Unsteady Heat Transfer to MHD Oscillatory Flow of Jeffrey fluid in a Channel Filled with Porous Material

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Abstract- The study on unsteady Heat Transfer to MHD Oscillatory flow of Jeffrey fluid through a porous medium under slip condition was investigated. The dimensionless governing equations are solved using perturbation technique. The analytical expressions for the velocity, temperature, skin friction or shear stress of the fluid have been obtained. The effects of flow parameters of Jeffrey fluid, Grashof number, Hartmann number, Slip parameter, porosity parameter, radiation parameter and frequency of the oscillation are carried out. The result obtained shows that velocity and skin friction decreases with increasing Hartmann number. It was also observed that the velocity and skin friction increases with increasing in Darcy number. The results obtained for Jeffrey fluid parameters was computed for velocity and skin friction shows that the velocity increases with increasing Jeffrey fluid parameter and skin friction decreases with increasing Jeffrey fluid parameter.

Index Terms- Jeffrey fluid, Heat transfers, MHD, Oscillatory flow, Porous medium, Slip condition,

I. INTRODUCTION

The study of non-Newtonian fluids has a variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. Jeffrey fluid is a type of non-Newtonian fluid that uses a relatively simpler linear model using time derivatives instead of convected derivatives, which are used by most fluid models. Jeffrey fluid is one of the rate type materials. It shows the linear viscoelastic effect of fluid which has many applications in polymer industries.

Many researchers have carried out studies in this area. Some of these include, El-Hakiem (2000) studied the MHD oscillatory flow on free convection radiation through a porous medium with constant velocity. Sahim (2008) investigated transient three dimensional flows through a porous medium with transverse permeability oscillating with time. The effect of viscous dissipative heat on three dimensional oscillatory flows with periodic suction velocity has been studied by Sahim (2010). Makinde and Mhone (2005) examined heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mehmood and Ali (2007) extended the work of Makinde and Mhone (2005) by considering the fluid slip at the lower wall. Kumar *et al*. (2010) extended the work of Mehmood and Ali (2007) by employing perturbation technique to the problem. The effect of slip conditions on the MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi (2006). Khaled and Vafai (2004) obtained the closed form solutions for steady periodic and transient velocity field under slip condition. Mansour *et al.* (2007) studied the free convection flow of micro polar fluid in slip flow regime through porous medium with periodic temperature and concentration. Mustafa *et al.* (2008) investigated unsteady MHD memory flow with oscillatory suction, variable free stream and heat sources. In addition, Abdul-Hakeem and Sathiyanathan (2009) presented analytical solution for two-dimensional oscillatory flow of an incompressible viscous fluid, through a highly porous medium bounded by an infinite vertical plate. Umavathi *et al.* (2009) studied the unsteady oscillatory flow and heat transfer in a horizontal composite channel. While Jha and Ajibade(2010) reported some interesting results on the free convective oscillatory flows induced by time dependent boundary conditions.

The effect of magnetic field on free convective flow of a viscous incompressible fluid past an infinite moving porous hot vertical plate in the presence of porous medium and radiation was analyzed by Singh *et al*. (2012). Uwanta and Hamza (2012) investigate unsteady heat transfer flow of a viscous, incompressible, electrically conductive fluid through porous medium with periodic suction and temperature oscillation. Kavita *et al.* (2012) investigate the influence of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel. In their work they observed that, the axial velocity increases with increasing of Jeffrey fluid. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid than that of Newtonian fluid. Aruna Kumari *et al.* (2012) studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel with slip effect at a lower wall where the expressions for the velocity and temperature are obtained analytically. Asadullah *et al.* (2013) consider the MHD flow of a Jeffrey fluid in converging and diverging channels. The flows between non parallel walls have a very significant role in physical and biological sciences. Idowa *et al.* (2013) studied the effect of heat and mass transfer on unsteady MHD Oscillatory flow of Jeffrey fluid in a horizontal channel with Chemical Reaction. Adesanya and Makinde (2014) studied MHD Oscillatory slip flow and heat transfer in a channel filled with porous medium. Al-Khafajy (2016) investigates the effect of heat transfer on MHD Oscillatory flow of Jeffrey fluid with variable viscosity through porous medium. Ahmad and Ishak (2017) studied steady two-dimensional mixed convection boundary layer flow and heat transfer of a Jeffrey fluid over a stretched sheet immersed in a porous medium in the presence of a transverse magnetic field.

