Unsteady Heat Transfer to MHD Oscillatory Flow of Jeffrey fluid in a Channel Filled with Porous Material

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Abstract- The study on unsteady Heat Transfer to MHD Oscillatory flow of Jeffrey fluid through a porous medium under slip condition was investigated. The dimensionless governing equations are solved using perturbation technique. The analytical expressions for the velocity, temperature, skin friction or shear stress of the fluid have been obtained. The effects of flow parameters of Jeffrey fluid, Grashof number, Hartmann number, Slip parameter, porosity parameter, radiation parameter and frequency of the oscillation are carried out. The result obtained shows that velocity and skin friction decreases with increasing Hartmann number. It was also observed that the velocity and skin friction increases with increasing in Darcy number. The results obtained for Jeffrey fluid parameters was computed for velocity and skin friction shows that the velocity increases with increasing Jeffrey fluid parameter and skin friction decreases with increasing Jeffrey fluid parameter.

Index Terms- Jeffrey fluid, Heat transfers, MHD, Oscillatory flow, Porous medium, Slip condition,

I. INTRODUCTION

The study of non-Newtonian fluids has a variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. Jeffrey fluid is a type of non-Newtonian fluid that uses a relatively simpler linear model using time derivatives instead of convected derivatives, which are used by most fluid models. Jeffrey fluid is one of the rate type materials. It shows the linear viscoelastic effect of fluid which has many applications in polymer industries.


The effect of magnetic field on free convective flow of a viscous incompressible fluid past an infinite moving porous hot vertical plate in the presence of porous medium and radiation was analyzed by Singh et al. (2012). Uwanta and Hamza (2012) investigate unsteady heat transfer flow of a viscous, incompressible, electrically conductive fluid through porous medium with periodic suction and temperature oscillation. Kavita et al. (2012) investigate the influence of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel. In their work they observed that, the axial velocity increases with increasing of Jeffrey fluid. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid than that of Newtonian fluid. Aruna Kumari et al. (2012) studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel with slip effect at a lower wall where the expressions for the velocity and temperature are obtained analytically. Asadullah et al. (2013) consider the MHD flow of a Jeffrey fluid in converging and diverging channels. The flows between non parallel walls have a very significant role in physical and biological sciences. Idowa et al. (2013) studied the effect of heat and mass transfer on unsteady MHD Oscillatory flow of Jeffrey fluid in a horizontal channel with Chemical Reaction. Adesanya and Makinde (2014) studied MHD Oscillatory slip flow and heat transfer in a channel filled with porous medium. Al-Khafaji (2016) investigates the effect of heat
transfer on MHD Oscillatory flow of Jeffrey fluid with variable viscosity through porous medium. Ahmad and Ishak (2017) studied steady two-dimensional mixed convection boundary layer flow and heat transfer of a Jeffrey fluid over a stretched sheet immersed in a porous medium in the presence of a transverse magnetic field.

But the work of Hamza et al. (2011) investigate the transient heat transfer to MHD oscillatory flow through porous medium under slip condition and oscillating temperature, but the authors has not addressed the influence of a Jeffrey fluid parameters in their work.

In the present paper, we investigate the influence of Jeffrey fluid, slip conditions, magnetic field and radiative heat transfer on unsteady flow of conducting optically thin fluid through a channel filled with porous medium and surface temperature oscillating.

II. MATHMATICAL FORMULATION

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has a small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system(x, y), where x lies along the center of the channel, y is the distance measured in the normal section. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

\[
\begin{align*}
\frac{\partial \text{u}}{\partial t} &= - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{(1+\beta')} \frac{\partial^2 \text{u}}{\partial y^2} - \frac{v}{k} \text{u}' - \frac{\sigma_e B_0^2}{\rho} \text{u}' + \frac{\beta(T' - T_0)}{\rho} 
\frac{\partial T'}{\partial t} &= \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial \text{u}'}{\partial y} 
\end{align*}
\]

(1)

With boundary conditions

\[
\text{u}' = 0, T' = T_0, \quad \text{on} \quad y' = 0 \quad \quad \quad \text{u}' = 0, T' = T'_{0} + (T'_{0} - T'_{0}) \cos \omega t', \quad \text{on} \quad y' = a
\]

(2)

Where \(\text{u}'\) is the axial velocity, \(t'\) is time, \(\omega'\) is frequency of the oscillation, \(T'\) the fluid temperature, \(p\) is the pressure gravitational force, \(c_p\) is the specific heat at constant pressure, \(k\) is the thermal conductivity, \(q\) is the radiative heat flux, \(\beta\) is the coefficient of volume expansion, \(k'\) is the porous medium permeability coefficient, \(B_0\) is the electromagnetic induction, \(\sigma_e\) is the conductivity of the fluid, \(\rho\) is the density of the fluid, \(v\) is the kinematics viscosity coefficient. It is assumed that walls temperature \(T'_{0}\), \(T'_{0}\) are high enough to induce radiative heat transfer, and \(y'\) is the dimensionless slip parameter. Following Makind and Mhone (2005), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by:

\[
\frac{\partial q_i}{\partial y'} = 4\alpha^2 (T'_{0} - T')
\]

(3)

Where \(\alpha\) is the mean radiation absorption coefficient.

