_{16})^{(2)} t} G_{16}\right] ds_{16} =
\]

\[1 + (a_{16}^{(2)}) t G_{17}^0 + \left(\frac{(a_{16}^{(2)}) (\bar{P}_{16})^{(2)}}{(\bar{Q}_{16})^{(2)}} e^{(\bar{Q}_{16})^{(2)} t} - 1\right)\]

From which it follows that

\[(G_{16}(t) - G_{16}^0) e^{-(\bar{Q}_{16})^{(2)} t} \leq \left(\frac{(a_{16}^{(2)}) (\bar{P}_{16})^{(2)}}{(\bar{Q}_{16})^{(2)}} e^{(\bar{Q}_{16})^{(2)} t} + (\bar{P}_{16})^{(2)}\right)\]

Analogous inequalities hold also for \(G_{17}, G_{18}, T_{16}, T_{17}, T_{18}\)
(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^{0} + \int_{0}^{t} \left[ (a_{20})^{(3)} \left( G_{21}^{0} + (P_{20})^{(3)} e^{(s_{20})^{(3)} x_{(20)}} \right) \right] ds_{(20)} = \left( 1 + (a_{20})^{(3)} t \right) G_{21}^{0} + \frac{(a_{20})^{(3)}(P_{20})^{(3)}}{(M_{20})^{(3)}} \left( e^{(s_{20})^{(3)} t} - 1 \right)$$

From which it follows that

$$(G_{20}(t) - G_{20}^{0}) e^{-(s_{20})^{(3)} t} \leq \frac{(a_{20})^{(3)}}{(M_{20})^{(3)}} \left[ \left( \hat{P}_{20}^{(3)} + G_{21}^{0} e^{(s_{20})^{(3)} t} \right) + (P_{20})^{(3)} \right]$$

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{24}(t) \leq G_{24}^{0} + \int_{0}^{t} \left[ (a_{24})^{(4)} \left( G_{25}^{0} + (P_{24})^{(4)} e^{(s_{24})^{(4)} x_{(24)}} \right) \right] ds_{(24)} = \left( 1 + (a_{24})^{(4)} t \right) G_{25}^{0} + \frac{(a_{24})^{(4)}(P_{24})^{(4)}}{(M_{24})^{(4)}} \left( e^{(s_{24})^{(4)} t} - 1 \right)$$

From which it follows that

$$(G_{24}(t) - G_{24}^{0}) e^{-(s_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(M_{24})^{(4)}} \left[ \left( \hat{P}_{24}^{(4)} + G_{25}^{0} e^{(s_{24})^{(4)} t} \right) + (P_{24})^{(4)} \right]$$

($G_{i}^{0}$) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{28}(t) \leq G_{28}^{0} + \int_{0}^{t} \left[ (a_{28})^{(5)} \left( G_{29}^{0} + (P_{28})^{(5)} e^{(s_{28})^{(5)} x_{(28)}} \right) \right] ds_{(28)} = \left( 1 + (a_{28})^{(5)} t \right) G_{29}^{0} + \frac{(a_{28})^{(5)}(P_{28})^{(5)}}{(M_{28})^{(5)}} \left( e^{(s_{28})^{(5)} t} - 1 \right)$$

From which it follows that

$$(G_{28}(t) - G_{28}^{0}) e^{-(s_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(M_{28})^{(5)}} \left[ \left( \hat{P}_{28}^{(5)} + G_{29}^{0} e^{(s_{28})^{(5)} t} \right) + (P_{28})^{(5)} \right]$$

($G_{i}^{0}$) is as defined in the statement of theorem 1

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself. Indeed it is obvious that

$$G_{32}(t) \leq G_{32}^{0} + \int_{0}^{t} \left[ (a_{32})^{(6)} \left( G_{33}^{0} + (P_{32})^{(6)} e^{(s_{32})^{(6)} x_{(32)}} \right) \right] ds_{(32)} = \left( 1 + (a_{32})^{(6)} t \right) G_{33}^{0} + \frac{(a_{32})^{(6)}(P_{32})^{(6)}}{(M_{32})^{(6)}} \left( e^{(s_{32})^{(6)} t} - 1 \right)$$

From which it follows that
From the hypotheses it follows where

\[ G(0) \] as defined in the statement of theorem 6

Analogous inequalities hold also for \( G_{25}, G_{26}, T_{24}, T_{25}, T_{26} \)

It is now sufficient to take \( \frac{(a_{ij})^{(1)}}{(M_{ij})^{(1)}} \) and \( \frac{(b_{ij})^{(1)}}{(M_{ij})^{(1)}} < 1 \) and to choose

\[ (\tilde{P}_{ij})^{(1)} \] and \( (\tilde{Q}_{ij})^{(1)} \) large to have

\[ \frac{(a_{ij})^{(1)}}{(M_{ij})^{(1)}} \left[ (\tilde{P}_{ij})^{(1)} + \left( (\tilde{P}_{ij})^{(1)} + G_{ij}^{0} \right) e^{-\frac{(P_{ij})^{(1)}}{\gamma_{ij}}} \right] \leq (\tilde{P}_{ij})^{(1)} \]

\[ \frac{(b_{ij})^{(1)}}{(M_{ij})^{(1)}} \left[ (\tilde{Q}_{ij})^{(1)} + T_{ij}^{0} e^{-\frac{(Q_{ij})^{(1)}}{\gamma_{ij}}} + (\tilde{Q}_{ij})^{(1)} \right] \leq (\tilde{Q}_{ij})^{(1)} \]

In order that the operator \( \mathcal{A}^{(1)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying \( \text{GLOBAL EQUATIONS} \) into itself

The operator \( \mathcal{A}^{(1)} \) is a contraction with respect to the metric

\[ d \left( (G^{(1)}, T^{(1)}), (G^{(2)}, T^{(2)}) \right) = \]

\[ \sup_{t \in \mathbb{R}^+} \max_{i \in \mathbb{I}} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-\gamma^{(1)}(t)} \max_{i \in \mathbb{I}} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-\gamma^{(1)}(t)} \]

Indeed if we denote

**Definition of \( \tilde{G}, \tilde{T} \):**

\[ (\tilde{G}, \tilde{T}) = \mathcal{A}^{(1)}(G, T) \]

It results

\[ |\tilde{G}_{ij} - \tilde{G}_{ij}^{(2)}| \leq \int_0^t (a_{ij})^{(1)} |G_{ij}^{14} - G_{ij}^{14} e^{-\gamma^{(1)}(t)x_{ij}} e^{\gamma^{(1)}(t)x_{ij}} ds_{ij} + \]

\[ \int_0^t (a_{ij}^{(1)} |G_{ij}^{14} - G_{ij}^{14} e^{-\gamma^{(1)}(t)x_{ij}} e^{\gamma^{(1)}(t)x_{ij}} + \]

\[ (a_{ij}^{(1)} |T_{ij}^{(1)}|, s_{ij}) |G_{ij}^{14} - G_{ij}^{14} e^{-\gamma^{(1)}(t)x_{ij}} e^{\gamma^{(1)}(t)x_{ij}} + \]

\[ G_{ij}^{14} |(a_{ij}^{n})^{(1)} (T_{ij}^{(1)} - s_{ij}) - (a_{ij}^{n})^{(1)} (T_{ij}^{(2)} - s_{ij}) | e^{-\gamma^{(1)}(t)x_{ij}} e^{\gamma^{(1)}(t)x_{ij}} ds_{ij} \]

Where \( s_{ij} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ |G^{(1)} - G^{(2)}| e^{-\gamma^{(1)}(t)} \leq \]

\[ \frac{1}{(M_{ij})^{(1)}} \left( (a_{ij})^{(1)} + (a_{ij}^{(1)}) + (a_{ij}^{(1)}) + (T_{ij})^{(1)} (K_{ij})^{(1)} \right) d \left( (G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}) \right) \]
And analogous inequalities for \( G_i \) and \( T_i \), Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \( (a_{13})^{(1)} \) and \( (b_{13})^{(1)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \( (P_{13})^{(1)} e^{(R_{13})^{(1)} t} \) and \( (Q_{13})^{(1)} e^{(R_{13})^{(1)} t} \) respectively of \( \mathbb{R}_+ \).

If instead of proving the existence of the solution on \( \mathbb{R}_+ \), we have to prove it only on a compact then it suffices to consider that \( (a_i)^{(1)} \) and \( (b_i)^{(1)} \), \( i = 13, 14, 15 \) depend only on \( T_{14} \) and respectively on \( G \) (and not on \( t \)) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \( t \) where \( G_i (t) = 0 \) and \( T_i (t) = 0 \)

From 19 to 24 it results

\[
G_i (t) \geq G_i^0 e^{\int \left[ -b_i' (a_i)^{(1)} - a_i'' \right] (t_1 (x_{13}), x_{13}) \text{d}x_{13}} 
\]

\[
T_i (t) \geq T_i^0 e^{-b_i' (a_i)^{(1)}} > 0 \text{ for } t > 0
\]

**Definition of** \( (\mathbf{M}_{13})^{(1)}_1, (\mathbf{M}_{13})^{(1)}_2 \) and \( (\mathbf{M}_{13})^{(1)}_3 \):

**Remark 3:** If \( G_{13} \) is bounded, the same property also \( G_{14} \) and \( G_{15} \) indeed if

\[
G_{13} < (\mathbf{M}_{13})^{(1)} \text{ it follows } \frac{dG_{14}}{dt} \leq (\mathbf{M}_{13})^{(1)} - (\mathbf{M}_{13})^{(1)} G_{14} \text{ and by integrating }
\]

\[
G_{14} \leq (\mathbf{M}_{13})^{(1)}_2 = G_{14} + 2 (a_{14})^{(1)} (\mathbf{M}_{13})^{(1)} / (a_{14})^{(1)}
\]

In the same way, one can obtain

\[
G_{15} \leq (\mathbf{M}_{13})^{(1)}_3 = G_{15} + 2 (a_{15})^{(1)} (\mathbf{M}_{13})^{(1)} / (a_{15})^{(1)}
\]

If \( G_{14} \) or \( G_{15} \) is bounded, the same property follows for \( G_{13} \), \( G_{15} \) and \( G_{13}, G_{14} \) respectively.

**Remark 4:** If \( G_{13} \) is bounded, from below, the same property holds for \( G_{14} \) and \( G_{15} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{14} \) is bounded from below.

**Remark 5:** If \( T_{13} \) is bounded from below and \( \lim_{t \to \infty} (b_i')^{(1)} (G(t), t) = (b_{14})^{(1)} \) then \( T_{14} \to \infty \).

**Definition of** \( (m)^{(1)} \) and \( \varepsilon_1 \):

Indeed let \( t_1 \) be so that for \( t > t_1 \)

\[
(b_{14})^{(1)} - (b_i')^{(1)} (G(t), t) < \varepsilon_1, T_{13} (t) > (m)^{(1)}
\]

Then \( \frac{dT_{14}}{dt} \geq (a_{14})^{(1)} (m)^{(1)} - \varepsilon_1 T_{14} \) which leads to

\[
T_{14} \geq \frac{(a_{14})^{(1)} (m)^{(1)}}{\varepsilon_1} \left( 1 - e^{-\varepsilon_1 t} \right) + T_{14}^0 e^{-\varepsilon_1 t} \text{ if we take } t \text{ such that } e^{-\varepsilon_1 t} = \frac{1}{2} \text{ it results }
\]

\[
T_{14} \geq \frac{(a_{14})^{(1)} (m)^{(1)}}{\varepsilon_1} \left( 1 - e^{-\varepsilon_1 t} \right) + T_{14}^0 e^{-\varepsilon_1 t}\right) \text{ if we take now } \varepsilon_1 \text{ sufficiently small one sees that } T_{14} \text{ is unbounded. The same property holds for } T_{15} \text{ if } \lim_{t \to \infty} (b_i'')^{(1)} (G(t), t) = (b_{15}'')^{(1)}
\]

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_i)^{(2)}}{\mathbf{M}_{16}^{(3)}} \), \( \frac{(b_i)^{(2)}}{\mathbf{M}_{16}^{(3)}} < 1 \) and to choose
\[
(\hat{P}_{16})^{(2)} \text{ and } (\hat{Q}_{16})^{(2)} \text{ large to have }
\]
\[
\frac{(a_j)^{(2)}}{(M_{16})^{(2)}} \left[ (\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0) e^{-\frac{(\hat{P}_{16})^{(2)} + G_j^0}{T_j}} \right] \leq (\hat{P}_{16})^{(2)}
\]
\[
\frac{(b_j)^{(2)}}{(M_{16})^{(2)}} \left[ ((\hat{Q}_{16})^{(2)} + T_j^0) e^{-\frac{(\hat{Q}_{16})^{(2)} + T_j^0}{T_j}} + (\hat{Q}_{16})^{(2)} \right] \leq (\hat{Q}_{16})^{(2)}
\]

In order that the operator \(A^{(2)}\) transforms the space of sextuples of functions \(G_i, T_i\) satisfying

The operator \(A^{(2)}\) is a contraction with respect to the metric
\[
d \left( (G_{16})^{(1)}, (T_{16})^{(1)}, (G_{16})^{(2)}, (T_{16})^{(2)} \right) =
\]
\[
\sup \left\{ \max_{i \in \mathbb{R}^+} \left[ |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-((G_{16})^{(2)})^t} \max_{i \in \mathbb{R}^+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-((G_{16})^{(2)})^t} \right] \right\}
\]

Indeed if we denote

**Definition of \(G_{16}, T_{16} \) :** \( (G_{16}, T_{16}) = A^{(2)}(G_{16}, T_{16}) \)

It results
\[
|G_{16}^{(1)} - G_i^{(2)}| \leq \int_0^t (a_{16})^{(2)} |G_{17}^{(1)} - G_i^{(2)}| e^{-((G_{16})^{(2)})^t} e^{((G_{16})^{(2)})^t} d s_{(16)} +
\]
\[
\int_0^t ((a_{16}^{(2)}(1) - G_{16}^{(2)}) e^{-((G_{16})^{(2)})^t} e^{((G_{16})^{(2)})^t} +
\]
\[
(a_{16}^{(2)}(2) (T_{16}^{(1)} - s_{(16)}) |G_{16}^{(1)} - G_i^{(2)}| e^{-((G_{16})^{(2)})^t} e^{((G_{16})^{(2)})^t} +
\]
\[
G_i^{(2)}(a_{16}^{(2)}(2) (T_{16}^{(1)} - s_{(16)}) - (a_{16}^{(2)}(2) (T_{16}^{(2)} - s_{(16)}) | e^{-((G_{16})^{(2)})^t} e^{((G_{16})^{(2)})^t} d s_{(16)}
\]

Where \(s_{(16)}\) represents integrand that is integrated over the interval \([0, t]\).

From the hypotheses it follows
\[
\left| (G_{16})^{(1)} - (G_{16})^{(2)} e^{-((G_{16})^{(2)})^t} \leq \frac{1}{(M_{16})^{(2)}} \left( (a_{16})^{(2)} + (a_{16}^{(2)}(1) + (\lambda_{16})^{(2)} + (\hat{P}_{16})^{(2)} (k_{16})^{(2)} d \left( ((G_{16})^{(1)}, (T_{16})^{(1)}, (G_{16})^{(2)}, (T_{16})^{(2)} \right) \right)
\]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{16})^{(2)}\) and \((b_{16})^{(2)}\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\hat{P}_{16})^{(2)} e^{((G_{16})^{(2)})^t}\) and \((\hat{Q}_{16})^{(2)} e^{((G_{16})^{(2)})^t}\) respectively on \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_{16})^{(2)}\) and \((b_{16})^{(2)}, i = 16, 17, 18\) depend only on \(T_{17}\) and respectively on \((G_{19})(\text{and not on } t)\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 19 to 24 it results
\[ G_i(t) \geq G_i^0 e^{-\int_{0}^{t}((a_i''(\tau))^2-(a_i''(\tau))^2(T_1(t),x(t))) d\tau(\tau)} \geq 0 \]

\[ T_i(t) \geq T_i^0 e^{-\int_{0}^{t}(b_i''(\tau))^2} > 0 \quad \text{for } t > 0 \]

**Definition of** \( (\mathcal{M}_{16}^{(2)})_1, (\mathcal{M}_{16}^{(2)})_2 \) and \( (\mathcal{M}_{16}^{(2)})_3 \):

**Remark 3**: if \( G_{16} \) is bounded, the same property have also \( G_{17} \) and \( G_{18} \). Indeed if \( G_{16} < (\mathcal{M}_{16}^{(2)})_1 \) it follows \( \frac{dG_{17}}{dt} \leq (\mathcal{M}_{16}^{(2)})_1 - (a_{17})^{(2)}G_{17} \) and by integrating

\[ G_{17} \leq (\mathcal{M}_{16}^{(2)})_1^2 \]

\[ G_{17} \leq (\mathcal{M}_{16}^{(2)})_2 = G_{17}^0 + 2(a_{17})^{(2)}(\mathcal{M}_{16}^{(2)})_1/(a_{17})^{(2)} \]

In the same way, one can obtain

\[ G_{18} \leq (\mathcal{M}_{16}^{(2)})_3 = G_{18}^0 + 2(a_{18})^{(2)}(\mathcal{M}_{16}^{(2)})_2/(a_{18})^{(2)} \]

If \( G_{17} \) or \( G_{18} \) is bounded, the same property follows for \( G_{16} \), \( G_{18} \) and \( G_{16} \), \( G_{17} \) respectively.

**Remark 4**: If \( G_{16} \) is bounded, from below, the same property holds for \( G_{17} \) and \( G_{18} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{17} \) is bounded from below.

**Remark 5**: If \( T_{16} \) is bounded from below and \( \lim_{t \to \infty}((b_i'')^{(2)}((G_{19})(t),t)) = (b_{17}'')^{(2)} \) then \( T_{17} \to \infty \).

