Simulation of Ziegler-Nichols PID Tuning for Position Control of DC Servo Motor

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Abstract- This paper presents the implementation of PID controller in position control of dc servo motor. Due to its simplicity, robustness and successful practical application, PID (Proportional-Integral-Derivative) controllers have become most widely used controller in the industry. There are several different methods through which the PID controller can generate automatic control efficiently. In this paper, the tuning method used for the proposed position control model of dc servo motor is Ziegler-Nichols (ZN) tuning algorithm. Here, a computer based model (using MATLAB SIMULINK) is furnished for obtaining the output during the position control of dc servo motor. Nowadays, dc servo motors have become the workhorse of the industrial sector due to their easy means of construction and maintenance. Therefore, the performance of the machine needs to be specified using computer aided programs and the control strategy best suited here is PID.

Index Terms- DC servo motor, Position control, PID tuning, Ziegler-Nichols method, MATLAB-Simulink

I. INTRODUCTION

The heart of many automatic control systems are servo motor drive. The dc servo motor has applications in automatic control systems, either speed or position control of the dc servo motor. The dc servo motor is basically like a transducer that converts electric energy into mechanical energy. The torque developed on the motor shaft is directly proportional to the field flux and the armature current. The dc servo motors are more expensive in comparison to ac servo motors because of brushes and commutators [1]. These motors have relatively low torque to volume and torque to inertia ratio, however the characteristic of dc servo motors are quite linear and are easy to control. Servos are commonly electrical or partially electronic in nature, using an electric motor as the primary means of creating mechanical force. Other types of servos use hydraulics, pneumatics, or magnetic principles. Usually, servos operate on the principle of negative feedback, where the control input is compared to the actual position of the mechanical system as measured by some sort of transducer at the output. Any difference between the actual and wanted values (error signal) is amplified and used to drive the system in the direction necessary to reduce or eliminate the error. Nowadays, servo motors are used in automatic machine tools, satellite tracking antennas, remote control airplanes, automatic navigation systems on boats and planes, and antiaircraft gun control systems. In the past decades, control theory has found several developments. Different intelligent control algorithms have been developed so far. However, the PID type controller is still the most widely used control strategy in industries [2]. Studies even indicate that approximately 90% of all industrial controllers are of the PID (Proportional-Integral-Derivative) type. The Ziegler-Nichols tuning method is a means of relating the process parameters- delay time, process gain and time constant to the controller parameters, controller gain and reset time. It has been developed for use on delay-followed-by-first-order-lag processes but can also adapt to real processes. Ziegler and Nichols presented two standard methods to tune a PID controller. These methods, due to their simplicity, are still widely used in different industrial and other tuning process. This paper is organized as follows. Section (II) describes about servo motor mathematical model. Section (III) addresses servo motor SIMULINK model. Section (IV) illustrates two kinds of Ziegler-Nichols. And section (V) concludes the whole results.

II. SERVO MOTOR MATHEMATICAL MODEL

The equivalent circuit diagram of servo motor is presented in Figure (1). The armature is modeled as a circuit with resistance, $R_A$ connected in series with an inductance, $L_A$ and a voltage source, $V_b(t)$ representing the back emf in the armature when the rotor rotates.

![Figure1. (a) A dc servo motor drives an inertia load](http://dx.doi.org/10.29322/IJSRP.9.07.2019.p9184)
(b) The equivalent circuit diagram of the dc servo motor

Mathematical modelling of dc Servo motor system:

\[ E_a(t) = \text{Input Voltage} \]
\[ i_a(t) = \text{Armature current} \]
\[ R_a = \text{Armature resistance (1 ohm)} \]
\[ L_a = \text{Armature inductance (29.79 \times 10^{-3} \text{Henry})} \]
\[ E_b(t) = \text{Back e.m.f} \]
\[ T_m = \text{Developed Torque} \]
\[ \omega_m = \text{Motor Angular Velocity} \]
\[ J = \text{Motor moment of Inertia (0.01Kg.m^2)} \]
\[ B = \text{Viscous friction coefficient (0.004N.m/rad/s)} \]
\[ K_b = \text{Back e.m.f constant (0.1V/rad/s)} \]
\[ K_T = \text{Torque constant (0.1 N.m/A)} \]

The differential equation of armature circuit is

\[ \frac{di_a}{dt} + \frac{1}{L_a} i_a + R_a i_a = E_a - E_b \quad (1) \]

The Torque equation is

\[ \frac{d\omega_m}{dt} = \frac{1}{J} T_m - B \omega_m \quad (2) \]

In order to create the block diagram of system; initial conditions are acquiescence zero and laplace transform is implemented to the equations.

\[ I_a(s) = \frac{E_a(s) - K_p \omega_m(s)}{R_a + L_a s} \quad (3) \]
\[ \omega_m(s) = \frac{K_i}{s} \]
\[ \dot{E}_a(s) = s^2 J_m L_a + s J_m R_a + K_b K_i \]
\[ \dot{\theta}_m(s) = \frac{s^3 J_m L_a + s^3 J_m R_a + K_b K_i}{s} \]

The overall transfer function of the servomotor is represented as

\[ G_p = \frac{0.1}{0.0002979 s^3 + 0.01012 s^2 + 0.014 s + 0.1} \quad (6) \]

III. SERVO MOTOR SIMULINK MODEL

In this paper, the software used for constructing the simulation model is MATLAB [3]. Firstly, we investigated to know the response of the dc servomotor. The output result is as shown in figure (2). The settling time came out to be 6.34 seconds with a maximum overshoot of 56.4 %.

