Switch Controller Design and H~ Performance Achievement

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DOI: 10.29322/IJSRP.9.07.2019.p91103
http://dx.doi.org/10.29322/IJSRP.9.07.2019.p91103

Abstract- This paper treats the problem of switching controller design which includes several linear time-invariant (LTI) controllers-all of them capable of stabilizing a specific LTI plant-in such a way that the H~ control performance of the closed-loop system is guaranteed for any switching sequence. It is demonstrated that there is a possibility to find realizations for these controller transfer matrices so that the closed-loop system remains H~ control performance under arbitrary switching.

Index Terms- H~ performance achievement, Switching control, Random switching, Realization theory.,

I. INTRODUCTION

Recently, we find an increasing interest from the scientific community in the study of linear switching system, which comprises a collection of subsystems described by linear dynamics (differential/difference equations), together with a switching rule that specifies the switching between the subsystems; see the survey papers [1-3], the recent book [4, 5] and the references cited therein [6-12]. With its strong engineering backgrounds, such systems can be used to construct a controller for a wide range of physical and engineering plants in practice. When the switching law has no given mode (or is arbitrary), one way to investigate analysis and synthesis problems of stability or L2 gain performance is to find a common Lyapunov function for all the switching models (c.f. [13]-[20]). Even though these conditions tend to be strong due to the existence of a common Lyapunov function that guarantees the stability or control performance of a system under all possible switching, those based on quadratic functions have been successful because of the development of efficient tools based on LMIs [21, 22].

For dealing with the control of complex systems where conflicting requirements make a single LTI controller unsuitable, reference [23] has provided a switching control strategy to achieve global stability under arbitrary switching. It has been proven that, given a single linear plant and a family of linear stabilizing compensators, there always exist (possibly non-minimal) realizations for all of them which assure global stability, no matter how we switch among the controller. This result is based on the Youla-Kucera parameterization [25],[26] of all stabilizing compensators. In a recent paper [24], given any arbitrary family of compensators, each of which is stabilizing the corresponding LTI plant, there exist suitable realizations for each of these compensators which assure stability of the closed-loop system under arbitrary switching.

On the other hand, the regular H~ problem could be solved by the famous Riccati equation based algorithm [27]. Meanwhile, it also provides a parameterization of all possible stabilizing controllers guaranteeing a bound of H~ performance of the closed-loop system [27]. In dealing with switching controllers design, a fundamental issue is how to guarantee the control performance under arbitrary switching. Our basic question is the following: given a strictly proper plant, under which conditions there exists a switching compensator which guarantees some H infinity performance under arbitrary switching? Till now, to the author's knowledge, switching between H~ performance controllers regardless the switching signal has not been explored fully yet.

The main idea of the present paper is to design a switching controller that includes several LTI controllers designed beforehand and independently-all of them capable of stabilizing a specific LTI plant-in such a way that H~ control performance of the closed-loop system is guaranteed for any switching sequence. That is, the switching signal is assumed to be arbitrary while a set of H~ controllers are implemented for the plant. The important step of the controller design is to select realizations for these individual parameters so that switching between them results in a stable time-varying system with a specific H~ performance criterion. Compared with previous results, there are two main contributions. First, given a switching system including a set of LTI systems guaranteeing corresponding H~ performances, it provides state space realizations of these LTI systems which guarantee that the switching system assures the H~ performance, whose bound is the maximum of H~ performance values of these LTI systems.

Second, given an LTI plant and a family of LTI controllers guaranteeing corresponding H~ performances for the LTI plant, respectively, state space realizations for these LTI controllers will be given, which assure not only an H~ performance of the overall closed-loop system under arbitrary switching but also the corresponding H~ performance of local closed-loop system at each switching point.