**Definition of** \( (m)^{(2)} \) and \( \varepsilon_2 \):

Indeed let \( t_2 \) be so that for \( t > t_2 \)

\[ (b_{17}'')^{(2)} - (b_i'')^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)} \]

Then \( \frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17} \) which leads to

\[ T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right) (1 - e^{-\varepsilon_2}) + T_{17}^0 e^{-\varepsilon_2} \] If we take \( t \) such that \( e^{-\varepsilon_2} = \frac{1}{2} \) it results

\[ T_{17} \geq \left( \frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2} \right), t = \log_{\varepsilon_2}^2 \] By taking now \( \varepsilon_2 \) sufficiently small one sees that \( T_{17} \) is unbounded. The same property holds for \( T_{18} \) if \( \lim_{t \to \infty}(b_{18}'')^{(2)}((G_{19})(t),t) = (b_{18}'')^{(2)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_{17})^{(3)}}{(M_{20}^{(2)})}, \frac{(b_{17})^{(3)}}{(M_{20}^{(2)})}, \frac{(b_{18})^{(3)}}{(M_{20}^{(2)})} < 1 \) and to choose

\( (P_{20})^{(3)}, (Q_{20})^{(3)} \) large to have

\[ \frac{(a_{17})^{(3)}}{(M_{20}^{(2)})} \left( P_{20}^{(3)} + (P_{20}^{(3)} + C_{17}^0 e^{-\left( \frac{(P_{20}^{(3)} + C_{17}^0)^2}{C_{17}^0} \right)^{2}}) \right) \leq (P_{20})^{(3)} \]

\[ \frac{(b_{17})^{(3)}}{(M_{20}^{(2)})} \left( (Q_{20}^{(3)} + T_{17}^0 e^{-\left( \frac{(Q_{20}^{(3)} + T_{17}^0)^2}{T_{17}^0} \right)^{2}}) + (Q_{20})^{(3)} \right) \leq (Q_{20})^{(3)} \]

In order that the operator \( \mathcal{A}^{(3)} \) transforms the space of sextuples of functions \( G_i, T_i \) into itself:

www.ijsrp.org
The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric
\[
d \left( ((G_{23})^{(1)}, (T_{23})^{(1)}), ((G_{23})^{(2)}, (T_{23})^{(2)}) \right) = \sup \left( \max_{i \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-((M_{20})^{(3)})(t)} \right) \max_i |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-((M_{20})^{(3)})(t)} \}
\]
Indeed if we denote
\[
\text{Definition of } G_{23}^{(3)} T_{23}^{(3)} : ( G_{23}^{(3)}, T_{23}^{(3)} ) = \mathcal{A}^{(3)}( G_{23}, T_{23} )
\]
It results
\[
\left| G_{20}^{(1)} - G_{20}^{(2)} \right| \leq \int_0^t \left( a_{20}^{(3)} - a_{20}^{(2)} \right) |G_{21}^{(1)} - G_{21}^{(2)}| e^{-((M_{20})^{(3)})(t)} e^{((M_{20})^{(3)})(t)} ds_{(20)} +
\int_0^t \left( a_{20}^{(3)} - a_{20}^{(2)} \right) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-((M_{20})^{(3)})(t)} e^{((M_{20})^{(3)})(t)} ds_{(20)} +
\]
\[
(a_{20}^{(3)}(T_{21}^{(3)}), s_{(20)}^{(3)})) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-((M_{20})^{(3)})(t)} e^{((M_{20})^{(3)})(t)} ds_{(20)} +
\]
\[
G_{20}^{(2)}(a_{20}^{(3)}(T_{21}^{(3)}), s_{(20)}^{(3)})) |G_{20}^{(1)} - G_{20}^{(2)}| e^{-((M_{20})^{(3)})(t)} e^{((M_{20})^{(3)})(t)} ds_{(20)}
\]
Where $s_{(20)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows
\[
\left| G_{20}^{(1)} - G_{20}^{(2)} \right| e^{-((M_{20})^{(3)})(t)} \leq \frac{1}{(M_{20})^{(3)}} ((a_{20}^{(3)} + a_{20}^{(2)} + (20^{(3)} + (20^{(3)}(20^{(3)})) + (20^{(3)})) ds_{(20)} +
\]
And analogous inequalities for $G_i$ and $T_i$. Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed $(a_{20}^{(3)})$ and $(b_{20}^{(3)})$ depending also on $t$ can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(20^{(3)}(t)) e^{((M_{20})^{(3)})(t)}$ and $(20^{(3)}(t)) e^{((M_{20})^{(3)})(t)}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_{i}^{(3)})$ and $(b_{i}^{(3)})$, $i = 20, 21, 22$ depend only on $T_{21}$ and respectively on $(G_{23})$ (and not on $t$) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any $t$ where $G_i(t) = 0$ and $T_i(t) = 0$

From 19 to 24 it results
\[
G_i(t) \geq G_0^i e^{-((M_{20})^{(3)})(t)} ds_{(20)} \geq 0
\]
\[
T_i(t) \geq T_0^i e^{-((M_{20})^{(3)})(t)} > 0 \text{ for } t > 0
\]

**Definition of** $(G_{20}^{(3)})_1$, $(G_{20}^{(3)})_2$ and $(G_{20}^{(3)})_3$ :

**Remark 3:** if $G_{20}$ is bounded, the same property have also $G_{21}$ and $G_{22}$. Indeed if
\[
G_{20} < (G_{20})^{(3)} \text{ it follows } \frac{dG_{21}}{dt} \leq ((M_{20})^{(3)})_1 - (a_{21})^{(3)} G_{21} \text{ and by integrating }
\]
\[
G_{21} \leq (G_{20})^{(3)}_1 = G_0^{(3)} + 2(a_{21})^{(3)}((M_{20})^{(3)})_1/(a_{21})^{(3)}
\]
In the same way, one can obtain

www.ijsrp.org
\[
G_{22} \leq (\mathcal{M}_{20})^{(3)}_3 = G_{22}^0 + 2(a_{22})^{(3)}((\mathcal{M}_{20})^{(3)}_2/(a_{22})^{(3)}
\]

If \( G_{21} \) or \( G_{22} \) is bounded, the same property follows for \( G_{20} \), \( G_{22} \) and \( G_{20} \), \( G_{21} \) respectively.

**Remark 4:** If \( G_{20} \) is bounded, from below, the same property holds for \( G_{21} \) and \( G_{22} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{21} \) is bounded from below.

**Remark 5:** If \( T_{20} \) is bounded from below and \( \lim_{t \to \infty} (b_i^{(3)}((G_{23})t, t)) = (b_i^{(3)}) \) then \( T_{21} \to \infty \).

**Definition of** \((m)^{(3)} \) and \( \varepsilon_3 \):

Indeed let \( t_3 \) be so that for \( t > t_3 \)

\[
(b_{21})^{(3)} - (b_i^{(3)}((G_{23})(t), t) < \varepsilon_3, T_{20}(t) > (m)^{(3)}
\]

Then \( \frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21} \) which leads to

\[
T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}
\]

If we take \( t \) such that \( e^{-\varepsilon_3 t} = \frac{1}{2} \) it results

\[
T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_3}
\]

By taking now \( \varepsilon_3 \) sufficiently small one sees that \( T_{21} \) is unbounded.

The same property holds for \( T_{22} \) if \( \lim_{t \to \infty} (b_{21}^{(3)}((G_{23})(t), t) = (b_{22}^{(3)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

It is now sufficient to take \( \frac{(a_i)^{(4)}}{(M_{24})^{(4)}} \), \( \frac{(b_{21})^{(4)}}{(M_{24})^{(4)}} \) \leq 1 \) and to choose

\( (\tilde{P}_{24})^{(4)} \) and \( (\tilde{Q}_{24})^{(4)} \) large to have

\[
\left\{ \frac{(a_i)^{(4)}}{(M_{24})^{(4)}} \left( \tilde{P}_{24}^{(4)} + \left( (\tilde{P}_{24})^{(4)} + G_{0}^{(4)} \right) e^{-\left( \frac{(\tilde{P}_{24})^{(4)} + G_{0}^{(4)}}{G_{0}^{(4)}} \right) t} \right) \right\} \leq (\tilde{P}_{24})^{(4)}
\]

\[
\left\{ \frac{(b_{21})^{(4)}}{(M_{24})^{(4)}} \left( (\tilde{Q}_{24}^{(4)} + \tau_{i}^{(4)}) e^{-\left( \frac{(\tilde{Q}_{24}^{(4)} + \tau_{i}^{(4)})}{\tau_{i}^{(4)}} \right) t} \right) \right\} \leq (\tilde{Q}_{24})^{(4)}
\]

In order that the operator \( \mathcal{A}^{(4)} \) transforms the space of sextuples of functions \( G_i, T_i \) satisfying IN to itself

The operator \( \mathcal{A}^{(4)} \) is a contraction with respect to the metric

\[
d \left( (G_{27})^{(1)}, (T_{27})^{(1)} \right), (G_{27})^{(2)}, (T_{27})^{(2)} \right) = \sup_{t \in \mathbb{R}} \max \left\{ G_i^{(1)}(t) - G_i^{(2)}(t) \left| e^{-\left( (N_{24})^{(4)} \right)_{\tau_{i}}^t} \right| \right\} \max_{t \in \mathbb{R}} \left| T_i^{(1)}(t) - T_i^{(2)}(t) \left| e^{-\left( (N_{24})^{(4)} \right)_{\tau_{i}}^t} \right| \right\}
\]

Indeed if we denote

**Definition of** \((\bar{G}_{24})(1), (\bar{T}_{24})(1) : (G_{24})^{(1)}, (T_{24})^{(1)} \) = \( \mathcal{A}^{(4)}((G_{27}), (T_{27})) \)

It results

\[
(\bar{G}_{24})^{(1)} - G_i^{(2)} \right\} \leq \int_{0}^{t} (a_{24})^{(4)} \left| G_{25}^{(1)} - G_{25}^{(2)} \left| e^{-\left( (N_{24})^{(4)} \right)_{\tau_{i}}^t} \right| e^{(N_{24})^{(4)}_{\tau_{i}}^t} \right| ds_{(24)} + \]

www.ijsrp.org
\[
\int_0^t (a_{24}^{(4)})^2 G_{24}^{(1)} - G_{24}^{(2)} e^{-\lambda_{24}^{(4)} s_{(24)}} e^{-\lambda_{24}^{(4)} x_{(24)}} +
\]
\[
(a_{24}^{(4)})^2 (T_{25}^{(1)}, s_{(24)}) |G_{24}^{(1)} - G_{24}^{(2)}| e^{-\lambda_{24}^{(4)} s_{(24)}} e^{\lambda_{24}^{(4)} x_{(24)}} +
\]
\[
G_{24}^{(2)} (a_{24}^{(4)} (T_{25}^{(1)}, s_{(24)})) - (a_{24}^{(4)} (T_{25}^{(2)}, s_{(24)})) | e^{-\lambda_{24}^{(4)} s_{(24)}} e^{\lambda_{24}^{(4)} x_{(24)}} dS_{(24)}
\]

Where \(s_{(24)}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows
\[
\left| (G_{27}^{(1)}) - (G_{27}^{(2)}) | e^{-\lambda_{24}^{(4)} t} \right| \leq \frac{1}{\lambda_{24}^{(4)}} \left| (a_{24}^{(4)}) + (a_{24}^{(4)}) (\widetilde{A}_{24}^{(4)} + (\widetilde{P}_{24}^{(4)} (\widetilde{R}_{24}^{(4)}) d \left( (G_{27}^{(1)}), (T_{27}^{(1)}), (G_{27}^{(2)}), (T_{27}^{(2)}) \right) \right)
\]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{24}^{(4)})^2\) and \((b_{24}^{(4)})\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((\widetilde{P}_{24}^{(4)} e^{\lambda_{24}^{(4)} t}\) and \((\widetilde{Q}_{24}^{(4)} e^{\lambda_{24}^{(4)} t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_i^{(4)})\) and \((b_i^{(4)})\), \(i = 24, 25, 26\) depend only on \(T_{25}\) and respectively on \((G_{27})\) and \((T_{25})\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 19 to 24 it results
\[
G_i(t) \geq 0 \quad T_i(t) > 0 \quad \text{for} \quad t > 0
\]

**Definition of** \((\overline{M}_{24}^{(4)}), (\overline{M}_{24}^{(4)})\) and \((\overline{M}_{24}^{(4)}):\)

**Remark 3:** If \(G_{24}\) is bounded, the same property have also \(G_{25}\) and \(G_{26}\). Indeed if
\[
G_{24} < (\overline{M}_{24}^{(4)})\text{ it follows } \frac{dG_{25}}{dt} \leq (\overline{M}_{24}^{(4)}) - (a_{25}^{(4)}) G_{25} \quad \text{and by integrating}
\]
\[
G_{25} \leq (\overline{M}_{24}^{(4)}) \text{ } 2 = G_{25} + 2(a_{25}^{(4)})(\overline{M}_{24}^{(4)}) / a_{25}^{(4)}
\]

In the same way, one can obtain
\[
G_{26} \leq (\overline{M}_{24}^{(4)}) \text{ } 3 = G_{26} + 2(a_{26}^{(4)})(\overline{M}_{24}^{(4)}) / a_{26}^{(4)}
\]

If \(G_{25}\) or \(G_{26}\) is bounded, the same property follows for \(G_{24}, G_{26}\) and \(G_{24}, G_{25}\) respectively.

**Remark 4:** If \(G_{24}\) is bounded, from below, the same property holds for \(G_{25}\) and \(G_{26}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{25}\) is bounded from below.

**Remark 5:** If \(T_{24}\) is bounded from below and \(\lim_{t \to \infty} ((b_i^{(4)}), ((G_{27})(t), t)) = (b_{25}^{(4)}), then \(T_{25} \to \infty\).
Definition of \( m^{(4)} \) and \( \varepsilon^{4} \): 

Indeed let \( t_{4} \) be so that for \( t > t_{4} \)

\[
(b_{25})^{(4)} - (b_{i})^{(4)}((G_{27})(t), t) < \varepsilon^{4}, T_{24} (t) > (m)^{(4)}
\]

Then \( \frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_{4}T_{25} \) which leads to

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_{4}} \right) (1 - e^{-\varepsilon_{4}t}) + T_{25}^{0} e^{-\varepsilon_{4}t} \text{ If we take } t \text{ such that } e^{-\varepsilon_{4}t} = \frac{1}{2} \text{ it results}
\]

\[
T_{25} \geq \left( \frac{(a_{25})^{(4)}(m)^{(4)}}{2} \right) \text{, } t = log \frac{2}{\varepsilon_{4}} \text{ By taking now } \varepsilon_{4} \text{ sufficiently small one sees that } T_{25} \text{ is unbounded.}
\]

The same property holds for \( T_{26} \) if \( \lim_{t \to \infty} (b_{26}''^{(4)}((G_{27})(t), t) = (b_{26})^{(4)} \)

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for \( G_{29}, G_{30}, T_{28}, T_{29}, T_{30} \)

It is now sufficient to take \( \frac{(a_{i})^{(5)}}{(M_{28})^{(5)}}, \frac{(b_{i})^{(5)}}{(M_{28})^{(5)}} < 1 \) and to choose

\( (P_{28})^{(5)} \) and \( (Q_{28})^{(5)} \) large to have

\[
\frac{(a_{i})^{(5)}}{(M_{28})^{(5)}} \left[ (P_{28})^{(5)} + ((P_{28})^{(5)} + G_{j}^{0}) e^{-\frac{(P_{28})^{(5)} + G_{j}^{0}}{c_{j}}}) \right] \leq (P_{28})^{(5)}
\]

\[
\frac{(b_{i})^{(5)}}{(M_{28})^{(5)}} \left[ ((Q_{28})^{(5)} + T_{j}^{0}) e^{-\frac{(Q_{28})^{(5)} + T_{j}^{0}}{t_{j}}}) + (Q_{28})^{(5)} \right] \leq (Q_{28})^{(5)}
\]

In order that the operator \( \mathcal{A}^{(5)} \) transforms the space of sextuples of functions \( G_{i}, T_{j} \) into itself
Remark 3: It results

\[ |G_{28}^{(1)} - G_{29}^{(2)}| \leq \int_0^t (a_{28}(\xi) |G_{29}^{(1)} - G_{29}^{(2)}| e^{-((R_{28})^{5}_{28}(\xi))s(28)} + \]

\[ \int_0^t (a_{28}(\xi) (G_{29}^{(1)} - G_{29}^{(2)})) e^{-((R_{28})^{5}_{28}(\xi))s(28)} + \]

\[ (a_{28}(\xi))^{(5)}(T_{29}^{(1)} - s_{28}) G_{29}^{(1)} - G_{29}^{(2)} | e^{-((R_{28})^{5}_{28}(\xi))s(28)} + \]

\[ G_{28}^{(2)}(\xi) e^{-((R_{28})^{5}_{28}(\xi))s(28)} + G_{28}^{(3)}(\xi) e^{-((R_{28})^{5}_{28}(\xi))s(28)}ds(28) \]

Where \( s_{28} \) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ |(G_{31})^{(1)} - (G_{31})^{(2)}| e^{-((R_{28})^{5}_{28})} \leq \]

\[ \frac{1}{(M_{28})^{5}}((a_{28}(\xi))^{(5)} + (a_{28}(\xi))^5 + (T_{28})^{(5)} + (P_{28})^{(5)}(\bar{k}_{28})^{(5)} + (\bar{P}_{28})^{(5)}(\bar{k}_{28})^{(5)})d\left((G_{31})^{(1)}, (T_{31})^{(1)}; (G_{31})^{(2)}, (T_{31})^{(2)}\right) \]

And analogous inequalities for \( G_i \) and \( T_i \). Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed \((a_{28}(\xi)^{(5)} \) and \((b_{28}(\xi)^{(5)} \) depending also on \( t \) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{28})^{(5)} e^{-((R_{28})^{5}_{28})t}\) and \((\bar{Q}_{28})^{(5)} e^{-((R_{28})^{5}_{28})t}\)

respectively of \( \mathbb{R}^+ \).

If instead of proving the existence of the solution on \( \mathbb{R}^+ \), we have to prove it only on a compact then it suffices to consider that \((a_{28}(\xi)^{(5)} \) and \((b_{28}(\xi)^{(5)} \), \( i = 28, 29, 30 \) depend only on \( T_{28} \) and respectively on \((G_{31})\) and \( not on t \) and hypothesis can be replaced by a usual Lipschitz condition.

Remark 2: There does not exist any \( t \) where \( G_i(t) = 0 \) and \( T_i(t) = 0 \)

From GLOBAL EQUATIONS it results

\[ G_i(t) \geq G_0 e^{-\int_0^t ([a_i(\xi)^{(5)} - a_{i(5)}(\bar{k}_{28}(\xi)) s(28))]ds(28)) \geq 0 \]

\[ T_i(t) \geq T_0 e^{-(b_i(\xi)^{(5)})t} > 0 \ for \ t > 0 \]

Definition of \((\bar{M}_{28})^{(5)}, (\bar{M}_{28})^{(5)}, 2 \) and \((\bar{M}_{28})^{(5)}\) :  

Remark 3: If \( G_{28} \) is bounded, the same property have also \( G_{28} \) and \( G_{30} \). Indeed if
\[ G_{28} < (\bar{M}_{28})^{(5)} \implies \frac{dG_{29}}{dt} \leq (\bar{M}_{28})^{(5)} - (a_{29}^{(5)})G_{29} \text{ and by integrating} \]
\[ G_{29} \leq (\bar{M}_{28})^{(5)} = G_{29}^{0} + 2(a_{29}^{(5)})(\bar{M}_{28})^{(5)}(a_{29}^{(5)}) \]

In the same way, one can obtain
\[ G_{30} \leq (\bar{M}_{28})^{(5)} = G_{30}^{0} + 2(a_{30}^{(5)})(\bar{M}_{28})^{(5)}(a_{30}^{(5)}) \]

If \( G_{29} \) or \( G_{30} \) is bounded, the same property follows for \( G_{28} \), \( G_{30} \) and \( G_{28}, G_{29} \) respectively.

**Remark 4:** If \( G_{28} \) is bounded, from below, the same property holds for \( G_{29} \) and \( G_{30} \). The proof is analogous with the preceding one. An analogous property is true if \( G_{29} \) is bounded from below.

**Remark 5:** If \( T_{28} \) is bounded from below and \( \lim_{t \to \infty} (b_{1}^{(5)}((G_{31})(t), t)) = (b_{29}^{(5)}) \) then \( T_{29} \to \infty \).