But the work of Hamza *et al*. (2011) investigate the transient heat transfer to MHD oscillatory flow through porous medium under slip condition and oscillating temperature, but the authors has not addressed the influence of a Jeffrey fluid parameters in their work.

In the present paper, we investigate the influence of Jeffrey fluid, slip conditions, magnetic field and radiative heat transfer on unsteady flow of conducting optically thin fluid through a channel filled with porous medium and surface temperature oscillating.

II. MATHEMATICAL FORMULATION

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has a small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system(x, y), where x lies along the center of the channel, y is the distance measured in the normal section. Then, assuming a Boussinesq incompressible fluid model, the equations governing the motion are given as:

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \frac{v}{(1+\beta_1)}\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{k'}u' - \frac{\sigma_e B_0^2 u'}{\rho} + {}^g\beta(T' - T'_0)$$
(1)

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q'}{\partial y'}$$
(2)

With boundary conditions

$$\left.\begin{aligned} u' - \gamma^*\frac{\partial u'}{\partial y'} = 0,\ T' = T'_0, \quad on\ y' = 0 \\ u' = 0, T' = T'_0 + (T'_\omega - T'_0)\cos\omega' t' \quad on\ y' = a \end{aligned}\right\}$$
(3)

Where u' is the axial velocity, t' is time, ω' is frequency of the oscillation, T' the fluid temperature, p is the pressure gravitational force, $c_p$is the specific heat at constant pressure, k is the thermal conductivity, q is the radiative heat flux, β is the coefficient of volume expansion, k' is the porous medium permeability coefficient, B_0 is the electromagnetic induction, σ_e is the conductivity of the fluid, ρ is the denity of the fluid, v is the kinematics viscosity coefficient. It is assumed that walls temperature T'_0, T'_ω are high enough to induce radiative heat transfer, and γ^* is the dimensionless slip parameter. Following Makindi and Mhone (2005), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by :

$$\frac{\partial q'}{\partial y'} = 4\alpha^2(T'_0 - T')$$
(4)

Where α is the mean radiation absorption coefficient.

The following dimensionless variables and parameters are introduced:

$$\left.\begin{aligned} x = \frac{x'}{a} \Rightarrow x' = xa, \quad y = \frac{y'}{a} \Rightarrow y' = ya \quad , u = \frac{u'}{U} \Rightarrow u' = uU, \\ w = \frac{w'a}{U} \Rightarrow w' = \frac{wU}{a}, t = \frac{t'U}{a} \Rightarrow t' = \frac{ta}{U}, \theta = \frac{T' - T'_0}{T'_\omega - T'_0} \Rightarrow T' = T'_0 + \theta(T'_\omega - T'_0), \\ p = \frac{ap'}{\rho v U} \Rightarrow p' = \frac{p\rho v U}{a},\ \gamma = \frac{\gamma^*}{a} \Rightarrow \gamma^* = \gamma a,\ S^S = \frac{1}{Da},\ Da = \frac{K'}{a^2} \Rightarrow k' = a^2 Da \\ N^2 = \frac{4\alpha^2 a^2}{k},\ H^2 = \frac{a^2\sigma_e B_0^2}{\rho v},\ Gr = \frac{g\beta(T'_\omega - T'_0)a^2}{vU},\ Pe = \frac{Ua\rho c_p}{k},\ Re = \frac{Ua}{v}, \end{aligned}\right\} \quad (5)$$

Where U is the flow mean velocity, the dimensionless governing equations together with appropriate boundary conditions can be written as:

$$Re\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{(1+\beta_1)}\frac{\partial^2 u}{\partial y^2} - (H^2 + S^2)u + Gr\theta$$
(6)

$$Pe\ \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + N^2\theta$$
(7)

Subject to the boundary condition

$$\left.\begin{aligned} u - \gamma\frac{\partial u}{\partial y} = 0, \theta = 0\ at\ y = 0 \\ u = 0, \theta = \cos\omega t \quad at\ y = 1 \end{aligned}\right\}$$
(8)

Where β_1 is the Jeffrey fluid, Gr is the thermal Grashof number, H is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, Da is the Darcy number, Pr is the Prandtl number, γ is the slip parameter and s is the porous medium shape factor.