II. DEFINITION AND PROBLEM STATEMENT

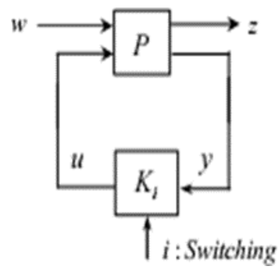

Fig.1 switching control

The state space realization of P is taken to be a simplified form as:

$$\begin{aligned} \dot{x} &= Ax + B_{1w} + B_{2u} \\ z &= C_{1x} + D_{12u} \\ y &= C_{2x} + D_{21w} \end{aligned} \tag{1}$$

With (A, B_1) controllable, (A, B_2) Stabilizable, (C_1, A) observable and (C_2, A) detectable, where $x \in \Re^n$ is the state. Where $w \in \Re^{m1}$ is the disturbance, $u \in \Re^{m2}$ is the control input, $z \in \Re^{p1}$ is the controlled output, $y \in \Re^{p2}$ is the state, respectively. Suppose further that the following regularity assumption hold:

$$D_{12}^T \begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}, D_{21} \begin{bmatrix} B_1^T & D_{21}^T \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$$

According to the concept of dual (see e.g. [29] for details), we define the dual of P as

$$p^* := \begin{cases} \dot{x} = A^T x + C_1^T w + C_2^T w \\ z = B_1^T x + D_{21}^T u \\ y = B_2^T x + D_{12}^T w \end{cases}$$

It is also assumed that there exists a family of linear switching controller transfer matrices $K_i(s)$, designed beforehand and independently for the plant (1) with corresponding H~ control performances γ_i. A state space realization of these switching controllers K_i are given as

$$K_i := \left[\begin{array}{c|c} A_{k,i} & B_{k,i} \\ \hline C_{k,i} & D_{k,i} \end{array} \right] \tag{2}$$

and meanwhile we assume the controller can switch arbitrarily, precisely that

$$i = i(t) \in \tau = \{1, 2, \ldots, r\} \tag{3}$$

That is, the method presented here to implement the switching controller guarantees an H~ control performance regardless of the algorithm used to command the switching between the controllers. Just as introduced in [24], the following assumptions will be considered.

Assumption 1: Non-Zenoness: The number of switching instants is finite on every finite interval.

Assumption 2: Zero Delay: There is no delay in the communication between the plant and the controller, which, at time t , knows the current $y(t)$ and configuration $i(t)$. Assumption 1 is not an essential restriction and avoids well-posedness issues. Conversely, Assumption 2, may be a restriction in practice, but fairly acceptable in most plants. The closed-loop system matrix achieved from (1) and (2) in

$$A_i^{cl} = \begin{bmatrix} A + B_2 D_{k,i} C_2 & B_2 C_{k,i} \\ B_{k,i} C_2 & A_{k,i} \end{bmatrix}, \quad B_i^{cl} = \begin{bmatrix} B_1 + B_2 D_{k,i} D_{21} \\ B_{k,i} D_{21} \end{bmatrix} \tag{4}$$

$$C_i^{cl} = \begin{bmatrix} C_1 + D_{12} D_{k,i} C_2 & D_{12} C_{k,i} \end{bmatrix}, \quad D_i^{cl} = D_{12} D_{k,i} D_{21} \tag{5}$$

Lemma 1: the following two properties are equivalent:

(i) Each of the closed-loop system in Fig.1 has H~ performance with a bound γ_i for fixed $i(t)$;

(ii) There exists a positive-definite Lyapunov matrix for system (4)-(5) satisfies

$$\begin{bmatrix} X_i A_i^{cl} + (A_i^{cl})^T X_i & X_i B_i^{cl} & (C_i^{cl})^T \\ * & -\gamma_i I & (D_i^{cl})^T \\ * & * & -\gamma_i I \end{bmatrix} \prec 0. \tag{6}$$

$$\begin{bmatrix} A_i^{cl} Y_i + Y_i (A_i^{cl})^T & B_i^{cl} & Y_i (C_i^{cl})^T \\ * & -\gamma_i I & (D_i^{cl})^T \\ * & * & -\gamma_i I \end{bmatrix} \prec 0. \tag{7}$$

The main problem is addressed in this paper as

Problem: Given LTI plant (1) and a family of transfer functions $K_i(s)$ assuring that the closed-loop system (4)-(5) is guaranteed H~ control performance γ_i for fixed i , does there exist realizations for the $K_i(s)$ such that the overall closed-loop system is guaranteed the H ~performance $\gamma := \max(\gamma_i)$ among arbitrary switching, and meanwhile, the corresponding H~ performance γ_i of each local closed-loop system is also maintained by each local controller $K_i(s)$ for fixed

3 PRELIMINARY

In this section, some useful properties among two Riccati equations and correspondent controller constructions for the standard H~ control problem will be overviewed as follows.