**Definition of** \((m)^{(5)}\) and \(\varepsilon_{5}\):

Indeed let \( t_{5} \) be so that for \( t > t_{5} \)

\[ (b_{29}^{(5)}) - (b_{1}^{(5)}((G_{31})(t), t)) < \varepsilon_{5}, T_{28}(t) > (m)^{(5)} \]

Then \( \frac{dT_{29}}{dt} \geq (a_{29}^{(5)})(m)^{(5)} - \varepsilon_{5}T_{29} \) which leads to
\[ T_{29} \geq \left(\frac{(a_{29}^{(5)})(m)^{(5)}}{\varepsilon_{5}}\right)(1 - e^{-\varepsilon_{5}t}) + T_{29}^{0} e^{-\varepsilon_{5}t} \]
If we take \( t \) such that \( e^{-\varepsilon_{5}t} = \frac{1}{2} \), it results
\[ T_{29} \geq \left(\frac{(a_{29}^{(5)})(m)^{(5)}}{2}\right), t = \log \left(\frac{2}{\varepsilon_{5}}\right) \]
By taking now \( \varepsilon_{5} \) sufficiently small one sees that \( T_{29} \) is unbounded.

The same property holds for \( T_{30} \) if \( \lim_{t \to \infty} (b_{30}^{(5)}((G_{31})(t), t)) = (b_{29}^{(5)}) \)

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for \( G_{33}, G_{34}, T_{32}, T_{33}, T_{34} \)

It is now sufficient to take \( \frac{(a_{32}^{(6)})(\bar{M}_{32})^{(6)}}{(a_{32}^{(6)})(\bar{M}_{32})^{(6)}} < 1 \) and to choose

\( (\bar{P}_{32})^{(6)} \) and \( (\bar{Q}_{32})^{(6)} \) large to have
\[ \frac{(a_{32}^{(6)})(\bar{M}_{32})^{(6)}}{(a_{32}^{(6)})(\bar{M}_{32})^{(6)}} \left( \bar{P}_{32}^{(6)} + \left( \bar{P}_{32}^{(6)} + \bar{G}_{32}^{0} \right) e^{-\frac{(\bar{P}_{32})^{(6)} + \bar{G}_{32}^{0}}{\bar{P}_{32}^{(6)}}} \right) \leq (\bar{P}_{32})^{(6)} \]
\[ \frac{(b_{32}^{(6)})(\bar{M}_{32})^{(6)}}{(a_{32}^{(6)})(\bar{M}_{32})^{(6)}} \left( (\bar{Q}_{32})^{(6)} + \bar{T}_{32}^{0} \right) e^{-\frac{(\bar{Q}_{32})^{(6)} + \bar{T}_{32}^{0}}{\bar{Q}_{32}^{(6)}}} + (\bar{Q}_{32})^{(6)} \right) \leq (\bar{Q}_{32})^{(6)} \]

In order that the operator \( \mathcal{A}^{(6)} \) transforms the space of sextuples of functions \( G_{i}, T_{i} \) into itself

The operator \( \mathcal{A}^{(6)} \) is a contraction with respect to the metric

www.ijsrp.org
\[ d \left( ((G_{35})^{(1)}, (T_{35})^{(1)}), ((G_{35})^{(2)}, (T_{35})^{(2)}) \right) = \]
\[ \sup_{t \in \mathbb{R}^+} \max |G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(M_{32})^{(6)} t}, \max |T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(M_{32})^{(6)} t} \]

Indeed if we denote

**Definition of** \((G_{35}), (T_{35}) : (G_{35}), (T_{35}) = \mathcal{A}^{(6)}((G_{35}), (T_{35}))\)

It results

\[ |\tilde{G}_i^{(1)} - \tilde{G}_i^{(2)}| \leq \int_0^t \left( a_{32}^{(6)} |G_i^{(1)} - G_i^{(2)}| e^{-(M_{32})^{(6)} t} \right) \] \[ + \int_0^t \left( (T_i^{(1)})^{(6)} |G_i^{(1)} - G_i^{(2)}| e^{-(M_{32})^{(6)} t} \right) \]

Where \(s_{32}\) represents integrand that is integrated over the interval \([0, t]\)

From the hypotheses it follows

\[ \left| (G_{35})^{(1)} - (G_{35})^{(2)} \right| e^{-(M_{32})^{(6)} t} \leq \]

And analogous inequalities for \(G_i\) and \(T_i\). Taking into account the hypothesis the result follows

**Remark 1:** The fact that we supposed \((a_{32}^{(6)})\) and \((b_{32}^{(6)})\) depending also on \(t\) can be considered as not conformal with the reality, however we have put this hypothesis, in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by \((P_{32})^{(6)} e^{(M_{32})^{(6)} t}\) and \((\tilde{Q}_{32})^{(6)} e^{(M_{32})^{(6)} t}\) respectively of \(\mathbb{R}_+\).

If instead of proving the existence of the solution on \(\mathbb{R}_+\), we have to prove it only on a compact then it suffices to consider that \((a_{i}^{(6)})\) and \((b_{i}^{(6)})\) depending on \(T_{33}\) and respectively on \((G_{35})\) and not on \(t\) and hypothesis can replaced by a usual Lipschitz condition.

**Remark 2:** There does not exist any \(t\) where \(G_i(t) = 0\) and \(T_i(t) = 0\)

From 69 to 32 it results

\[ G_i(t) \geq G_i^0 e^{-\int_0^t (a_i^{(6)} - a_{i}^{(6)}(T_3, s_{32})) e^{(M_{32})^{(6)} t}} \] \[ T_i(t) \geq T_i^0 e^{-(b_i^{(6)} t)} > 0 \text{ for } t > 0 \]

**Definition of** \((M_{32})^{(6)} : (M_{32})^{(6)}_1\) and \((M_{32})^{(6)}_2\) and \((M_{32})^{(6)}_3\):

**Remark 3:** If \(G_{32}\) is bounded, the same property have also \(G_{33}\) and \(G_{34}\). indeed if \(G_{32} < (M_{32})^{(6)}\) it follows \(\frac{dG_{33}}{dt} \leq \left( (M_{32})^{(6)}_1 - (a_{33}^{(6)}) G_{33} \right)\) and by integrating
\[ G_{33} = (\tilde{M}_{32}(6))_2 = G_{33}^0 + 2(a_{33}^{(6)})(\tilde{M}_{32}(6))_1/(a_{33}^{(6)}) \]

In the same way, one can obtain
\[ G_{34} = (\tilde{M}_{32}(6))_3 = G_{34}^0 + 2(a_{34}^{(6)})(\tilde{M}_{32}(6))_2/(a_{34}^{(6)}) \]

If \(G_{33}\) or \(G_{34}\) is bounded, the same property follows for \(G_{32}\), \(G_{34}\) and \(G_{32}\), \(G_{33}\) respectively.

**Remark 4:** If \(G_{32}\) is bounded, from below, the same property holds for \(G_{33}\) and \(G_{34}\). The proof is analogous with the preceding one. An analogous property is true if \(G_{33}\) is bounded from below.

**Remark 5:** If \(T_{32}\) is bounded from below and \(\lim_{t \to \infty} ((b_i^{(6)} ((G_{35}(t), t))) = (b_i^{(6)}) = T_{33} \to \infty.\)

**Definition of \((m)^{(6)}\) and \(\varepsilon_6:\**

Indeed let \(t_6\) be so that for \(t > t_6\)
\[
(b_{33}^{(6)}) - (b_{13}^{(6)} ((G_{35}(t), t)) < \varepsilon_6, T_{32}(t) > (m)^{(6)}
\]

Then \(\frac{dT_{33}}{dt} \geq (a_{33}^{(6)})(m)^{(6)} - \varepsilon_6 T_{33}\), which leads to
\[
T_{33} \geq \left(\frac{(a_{33}^{(6)})(m)^{(6)})}{\varepsilon_6}\right) (1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}
\]

If we take \(t\) such that \(e^{-\varepsilon_6 t} = \frac{1}{2}\), it results
\[
T_{33} \geq \left(\frac{(a_{33}^{(6)})(m)^{(6)})}{2}\right) \cdot t = log \frac{2}{\varepsilon_6}
\]

By taking now \(\varepsilon_6\) sufficiently small one sees that \(T_{33}\) is unbounded.

The same property holds for \(T_{34}\) if \(\lim_{t \to \infty} (b_{34}^{(6)} ((G_{35}(t), t))) = (b_{34}^{(6)}) = (b_{34}^{(6)})\).

We now state a more precise theorem about the behaviors at infinity of the solutions

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}:\**

(a) \(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}\) four constants satisfying
\[-(\sigma_2)^{(1)} \leq -a_{13}^{(1)}(1) + (a_{13}^{(1)} - (a_{13}^{(1)}))T_{14}(t) + (a_{14}^{(1)})(T_{14}, t) \leq -(\sigma_1)^{(1)}\]
\[-(\tau_2)^{(1)} \leq -(b_{13}^{(1)}(1)) + (b_{13}^{(1)} - (b_{13}^{(1)}))G(t) - (b_{14}^{(1)})(G, t) \leq -(\tau_1)^{(1)}\]

**Definition of \((v_1)^{(1)}, (v_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, v^{(1)}, u^{(1)}:\**

(b) By \((v_1)^{(1)} > 0, (v_2)^{(1)} < 0\) and respectively \((u_1)^{(1)} > 0, (u_2)^{(1)} < 0\) the roots of \((a_{13}^{(1)}(1)v^{(1)})^2 + (\sigma_1)^{(1)}v^{(1)} - (a_{13}^{(1)}) = 0\) and \((b_{14}^{(1)})(u^{(1)})^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13}^{(1)}) = 0\)

**Definition of \((\tilde{v}_1)^{(1)}, (\tilde{v}_2)^{(1)}, (\tilde{u}_1)^{(1)}, (\tilde{u}_2)^{(1)}:\**

By \((\tilde{v}_1)^{(1)} > 0, (\tilde{v}_2)^{(1)} < 0\) and respectively \((\tilde{u}_1)^{(1)} > 0, (\tilde{u}_2)^{(1)} < 0\) the roots of the equations
\[ (a_{14}^{(1)})(v^{(1)})^2 + (\sigma_2)^{(1)}v^{(1)} - (a_{13}^{(1)}) = 0 \text{ and } \quad (b_{14}^{(1)})(u^{(1)})^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13}^{(1)}) = 0 \]

**Definition of \((m_1)^{(1)}), (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (v_0)^{(1)}:\** -
(c) If we define \((m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}\) by

\[
(m_2)^{(1)} = (v_0)^{(1)}, (m_1)^{(1)} = (v_1)^{(1)}, \text{ if } (v_0)^{(1)} < (v_1)^{(1)}
\]

\[
(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (\bar{v}_1)^{(1)}, \text{ if } (v_1)^{(1)} < (v_0)^{(1)} < (\bar{v}_1)^{(1)},
\]

and \((v_0)^{(1)} = \frac{c_{13}^1}{c_{14}^1}\)

\[
(m_2)^{(1)} = (v_1)^{(1)}, (m_1)^{(1)} = (v_0)^{(1)}, \text{ if } (\bar{v}_1)^{(1)} < (v_0)^{(1)}
\]

and analogously

\[
(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)}, \text{ if } (u_0)^{(1)} < (u_1)^{(1)}
\]

\[
(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, \text{ if } (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)},
\]

and \((u_0)^{(1)} = \frac{c_{13}^1}{c_{14}^1}\)

\[
(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, \text{ if } (\bar{u}_1)^{(1)} < (u_0)^{(1)} \text{ where } (u_1)^{(1)}, (\bar{u}_1)^{(1)}
\]

are defined respectively

Then the solution satisfies the inequalities

\[
G_{13}^0 e^{(s_1)^{(1)}-(p_{13})^{(1)}}t \leq G_{13}^0 e^{(s_1)^{(1)}}t
\]

where \((p_i)^{(1)}\) is defined

\[
\frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(s_1)^{(1)}-(p_{13})^{(1)}}t \leq G_{14}^0 \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(s_1)^{(1)}+t}
\]

\[
\frac{(a_{13})^{(1)}G_{13}^0}{(m_2)^{(1)}(s_1)^{(1)}-(s_{15})^{(1)}+e^{(s_{15})^{(1)}+t}} e^{(s_1)^{(1)}-(p_{13})^{(1)}+t} - e^{-((s_{15})^{(1)}+t)} + G_{15}^0 e^{-(s_{15})^{(1)}+t} \leq G_{15}^0 e^{-(s_{15})^{(1)}+t}
\]

\[
T_{13}^0 e^{(r_1)^{(1)}+t} \leq T_{13}^0 \leq T_{13}^0 e^{((r_1)^{(1)}+(r_{13})^{(1)})+t}
\]

\[
\frac{1}{(m_2)^{(1)}} T_{13}^0 e^{(r_1)^{(1)}+t} \leq T_{13}^0 \leq \frac{1}{(m_2)^{(1)}} T_{13}^0 e^{((r_1)^{(1)}+(r_{13})^{(1)})+t}
\]

\[
\frac{(b_{13})^{(1)}T_{13}^0}{(m_2)^{(1)}((r_1)^{(1)}+(r_{13})^{(1)})} \left[ e^{(r_1)^{(1)}+t} - e^{-(r_{13})^{(1)}+t} \right] + T_{13}^0 e^{-(r_{13})^{(1)}+t} \leq T_{13}^0 \leq T_{13}^0 e^{-(r_{13})^{(1)}+t}
\]

**Definition of \((S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}\):**

Where \((S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a_{13})^{(1)}\)

\[
(S_2)^{(1)} = (a_{15})^{(1)} - (p_{13})^{(1)}
\]

\[
(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b_{13})^{(1)}
\]

\[
(R_2)^{(1)} = (b_{13})^{(1)} - (r_{13})^{(1)}
\]
Behavior of the solutions

If we denote and define

Definition of \((\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}\):

(d) \((\sigma_1)^{(2)},(\sigma_2)^{(2)},(\tau_1)^{(2)},(\tau_2)^{(2)}\) four constants satisfying

\[-(\sigma_2)^{(2)} \leq -(a_{16}^{(2)}) + (a_{17}^{(2)}) - (a_{16}^{(2)})(T_{17},t) + (a_{17}^{(2)})(T_{17},t) \leq -(\sigma_1)^{(2)}\]

\[-(\tau_2)^{(2)} \leq -(b_{16}^{(2)}) + (b_{17}^{(2)}) - (b_{16}^{(2)})(G_{19},t) - (b_{17}^{(2)})(G_{19},t) \leq -(\tau_1)^{(2)}\]

Definition of \((v_1)^{(2)},(v_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}\):

By \((v_1)^{(2)} > 0,(v_2)^{(2)} < 0\) and respectively \((u_1)^{(2)} > 0,(u_2)^{(2)} < 0\) the roots

(e) of the equations 
\[(a_{17}^{(2)})(v^{(2)})^2 + (\sigma_1)^{(2)}v^{(2)} - (a_{16}^{(2)}) = 0\]

and 
\[(b_{14}^{(2)})(u^{(2)})^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16}^{(2)}) = 0\]

Definition of \((\bar{v}_1)^{(2)},(\bar{v}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}\):

By \((\bar{v}_1)^{(2)} > 0,(\bar{v}_2)^{(2)} < 0\) and respectively \((\bar{u}_1)^{(2)} > 0,(\bar{u}_2)^{(2)} < 0\) the roots of the equations 
\[(a_{17}^{(2)})(v^{(2)})^2 + (\sigma_2)^{(2)}v^{(2)} - (a_{16}^{(2)}) = 0\]

and 
\[(b_{17}^{(2)})(u^{(2)})^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16}^{(2)}) = 0\]

Definition of \((m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}\):

(f) If we define \((m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}\) by

\[(m_2)^{(2)} = (v_0)^{(2)},(m_1)^{(2)} = (v_1)^{(2)}, \text{ if } (v_0)^{(2)} < (v_1)^{(2)}\]

\[(m_2)^{(2)} = (v_1)^{(2)},(m_1)^{(2)} = (\bar{v}_1)^{(2)}, \text{ if } (v_1)^{(2)} < (v_0)^{(2)} < (\bar{v}_1)^{(2)}\]

and

\[
(v_0)^{(2)} = \frac{v_{16}^{(2)}}{v_{17}^{(2)}}
\]

\[(m_2)^{(2)} = (v_1)^{(2)},(m_1)^{(2)} = (v_0)^{(2)}, \text{ if } (\bar{v}_1)^{(2)} < (v_0)^{(2)}\]

and analogously

\[(\mu_2)^{(2)} = (u_0)^{(2)},(\mu_1)^{(2)} = (u_1)^{(2)}, \text{ if } (u_0)^{(2)} < (u_1)^{(2)}\]

\[(\mu_2)^{(2)} = (u_1)^{(2)},(\mu_1)^{(2)} = (\bar{u}_1)^{(2)}, \text{ if } (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)}\]

and

\[
(u_0)^{(2)} = \frac{T_{16}^{(2)}}{T_{17}^{(2)}}
\]

\[(\mu_2)^{(2)} = (u_1)^{(2)},(\mu_1)^{(2)} = (u_0)^{(2)}, \text{ if } (\bar{u}_1)^{(2)} < (u_0)^{(2)}\]

Then the solution satisfies the inequalities

\[G_{16}^{0}e^{(S_1)^{(2)} - (P_{16})^{(2)}t} \leq G_{16}(t) \leq G_{16}^{0}e^{(S_1)^{(2)}t}\]

\((p)^{(2)}\) is defined

www.ijsrp.org
\[
\frac{1}{(m_1)^2} G_{16}^0 e^{((S_1)^2-(P_{16})^2) t} \leq G_{17}(t) \leq \frac{1}{(m_2)^2} G_{16}^0 e^{((S_1)^2) t}
\]

\[
\left(\left(\frac{(a_{16})^2 G_{16}^0}{(m_1)^2} \right) e^{((S_1)^2-(P_{16})^2-(S_2)^2) t} - e^{-(S_2)^2 t} \right) + \left(\left(\frac{(a_{16})^2 G_{16}^0}{(m_2)^2} \right) e^{((S_1)^2-(a_{18})^2) t} - e^{-(a_{18})^2 t} \right) \leq G_{18}(t) \leq 1
\]

\[
T_{16}^0 e^{(R_{12})^2 t} \leq T_{16}^0 e^{((R_{12})^2+(R_{18})^2) t}
\]

\[
\frac{1}{(\mu_1)^2} T_{16}^0 e^{(R_{12})^2 t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^2} T_{16}^0 e^{((R_{12})^2+(R_{18})^2) t}
\]

\[
\left(\left(\frac{(b_{18})^2 T_{16}^0}{(\mu_1)^2} \right) e^{(R_{12})^2 t} - e^{-(R_{18})^2 t} \right) + T_{18}^0 e^{-(R_{18})^2 t} \leq T_{18}(t) \leq 1
\]

\[
\left(\left(\frac{(b_{18})^2 T_{16}^0}{(\mu_2)^2} \right) e^{(R_{12})^2+(R_{18})^2+(R_{22})^2} - e^{(R_{22})^2 t} \right) + T_{22}^0 e^{-(R_{22})^2 t}
\]