The following dimensionless variables and parameters are introduced:

\[
\begin{align*}
\frac{x}{a} &= x, \quad \frac{y}{a} = y, \quad \text{u} = \frac{u}{U}, \quad \text{u}' = \frac{u'}{U},
\frac{w}{w} &= \frac{w}{U}, \quad \frac{t}{T} = \frac{t}{T}, \quad \frac{t'}{T} = \frac{t'}{T}, \quad \frac{\beta}{\beta_1} = \frac{\beta(T' - T_0)}{T'_{0} - T'_{0}},
\frac{p}{\rho u v} &= \frac{p}{\rho u v}, \quad \gamma = \frac{\gamma}{\beta_1}, \quad \text{S}_3 = \frac{1}{D_a}, \quad \text{Da} = \frac{K}{a^2}, \quad k' = a^2 D_a
\end{align*}
\]

(4)

Where \(U\) is the flow mean velocity, the dimensionless governing equations together with appropriate boundary conditions can be written as:

\[
\begin{align*}
\Re \frac{\partial \text{u}}{\partial t} &= - \frac{\partial p}{\partial x} + \frac{1}{(1+\beta')} \frac{\partial^2 \text{u}}{\partial y^2} - (H^2 + S^2) \text{u} + Gr \theta
\Pr \frac{\partial \theta}{\partial t} &= \frac{\partial^2 \theta}{\partial y^2} + N^2 \theta
\end{align*}
\]

(5)

Subject to the boundary condition

\[
\begin{align*}
\text{u} - \gamma \frac{\partial \text{u}}{\partial y} &= 0, \quad \theta = 0 \quad \text{at} \quad y = 0 \\
\text{u} &= 0, \quad \theta = \cos \omega t \quad \text{at} \quad y = 1
\end{align*}
\]

(6)

Where \(\beta_1\) is the Jeffrey fluid, \(Gr\) is the thermal Grashof number, \(H\) is the Hartmann number, \(N\) is the radiation parameter, \(\Pr\) is the Peclet number, \(\Re\) is the Reynolds number, \(\text{Da}\) is the Darcy number, \(\Pr\) is the Prandtl number, \(\gamma\) is the slip parameter and \(s\) is the porous medium shape factor.

III. METHOD OF SOLUTION

In order to solve equations (6), (7) and (8) for purely oscillatory flow, let the pressure gradient, fluid velocity and temperature be:

\[
\text{u}(y, t) = u_0(y)e^{i\omega t} + U_1(y)e^{-i\omega t}
\]

(9)

\[
\theta(y, t) = \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t}
\]

(10)

Assume that \(-\frac{\partial p}{\partial x} = \lambda e^{i\omega t} + e^{-i\omega t}\)

Where \(\lambda < 0\) for favorable pressure, \(\omega\) is the frequency of the oscillation. Substituting the above expressions: (9) and (10) into (6), (7) and (8), we obtained.
\( \theta_0 + N_2 \theta_0 = 0 \)

(11)

\( \theta_1 + N_2 \theta_1 = 0 \)

(12)

\( u_0 - M_1 u_0 = -a\alpha - aGr\theta_0 \)

(13)

\( u_1 - M_1 u_1 = -a\alpha - aGr \theta_1 \)

(14)

\( u_0 - \gamma u_0 = 0, u_1 - \gamma u_1 = 0, \theta_0 = 0, \theta_1 = 0, a, t = y = 0 \)

\( u_0 = 0, u_1 = 0, \theta_0 = \frac{1}{2}, \theta_1 = \frac{1}{2} \)

at \( y = 0 \}

(15)

Where \( \alpha = 1 + \beta_1 \)

\( N_1^2 = (N^2 - Pei\omega), N_2^2 = (N^2 + Pei\omega) \)

\( M_1 = (H^2 + S^2 + Rei\omega), M_2 = (H^2 + S^2 - Rei\omega) \)

Equations (11) to (15) are solved and the solution for fluid temperature and velocity are given as follows:

\[
\theta(y, t) = \frac{1}{2} \sin\frac{N_1 y}{N_1} e^{i\omega t} + \sin\frac{N_2 y}{N_2} e^{-i\omega t}
\]