To simplify notation, we define some equations as

$$\psi(X) = A^T X + XA + X(\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X + C_1^T C_1 \tag{8}$$

$$\psi(Y) = A_{tmp} Y + Y A_{tmp}^T + Y(\gamma^{-2} X B_2 B_2^T X - C_2^T C_2) Y + B_1 B_1^T \tag{9}$$

Where $A_{tmp} = A + \gamma^{-2} B_1 B_1^T X$.

Lemma 2: There exists an admissible γ -suboptimal controller if and only if the two conditions below hold

(i) There exists a symmetric nonnegative stabilizing solution X to the Riccati equation $\psi(X) = 0$.

(ii) For the X given in (i), there exits a symmetric nonnegative stabilizing solution Y to Riccati equation $\psi(Y) = 0$.

Moreover, when two conditions hold, one such controller is

$$K_{sub1} = \left[\begin{array}{c|c} \hat{A} + \hat{B}_2 F + LC_2 & -L \\ \hline F & 0 \end{array}\right] \tag{10}$$

Where $\hat{A} = A + \gamma^{-2} B_1 B_1^T X + \gamma^{-2} YXB_2 B_2^T X$, $\hat{B}_2 = B_2 + \gamma^{-2} YXB_2$, $L = -YC_2^T$ *and* $F = -B_2^T X$

Lemma 3: If conditions (i) and (ii) of lemma (2) are satisfied, the set of all admissible controllers such that $\|T_{zw}\|_\infty \prec \gamma$ equals the set of all transfer matrices from y to u as

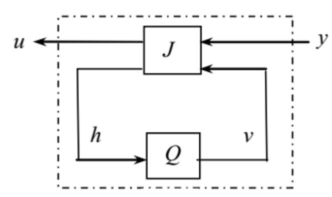

Fig.2. parameterization of γ -suboptimal controller with Q .

Where $Q \in H_\infty$ and $\|Q\|_\infty \prec \gamma$

$$J := \left[\begin{array}{c|cc} \hat{A} + \hat{B}_2 F + LC_2 & -L & \hat{B}_2 \\ \hline F & 0 & I \\ -C_2 & I & 0 \end{array}\right] \tag{11}$$

It is a very useful construction, since the free parameter Q increase adjustable parameter for control systems.

Remark: Any stabilizing controller $K(S)$ is expressed with a quadratically stable Q_0 as $K = F_l(J, Q_0)$. Considering a function $Q_0 = F_l(\hat{J}, K)$, where

$$\hat{J} := \left[\begin{array}{c|cc} \hat{A} & YC_2^T & \hat{B}_2 \\ \hline B_2^T X & 0 & I \\ C_2 & I & 0 \end{array}\right] = \left[\begin{array}{c|cc} \hat{A} & -L & \hat{B}_2 \\ \hline -F & 0 & I \\ C_2 & I & 0 \end{array}\right] \tag{12}$$

$$F_l(J, Q_0) = F_l(J, F_l(\hat{J}, K)) = F_l(J_{tmp}, K), \tag{13}$$

Where J_{tmp} can be obtained by using the state space star product formula

$$J_{tmp} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

Hence $F_l(J, Q_0) = F_l(J_{tmp}, K) = K$

In this setup, the following lemma (similar to [27, lemma 16.8]) is utilized in the sequel

Lemma 4: Assume conditions (i) and (ii) of lemma (2) are satisfied. Then the controller $K(s)$ is admissible for $P(s)$ and $\|F_l(P, K)\|_\infty \prec \gamma$.