**Definition of \((S_1)^2, (S_2)^2, (R_1)^2, (R_2)^2)\):**

Where \((S_1)^2 = (a_{16})^2 (m_1^2) - (a_{18})^2, (R_2)^2 = (b_{18})^2 (m_2^2) - (b_{18})^2\)

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}):**

(a) \((\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}\) four constants satisfying

\[-(\sigma_1)^{(3)} \leq -(a_{20})^{(3)} + (a_{21})^{(3)} - (a_{20})^{(3)}(T_{21}, t) + (a_{21})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}\]

\[-(\tau_2)^{(3)} \leq -(b_{20})^{(3)} + (b_{21})^{(3)} - (b_{20})^{(3)}(G, t) - (b_{21})^{(3)}(G, t) \geq -(\tau_1)^{(3)}\]

**Definition of \((v_1)^{(3)}, (v_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}):**

(b) By \((v_1)^{(3)} > 0, (v_2)^{(3)} < 0\) and respectively \((u_1)^{(3)} > 0, (u_2)^{(3)} < 0\) the roots of the equations \((a_{21})^{(3)}(v)^2 + (\sigma_1)^{(3)}v - (a_{20})^{(3)} = 0\)

and \((b_{21})^{(3)}(u)^2 + (\tau_1)^{(3)}u - (b_{20})^{(3)} = 0\) and

By \((\bar{v}_1)^{(3)} > 0, (\bar{v}_2)^{(3)} < 0\) and respectively \((\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0\) the roots of the equations \((a_{21})^{(3)}(v)^2 + (\sigma_2)^{(3)}v - (a_{20})^{(3)} = 0\)

and \((b_{21})^{(3)}(u)^2 + (\tau_2)^{(3)}u - (b_{20})^{(3)} = 0\)

**Definition of \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}):**

(c) If we define \((m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}\) by
\( (m_2)^{(3)} = (v_0)^{(3)}, (m_1)^{(3)} = (v_1)^{(3)}, \text{ if } (v_0)^{(3)} < (v_1)^{(3)} \)

\( (m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (\bar{v}_1)^{(3)}, \text{ if } (v_1)^{(3)} < (v_0)^{(3)} < (\bar{v}_1)^{(3)}, \)

and \( (v_0)^{(3)} = \frac{\sigma_{0}}{\tilde{\sigma}_{21}} \)

\( (m_2)^{(3)} = (v_1)^{(3)}, (m_1)^{(3)} = (v_0)^{(3)}, \text{ if } (\bar{v}_1)^{(3)} < (v_0)^{(3)} \)

and analogously

\( (\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \text{ if } (u_0)^{(3)} < (u_1)^{(3)} \)

\( (\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \text{ if } (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)}, \)

and \( (u_0)^{(3)} = \frac{\mu_{0}}{\tilde{\mu}_{21}} \)

Then the solution satisfies the inequalities

\[
G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{((S_1)^{(3)})t} \\
(p_j)^{(3)} \text{ is defined}
\]

\[
\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{((S_1)^{(3)})t}
\]

\[
\frac{(a_{22})^{(3)} G_{20}^0}{(m_1)^{(3)}} e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{((S_1)^{(3)})t} \leq G_{22}(t) \leq \frac{(a_{22})^{(3)} G_{20}^0}{(m_2)^{(3)}} e^{((S_1)^{(3)}-(a_{22})^{(3)})t} + G_{22}^0 e^{-(a_{22})^{(3)}t}
\]

\[
T_{20}^0 e^{((R_1)^{(3)})t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}
\]

\[
\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{((R_1)^{(3)})t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}
\]

\[
\frac{(b_{22})^{(3)} T_{20}^0}{(\mu_1)^{(3)}} e^{((R_1)^{(3)}-(b_{22})^{(3)})t} \leq T_{22}(t) \leq \frac{(b_{22})^{(3)} T_{20}^0}{(\mu_2)^{(3)}} e^{((R_1)^{(3)}+(b_{22})^{(3)})t} + T_{22}^0 e^{-(b_{22})^{(3)}t}
\]

**Definition of** \((S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}: \)

Where \((S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{10})^{(3)}\)

\((S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}\)

\((R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{10})^{(3)}\)

\((R_2)^{(3)} = (b_{22})^{(3)} - (r_{22})^{(3)}\)

**Behavior of the solutions**

If we denote and define

**Definition of** \((\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}: \)
(d) \((\sigma_{1})^{(4)}, (\sigma_{2})^{(4)}, (\tau_{1})^{(4)}, (\tau_{2})^{(4)}\) four constants satisfying
\[-(\sigma_{2})^{(4)} \leq -(a_{24}')^{(4)} + (a_{25}')^{(4)} - (a_{24}'')^{(4)}(T_{25}, t) + (a_{25}'')^{(4)}(T_{25}, t) \leq -(\sigma_{1})^{(4)}\]
\[-(\tau_{2})^{(4)} \leq -(b_{24}')^{(4)} + (b_{25}')^{(4)} - (b_{24}'')^{(4)}(G_{27}, t) \leq -(\tau_{1})^{(4)}\]

**Definition of** \((v_{1})^{(4)}, (v_{2})^{(4)}, (u_{1})^{(4)}, (u_{2})^{(4)}, v^{(4)}, u^{(4)}\)

\((e)\) By \((v_{1})^{(4)} > 0, (v_{2})^{(4)} < 0\) and respectively \((u_{1})^{(4)} > 0, (u_{2})^{(4)} < 0\) the roots of the equations
\[(a_{25}^{(4)}(v^{(4)})^{2} + (\sigma_{1})^{(4)}v^{(4)} - (a_{24}^{(4)} = 0)\]
and \[(b_{25}^{(4)}(u^{(4)})^{2} + (\tau_{1})^{(4)}u^{(4)} - (b_{24}^{(4)} = 0)\]

**Definition of** \((\bar{v}_{1})^{(4)}, (\bar{v}_{2})^{(4)}, (\bar{u}_{1})^{(4)}, (\bar{u}_{2})^{(4)}\)

By \((\bar{v}_{1})^{(4)} > 0, (\bar{v}_{2})^{(4)} < 0\) and respectively \((\bar{u}_{1})^{(4)} > 0, (\bar{u}_{2})^{(4)} < 0\) the roots of the equations
\[(a_{25}^{(4)}(v^{(4)})^{2} + (\sigma_{2})^{(4)}v^{(4)} - (a_{24}^{(4)} = 0)\]
and \[(b_{25}^{(4)}(u^{(4)})^{2} + (\tau_{2})^{(4)}u^{(4)} - (b_{24}^{(4)} = 0)\]

**Definition of** \((m_{1})^{(4)}, (m_{2})^{(4)}, (\mu_{1})^{(4)}, (\mu_{2})^{(4)}, (v_{0})^{(4)}\)

(f) If we define \((m_{1})^{(4)}, (m_{2})^{(4)}, (\mu_{1})^{(4)}, (\mu_{2})^{(4)}\) by
\[(m_{2})^{(4)} = (v_{0})^{(4)}, (m_{1})^{(4)} = (v_{1})^{(4)}\]
\[(m_{2})^{(4)} = (v_{4})^{(4)}, (m_{1})^{(4)} = (\bar{v}_{4})^{(4)}\]
and
\[\left(\frac{v_{0}}{\sigma_{2}} \right) \left(\frac{v_{1}}{\bar{v}_{4}} \right) = \frac{c_{24}}{c_{25}}\]

\[(m_{2})^{(4)} = (v_{1})^{(4)}, (m_{1})^{(4)} = (v_{0})^{(4)}\]
and analogously
\[(\mu_{2})^{(4)} = (u_{0})^{(4)}, (\mu_{1})^{(4)} = (u_{1})^{(4)}\]
\[(\mu_{2})^{(4)} = (u_{4})^{(4)}, (\mu_{1})^{(4)} = (\bar{u}_{4})^{(4)}\]
and
\[\left(\frac{u_{0}}{\tau_{2}} \right) \left(\frac{u_{1}}{\bar{u}_{4}} \right) = \frac{c_{24}}{c_{25}}\]

Then the solution satisfies the inequalities
\[G^{0}_{24}e^{((S_{1})^{(4)} - (p_{24}))t} \leq G_{24}(t) \leq G^{0}_{24}e^{((S_{1})^{(4)}t}\]

where \((p_{i})^{(4)}\) is defined

\[\frac{1}{(m_{2})^{(4)}} G^{0}_{24}e^{((S_{1})^{(4)} - (p_{24}))t} \leq G_{25}(t) \leq \frac{1}{(m_{2})^{(4)}} G^{0}_{24}e^{((S_{1})^{(4)}t}\]

\[\left(\frac{(a_{25})^{(4)}G_{25}^{0}}{(m_{1})^{(4)}(S_{1})^{(4)} - (p_{24}))} - \frac{e^{((S_{1})^{(4)} - (p_{24}))t} - e^{-(S_{2})^{(4)}t}}{G_{26}^{0}e^{-(S_{2})^{(4)}t}}\right) + G_{26}^{0}e^{-(S_{2})^{(4)}t} \leq G_{26}(t) \leq \]

\[\frac{1}{(m_{2})^{(4)}(S_{1})^{(4)} - (a_{26})^{(4)}} \left( e^{(S_{1})^{(4)t} - e^{-(a_{26})^{(4)}t}} + G_{26}^{0}e^{-(a_{26})^{(4)}t}\right)\]

www.ijsrp.org
\[
\begin{align*}
\frac{T_{24}^0 e^{(R_1^4 t)}}{(\mu_1^4)^2 T_{24}^0 e^{(R_1^4 + R_2^4) t}} \leq T_{24}(t) & \leq \frac{T_{24}^0 e^{(R_1^4 + R_2^4) t}}{(\mu_2^4)^2 T_{24}^0 e^{(R_1^4 + R_2^4) t}} \\
\frac{(b_{26}^4)^{r_{26}} T_{24}^0 e^{(R_1^4 - b_{26}^4) t}}{(\mu_1^4)^2 \tilde{T}_{24}^2 e^{(R_1^4 - b_{26}^4) t}} \leq T_{26}(t) & \leq \frac{(b_{26}^4)^{r_{26}} T_{24}^0 e^{(R_1^4) t}}{(\mu_2^4)^2 \tilde{T}_{24}^2 e^{(R_1^4) t}} + T_{26}^0 e^{-(b_{26}^4)^{r_{26}} t} \leq T_{26}(t) \leq \\
\frac{(a_{26}^4)^{r_{26}} T_{24}^0 e^{(R_1^4) t}}{(\mu_2^4)^2 \tilde{T}_{24}^2 e^{(R_1^4 + R_2^4) t}} \leq T_{26}(t) & \leq \frac{(a_{26}^4)^{r_{26}} T_{24}^0 e^{(R_1^4) t}}{(\mu_2^4)^2 \tilde{T}_{24}^2 e^{(R_1^4) t}} + T_{26}^0 e^{-(r_{26}^4)^{r_{26}} t} 
\end{align*}
\]

**Definition of** \( (S_1^4), (S_2^4), (R_1^4), (R_2^4) \):

Where \( (S_1^4) = (a_{24}^4) (m_2^4) - (a_{24}^4)^4 \)

\( (S_2^4) = (a_{26}^4) - (p_{26}^4)^4 \)

\( (R_1^4) = (b_{24}^4) (m_2^4) - (b_{24}^4)^4 \)

\( (R_2^4) = (b_{26}^4) - (r_{26}^4)^4 \)

**Behavior of the solutions**

If we denote and define

**Definition of** \( (\sigma_1^5), (\sigma_2^5), (\tau_1^5), (\tau_2^5) \):

\( (\sigma_1^5), (\sigma_2^5), (\tau_1^5), (\tau_2^5) \) four constants satisfying

\(-\sigma_1^5 \leq -(a_{28}^5)^2 + (a_{28}^5)^5 - (a_{28}^5)^2 T_{29} + (a_{28}^5)^2 T_{29} \leq -\sigma_1^5 \)

\(-\tau_1^5 \leq -(b_{28}^5)^2 + (b_{28}^5)^5 - (b_{28}^5)^2 (G_{31}, t) - (b_{28}^5)^2 (G_{31}, t) \leq -\tau_1^5 \)

**Definition of** \( (v_1^5), (v_2^5), (u_1^5), (u_2^5), (v^5), (u^5) \):

\( (v_1^5) > 0, (v_2^5) < 0 \) and respectively \( (u_1^5) > 0, (u_2^5) < 0 \) the roots of the equations

\( (a_{28}^5) (v^5)^2 + (a_1^5 v^5) - (a_{28}^5) = 0 \)

and \( (b_{28}^5) (u^5)^2 + (\tau_1^5 u^5) - (b_{28}^5) = 0 \)

**Definition of** \( (\tilde{v_1}^5), (\tilde{v_2}^5), (\tilde{u_1}^5), (\tilde{u_2}^5), (\tilde{v}^5), (\tilde{u}^5) \):

By \( (\tilde{v_1}^5) > 0, (\tilde{v_2}^5) < 0 \) and respectively \( (\tilde{u_1}^5) > 0, (\tilde{u_2}^5) < 0 \) the roots of the equations

\( (a_{28}^5) (v^5)^2 + (a_1^5 v^5) - (a_{28}^5) = 0 \)

and \( (b_{28}^5) (u^5)^2 + (\tau_1^5 u^5) - (b_{28}^5) = 0 \)

**Definition of** \( (m_1^5), (m_2^5), (\mu_1^5), (\mu_2^5), (v_0^5) \):

If we define \( (m_1^5), (m_2^5), (\mu_1^5), (\mu_2^5), (v_0^5) \) by

\( (m_2^5) = (v_0^5), (m_1^5) = (v_1^5), \text{ if } (v_0^5) < (v_1^5) \)

\( (m_2^5) = (v_1^5), (m_1^5) = (v_0^5), \text{ if } (v_1^5) < (v_0^5) \)

\( (m_2^5) = (v_1^5), (m_1^5) = (v_0^5), \text{ if } (v_1^5) < (v_0^5) \)

www.ijsrp.org
and analogously
\[
(\mu_2)^{(5)} = (u_0)^{(5)}, \quad (\mu_1)^{(5)} = (u_1)^{(5)}, \quad \text{if} \quad (u_0)^{(5)} < (u_1)^{(5)}
\]
\[
(\mu_2)^{(5)} = (u_1)^{(5)}, \quad (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \quad \text{if} \quad (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},
\]
and
\[
(u_0)^{(5)} = \frac{T_{28}}{T_{29}}
\]
\[
(\mu_2)^{(5)} = (u_1)^{(5)}, \quad (\mu_1)^{(5)} = (u_0)^{(5)}, \quad \text{if} \quad (u_0)^{(5)} < (u_1)^{(5}) \quad \text{where} \quad (u_1)^{(5)}, \quad (\bar{u}_1)^{(5)}
\]
are defined respectively

Then the solution satisfies the inequalities
\[
G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)}) t} \leq G_{28}^0 e^{(S_1)^{(5)} t}
\]
where \((p_i)^{(5)}\) is defined
\[
\frac{1}{(m_3)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)}) t} \leq G_{28}^0 e^{(S_1)^{(5)} t} \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}
\]
\[
\left( \frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}} \right) \left( (S_1)^{(5)} - (p_{28})^{(5)} \right) \leq e^{((S_1)^{(5)} - (p_{28})^{(5)}) t} - e^{-(S_2)^{(5)} t} + G_{30}^0 e^{-(S_2)^{(5)} t} \leq G_{30}^0 \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)} t}
\]
\[
T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t} \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t}
\]
\[
\frac{1}{(mu_1)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t} \leq T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t} \leq \frac{1}{(mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t}
\]
\[
\left( \frac{(a_{30})^{(5)} T_{28}^0}{(mu_1)^{(5)}} \right) \left( (R_1)^{(5)} + (r_{28})^{(5)} \right) \leq e^{((R_1)^{(5)} + (r_{28})^{(5)}) t} - e^{-(R_2)^{(5)} t} + T_{30}^0 e^{-(R_2)^{(5)} t} \leq T_{30}^0 \leq \frac{1}{(mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)}) t} + T_{30}^0 e^{-(R_2)^{(5)} t}
\]

**Definition of \((S_1)^{(5)}\), \((S_2)^{(5)}\), \((R_1)^{(5)}\), \((R_2)^{(5)}\):**

Where \((S_1)^{(5)} = (a_{28})^{(5)} (m_2)^{(5)} - (a'_{28})^{(5)}\)
\[
(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}
\]
\[
(R_1)^{(5)} = (b_{28})^{(5)} (\mu_2)^{(5)} - (b'_{28})^{(5)}
\]
\[
(R_2)^{(5)} = (b_{30})^{(5)} - (r_{30})^{(5)}
\]

**Behavior of the solutions**

If we denote and define

**Definition of \((\sigma_1)^{(6)}\), \((\sigma_2)^{(6)}\), \((r_1)^{(6)}\), \((r_2)^{(6)}\):**

\[
\left( \begin{array}{c}
(\sigma_1)^{(6)} \\
(\sigma_2)^{(6)} \\
(r_1)^{(6)} \\
(r_2)^{(6)}
\end{array} \right) \text{ four constants satisfying}
\]
\[
- (\sigma_2)^{(6)} \leq - (a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)} (T_{33}, t) + (a''_{33})^{(6)} T_{33}, t \leq -(\sigma_1)^{(6)}
\]
\[
- (r_2)^{(6)} \leq - (b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)} (G_{33}, t) - (b''_{33})^{(6)} (G_{35}, t) \leq -(r_1)^{(6)}
\]

www.ijsrp.org
Definition of \((v_1)^{(6)}, (v_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, v^{(6)}, u^{(6)}\):

\((k)\) By \((v_1)^{(6)} > 0, (v_2)^{(6)} < 0\) and respectively \((u_1)^{(6)} > 0, (u_2)^{(6)} < 0\) the roots of the equations
\[ (a_{32})^{(6)}(v^{(6)})^2 + (\sigma_{(6)}v^{(6)} - (a_{32})^{(6)} = 0 \]
and \((b_{32})^{(6)}(u^{(6)})^2 + (\tau_{(6)}u^{(6)} - (b_{32})^{(6)} = 0 \)
and analogously

Definition of \((\bar{v}_1)^{(6)}, (\bar{v}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}\):

By \((\bar{v}_1)^{(6)} > 0, (\bar{v}_2)^{(6)} < 0\) and respectively \((\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0\) the roots of the equations
\[ (a_{33})^{(6)}(v^{(6)})^2 + (\sigma_{(6)}v^{(6)} - (a_{33})^{(6)} = 0 \]
and \((b_{33})^{(6)}(u^{(6)})^2 + (\tau_{(6)}u^{(6)} - (b_{33})^{(6)} = 0 \)

Definition of \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (v_0)^{(6)}\) :