III. METHOD OF SOLUTION

In order to solve equations (6), (7) and (8) for purely oscillatory flow, let the pressure gradient, fluid velocity and temperature be:

$$u(y,t) = u_0(y)e^{i\omega t} + u_1(y)e^{-i\omega t}$$
(9)

$$\theta(y,t) = \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t}$$
(10)

Assume that $-\frac{\partial P}{\partial x} = \lambda[e^{i\omega t} + e^{-i\omega t}]$

Where λ <0 for favorable pressure, ω is the frequency of the oscillation. Substituting the above expressions: (9) and (10) into (6), (7) and (8), we obtained

$\theta_0'' + N_1^2\theta_0 = 0$
(11)
$\theta_1'' + N_2^2\theta_1 = 0$
(12)
$u_0'' - M_1^2 u_0 = -\alpha\lambda\text{-}\alpha Gr\theta_0$
(13)
$u_1'' - M_2^2 u_1 = -\alpha\lambda\text{-}\alpha Gr\theta_1$
(14)

$$\left.\begin{array}{l} u_0 - \gamma u_0' = 0, u_1 - \gamma u_1' = 0, \theta_0 = 0, \theta_1 = 0, at\ y = 0 \\ u_0 = 0,\ \ u_1 = 0,\ \ \theta_0 = \frac{1}{2},\ \ \theta_1 = \frac{1}{2} \qquad\qquad at\ y = 0 \end{array}\right\}$$

(15)
Where $\alpha = 1 + \beta_1$
$N_1^2 = (N^2 - Pei\omega)$, $N_2^2 = (N^2 + Pei\omega)$,
$M_1^2 = (H^2 + S^2 + Rei\omega)$ And $M_2^2 = (H^2 + S^2 - Rei\omega)$

Equations (11) to (15) are solved and the solution for fluid temperature and velocity are given as follows:

$\theta(y,t) = \frac{1}{2}\left[\frac{\sin N_1 y}{2\sin N_1}e^{i\omega t} + \frac{\sin N_2 y}{2\sin N_2}e^{-i\omega t}\right]$
(16)
$u(y,t) = [A_5\cosh M_1 y + A_6\sinh M_1 y + F + G_2\sin N_1 y]e^{i\omega t} + [A_7\cosh M_2 y + A_8\sinh M_2 y + K + G_4\sin N_2 y]e^{-i\omega t}$ (17)

Skin friction or shear stress
$\tau_0 = \frac{d\theta}{dy}/_{y=0} = \frac{1}{2}\left[\frac{N_1}{\sin N_1}e^{i\omega t} + \frac{N_2}{\sin N_2}e^{-i\omega t}\right]$
(18)
$\tau_1 = \frac{d\theta}{dy}/_{y=1} = \frac{1}{2}(N_1\cot N_1 e^{i\omega t} + N_2\cot N_2)e^{-i\omega t}$
(19)
Nusselt number or rate of heat transfer is given by;
$Nu_0 = \frac{du}{dy}/_{y=0} = [A_6 M_1 + G_2 N_1]e^{i\omega t} + [A_8 M_2 + G_4 N_2]e^{-i\omega t}$
(20)

IV. GRAPHICAL RESULTS AND DISCUSSION

To study the MHD Oscillatory flow of Jeffrey fluid in a channel filled with porous material, the velocity u, temperature θ and skin friction ($\tau_0 and \tau_1$) profiles are depicted graphically against y for different values of different parameters; Jeffrey fluid β_1, thermal Grashof number Gr, Hartmann number H, radiation parameter N, Peclet number Pe, Reynolds number Re, Darcy number *Da*, Prandtl number *Pr*, slip parameter γ *and* porous medium shape factor s. We made use of the following parameter values except otherwise indicated, $\beta_1 = 0.01$, Re = 1, S = 1, Ha = 1, Gr = 10, Pe = 1, N = 1, λ=−1, $\gamma = 0.5$ and $\omega = 1$.