According to the lemma above, we can choose a proper parameter $Q = F_l(\hat{J}, K)$, whose state space realization could be derived with the controller $K := \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right]$ as

$$Q := \left[\begin{array}{c|c} A_Q & B_Q \\ \hline C_Q & D_Q \end{array}\right] = \left[\begin{array}{cc|c} \hat{A} + \hat{B}_2 D_k C_2 & \hat{B}_2 C_k & \hat{B}_2 D_k - L \\ B_k C_2 & K_k & B_k \\ \hline D_k C_2 - F & C_k & D_k \end{array}\right] \tag{14}$$

4 MAIN RESULTS

In this section, the proposed control structure and the free parameter Q_0 are shown in Fig.4. According to lemma 3, the free parameter composed of the controllers K_i and $\hat{J}$ is proper stable transfer function that has an H ~ control performance.

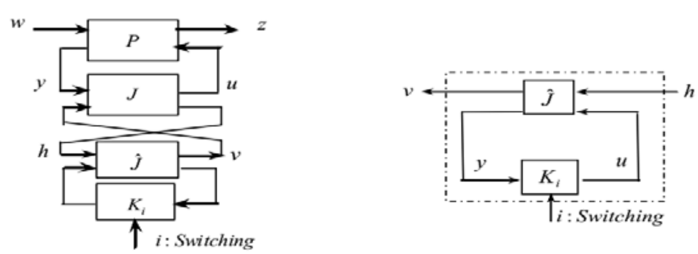

Fig.3. The proposed control structure and $Q_0 = F_l(\hat{J}, K_i)$

First, H~ performance realization of the switching system with transfer function expression will be considered here. Similar with the results presented in [24] for the case of linear switching plant, the free parameter Q_0 must be properly realized.

Theorem 1: As long as each system $Q_{0,i} = \left[\begin{array}{c|c} A_{Q_{0,i}} & B_{Q_{0,i}} \\ \hline C_{Q_{0,i}} & D_{Q_{0,i}} \end{array}\right]$ of the linear switching system Q_0 has H~ performance with a bound γ_i for fixed i , there always exists H~ performance realization with a bound

$\gamma := \max(\gamma_i),\ i = i(t) \in \tau = \{1,2,...,r\}$

for the switching system Q_0.

Proof: To prove that there exists a realization that remains an H~ performance for the switching system Q_0 , since each system $Q_{0,i}$ has H~ performance with a bound γ_i for fixed i , the performance criterion could be chosen as $\gamma := \max(\gamma_i)$. According to lemma 1, there always exist r linear matrix inequalities for the performance criterion γ as

$$\begin{bmatrix} X_i A_{Q_{0,i}} + (A_{Q_{0,i}})^T X_i & X_i B_{Q_{0,i}} & (C_{Q_{0,i}})^T \\ * & -\gamma I & (D_{Q_{0,i}})^T \\ * & * & -\gamma I \end{bmatrix} \prec 0 \tag{15}$$

If we multiply (15) by $T_i = \begin{bmatrix} R_i^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ on the left and T_i^T on the right, where R_i is an upper triangular matrix (Cholesky's decomposition) of $X_i = R_i^T R_i$ since T is full row rank, we can get

$$\begin{bmatrix} \left[\tilde{A}_{Q_{0,i}} + (\tilde{A}_{Q_{0,i}})^T\right] & \tilde{B}_{Q_{0,i}} & (\tilde{C}_{Q_{0,i}})^T \\ * & -\gamma I & (\tilde{D}_{Q_{0,i}})^T \\ * & * & -\gamma I \end{bmatrix} \prec 0 \tag{16}$$

Where $\tilde{A}_{Q_{0,i}} = R_i A_{Q_{0,i}} R_i^{-1},\ \tilde{B}_{Q_{0,i}} = R_i B_{Q_{0,i}},\ \tilde{C}_{Q_{0,i}} = C_{Q_{0,i}} R_i^T,\ and\ \tilde{D}_{Q_{0,i}} = D_{Q_{0,i}}$.