\((l)\) If we define \((m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}\) by

\[ (m_2)^{(6)} = (v_0)^{(6)}, (m_1)^{(6)} = (v_1)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)} \]
and
\[ (v_0)^{(6)} = \frac{\mu_2}{\mu_1} \]

\[ (m_2)^{(6)} = (v_1)^{(6)}, (m_1)^{(6)} = (v_0)^{(6)}, \text{ if } (v_0)^{(6)} < (v_1)^{(6)} \]
and analogously

\[ (\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)} \]
and
\[ (u_0)^{(6)} = \frac{\mu_2}{\mu_1} \]

\[ (\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \text{ if } (u_0)^{(6)} < (u_1)^{(6)} \]

Then the solution satisfies the inequalities

\[ G^{0}_{32}e_{[\mu_1]^{(6)}(p_{32})^{(6)}]t} \leq G^{0}_{32}e_{[\mu_1]^{(6)}(p_{32})^{(6)}]t} \leq G^{0}_{32}e_{(S_1)^{(6)}]t} \]

where \((p_{j})^{(6)}\) is defined

\[ \frac{1}{(m_1)^{(6)}}G^{0}_{32}e_{[\mu_1]^{(6)}(p_{32})^{(6)}]t} \leq G^{0}_{32}e_{[\mu_1]^{(6)}(p_{32})^{(6)}]t} \leq \frac{1}{(m_2)^{(6)}}G^{0}_{32}e_{(S_1)^{(6)}]t} \]

\[ G^{0}_{32}e_{(R_1)^{(6)}]t} \leq G^{0}_{32}e_{(R_1)^{(6)}]t} \leq G^{0}_{32}e_{(R_1)^{(6)}]t} \]

\[ T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \]

\[ \frac{1}{(m_1)^{(6)}}T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \leq \frac{1}{(m_2)^{(6)}}T^{0}_{32}e_{(R_1)^{(6)}]t} \]

\[ T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \]

\[ \frac{1}{(m_1)^{(6)}}(b_{34})^{(6)}T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \leq \frac{1}{(m_2)^{(6)}}(b_{34})^{(6)}T^{0}_{32}e_{(R_1)^{(6)}]t} \]

\[ T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \leq T^{0}_{32}e_{(R_1)^{(6)}]t} \]

www.ijsrp.org
\[
\frac{(a_{34})^6 t^0}{(\mu_{32})^6 ((R_1)^6 + (R_2)^6)} \left[ e^{((R_1)^6 + (R_2)^6)t} - e^{-(R_2)^6 t} \right] + T_{34}^0 e^{-(R_2)^6 t}
\]

**Definition of** \((S_1)^6, (S_2)^6, (R_1)^6, (R_2)^6\):

Where \((S_1)^6 = (a_{32})^6 (m_2)^6 - (a_{32})^6\)
\((S_2)^6 = (a_{34})^6 - (p_{34})^6\)
\((R_1)^6 = (b_{32})^6 (\mu_2)^6 - (b_{32})^6\)
\((R_2)^6 = (b_{34})^6 - (r_{34})^6\)

**Proof**: From GLOBAL EQUATIONS we obtain
\[
\frac{dv^{(1)}}{dt} = (a_{13})^{(1)} - \left( (a_{13})^{(1)} - (a_{13})^{(1)} + (a_{13})^{(1)} (T_{14}, t) \right) - (a_{14})^{(1)} (T_{14}, t) v^{(1)} - (a_{13})^{(1)}
\]

**Definition of** \(v^{(1)}\) :
\[
v^{(1)} = \frac{\partial_{12}}{\partial_{14}}
\]

It follows
\[
- (a_{14})^{(1)} v^{(1)} + \sigma_1 v^{(1)} - (a_{13})^{(1)} \leq \frac{dv^{(1)}}{dt} \leq - (a_{14})^{(1)} v^{(1)} + \sigma_1 v^{(1)} - (a_{13})^{(1)}
\]

From which one obtains

**Definition of** \((\bar{v}_1)^{(1)}, (v_0)^{(1)}\) :

(a) For \(0 < \left(\frac{v_0^{(1)}}{v_1^{(1)}}\right)^{(1)} = \frac{\partial_{12}}{\partial_{14}} < \left(\frac{v_1^{(1)}}{v_1^{(1)}}\right)^{(1)} < \left(\frac{v_1^{(1)}}{v_1^{(1)}}\right)^{(1)}\)

\[
v^{(1)}(t) \geq \frac{v^{(1)}(t) + (C^{(1)})^{(1)} (v_0^{(1)})^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)})^{(1)} - (v_0^{(1)})^{(1)} t]}{1 + (C^{(1)})^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)})^{(1)} - (v_0^{(1)})^{(1)} t}}
\]

it follows \((v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}\)

In the same manner, we get

\[
v^{(1)}(t) \leq \frac{v^{(1)}(t) + (C^{(1)})^{(1)} (v_0^{(1)})^{(1)}}{1 + (C^{(1)})^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)})^{(1)} - (v_0^{(1)})^{(1)} t}} \leq v^{(1)}(t) \leq \frac{(C^{(1)})^{(1)}}{(v_0^{(1)})^{(1)} - (v_2^{(1)})^{(1)}}
\]

From which we deduce \((v_0)^{(1)} \leq v^{(1)}(t) \leq (v_1)^{(1)}\)

(b) If \(0 < \left(\frac{v_0^{(1)}}{v_1^{(1)}}\right)^{(1)} = \frac{\partial_{12}}{\partial_{14}} < \left(\frac{v_1^{(1)}}{v_1^{(1)}}\right)^{(1)}\) we find like in the previous case,

\[
\left(\frac{v_0^{(1)}}{v_1^{(1)}}\right)^{(1)} = \frac{v^{(1)}(t) + (C^{(1)})^{(1)} (v_2^{(1)})^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)})^{(1)} - (v_2^{(1)})^{(1)} t]}{1 + (C^{(1)})^{(1)} e^{-(a_{14})^{(1)} (v_1^{(1)})^{(1)} - (v_2^{(1)})^{(1)} t}} \leq v^{(1)}(t) \leq \frac{(C^{(1)})^{(1)}}{(v_0^{(1)})^{(1)} - (v_2^{(1)})^{(1)}}
\]

www.ijrsrp.org
\[
\frac{(v_1^{(1)}) + (Cv_2^{(1)})e^{-[\sigma_{14}(1)(v_{11}^{(1)} - v_{21}^{(1)})t]}}{1 + (Cv_1^{(1)})e^{-[\sigma_{14}(1)(v_{11}^{(1)} - v_{21}^{(1)})t]}} \leq (\bar{v}_1^{(1)})
\]

(c) If \(0 < (v_1^{(1)}) \leq (\bar{v}_2^{(1)}) \leq (v_0^{(1)}) = \frac{G_{13}}{G_{14}}\), we obtain

\[
(v_1^{(1)}) \leq v^{(1)}(t) \leq \frac{(v_1^{(1)}) + (Cv_2^{(1)})e^{-[\sigma_{14}(1)(v_{11}^{(1)} - v_{21}^{(1)})t]}}{1 + (Cv_1^{(1)})e^{-[\sigma_{14}(1)(v_{11}^{(1)} - v_{21}^{(1)})t]}} \leq (v_0^{(1)})
\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(1)}(t) :-\)

\[
(m_2^{(1)}) \leq v^{(1)}(t) \leq (m_1^{(1)}), \quad v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}
\]

In a completely analogous way, we obtain

**Definition of** \(u^{(1)}(t) :-\)

\[
(\mu_2^{(1)}) \leq u^{(1)}(t) \leq (\mu_1^{(1)}), \quad u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}
\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \((a_{13}^{(1)}) = (a_{14}^{(1)}), \text{ then } (\sigma_1^{(1)}) = (\sigma_2^{(1)})\) and in this case \((v_1^{(1)}) = (\bar{v}_1^{(1)})\) if in addition \((v_0^{(1)}) = (v_1^{(1)})\) then \(v^{(1)}(t) = (v_0^{(1)})\) and as a consequence \(G_{13}(t) = (v_0^{(1)})G_{14}(t)\) this also defines \((v_0^{(1)})\) for the special case

Analogously if \((b_{13}^{(1)}) = (b_{14}^{(1)}), \text{ then } (\tau_1^{(1)}) = (\tau_2^{(1)})\) and then

\((u_1^{(1)}) = (\bar{u}_1^{(1)})\) if in addition \((u_0^{(1)}) = (u_1^{(1)})\) then \(T_{13}(t) = (u_0^{(1)})T_{14}(t)\) This is an important conclusion of the relationship between \((v_1^{(1)})\) and \((\bar{v}_1^{(1)})\), and definition of \((u_0^{(1)})\).

we obtain

\[
\frac{dv^{(2)}}{dt} = (a_{16})^{(2)} - (a_{16}^{(2)} - (a_{17}^{(2)}) + (a_{16}^{(2)}(T_{17}t))) - (a_{17}^{(2)}(T_{17}t)v^{(2)} - (a_{17}^{(2)}v^{(2)})
\]

**Definition of** \(v^{(2)} :-\)

\[
\quad v^{(2)} = \frac{G_{16}}{G_{17}}
\]

It follows

\[
-(a_{17}^{(2)}v^{(2)})^2 + (\sigma_2^{(2)}v^{(2)} - (a_{16}^{(2)}) \leq \frac{dv^{(2)}}{dt} \leq -(a_{17}^{(2)}v^{(2)})^2 + (\sigma_1^{(2)}v^{(2)} - (a_{16}^{(2)})^2)
\]

From which one obtains

**Definition of** \((\bar{v}_1^{(2)}), (v_0^{(2)}) :-\)

(d) For \(0 < (v_0^{(2)}) = \frac{G_0}{G_{17}} < (v_1^{(2)}) < (\bar{v}_1^{(2)})
\]

www.ijsrp.org
\[ v^{(2)}(t) \geq \left( \frac{v_{1}^{(2)} + G(2)(v_{2}^{(2)})}{1 + C(2)k} \right) e^{-[a_{17}^{(2)}(v_{1}^{(2)} - v_{0}^{(2)})]t} \]

it follows \((v_{0}^{(2)}) \leq v^{(2)}(t) \leq (v_{1}^{(2)})\)

In the same manner, we get

\[ v^{(2)}(t) \leq \left( \frac{\varphi_{1}^{(2)} + G(2)(v_{2}^{(2)})}{1 + C(2)k} \right) e^{-[a_{17}^{(2)}(v_{1}^{(2)} - v_{0}^{(2)})]t} \leq \frac{v^{(2)}(t)}{\varphi_{1}^{(2)}} \]

From which we deduce \((v_{0}^{(2)}) \leq v^{(2)}(t) \leq (\tilde{v}_{1}^{(2)})\)

(e) If \(0 < (v_{1}^{(2)}) < (v_{0}^{(2)}) = \frac{G_{16}}{G_{17}} \) we find like in the previous case,

\[ (v_{1}^{(2)}) \leq \left( \frac{\varphi_{1}^{(2)} + G(2)(v_{2}^{(2)})}{1 + C(2)k} \right) e^{-[a_{17}^{(2)}(v_{1}^{(2)} - v_{0}^{(2)})]t} \leq \frac{v^{(2)}(t)}{\varphi_{1}^{(2)}} \leq \frac{v^{(2)}(t)}{(v_{0}^{(2)})} \]

(f) If \(0 < (v_{1}^{(2)}) \leq (\tilde{v}_{1}^{(2)}) \leq (v_{0}^{(2)}) = \frac{G_{16}}{G_{17}} \), we obtain

\[ (v_{1}^{(2)}) \leq v^{(2)}(t) \leq (v_{0}^{(2)}) \leq (v_{1}^{(2)}) \]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(2)}(t)\) :

\[ (m_{2}^{(2)}) \leq v^{(2)}(t) \leq (m_{1}^{(2)}) \]

\[ v^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)} \]

In a completely analogous way, we obtain

**Definition of** \(u^{(2)}(t)\) :

\[ (\mu_{2}^{(2)}) \leq u^{(2)}(t) \leq (\mu_{1}^{(2)}) \]

\[ u^{(2)}(t) = \frac{G_{15}(t)}{G_{17}(t)} \]

**Particular case:**

If \((a_{16}^{(2)}) = (a_{17}^{(2)})\) then \((\sigma_{1}^{(2)}) = (\sigma_{2}^{(2)})\) and in this case \((v_{1}^{(2)}) = (v_{0}^{(2)})\) if in addition \((v_{0}^{(2)}) = (v_{1}^{(2)})\) then \(v^{(2)}(t) = (v_{0}^{(2)})\) and as a consequence \(G_{16}(t) = (v_{0}^{(2)})G_{17}(t)\)

Analogously if \((b_{16}^{(2)}) = (b_{17}^{(2)})\), then \((\tau_{1}^{(2)} = (\tau_{2}^{(2)})\) and then

\(u(1)^{(2)} = (\tilde{u}_{1}^{(2)})\) if in addition \((u_{0}^{(2)}) = (u_{1}^{(2)})\) then \(T_{16}(t) = (u_{0}^{(2)})T_{17}(t)\) This is an important consequence of the relation between \((v_{1}^{(2)})\) and \((\tilde{v}_{1}^{(2)})\)

From GLOBAL EQUATIONS we obtain

\[ \frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left( (a_{20})^{(3)} - (a_{21})^{(3)} + (a_{20})^{(3)}(T_{21}, t) \right) - (a_{21})^{(3)}(T_{21}, t)v^{(3)} - (a_{21})^{(3)}v^{(3)} \]
Definition of $\nu^{(3)}$ :-

$$\nu^{(3)} = \frac{g_0}{g_21}$$

It follows

$$-\left((a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{g_0}{g_21} < (\nu_1)^{(3)} < (\nu_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}}{1 + (C)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}} \cdot (C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner, we get

$$\nu^{(3)}(t) \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}}{1 + (C)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}} \cdot (\nu_1)^{(3)} - (\nu_2)^{(3)}$$

Definition of $(\nu_1)^{(3)}$ :-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{g_0}{g_21} < (\nu_1)^{(3)}$ we find like in the previous case,

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}}{1 + (C)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}} \leq \nu^{(3)}(t) \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}}{1 + (C)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}} \cdot \frac{(\nu_1)^{(3)} - (\nu_2)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}$$

(c) If $0 < (\nu_1)^{(3)} \leq (\nu_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{g_0}{g_21}$, we obtain

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}}{1 + (C)^{(3)}e^{-\left((a_{21})^{(3)}(\nu_1)^{(3)} - (\nu_0)^{(3)}\right)}t}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c), we have

Definition of $\nu^{(3)}(t)$ :-

$$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$$

$$\nu^{(3)}(t) = \frac{g_0}{g_21(t)}$$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$ :-

$$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$$

$$u^{(3)}(t) = \frac{g_20(t)}{g_21(t)}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the
theorem.

Particular case : 
If \( (a_{20}^v)^{(3)} = (a_{21}^v)^{(3)} \) then \( (a_1^v)^{(3)} = (\sigma_2)^{(3)} \) and in this case \( (v_1)^{(3)} = (\bar{v}_2)^{(3)} \) if in addition \( (v_0)^{(3)} = (v_1)^{(3)} \) then \( v^{(3)}(t) = (v_0)^{(3)} \) and as a consequence \( g_{20}(t) = (v_0)^{(3)} \) \( g_{21}(t) \)

Analogously if \( (b_{20}^v)^{(3)} = (b_{21}^v)^{(3)} \), then \( (\tau_1)^{(3)} = (\tau_2)^{(3)} \) and then

\( (u_1)^{(3)} = (\bar{u}_1)^{(3)} \) if in addition \( (u_0)^{(3)} = (u_1)^{(3)} \) then \( T_{20}(t) = (u_0)^{(3)} T_{21}(t) \). This is an important consequence of the relation between \( (v_1)^{(3)} \) and \( (\bar{v}_1)^{(3)} \).

From GLOBAL EQUATIONS we obtain

\[
\frac{dv^{(4)}}{dt} = (24a)^{(4)} - (24a)^{(4)} + (24a)^{(4)}(25t,t) - (25a)^{(4)}(25t,t)v^{(4)} - (25a)^{(4)}v^{(4)}
\]

**Definition of \( v^{(4)} \) :**  
\[ v^{(4)} = \frac{\partial}{\partial t} \]

It follows that

\[
-(25a)^{(4)}(v^{(4)})^2 + (24a)^{(4)}v^{(4)} - (24a)^{(4)} \leq \frac{dv^{(4)}}{dt} \leq -(25a)^{(4)}(v^{(4)})^2 + (24a)^{(4)}v^{(4)} - (24a)^{(4)}
\]

From which one obtains

**Definition of \( (\bar{v}_1)^{(4)} \), \( (v_0)^{(4)} \) :**

(d) For \( 0 < (v_0)^{(4)} = \frac{\partial}{\partial 25} < (v_1)^{(4)} < (\bar{v}_1)^{(4)} \)

\[ v^{(4)}(t) \geq \frac{4(v_1)^{(4)} + (v_0)^{(4)}(v_2)^{(4)}[e^{-25a(t)}(v_1)^{(4)} - (v_0)^{(4)}]t}{(v_0)^{(4)} - (v_2)^{(4)}} \]

it follows \( (v_0)^{(4)} \leq v^{(4)}(t) \leq (v_1)^{(4)} \)

In the same manner, we get

\[ v^{(4)}(t) \leq \frac{(v_1)^{(4)} + (v_0)^{(4)}(v_2)^{(4)}[e^{-25a(t)}(v_1)^{(4)} - (v_0)^{(4)}]t}{(v_0)^{(4)} - (v_2)^{(4)}} \]

From which we deduce \( (v_0)^{(4)} \leq v^{(4)}(t) \leq (\bar{v}_1)^{(4)} \)

(e) If \( 0 < (v_1)^{(4)} < (v_0)^{(4)} = \frac{\partial}{\partial 25} < (\bar{v}_1)^{(4)} \) we find like in the previous case,

\[ (v_1)^{(4)} \leq \frac{(v_1)^{(4)} + (v_0)^{(4)}(v_2)^{(4)}[e^{-25a(t)}(v_1)^{(4)} - (v_0)^{(4)}]t}{1 + (v_0)^{(4)}[e^{-25a(t)}(v_1)^{(4)} - (v_0)^{(4)}]t} \leq (v_1)^{(4)} \leq \]

\[ (v_1)^{(4)} + (v_0)^{(4)}(v_2)^{(4)}[e^{-25a(t)}(v_1)^{(4)} - (v_0)^{(4)}]t \]

(f) If \( 0 < (v_1)^{(4)} < (\bar{v}_1)^{(4)} \leq (v_0)^{(4)} = \frac{\partial}{\partial 25} \), we obtain
\[(v_t)^{(4)} \leq v^{(4)}(t) \leq \frac{\left[G_{24}^{(4)} + (v_{24})^{(4)}e^{-\left[\left(\nu_{25}\right)^{(4)}(v_t)^{(4)} - (v_25)^{(4)}\right]\cdot t}\right]}{1 + (v_{24})^{(4)}e^{-\left[\left(\nu_{25}\right)^{(4)}(v_t)^{(4)} - (v_25)^{(4)}\right]\cdot t}} \leq (v_0)^{(4)}\]

And so with the notation of the first part of condition (c), we have

**Definition of** \(v^{(4)}(t)\) :

\[(m_2)^{(4)} \leq v^{(4)}(t) \leq (m_1)^{(4)}, \quad v^{(4)}(t) = \frac{G_{24}^{(4)}}{G_{25}^{(4)}}\]

In a completely analogous way, we obtain

**Definition of** \(u^{(4)}(t)\) :

\[(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad u^{(4)}(t) = \frac{T_{24}^{(4)}}{T_{25}^{(4)}}\]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If \((a_25)^{(4)} = (a_25)^{(4)}\), then \((\sigma_1)^{(4)} = (\sigma_2)^{(4)}\) and in this case \((v_1)^{(4)} = (\tilde{v}_1)^{(4)}\) if in addition \((v_0)^{(4)} = (v_1)^{(4)}\) then \(v^{(4)}(t) = (v_0)^{(4)}\) and as a consequence \(G_{24}^{(4)}(t) = (v_0)^{(4)}G_{25}^{(4)}(t)\) this also defines \((v_0)^{(4)}\) for the special case.