(16)

\[
u(y, t) = [A_5 \cosh M_1 y + A_6 \sinh M_1 y + F + G_2 \sin N_1 y]e^{i\omega t} + [A_7 \cosh M_2 y + A_8 \sinh M_2 y + K + G_4 \sin N_2 y]e^{-i\omega t}
\]

(17)

Skin friction or shear stress

\[
\tau_0 = \frac{d\theta}{dy} / \gamma = 0 = \frac{1}{2} \left( \frac{N_1}{\sin N_1} e^{i\omega t} + \frac{N_2}{\sin N_2} e^{-i\omega t} \right)
\]

(18)

\[
\tau_1 = \frac{d\theta}{dy} / \gamma = 1 = \frac{1}{2} (N_1 \cot N_1 e^{i\omega t} + N_2 \cot N_2) e^{-i\omega t}
\]

(19)

Nusselt number or rate of heat transfer is given by:

\[
Nu_0 = \frac{du}{dy} / \gamma = 0 = [A_6 M_1 + G_2 N_1]e^{i\omega t} + [A_8 M_2 + G_4 N_2]e^{-i\omega t}
\]

(20)

IV. GRAPHICAL RESULTS AND DISCUSSION

To study the MHD Oscillatory flow of Jeffrey fluid in a channel filled with porous material, the velocity \( u \), temperature \( \theta \) and skin friction \( \tau_0 and \tau_1 \) profiles are depicted graphically against \( y \) for different values of different parameters: Jeffrey fluid\( \beta_1 \), thermal Grashof number \( Gr \), Hartmann number \( H \), radiation parameter \( N \), Peclet number \( Pe \), Reynolds number \( Re \), Darcy number \( Da \), Prandtl number \( Pr \), slip parameter \( \gamma \) and porous medium shape factor \( s \). We made use of the following parameter values except otherwise indicated, \( \beta_1 = 0.01 \), \( Re = 1 \), \( S = 1 \), \( Ha = 1 \), \( Gr = 10 \), \( Pe = 1 \), \( N = 1 \), \( \lambda = -1 \), \( \gamma = 0.5 \) and \( \omega = 1 \).

The temperature profiles for different values of the Peclet number \( (Pe = 1, 2, 3, 4) \) are shown in Figures 1. It is observed that the temperature increases with increasing Peclet number.

The velocity profiles have been studied and presented in Figures 2 to 5. The velocity profiles for different values of the Hartmann number \( (Ha = 1, 5, 10, 15) \) is shown in fig. 2. It observed that the velocity decreases with increasing Hartmann number. The velocity profiles for different values of Darcy number \( (Da = 0.01, 0.03, 0.05, 0.07) \), Grashof number \( (Gr = 1, 2, 3, 4) \) and Jeffrey fluid parameter \( \beta_1 = 0.1, 1, 5) \) are shown in Figures 3, 4 and 5 respectively. It is observed that the velocity decrease with increasing Darcy number, Grashof number and Jeffrey fluid parameter.

The variation of the skin friction \( \tau_0 and \tau_1 \) on the porous plate with material parameters are shown in fig. 6-8. The skin friction \( \tau_0 and \tau_1 \) profiles for different values of Darcy number \( (Da = 0.01, 0.02, 0.03, 0.04) \) is shown in fig. 6. It is observed that skin friction increases with increasing Darcy number. The skin friction \( \tau_0 and \tau_1 \) profiles for different values of Jeffrey fluid parameter \( \beta_1 = 0.1, 0.3, 0.5, 0.7 \) and the Hartmann number \( (Ha = 1, 3, 5, 7) \) is shown in fig. 7 and 8 respectively. It is observed that skin friction \( \tau_0 and \tau_1 \) decreases with increasing Jeffrey fluid parameter and Hartmann number.
V. CONCLUSIONS

This paper investigates the heat transfer to MHD Oscillatory flow of Jeffrey fluid in a channel filled with porous material. The velocity, temperature and skin friction profiles are obtained analytically. The effect of different parameters namely, the radiation parameter, Grashof number, Hartmann number, Jeffrey fluid parameter, Porosity parameter, Slip parameter, frequency of the oscillation and Peclet number are studied. The conclusions of the study are as follows;

(i) It is observed that the temperature increases with increasing Peclet number.
(ii) It observed that the velocity decreases with increasing Hartmann number.
(iii) It is observed that the velocity decrease with increasing Darcy number, Grashof number and Jeffrey fluid parameter.
(iv) It is observed that skin friction increase with increasing Darcy number.
(v) It is observed that skin friction decreases with increasing of Jeffrey fluid parameter and Hartmann number.

REFERENCES