The temperature profiles for different values of the Peclet number (Pe = 1, 2, 3, 4) are shown in Figures 1. It is observed that the temperature increases with increasing Peclet number.

The velocity profiles have been studied and presented in Figures 2 to 5. The velocity profiles for different values of the Hartmann number (Ha = 1, 5, 10, 15) is shown in fig. 2. It observed that the velocity decreases with increasing Hartmann number. The velocity profiles for different values of Dacy number (Da = 0.01, 0.03, 0.05, 0.07), Grashof number (Gr = 1, 2, 3, 4) and Jeffrey fluid parameter (β_1=0.1,1,5) are shown in Figures 3, 4 and 5 respectively. It is observed that the velocity decrease with increasing Dacy number, Grashof number and Jeffrey fluid parameter.

The variation of the skin friction ($\tau_0 and \tau_1$) on the porous plate with material parameters are shown in fig.6-8. The skin friction ($\tau_0 and \tau_1$) profiles for different values of Darcy number (Da= 0.01, 0.02, 0.03, 0.04) is shown in fig. 6. It is observed that skin friction increase with increasing Darcy number. The skin friction ($\tau_0 and \tau_1$) profiles for different values of Jeffrey fluid parameter (β_1=0.1, 0.3, 0.5, 0.7) and the Hartmann number (Ha = 1, 3, 5, 7) is shown in fig.7 and 8 respectively. It is observed that skin friction ($\tau_0 and \tau_1$) decreases with increasing of Jeffrey fluid parameter and Hartmann number.