Analogously if \((b_{24}^{'(4)} = (b_{25}^{'(4)}, then \((\tau_1)^{(4)} = (\tau_2)^{(4)}\) and then \((u_1)^{(4)} = (\tilde{u}_1)^{(4)}\) if in addition \((u_0)^{(4)} = (u_1)^{(4)}\) then \(T_{24}^{(4)}(t) = (u_0)^{(4)}T_{25}^{(4)}(t)\) This is an important consequence of the relation between \((v_1)^{(4)}\) and \((\tilde{v}_1)^{(4)}\), and definition of \((u_0)^{(4)}\).

From GLOBAL EQUATIONS we obtain

\[\frac{dv^{(5)}}{dt} = (a_{28})^{(5)} - \left[\left(a_{28}^{'}\right)^{(5)} - \left(a_{29}^{''}\right)^{(5)} + \left(a_{28}^{''}\right)^{(5)}(T_{29}, t)\right] \leq \frac{dv^{(5)}}{dt} \leq - (a_{28})^{(5)}(T_{29}, t)\]

**Definition of** \(v^{(5)}\) :

\[v^{(5)} = \frac{G_{28}^{(5)}}{G_{29}^{(5)}}\]

It follows

\[-(a_{28})^{(5)}(v^{(5)})^2 + (\sigma_2)^{(5)}v^{(5)} - (a_{28})^{(5)} \leq \frac{dv^{(5)}}{dt} \leq -(a_{28})^{(5)}(v^{(5)})^2 + (\sigma_1)^{(5)}v^{(5)} - (a_{28})^{(5)}\]

From which one obtains

**Definition of** \((\tilde{v}_1)^{(5)}, (v_0)^{(5)}\) :

\[\begin{align*}
\text{(g) For } & 0 < v_0^{(5)} = \frac{G_{28}^{(5)}}{G_{29}^{(5)}} < v_1^{(5)} < (\tilde{v}_1)^{(5)} \\
\frac{v^{(5}(t))}{(v^{(5}(t))} & \leq \frac{(v_1)^{(5)} + (\sigma_2)^{(5)}e^{-\left[\left(\nu_{25}\right)^{(5)}(v_t)^{(5)} - (v_25)^{(5)}\right]\cdot t}}{1 + (v_{24})^{(4)}e^{-\left[\left(\nu_{25}\right)^{(5)}(v_t)^{(5)} - (v_25)^{(5)}\right]\cdot t}} \leq (v_0)^{(5)}
\end{align*}\]

it follows \((v_0)^{(5)} \leq v^{(5}(t)) \leq (v_1)^{(5)}\)

In the same manner, we get

www.ijsrp.org
\[ \nu^{(5)}(t) \leq \frac{(v_1^{(5)} + \tau_1^{(5)})(v_2^{(5)} + \tau_2^{(5)})}{5 + \tau_1^{(5)}} \left[ \frac{-(\nu_2^{(5)})(\nu_1^{(5)} - \nu_2^{(5)})}{1 + \tau_1^{(5)}} \right] \]

From which we deduce \( (v_0^{(5)}) \leq \nu^{(5)}(t) \leq (\bar{v}_1^{(5)}) \)

(h) If \( 0 < (v_1^{(5)}) < (v_0^{(5)}) = \frac{g_2^{(5)}}{g_2^{(5)}} < (\bar{v}_1^{(5)}) \) we find like in the previous case,

\[ \left( \frac{v_1^{(5)} + \tau_1^{(5)} + \nu_2^{(5)} + \tau_2^{(5)}}{5 + \tau_1^{(5)}} \right) \left[ \frac{-(\nu_2^{(5)})(\nu_1^{(5)} - \nu_2^{(5)})}{1 + \tau_1^{(5)}} \right] \leq \nu^{(5)}(t) \leq \left( \frac{v_1^{(5)} + \tau_1^{(5)} + \nu_2^{(5)} + \tau_2^{(5)}}{5 + \tau_1^{(5)}} \right) \left[ \frac{-(\nu_2^{(5)})(\nu_1^{(5)} - \nu_2^{(5)})}{1 + \tau_1^{(5)}} \right] \]

(i) If \( 0 < (v_1^{(5)}) \leq (\bar{v}_1^{(5)}) \leq \left( \frac{v_0^{(5)} = \frac{g_2^{(5)}}{g_2^{(5)}}}{5 + \tau_1^{(5)}} \right) \), we obtain

\[ \left( \frac{v_1^{(5)} + \tau_1^{(5)} + \nu_2^{(5)} + \tau_2^{(5)}}{5 + \tau_1^{(5)}} \right) \left[ \frac{-(\nu_2^{(5)})(\nu_1^{(5)} - \nu_2^{(5)})}{1 + \tau_1^{(5)}} \right] \leq \nu^{(5)}(t) \leq \left( \frac{v_1^{(5)} + \tau_1^{(5)} + \nu_2^{(5)} + \tau_2^{(5)}}{5 + \tau_1^{(5)}} \right) \left[ \frac{-(\nu_2^{(5)})(\nu_1^{(5)} - \nu_2^{(5)})}{1 + \tau_1^{(5)}} \right] \]

And so with the notation of the first part of condition (c), we have Definition of \( \nu^{(5)}(t) \) :

\[ (m_2^{(5)}) \leq \nu^{(5)}(t) \leq (m_1^{(5)}) \quad \nu^{(5)}(t) = \frac{g_2^{(5)}}{g_2^{(5)}} \]

In a completely analogous way, we obtain Definition of \( \mu^{(5)}(t) \) :

\[ (\mu_2^{(5)}) \leq \mu^{(5)}(t) \leq (\mu_1^{(5)}) \quad \mu^{(5)}(t) = \frac{T_2^{(5)}}{T_2^{(5)}} \]

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case:**

If \( (a_2^{(5)}) = (a_2^{(5)}) \), then \( (\sigma_1^{(5)}) = (\sigma_2^{(5)}) \) and in this case \( (v_1^{(5)}) = (\bar{v}_1^{(5)}) \) if in addition \( (v_0^{(5)}) = (v_3^{(5)}) \), then \( \nu^{(5)}(t) = (v_0^{(5)}) \) and as a consequence \( G_2^{(5)}(t) = (v_0^{(5)})G_2^{(5)}(t) \) this also defines \( (v_0^{(5)}) \) for the special case.

Analogously if \( (b_2^{(5)}) = (b_2^{(5)}) \), then \( (\tau_1^{(5)}) = (\tau_2^{(5)}) \) and then \( (u_1^{(5)}) = (\bar{u}_1^{(5)}) \) if in addition \( (u_0^{(5)}) = (u_1^{(5)}) \) and as \( T_2^{(5)}(t) = (u_0^{(5)})T_2^{(5)}(t) \) this is an important consequence of the relation between \( (v_1^{(5)}) \) and \( (\bar{u}_1^{(5)}) \), and definition of \( (u_0^{(5)}) \).

we obtain

\[ \frac{d\nu^{(6)}}{dt} = (a_2^{(6)}) - \left( (a_2^{(6)}) - (a_2^{(6)}) + (a_2^{(6)})(T_3^{(6)} - t) \right) - (a_2^{(6)})(T_3^{(6)} - t)\nu^{(6)} - (a_2^{(6)})(6)\nu^{(6)} \]

**Definition of \( \nu^{(6)} \) :**

\[ \nu^{(6)} = \frac{g_2^{(6)}}{g_3^{(6)}} \]

It follows

\[ -\left( (a_2^{(6)})(\nu^{(6)})^2 + (\sigma_2^{(6)})\nu^{(6)} - (a_2^{(6)}) \right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left( (a_3^{(6)})(\nu^{(6)})^2 + (\sigma_1^{(6)})\nu^{(6)} - (a_2^{(6)}) \right) \]

www.ijsrp.org
From which one obtains

**Definition of** $(\bar{v}_1)^{(6)}, (\bar{v}_0)^{(6)}$ :  

(j) For $0 < (v_0)^{(6)} = \frac{g_{22}}{g_{33}} < (v_1)^{(6)} < (\bar{v}_1)^{(6)}$

$$v^{(6)}(t) \geq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \geq v^{(6)}(t) \leq \frac{(v_1)^{(6)} - (v_2)^{(6)} - g_{32}(t)}{g_{33}(t)},$$

it follows $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

In the same manner, we get

$$v^{(6)}(t) \leq \frac{(v_2)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq \frac{(v_2)^{(6)} - (v_2)^{(6)} - g_{32}(t)}{g_{33}(t)},$$

From which we deduce $(v_0)^{(6)} \leq v^{(6)}(t) \leq (v_1)^{(6)}$

(k) If $0 < (v_1)^{(6)} < (v_0)^{(6)} = \frac{g_{32}}{g_{33}} < (\bar{v}_1)^{(6)}$, we find like in the previous case,

$$v^{(6)}(t) \leq \frac{(v_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq (\bar{v}_1)^{(6)}.$$

(l) If $0 < (v_1)^{(6)} \leq (\bar{v}_1)^{(6)} \leq (v_0)^{(6)} = \frac{g_{32}}{g_{33}}$, we obtain

$$v^{(6)}(t) \leq \frac{(\bar{v}_1)^{(6)} + (C)^{(6)}(v_2)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}}{1 + (C)^{(6)}e^{-[a_{33}]^{(6)}(v_1)^{(6)} - (v_2)^{(6)}]t}} \leq (v_0)^{(6)}.$$

And so with the notation of the first part of condition (c), we have

**Definition of** $v^{(6)}(t)$ :

$$(m_2)^{(6)} \leq v^{(6)}(t) \leq (m_1)^{(6)}, \quad v^{(6)}(t) = \frac{g_{32}(t)}{g_{33}(t)}.$$

In a completely analogous way, we obtain

**Definition of** $u^{(6)}(t)$ :

$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad u^{(6)}(t) = \frac{g_{32}(t)}{g_{33}(t)}.$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

**Particular case**:

If $(a_{32})^{(6)} = (a_{33})^{(6)}$ then $(\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(v_1)^{(6)} = (\bar{v}_1)^{(6)}$ if in addition $(v_0)^{(6)} = (v_1)^{(6)}$ then $v^{(6)}(t) = (v_0)^{(6)}$ and as a consequence $G_{32}(t) = (v_0)^{(6)}G_{33}(t)$ this also defines $(v_0)^{(6)}$ for the special case.

Analogously if $(b_{32})^{(6)} = (b_{33})^{(6)}$, then $(\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
If $u(t) = (\bar{u})^T(\bar{t})$ if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$. This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, and definition of $(u_0)^{(6)}$.

We can prove the following

**Theorem 3:** If $(a_i)^{(1)}$ and $(b_i)^{(1)}$ are independent on $t$, and the conditions

$$
(a_{i3}^{(1)})^2 - (a_{i4}^{(1)})^2 < 0
$$

$$
(a_{i3}^{(1)})^2 - (a_{i4}^{(1)})^2 + (a_{i3}^{(1)})(a_{i4}^{(1)}) > 0
$$

$$
(b_{i3}^{(1)})^2 - (b_{i4}^{(1)})^2 > 0
$$

$$
(b_{i3}^{(1)})(b_{i4}^{(1)}) - (b_{i3}^{(1)})(b_{i4}^{(1)}) > 0
$$

with $(p_{i13}^{(1)}, (r_{i14}^{(1)})$ as defined, then the system

If $(a_i^{(2)})$ and $(b_i^{(2)})$ are independent on $t$, and the conditions

$$
(a_{i6}^{(2)})^2 - (a_{i7}^{(2)})^2 < 0
$$

$$
(a_{i6}^{(2)})^2 - (a_{i7}^{(2)})^2 + (a_{i6}^{(2)})(a_{i7}^{(2)}) > 0
$$

$$
(b_{i6}^{(2)})^2 - (b_{i7}^{(2)})^2 > 0
$$

$$
(b_{i6}^{(2)})(b_{i7}^{(2)}) - (b_{i6}^{(2)})(b_{i7}^{(2)}) > 0
$$

with $(p_{i16}^{(2)}, (r_{i17}^{(2)})$ as defined are satisfied, then the system

If $(a_i^{(3)})$ and $(b_i^{(3)})$ are independent on $t$, and the conditions

$$
(a_{i20}^{(3)})^2 - (a_{i21}^{(3)})^2 < 0
$$

$$
(a_{i20}^{(3)})^2 - (a_{i21}^{(3)})^2 + (a_{i20}^{(3)})(a_{i21}^{(3)}) > 0
$$

$$
(b_{i20}^{(3)})^2 - (b_{i21}^{(3)})^2 > 0
$$

$$
(b_{i20}^{(3)})(b_{i21}^{(3)}) - (b_{i20}^{(3)})(b_{i21}^{(3)}) > 0
$$

with $(p_{i20}^{(3)}, (r_{i21}^{(3)})$ as defined are satisfied, then the system

If $(a_i^{(4)})$ and $(b_i^{(4)})$ are independent on $t$, and the conditions

$$
(a_{i24}^{(4)})^2 - (a_{i25}^{(4)})^2 < 0
$$

$$
(a_{i24}^{(4)})^2 - (a_{i25}^{(4)})^2 + (a_{i24}^{(4)})(a_{i25}^{(4)}) > 0
$$

$$
(b_{i24}^{(4)})^2 - (b_{i25}^{(4)})^2 > 0
$$

$$
(b_{i24}^{(4)})(b_{i25}^{(4)}) - (b_{i24}^{(4)})(b_{i25}^{(4)}) > 0
$$

with $(p_{i24}^{(4)}, (r_{i25}^{(4)})$ as defined are satisfied, then the system

If $(a_i^{(5)})$ and $(b_i^{(5)})$ are independent on $t$, and the conditions

$$
(a_{i28}^{(5)})^2 - (a_{i29}^{(5)})^2 < 0
$$

www.ijsrp.org
\begin{align*}
(a_{20})'(5)(a_{29})'(5) - (a_{20})'(5)(a_{29})'(5) + (a_{20})'(5)(p_{20})'(5) + (a_{29})'(5)(p_{29})'(5) + (p_{20})'(5)(p_{29})'(5) > 0 \\
(b_{20})'(5)(b_{29})'(5) - (b_{20})'(5)(b_{29})'(5) > 0,
\end{align*}

if \((a_i)^{(6)}\) and \((b_i)^{(6)}\) are independent on \(t\), and the conditions

\begin{align*}
(a_{32})'(6)(a_{33})'(6) - (a_{32})'(6)(a_{33})'(6) < 0 \\
(a_{32})'(6)(a_{33})'(6) - (a_{32})'(6)(a_{33})'(6) + (a_{32})'(6)(p_{32})'(6) + (a_{33})'(6)(p_{33})'(6) + (p_{32})'(6)(p_{33})'(6) > 0 \\
(b_{32})'(6)(b_{33})'(6) - (b_{32})'(6)(b_{33})'(6) > 0,
\end{align*}

if \((p_{32})'(6), (r_{32})'(6)\) as defined are satisfied, then the system

\begin{align*}
(a_{13})'(1)G_{14} - [(a_{13})'(1) + (a_{13})''(1)T_{14}]G_{13} = 0 \\
(a_{14})'(1)G_{13} - [(a_{14})'(1) + (a_{14})''(1)T_{14}]G_{14} = 0 \\
(a_{15})'(1)G_{14} - [(a_{15})'(1) + (a_{15})''(1)T_{14}]G_{15} = 0 \\
(b_{13})'(1)T_{14} - [(b_{13})'(1) - (b_{13})''(1)G]T_{13} = 0 \\
(b_{14})'(1)T_{13} - [(b_{14})'(1) - (b_{14})''(1)G]T_{14} = 0 \\
(b_{15})'(1)T_{14} - [(b_{15})'(1) - (b_{15})''(1)G]T_{15} = 0
\end{align*}

has a unique positive solution, which is an equilibrium solution for the system

\begin{align*}
(a_{16})'(2)G_{17} - [(a_{16})'(2) + (a_{16})''(2)T_{17}]G_{16} = 0 \\
(a_{17})'(2)G_{16} - [(a_{17})'(2) + (a_{17})''(2)T_{17}]G_{17} = 0 \\
(a_{18})'(2)G_{17} - [(a_{18})'(2) + (a_{18})''(2)T_{17}]G_{18} = 0 \\
(b_{16})'(2)T_{17} - [(b_{16})'(2) - (b_{16})''(2)G_{19}]T_{16} = 0 \\
(b_{17})'(2)T_{16} - [(b_{17})'(2) - (b_{17})''(2)G_{19}]T_{17} = 0 \\
(b_{18})'(2)T_{17} - [(b_{18})'(2) - (b_{18})''(2)G_{19}]T_{18} = 0
\end{align*}

has a unique positive solution, which is an equilibrium solution for

\begin{align*}
(a_{20})'(3)G_{21} - [(a_{20})'(3) + (a_{20})''(3)T_{21}]G_{20} = 0 \\
(a_{21})'(3)G_{20} - [(a_{21})'(3) + (a_{21})''(3)T_{21}]G_{21} = 0 \\
(a_{22})'(3)G_{21} - [(a_{22})'(3) + (a_{22})''(3)T_{21}]G_{22} = 0 \\
(b_{20})'(3)T_{21} - [(b_{20})'(3) - (b_{20})''(3)G_{23}]T_{20} = 0
\end{align*}
\[(b_{21})^{(3)}T_{20} - [(b_{21})^{(3)} - (b_{21}^{''})^{(3)}(G_{23})]T_{21} = 0\]

\[(b_{22})^{(3)}T_{21} - [(b_{22}^{''})^{(3)}(G_{23})]T_{22} = 0\]

has a unique positive solution, which is an equilibrium solution

\[(a_{24})^{(4)}G_{25} - [(a_{24})^{(4)} + (a_{24}^{''})^{(4)}(T_{25})]G_{24} = 0\]

\[(a_{25})^{(4)}G_{24} - [(a_{25})^{(4)} + (a_{25}^{''})^{(4)}(T_{25})]G_{25} = 0\]

\[(a_{26})^{(4)}G_{25} - [(a_{26})^{(4)} + (a_{26}^{''})^{(4)}(T_{25})]G_{26} = 0\]

\[(b_{24})^{(4)}T_{25} - [(b_{24}^{''})^{(4)}(G_{27})]T_{24} = 0\]

\[(b_{25})^{(4)}T_{24} - [(b_{25}^{''})^{(4)}(G_{27})]T_{25} = 0\]