IV. ZIEGLER-NICHOLS PID TUNING METHOD

Tuning methods of controller describe the controller parameters in the form of formulae or algorithms. They ensure that the resultant process control system would be stable and would achieve the desired objectives. In literature, a wide variety of PID controller tuning methods are proposed [4]. These are broadly classified into three categories and these are

- Closed loop methods
  - Ziegler-Nichols method
  - Modified Ziegler-Nichols method
  - Tyreus-Luyben method
  - Damped oscillation method
- Open loop methods
  - Cohen-Coon method
  - Fertik method
  - IMC method
  - Minimum error criteria (IA)

In closed loop tuning methods the plant is operating in closed loop and controller tuning is performed during automatic state. In contrast the open loop techniques operate the plant in open loop and the controller tuning is done in manual state. In this paper, the tuning methods considered for simulation is Ziegler-Nichols method.

A proportional-integral-derivative controller (PID) is a generic control loop feedback mechanism widely used in industrial control systems [6]. A PID controller will correct the error between the output and the desired input or set point by calculating and give an output of correction that will adjust the process accordingly.

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt} \quad (7) \]

Where \( K_p \) is proportional gain, \( K_i \) is the integral gain, and \( K_d \) is the derivative gain. The Proportional value determines the reaction to the current error, the Integral determines the reaction based on the sum of recent errors and the Derivative determines the reaction to the rate at which the error has been changing.
The PID controller has the transfer function:

\[ G_c(s) = K_p + \frac{K_i}{s} + K_ds \]  

(8)

\[ G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_D s\right) \]  

(9)

The value of Kp that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh’s stability criterion [5].

Ziegler-Nichols method: The Ziegler-Nichols method is commonly used in tuning of PID controllers. The proposed ruled of Ziegler-Nichols method is for determining values of the proportional Kp, integral time Ti, and derivative time Td based on the transient response characteristics of the plant. Such as the determination of the parameters of PID controllers or tuning of PID controller can be made on site by experiment on plant. In this method, it described simple mathematical procedures, the first and second methods respectively for tuning PID controllers. These procedures are accepted as standard in control system practices.

A. Ziegler-Nichols (first method)

If the plant involves neither integrator nor dominant complex-conjugate poles, then that step response exhibits as S-shape curve, the first method can be applied. Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

B. Ziegler-Nichols (second method)

We first set \( T_i = \infty \) and \( T_d = 0 \), use the proportional control action only. And increase \( K_p \) from 0 to a critical value \( K_{cr} \) at which the output first exhibits sustained oscillations. And then consider the \( T_i \) and \( T_d \) according to table II.

### Table I

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>( K_p )</th>
<th>( T_i )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( \frac{T}{L} )</td>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>( 0.9 \frac{T}{L} )</td>
<td>( \frac{L}{0.3} )</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>( 1.2 \frac{T}{L} )</td>
<td>( 2L )</td>
<td>0.5L</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Type of controller</th>
<th>( K_p )</th>
<th>( T_i )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>( 0.5K_{cr} )</td>
<td>( \infty )</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>( 0.45K_{cr} )</td>
<td>( \frac{1}{1.2} \frac{P_{cr}}{P_{cr}} )</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>( 0.6K_{cr} )</td>
<td>( 0.5P_{cr} )</td>
<td>0.125P_{cr}</td>
</tr>
</tbody>
</table>

We determine the parameters of the PID controller by second method as follows:

\[ K_p = 2.253575025 \]

\[ T_i = 0.458269661 \]

\[ T_d = 0.114567415 \]
The transfer function of the PID controller is thus

\[
G_c(s) = \frac{1}{0.45826966s + 0.114567415s + 2.253575025}
\]

In the second model, with Ziegler-Nichols tuning method was used, the response is shown in the figure (6). Here, the value of settling time came out to be 1.03 seconds and a maximum overshoot of 5.09 %.

V. CONCLUSION

In this paper, we investigated about mathematic model and SIMULINK model of a dc servo motor. And Ziegler-Nichols tuning method is utilized for PID control of a dc servo motor. First analyzed Ziegler-Nichols tuning gain by second method and then continued in MATLAB SIMULINK. Initial settle time of dc servo motor is around 6.34 seconds. After used PID with Ziegler-Nichols closed-loop tuning algorithm, the overshoot of the system gets reduced drastically. The settling time of the second simulation was also very less as compared to first one that is the conventional method, that didn’t meet the design requirement.

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