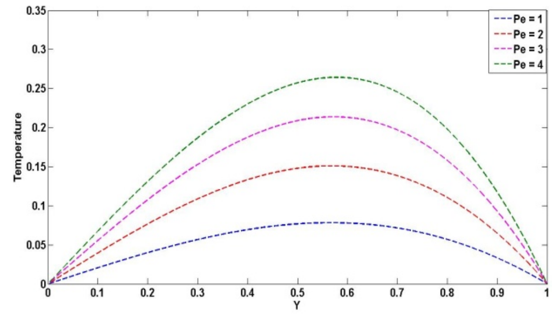

Figure 1: Temperature profiles for different values of the peclet number

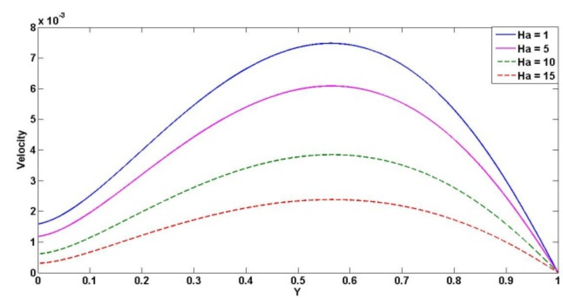

Fig. 2: Velocity profiles for different values of the hartmann number

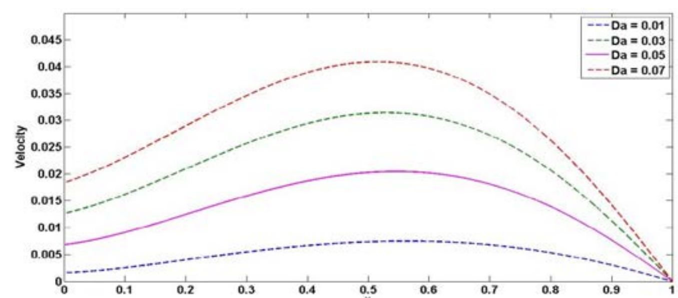

Fig. 3: Velocity profiles for different values of Darcy number

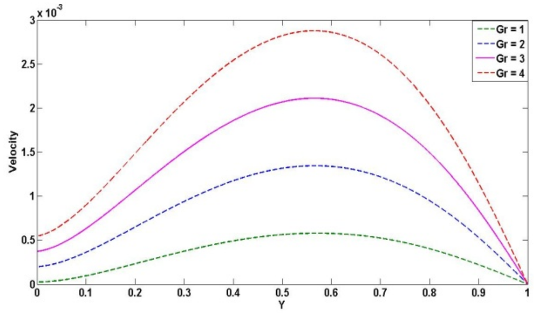

Fig. 4: Velocity profiles for different values of grashof number Gr

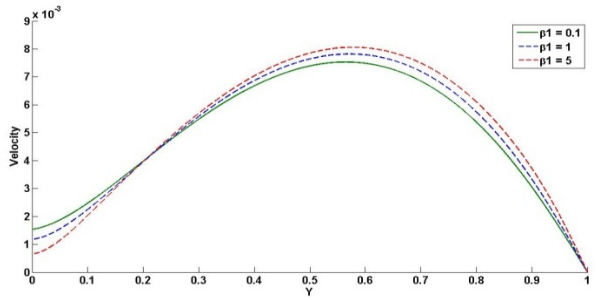

Fig. 5: Velocity profiles for different values of Jeffrey fluid parameter

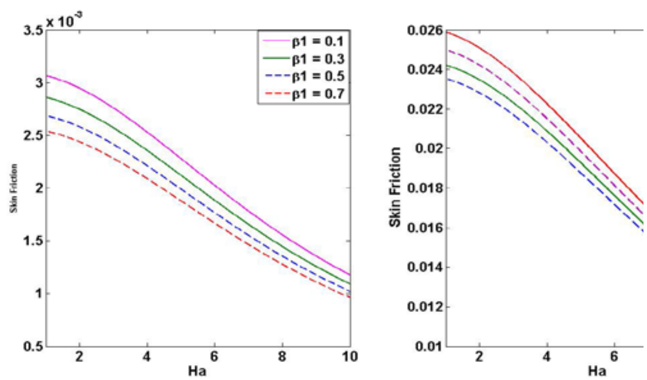

Fig. 6: Skin friction for different values β_1

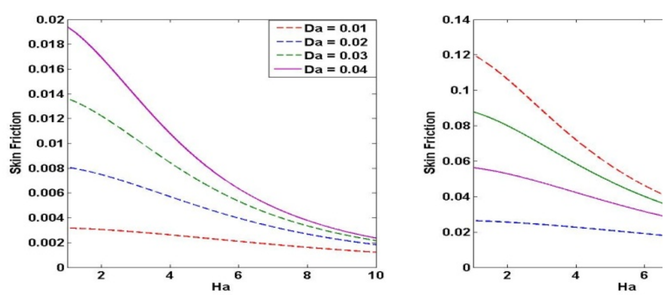

Fig. 7: Skin friction for different values of Da

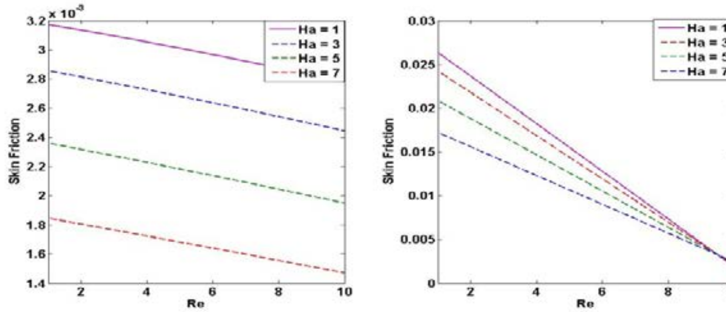

Fig. 8: Skin friction for different values of Ha

V. CONCLUSIONS

This paper investigates the heat transfer to MHD Oscillatory flow of Jeffrey fluid in a channel filled with porous material. The velocity, temperature and skin friction profiles are obtained analytically. The effect of different parameters namely, the radiation parameter, Grashof number, Hartmann number, Jeffrey fluid parameter, Porosity parameter, Slip parameter, frequency of the oscillation and Peclet number are studied. The conclusions of the study are as follows;

(i) It is observed that the temperature increases with increasing Peclet number.

(ii) It observed that the velocity decreases with increasing Hartmann number.

(iii) It is observed that the velocity decrease with increasing Dacy number, Grashof number and Jeffrey fluid parameter.

(iv) It is observed that skin friction increase with increasing Darcy number.

(v) It is observed that skin friction decreases with increasing of Jeffrey fluid parameter and Hartmann number.

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