\[(b_{26})^{(4)}T_{25} - [(b_{26}^{''})^{(4)}(G_{27})]T_{26} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{28})^{(5)}G_{29} - [(a_{28})^{(5)} + (a_{28}^{''})^{(5)}(T_{29})]G_{28} = 0\]

\[(a_{29})^{(5)}G_{28} - [(a_{29})^{(5)} + (a_{29}^{''})^{(5)}(T_{29})]G_{29} = 0\]

\[(a_{30})^{(5)}G_{29} - [(a_{30})^{(5)} + (a_{30}^{''})^{(5)}(T_{29})]G_{30} = 0\]

\[(b_{28})^{(5)}T_{29} - [(b_{28}^{''})^{(5)}(G_{31})]T_{28} = 0\]

\[(b_{29})^{(5)}T_{28} - [(b_{29}^{''})^{(5)}(G_{31})]T_{29} = 0\]

\[(b_{30})^{(5)}T_{29} - [(b_{30}^{''})^{(5)}(G_{31})]T_{30} = 0\]

has a unique positive solution, which is an equilibrium solution for the system

\[(a_{32})^{(6)}G_{33} - [(a_{32})^{(6)} + (a_{32}^{''})^{(6)}(T_{33})]G_{32} = 0\]

\[(a_{33})^{(6)}G_{32} - [(a_{33})^{(6)} + (a_{33}^{''})^{(6)}(T_{33})]G_{33} = 0\]

\[(a_{34})^{(6)}G_{33} - [(a_{34})^{(6)} + (a_{34}^{''})^{(6)}(T_{33})]G_{34} = 0\]

\[(b_{32})^{(6)}T_{33} - [(b_{32})^{''}^{(6)}(G_{35})]T_{32} = 0\]

\[(b_{33})^{(6)}T_{32} - [(b_{33})^{''}^{(6)}(G_{35})]T_{33} = 0\]

\[(b_{34})^{(6)}T_{33} - [(b_{34})^{''}^{(6)}(G_{35})]T_{34} = 0\]

has a unique positive solution, which is an equilibrium solution for the system
(a) Indeed the first two equations have a nontrivial solution \( G_{13}, G_{14} \) if

\[
F(T) = (a'_{13})^{(1)}(a_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a_{13})^{(1)}(T_{14}) + (a'_{13})^{(1)}(T_{14})(a_{14})^{(1)}(T_{14}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{16}, G_{17} \) if

\[
F(T_{16}) = (a'_{16})^{(2)}(a_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a'_{16})^{(2)}(a_{17})^{(2)}(T_{17}) + (a'_{17})^{(2)}(a_{16})^{(2)}(T_{17}) + (a'_{16})^{(2)}(T_{17})(a_{17})^{(2)}(T_{17}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{20}, G_{21} \) if

\[
F(T_{20}) = (a_{20})^{(3)}(a_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a'_{20})^{(3)}(a_{21})^{(3)}(T_{21}) + (a'_{21})^{(3)}(a_{20})^{(3)}(T_{21}) + (a'_{20})^{(3)}(T_{21})(a_{21})^{(3)}(T_{21}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{24}, G_{25} \) if

\[
F(T_{27}) = (a_{24})^{(4)}(a_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a_{24})^{(4)}(T_{25}) + (a'_{24})^{(4)}(T_{25})(a_{25})^{(4)}(T_{25}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{28}, G_{29} \) if

\[
F(T_{31}) = (a_{28})^{(5)}(a_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a_{28})^{(5)}(T_{29}) + (a'_{28})^{(5)}(T_{29})(a_{29})^{(5)}(T_{29}) = 0
\]

(a) Indeed the first two equations have a nontrivial solution \( G_{32}, G_{33} \) if

\[
F(T_{35}) = (a_{32})^{(6)}(a_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a_{32})^{(6)}(T_{33}) + (a'_{32})^{(6)}(T_{33})(a_{33})^{(6)}(T_{33}) = 0
\]

**Definition and uniqueness of \( T_{14}^* \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{14})^{(1)}(T_{14}) \) being increasing, it follows that there exists a unique \( T_{14}^* \) for which \( f(T_{14}^*) = 0 \). With this value, we obtain from the three first equations

\[
G_{13} = \frac{(a_{13})^{(1)}G_{14}}{(a_{13})^{(1)} + (a_{13})^{(1)}(T_{14})}, \quad G_{15} = \frac{(a_{15})^{(1)}G_{14}}{(a_{15})^{(1)} + (a_{15})^{(1)}(T_{14})}
\]

**Definition and uniqueness of \( T_{17}^* \):**

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_{17})^{(2)}(T_{17}) \) being increasing, it follows that there
exists a unique \( T_{17} \) for which \( f(T_{17}) = 0 \). With this value, we obtain from the three first equations

\[
G_{16} = \frac{(a_{16})^2 G_{17}}{[(a_{16})^3 + (a_{16})^2(T_{17})]}, \quad G_{18} = \frac{(a_{18})^2 G_{17}}{[(a_{18})^3 + (a_{18})^2(T_{17})]}
\]

**Definition and uniqueness of** \( T_{21} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_i)^{(1)}(T_{21}) \) being increasing, it follows that there exists a unique \( T_{21} \) for which \( f(T_{21}) = 0 \). With this value, we obtain from the three first equations

\[
G_{20} = \frac{(a_{20})^3(G_{21})}{[(a_{20})^3 + (a_{20})^3(T_{21})]}, \quad G_{22} = \frac{(a_{22})^3(G_{21})}{[(a_{22})^3 + (a_{22})^3(T_{21})]}
\]

**Definition and uniqueness of** \( T_{25} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_i)^{(5)}(T_{25}) \) being increasing, it follows that there exists a unique \( T_{25} \) for which \( f(T_{25}) = 0 \). With this value, we obtain from the three first equations

\[
G_{24} = \frac{(a_{28})^5(G_{25})}{[(a_{28})^5 + (a_{28})^5(T_{25})]}, \quad G_{26} = \frac{(a_{30})^5(G_{25})}{[(a_{30})^5 + (a_{30})^5(T_{25})]}
\]

**Definition and uniqueness of** \( T_{29} \):

After hypothesis \( f(0) < 0, f(\infty) > 0 \) and the functions \( (a_i)^{(6)}(T_{29}) \) being increasing, it follows that there exists a unique \( T_{29} \) for which \( f(T_{29}) = 0 \). With this value, we obtain from the three first equations

\[
G_{28} = \frac{(a_{32})^6(G_{29})}{[(a_{32})^6 + (a_{32})^6(T_{29})]}, \quad G_{30} = \frac{(a_{34})^6(G_{29})}{[(a_{34})^6 + (a_{34})^6(T_{29})]}
\]

(c) By the same argument, the equations 92, 93 admit solutions \( G_{13}, G_{14} \) if

\[
\varphi(G) = (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -
\left[(b_{13})^{(1)}(b_{14})^{(1)}(G) + (b_{13})^{(1)}(b_{13})^{(1)}(G) + (b_{14})^{(1)}(b_{14})^{(1)}(G) = 0
\right]
\]

Where in \( G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{14} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{14} \) such that \( \varphi(G^*) = 0 \)

(f) By the same argument, the equations 92, 93 admit solutions \( G_{16}, G_{17} \) if

\[
\varphi(G_{19}) = (b_{16})^{(2)}(b_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -
\left[(b_{16})^{(2)}(b_{17})^{(2)}(G_{19}) + (b_{16})^{(2)}(b_{16})^{(2)}(G_{19}) + (b_{16})^{(2)}(G_{19})(b_{17})^{(2)}(G_{19}) = 0
\right]
\]

Where in \( G(G_{19}, G_{17}, G_{18}), G_{16}, G_{18} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{17} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there
exists a unique \( G_{14}^* \) such that \( \varphi(G_{19}^*) = 0 \)

(g) By the same argument, the concatenated equations admit solutions \( G_{20}, G_{21} \) if

\[
\varphi(G_{23}) = (b_{20})^3(b_{21})^3 - (b_{20})^3(b_{21})^3 - \\
[(b_{20})^3(b_{21})^3] (G_{23}) + (b_{21})^3(b_{20})^3 (G_{23}) + (b_{20})^3(b_{21})^3 (G_{23}) = 0
\]

Where in \( G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{21} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{21}^* \) such that \( \varphi(G_{21}^*) = 0 \)

(h) By the same argument, the equations of modules admit solutions \( G_{24}, G_{25} \) if

\[
\varphi(G_{27}) = (b_{24})^4(b_{25})^4 - (b_{24})^4(b_{25})^4 - \\
[(b_{24})^4(b_{25})^4] (G_{27}) + (b_{25})^4(b_{24})^4 (G_{27}) + (b_{24})^4(b_{25})^4 (G_{27}) = 0
\]

Where in \( G_{27}(G_{24}, G_{25}, G_{26}), G_{24}, G_{26} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{25} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{25}^* \) such that \( \varphi(G_{25}^*) = 0 \)

(i) By the same argument, the equations (modules) admit solutions \( G_{26}, G_{29} \) if

\[
\varphi(G_{31}) = (b_{28})^5(b_{29})^5 - (b_{28})^5(b_{29})^5 - \\
[(b_{28})^5(b_{29})^5] (G_{31}) + (b_{29})^5(b_{28})^5 (G_{31}) + (b_{28})^5(b_{29})^5 (G_{31}) = 0
\]

Where in \( G_{31}(G_{28}, G_{29}, G_{30}), G_{28}, G_{30} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{29} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{29}^* \) such that \( \varphi(G_{31}^*) = 0 \)

(j) By the same argument, the equations (modules) admit solutions \( G_{32}, G_{33} \) if

\[
\varphi(G_{35}) = (b_{32})^6(b_{33})^6 - (b_{32})^6(b_{33})^6 - \\
[(b_{32})^6(b_{33})^6] (G_{35}) + (b_{33})^6(b_{32})^6 (G_{35}) + (b_{32})^6(b_{33})^6 (G_{35}) = 0
\]

Where in \( G_{35}(G_{32}, G_{33}, G_{34}), G_{32}, G_{34} \) must be replaced by their values from 96. It is easy to see that \( \varphi \) is a decreasing function in \( G_{33} \) taking into account the hypothesis \( \varphi(0) > 0, \varphi(\infty) < 0 \) it follows that there exists a unique \( G_{33}^* \) such that \( \varphi(G_{33}^*) = 0 \)

Finally we obtain the unique solution of 89 to 94

\( G_{14}^* \) given by \( \varphi(G^*) = 0 \), \( T_{14}^* \) given by \( f(T_{14}^*) = 0 \) and

\[
G_{13}^* = \frac{(a_{13})^3(G_{14})}{[a_{13}^3 + (a_{13})^3(T_{14}^*)]} , \quad G_{15}^* = \frac{(a_{15})^3(G_{14})}{[a_{15}^3 + (a_{15})^3(T_{14}^*)]} \\
T_{13}^* = \frac{(b_{13})^3(G_{14})}{[b_{13}^3 - (b_{13})^3(G^*)]} , \quad T_{15}^* = \frac{(b_{15})^3(G_{14})}{[b_{15}^3 - (b_{15})^3(G^*)]}
\]

Obviously, these values represent an equilibrium solution
Finally we obtain the unique solution

\[ G_{17} \text{ given by } \varphi((G_{19})^*) = 0, \quad T_{17}^* \text{ given by } f(T_{17}^*) = 0 \text{ and } \]

\[ G_{16} = \frac{(a_{16})^2 G_{17}}{[a_{16}]^2 + (a_{16})^2 (T_{17}^*)], \quad G_{18} = \frac{(a_{18})^2 G_{17}}{[a_{18}]^2 + (a_{18})^2 (T_{17}^*)} \]

\[ T_{16}^* = \frac{(a_{16})^2 T_{17}^*}{[b_{16}]^2 + (a_{16})^2 ((G_{19})^*)}, \quad T_{18}^* = \frac{(b_{18})^2 T_{17}^*}{[b_{18}]^2 + (b_{18})^2 ((G_{19})^*)} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{21} \text{ given by } \varphi((G_{23})^*) = 0, \quad T_{21}^* \text{ given by } f(T_{21}^*) = 0 \text{ and } \]

\[ G_{20} = \frac{(a_{20})^3 G_{21}}{[a_{20}]^3 + (a_{20})^3 (T_{21}^*)], \quad G_{22} = \frac{(a_{22})^3 G_{21}}{[a_{22}]^3 + (a_{22})^3 (T_{21}^*)} \]

\[ T_{20}^* = \frac{(b_{20})^3 T_{21}^*}{[b_{20}]^3 + (b_{20})^3 (G_{23}^*)}, \quad T_{22}^* = \frac{(b_{22})^3 T_{21}^*}{[b_{22}]^3 + (b_{22})^3 (G_{23}^*)} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{25} \text{ given by } \varphi(G_{27}) = 0, \quad T_{25}^* \text{ given by } f(T_{25}^*) = 0 \text{ and } \]

\[ G_{24} = \frac{(a_{24})^4 G_{25}}{[a_{24}]^4 + (a_{24})^4 (T_{25}^*)], \quad G_{26} = \frac{(a_{26})^4 G_{25}}{[a_{26}]^4 + (a_{26})^4 (T_{25}^*)} \]

\[ T_{24}^* = \frac{(b_{24})^4 T_{25}^*}{[b_{24}]^4 + (b_{24})^4 ((G_{27})^*)}, \quad T_{26}^* = \frac{(b_{26})^4 T_{25}^*}{[b_{26}]^4 + (b_{26})^4 ((G_{27})^*)} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{29} \text{ given by } \varphi((G_{31})^*) = 0, \quad T_{29}^* \text{ given by } f(T_{29}^*) = 0 \text{ and } \]

\[ G_{28} = \frac{(a_{28})^5 G_{29}}{[a_{28}]^5 + (a_{28})^5 (T_{29}^*)], \quad G_{30} = \frac{(a_{30})^5 G_{29}}{[a_{30}]^5 + (a_{30})^5 (T_{29}^*)} \]

\[ T_{28}^* = \frac{(b_{28})^5 T_{29}^*}{[b_{28}]^5 + (b_{28})^5 ((G_{31})^*)}, \quad T_{30}^* = \frac{(b_{30})^5 T_{29}^*}{[b_{30}]^5 + (b_{30})^5 ((G_{31})^*)} \]

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution

\[ G_{33} \text{ given by } \varphi((G_{35})^*) = 0, \quad T_{33}^* \text{ given by } f(T_{33}^*) = 0 \text{ and } \]

\[ G_{32} = \frac{(a_{32})^6 G_{33}}{[a_{32}]^6 + (a_{32})^6 (T_{33}^*)], \quad G_{34} = \frac{(a_{34})^6 G_{33}}{[a_{34}]^6 + (a_{34})^6 (T_{33}^*)} \]

\[ T_{32}^* = \frac{(b_{32})^6 T_{33}^*}{[b_{32}]^6 + (b_{32})^6 ((G_{35})^*)}, \quad T_{34}^* = \frac{(b_{34})^6 T_{33}^*}{[b_{34}]^6 + (b_{34})^6 ((G_{35})^*)} \]

Obviously, these values represent an equilibrium solution
ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions \((a^i_\alpha)^{(1)}\) and \((b^i_\alpha)^{(2)}\) belong to \(C^1(\mathbb{R}_+)^n\), then the above equilibrium point is asymptotically stable.

Proof: Denote

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{14})^{(1)}}{\partial T_{14}} (T_{14}) = (q_{14})^{(1)}, \quad \frac{\partial (b_{19})^{(2)}}{\partial G_j} (G^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{13}}{dt} = -((a_{13})^{(1)} + (p_{13})^{(1)})G_{13} + (a_{13})^{(1)}G_{14} - (q_{13})^{(1)}G_{13}T_{14}
\]

\[
\frac{dG_{14}}{dt} = -((a_{14})^{(1)} + (p_{14})^{(1)})G_{14} + (a_{14})^{(1)}G_{13} - (q_{14})^{(1)}G_{14}T_{14}
\]

\[
\frac{dG_{15}}{dt} = -((a_{15})^{(1)} + (p_{15})^{(1)})G_{15} + (a_{15})^{(1)}G_{14} - (q_{15})^{(1)}G_{15}T_{14}
\]

\[
\frac{dT_{13}}{dt} = -((b_{13})^{(1)} - (r_{13})^{(1)})T_{13} + (b_{13})^{(1)}T_{14} + \sum_{j=13}^{15} s_{13(j)}G_{1j}T_{1j}G_j
\]

\[
\frac{dT_{14}}{dt} = -((b_{14})^{(1)} - (r_{14})^{(1)})T_{14} + (b_{14})^{(1)}T_{13} + \sum_{j=13}^{15} s_{14(j)}G_{1j}T_{1j}G_j
\]

\[
\frac{dT_{15}}{dt} = -((b_{15})^{(1)} - (r_{15})^{(1)})T_{15} + (b_{15})^{(1)}T_{14} + \sum_{j=13}^{15} s_{15(j)}G_{1j}T_{1j}G_j
\]

If the conditions of the previous theorem are satisfied and if the functions \((a^i_\alpha)^{(2)}\) and \((b^i_\alpha)^{(2)}\) belong to \(C^2(\mathbb{R}_+)^n\), then the above equilibrium point is asymptotically stable.

Denote

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{12})^{(2)}}{\partial T_{17}} (T_{17}) = (q_{17})^{(2)}, \quad \frac{\partial (b_{19})^{(2)}}{\partial G_j} (G^*) = s_{ij}
\]

Taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{16}}{dt} = -((a_{16})^{(2)} + (p_{16})^{(2)})G_{16} + (a_{16})^{(2)}G_{17} - (q_{16})^{(2)}G_{16}T_{17}
\]

\[
\frac{dG_{17}}{dt} = -((a_{17})^{(2)} + (p_{17})^{(2)})G_{17} + (a_{17})^{(2)}G_{16} - (q_{17})^{(2)}G_{17}T_{17}
\]

\[
\frac{dG_{18}}{dt} = -((a_{18})^{(2)} + (p_{18})^{(2)})G_{18} + (a_{18})^{(2)}G_{17} - (q_{18})^{(2)}G_{18}T_{17}
\]

\[
\frac{dT_{16}}{dt} = -((b_{16})^{(2)} - (r_{16})^{(2)})T_{16} + (b_{16})^{(2)}T_{17} + \sum_{j=16}^{18} s_{16(j)}G_{1j}T_{1j}G_j
\]

\[
\frac{dT_{17}}{dt} = -((b_{17})^{(2)} - (r_{17})^{(2)})T_{17} + (b_{17})^{(2)}T_{16} + \sum_{j=16}^{18} s_{17(j)}T_{1j}G_j
\]

\[
\frac{dT_{18}}{dt} = -((b_{18})^{(2)} - (r_{18})^{(2)})T_{18} + (b_{18})^{(2)}T_{17} + \sum_{j=16}^{18} s_{18(j)}T_{1j}G_j
\]
If the conditions of the previous theorem are satisfied and if the functions \( (a_i''(3)) \) and \( (b_i''(3)) \) Belong to \( C^{(3)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \( \mathcal{G}_i, T_i \) :

\[
G_i = G_i^* + \mathcal{G}_i \quad , T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{21}(3))}{\partial T_{21}} (T_{21}^*) = (q_{21}) (3) \quad , \quad \frac{\partial (b_{i}(3))}{\partial G_j} (G_{23})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
d \frac{d G_{20}}{d t} = -((a_{20}''(3)) + (p_{20}''(3)))G_{20} + (a_{20}''(3))G_{21} - (q_{20}''(3))G_{20}^* T_{21}
\]

\[
d \frac{d G_{21}}{d t} = -((a_{21}''(3)) + (p_{21}''(3)))G_{21} + (a_{21}''(3))G_{20} - (q_{21}''(3))G_{21}^* T_{21}
\]

\[
d \frac{d G_{22}}{d t} = -((a_{22}''(3)) + (p_{22}''(3)))G_{22} + (a_{22}''(3))G_{21} - (q_{22}''(3))G_{22}^* T_{21}
\]

\[
d \frac{d T_{20}}{d t} = -((b_{20}''(3)) - (r_{20}''(3)))T_{20} + (b_{20}''(3))T_{21} + \sum_{j=20}^{22} (s_{(20)(j)} T_{20} G_j)
\]

\[
d \frac{d T_{21}}{d t} = -((b_{21}''(3)) - (r_{21}''(3)))T_{21} + (b_{21}''(3))T_{20} + \sum_{j=20}^{22} (s_{(21)(j)} T_{21} G_j)
\]

\[
d \frac{d T_{22}}{d t} = -((b_{22}''(3)) - (r_{22}''(3)))T_{22} + (b_{22}''(3))T_{21} + \sum_{j=20}^{22} (s_{(22)(j)} T_{22} G_j)
\]

If the conditions of the previous theorem are satisfied and if the functions \( (a_i''(4)) \) and \( (b_i''(4)) \) Belong to \( C^{(4)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \( \mathcal{G}_i, T_i \) :

\[
G_i = G_i^* + \mathcal{G}_i \quad , T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_{25}''(4))}{\partial T_{25}} (T_{25}^*) = (q_{25}) (4) \quad , \quad \frac{\partial (b_{i}''(4))}{\partial G_j} (G_{27})^* = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
d \frac{d G_{24}}{d t} = -((a_{24}''(4)) + (p_{24}''(4)))G_{24} + (a_{24}''(4))G_{25} - (q_{24}''(4))G_{24}^* T_{25}
\]

\[
d \frac{d G_{25}}{d t} = -((a_{25}''(4)) + (p_{25}''(4)))G_{25} + (a_{25}''(4))G_{24} - (q_{25}''(4))G_{25}^* T_{25}
\]

\[
d \frac{d G_{26}}{d t} = -((a_{26}''(4)) + (p_{26}''(4)))G_{26} + (a_{26}''(4))G_{25} - (q_{26}''(4))G_{26}^* T_{25}
\]

\[
d \frac{d T_{24}}{d t} = -((b_{24}''(4)) - (r_{24}''(4)))T_{24} + (b_{24}''(4))T_{25} + \sum_{j=24}^{26} (s_{(24)(j)} T_{24} G_j)
\]

\[
d \frac{d T_{25}}{d t} = -((b_{25}''(4)) - (r_{25}''(4)))T_{25} + (b_{25}''(4))T_{24} + \sum_{j=24}^{26} (s_{(25)(j)} T_{25} G_j)
\]

\[
d \frac{d T_{26}}{d t} = -((b_{26}''(4)) - (r_{26}''(4)))T_{26} + (b_{26}''(4))T_{25} + \sum_{j=24}^{26} (s_{(26)(j)} T_{26} G_j)
\]
If the conditions of the previous theorem are satisfied and if the functions \( (a_i''')^{(5)} \) and \( (b_i''')^{(5)} \) Belong to \( C^{(5)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \( G_i, T_i \) :

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_i^{(5)})}{\partial T_29} (T_29^*) = (q_29)^{(5)} \quad , \quad \frac{\partial (b_i^{(5)})}{\partial G_j} ((G_{31})^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{28}}{dt} = -((a_28^{(5)} + (p_{28})^{(5)})G_{28} + (a_28^{(5)}G_{29} - (q_{28})^{(5)}G_{28}^* T_{29})
\]

\[
\frac{dG_{29}}{dt} = -((a_29^{(5)} + (p_{29})^{(5)})G_{29} + (a_29^{(5)}G_{28} - (q_{29})^{(5)}G_{29}^* T_{29})
\]

\[
\frac{dG_{30}}{dt} = -((a_30^{(5)} + (p_{30})^{(5)})G_{30} + (a_30^{(5)}G_{29} - (q_{30})^{(5)}G_{30}^* T_{29})
\]

\[
\frac{dT_{29}}{dt} = -((b_{29}^{(5)} - (r_{29})^{(5)})T_{29} + (b_{29}^{(5)}T_{29} + \sum_{j=29}^{30}(s_{29}(i)T_{29} G_j)
\]

\[
\frac{dT_{30}}{dt} = -((b_{30}^{(5)} - (r_{30})^{(5)})T_{30} + (b_{30}^{(5)}T_{30} + \sum_{j=30}^{31}(s_{30}(i)T_{30} G_j)
\]

If the conditions of the previous theorem are satisfied and if the functions \( (a_i''')^{(6)} \) and \( (b_i''')^{(6)} \) Belong to \( C^{(6)}(\mathbb{R}_+) \) then the above equilibrium point is asymptotically stable.

Denote

**Definition of** \( G_i, T_i \) :

\[
G_i = G_i^* + G_i, \quad T_i = T_i^* + T_i
\]

\[
\frac{\partial (a_i^{(6)})}{\partial T_{33}} (T_{33}^*) = (q_{33})^{(6)} \quad , \quad \frac{\partial (b_i^{(6)})}{\partial G_j} ((G_{35})^*) = s_{ij}
\]

Then taking into account equations (global) and neglecting the terms of power 2, we obtain

\[
\frac{dG_{32}}{dt} = -((a_32^{(6)} + (p_{32})^{(6)})G_{32} + (a_32^{(6)}G_{33} - (q_{32})^{(6)}G_{32}^* T_{33})
\]

\[
\frac{dG_{33}}{dt} = -((a_33^{(6)} + (p_{33})^{(6)})G_{33} + (a_33^{(6)}G_{32} - (q_{33})^{(6)}G_{33}^* T_{33})
\]

\[
\frac{dG_{34}}{dt} = -((a_34^{(6)} + (p_{34})^{(6)})G_{34} + (a_34^{(6)}G_{33} - (q_{34})^{(6)}G_{34}^* T_{33})
\]

\[
\frac{dT_{32}}{dt} = -((b_32^{(6)} - (r_{32})^{(6)})T_{32} + (b_32^{(6)}T_{32} + \sum_{j=32}^{34}(s_{32}(i)T_{32} G_j)
\]

\[
\frac{dT_{33}}{dt} = -((b_33^{(6)} - (r_{33})^{(6)})T_{33} + (b_33^{(6)}T_{33} + \sum_{j=33}^{34}(s_{33}(i)T_{33} G_j)
\]

www.ijsrp.org
\[ \frac{dT_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})T_{34} + (b_{34})^{(6)}T_{33} + \sum_{j=32}^{34}(s_{34}(j)T_{34}G_j) \]

The characteristic equation of this system is

\[ (\lambda)^{1(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)}\{((\lambda)^{1(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)}) \}
\]

\[ \{((\lambda)^{1(1)} + (a'_{13})^{(1)} + (p_{13})^{(1)})q_{14}\}G'_{14} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13} \}
\]

\[ ((\lambda)^{1(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(14)}T_{14} + (b_{14})^{(1)}S_{(13),(14)}T_{14} \]

\[ + \{((\lambda)^{1(1)} + (a_{14})^{(1)} + (p_{14})^{(1)})(q_{13})^{(1)}G'_{13} + (a_{13})^{(1)}(q_{14})^{(1)}G_{14} \}
\]

\[ ((\lambda)^{1(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(13)}T_{14} + (b_{14})^{(1)}S_{(13),(13)}T_{13} \]

\[ ((\lambda)^{1(1)})^2 + ((a'_{13})^{(1)} + (a'_{14})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)})\lambda^{1(1)}\]

\[ ((\lambda)^{1(1)})^2 + (b'_{13})^{(1)} - (r_{13})^{(1)} + (r_{13})^{(1)}(\lambda)^{1(1)}\]

\[ + (\lambda)^{1(1)}(a'_{13})^{(1)} + (p_{13})^{(1)}(a_{13})^{(1)}(q_{14})^{(1)}G_{13} + (a_{14})^{(1)}(q_{13})^{(1)}G_{13} \]

\[ ((\lambda)^{1(1)} + (b'_{13})^{(1)} - (r_{13})^{(1)})S_{(14),(15)}T_{14} + (b_{14})^{(1)}S_{(13),(15)}T_{13} \]

\[ = 0 \]

\[ + \]

\[ (\lambda)^{2(2)} + (b'_{16})^{(2)} - (r_{18})^{(2)}\{((\lambda)^{2(2)} + (a'_{16})^{(2)} + (p_{18})^{(2)}) \}
\]

\[ \{((\lambda)^{2(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})q_{17}\}G_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G_{16} \}
\]

\[ ((\lambda)^{2(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(17)}T_{17} + (b_{17})^{(2)}S_{(16),(17)}T_{17} \]

\[ + ((\lambda)^{2(2)} + (a_{17})^{(2)} + (p_{17})^{(2)})q_{16}\}G_{16} + (a_{16})^{(2)}(q_{17})^{(2)}G_{17} \]

\[ ((\lambda)^{2(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(16)}T_{17} + (b_{17})^{(2)}S_{(16),(16)}T_{16} \]

\[ ((\lambda)^{2(2)})^2 + ((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)})\lambda^{2(2)}\]

\[ ((\lambda)^{2(2)})^2 + (b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}(\lambda)^{2(2)}\]

\[ + ((\lambda)^{2(2)} + (a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)})\lambda^{2(2)}G_{18} \]

\[ + ((\lambda)^{2(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)})q_{17}\}G_{17} + (a_{17})^{(2)}(q_{16})^{(2)}G_{16} \]

\[ ((\lambda)^{2(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)})S_{(17),(18)}T_{17} + (b_{17})^{(2)}S_{(16),(18)}T_{16} \]

\[ = 0 \]
\[ ((\lambda^{(3)}) + (b_{22}^{(3)}) - (r_{22}^{(3)}))((\lambda^{(3)}) + (a_{22}^{(3)}) + (p_{22}^{(3)})) \]
\[ (((\lambda^{(3)}) + (a_{20}^{(3)}) + (p_{20}^{(3)}))(q_{21}^{(3)})G_{21}^{(3)} + (a_{21}^{(3)})(q_{20}^{(3)})G_{20}^{(3)})) \]
\[ ((\lambda^{(3)}) + (b_{20}^{(3)}) - (r_{20}^{(3)}))s_{(21),(21)}T_{21}^{(3)} + (b_{21}^{(3)})s_{(20),(21)}T_{21}^{(3)} \]
\[ + ((\lambda^{(3)}) + (a_{21}^{(3)}))q_{20}^{(3)}G_{20}^{(3)} + (a_{20}^{(3)})(q_{21}^{(3)})G_{21}^{(3)} \]
\[ ((\lambda^{(3)}) + (b_{20}^{(3)}) - (r_{20}^{(3)}))s_{(21),(20)}T_{21}^{(3)} + (b_{21}^{(3)})s_{(20),(20)}T_{21}^{(3)} \]
\[ ((\lambda^{(3)})^{2} + ((a_{20}^{(3)}) + (a_{21}^{(3)}) + (p_{20}^{(3)}) + (p_{21}^{(3)}))((\lambda^{(3)}) \]
\[ ((\lambda^{(3)})^{2} + ((b_{20}^{(3)}) + (b_{21}^{(3)}) - (r_{20}^{(3)}) + (r_{21}^{(3)})((\lambda^{(3)}) \]
\[ + ((\lambda^{(3)})^{2} + ((a_{20}^{(3)}) + (a_{21}^{(3)}) + (p_{20}^{(3)}) + (p_{21}^{(3)}))((\lambda^{(3)}) \]
\[ ((\lambda^{(3)}) + (b_{20}^{(3)}) - (r_{20}^{(3)}))s_{(21),(22)}T_{21}^{(3)} + (b_{21}^{(3)})s_{(20),(22)}T_{21}^{(3)} \]
\[ = 0 \]

\[ ((\lambda^{(4)}) + (b_{26}^{(4)}) - (r_{26}^{(4)}))((\lambda^{(4)}) + (a_{26}^{(4)}) + (p_{26}^{(4)}) \]
\[ (((\lambda^{(4)}) + (a_{24}^{(4)}) + (p_{24}^{(4)}))(q_{25}^{(4)})G_{25}^{(4)} + (a_{25}^{(4)})(q_{24}^{(4)})G_{24}^{(4)})) \]
\[ ((\lambda^{(4)}) + (b_{24}^{(4)}) - (r_{24}^{(4)}))s_{(25),(25)}T_{25}^{(4)} + (b_{25}^{(4)})s_{(24),(25)}T_{25}^{(4)} \]
\[ + ((\lambda^{(4)}) + (a_{25}^{(4)}) + (p_{25}^{(4)}))(q_{24}^{(4)})G_{24}^{(4)} + (a_{24}^{(4)})(q_{25}^{(4)})G_{25}^{(4)} \]
\[ ((\lambda^{(4)}) + (b_{24}^{(4)}) - (r_{24}^{(4)}))s_{(25),(24)}T_{25}^{(4)} + (b_{25}^{(4)})s_{(24),(24)}T_{25}^{(4)} \]
\[ ((\lambda^{(4)})^{2} + ((a_{24}^{(4)}) + (a_{25}^{(4)}) + (p_{24}^{(4)}) + (p_{25}^{(4)}))((\lambda^{(4)}) \]
\[ ((\lambda^{(4)})^{2} + ((b_{24}^{(4)}) + (b_{25}^{(4)}) - (r_{24}^{(4)}) + (r_{25}^{(4)}))((\lambda^{(4)}) \]
\[ + ((\lambda^{(4)})^{2} + ((a_{24}^{(4)}) + (a_{25}^{(4)}) + (p_{24}^{(4)}) + (p_{25}^{(4)}))((\lambda^{(4)}) \]
\[ ((\lambda^{(4)}) + (b_{24}^{(4)}) - (r_{24}^{(4)}))s_{(25),(26)}T_{25}^{(4)} + (b_{25}^{(4)})s_{(24),(26)}T_{25}^{(4)} \]
\[ = 0 \]

www.ijsrp.org
\[
\begin{align*}
&(\lambda)^{(5)} + (b_{30})^{(5)} - (r_{30})^{(5)}((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}) \\
&\quad + ((\lambda)^{(5)} + (a_{28})^{(5)} + (p_{28})^{(5)})(q_{29})^{(5)}G_{29}^{*} + (a_{28})^{(5)}(q_{28})^{(5)}G_{28}^{*} \\
&\quad + ((\lambda)^{(5)} + (b_{28})^{(5)} - (r_{28})^{(5)})s_{(29),(29)}T_{29}^{*} + (b_{29})^{(5)}s_{(28),(28)}T_{28}^{*} \\
&\quad + ((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(q_{29})^{(5)}G_{29}^{*} + (a_{29})^{(5)}(q_{28})^{(5)}G_{28}^{*} \\
&\quad + ((\lambda)^{(5)} + (b'_{29})^{(5)} - (r_{28})^{(5)})s_{(29),(30)}T_{29}^{*} + (b_{29})^{(5)}s_{(28),(30)}T_{28}^{*} \\
&\quad + ((\lambda)^{(5)} + (a'_{29})^{(5)} + (p_{29})^{(5)})(a_{30})^{(5)}(q_{29})^{(5)}G_{29}^{*} + (a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^{*} \\
&\quad = 0
\end{align*}
\]

+ 

\[
\begin{align*}
&(\lambda)^{(6)} + (b_{34})^{(6)} - (r_{34})^{(6)}((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)}) \\
&\quad + ((\lambda)^{(6)} + (a_{32})^{(6)} + (p_{32})^{(6)})(q_{33})^{(6)}G_{33}^{*} + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*} \\
&\quad + ((\lambda)^{(6)} + (b_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(33)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(33)}T_{33}^{*} \\
&\quad + ((\lambda)^{(6)} + (a_{33})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)}G_{32}^{*} + (a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*} \\
&\quad + ((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(32)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(32)}T_{32}^{*} \\
&\quad + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (a_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)})(\lambda)^{(6)} \\
&\quad + ((\lambda)^{(6)} + (b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)})(\lambda)^{(6)} \\
&\quad + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (a_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)})(\lambda)^{(6)}(q_{34})^{(6)}G_{34} \\
&\quad + ((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)})(a_{34})^{(6)}G_{34}^{*} + (a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*} \\
&\quad = 0
\end{align*}
\]

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.
Acknowledgments:
The introduction is a collection of information from various articles, Books, News Paper reports, Home Pages Of authors, Journal Reviews, Nature 's Letters, Article Abstracts, Research papers, Abstracts Of Research Papers, Stanford Encyclopedia, Web Pages, Ask a Physicist Column, Deliberations with Professors, the internet including Wikipedia. We acknowledge all authors who have contributed to the same. In the eventuality of the fact that there has been any act of omission on the part of the authors, we regret with great deal of compunction, contrition, regret, trepidiation and remorse. As Newton said, it is only because erudite and eminent people allowed one to piggy ride on their backs; probably an attempt has been made to look slightly further. Once again, it is stated that the references are only illustrative and not comprehensive.

REFERENCES


First Author: 1Mr. K. N. Prasanna Kumar has three doctorates one each in Mathematics, Economics, Political Science. Thesis was based on Mathematical Modeling. He was recently awarded D.litt. for his work on ‘Mathematical Models in Political Science’--- Department of studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India Corresponding Author: drknpkumar@gmail.com

Second Author: 2Prof. B.S Kiranagi is the Former Chairman of the Department of Studies in Mathematics, Manasa Gangotri and present Professor Emeritus of UGC in the Department. Professor Kiranagi has guided over 25 students and he has received many encomiums and laurels for his contribution to Co homology Groups and Mathematical Sciences. Known for his prolific writing, and one of the senior most Professors of the country, he has over 150 publications to his credit. A prolific writer and a prodigious thinker, he has to his credit several books on Lie Groups, Co Homology Groups, and other mathematical application topics, and excellent publication history.-- UGC Emeritus Professor (Department of studies in Mathematics), Manasagangotri, University of Mysore, Karnataka, India
Third Author: Prof. C.S. Bagewadi is the present Chairman of Department of Mathematics and Department of Studies in Computer Science and has guided over 25 students. He has published articles in both national and international journals. Professor Bagewadi specializes in Differential Geometry and its wide-ranging ramifications. He has to his credit more than 159 research papers. Several Books on Differential Geometry, Differential Equations are coauthored by him--- Chairman, Department of studies in Mathematics and Computer science, Jnanasahyadri Kuvempu University, Shankarghatta, Shimoga district, Karnataka, India

www.ijsrp.org