Since $Q_{0,i}(s) = \tilde{C}_{Q_{0,i}}\left[sI - \tilde{A}_{Q_{0,i}}\right]^{-1}\tilde{B}_{Q_{0,i}} + \tilde{D}_{Q_{0,i}} = C_{Q_{0,i}}\left[sI - A_{Q_{0,i}}\right]^{-1}B_{Q_{0,i}} + D_{Q_{0,i}}$, $i = i(t) \in \tau = \{1,2,...,r\}$ and there exists identity matrix, it means the state space realization $\left\{\tilde{A}_{Q_{0,i}}, \tilde{B}_{Q_{0,i}}, \tilde{C}_{Q_{0,i}}, \tilde{D}_{Q_{0,i}}\right\}$ has H~ performance with the bound γ for any $i = i(t) \in \tau = \{1,2,...,r\}$

The following procedure provides the H~performance realization with the bound γ for the switching system Q_0.

Procedure:

1) Take any realization $\left\{A_{Q_{0,i}}, B_{Q_{0,i}}, C_{Q_{0,i}}, D_{Q_{0,i}}\right\}$ of each system $Q_{0,i}$, so that $Q_{0,i}(s) = C_{Q_{0,i}}\left[sI - A_{Q_{0,i}}\right]^{-1} A_{Q_{0,i}} + D_{Q_{0,i}}$ which is H~ performance with a bound γ_i
2) Compute the positive-definite solution X_i of the following inequalities as

$$\begin{bmatrix} X_i A_{Q_{0,i}} + A_{Q_{0,i}}^T X_i & X_i B_{Q_{0,i}} & C_{Q_{0,i}}^T \\ * & -\gamma I & D_{Q_{0,i}}^T \\ * & * & -\gamma I \end{bmatrix} \prec 0, \quad (17)$$

Where $\gamma := \max(\gamma_i)$

3) Factorize X_i as $X_i = R_i^T$ where R is an upper matrix triangular matrix (Cholesky's decomposition).

4) Realize the given function as

$$\begin{aligned} \dot{Z}(t) &= \tilde{A}_{Q_{0,i}} Z(t) + \tilde{B}_{Q_{0,i}} v(t) \\ h(t) &= \tilde{C}_{Q_{0,i}} z(t) + \tilde{D}_{Q_{0,i}} v(t) \end{aligned} \quad (18)$$

Where $\tilde{A}_{Q_{0,i}} = R_i A_{Q_{0,i}} R_i^{-1}$, $\tilde{B}_{Q_{0,i}} = R_i B_{Q_{0,i}}$, $\tilde{C}_{Q_{0,i}} R_i^{-1}$, *and* $\tilde{D}_{Q_{0,i}} = D_{Q_{0,i}}$

Henceforth, two Riccati equations (8) and (9) of the standard H~ control problem always have symmetric nonnegative stabilizing solutions for $\gamma := \max(\gamma_i)$. The main result of this subsection is stated as follows.

Theorem 2: Given LTI plant (1) and a family of transfer functions $K_i(s)$, $i = i(t) \in \tau = \{1, 2, ..., r\}$, each stabilizing plant (1) with corresponding H~ performance γ_i, and then Riccati equations (8) and (9) of the standard H~ control problem have symmetric nonnegative stabilizing solutions X and Y for the criterion $\gamma := \max(\gamma_i)$, there exists non-minimal realizations of these switching transfer functions.

$K_i(s)$ shown in Fig.4 that guarantees H~ performance of the closed-loop system for every i , and a set of state space realizations of $K_i(s)$ are given as

$$A_{k,i} = \begin{bmatrix} \hat{A} + \hat{B}_2 F + LC_2 - \hat{B}_2 D_{k,i} C_2 & \hat{B}_2 \tilde{C}_{Q_{0,i}} \\ -\tilde{B}_{Q_{0,i}} C_2 & \tilde{A}_{Q_{0,i}} \end{bmatrix},$$

$$B_{k,i} = \begin{bmatrix} \hat{B}_2 D_{k,i} - L \\ \tilde{B}_{Q_{0,i}} \end{bmatrix}, \quad (19)$$

$$C_{k,i} = \begin{bmatrix} F - D_{k,i} & \tilde{C}_{Q_{0,i}} \end{bmatrix}, \; D_{k,i} = D_{k,i} \quad (20)$$

Where $\hat{A} = A + \gamma^{-2} B_1 B_1^T X + \gamma^{-2} YXB_2 B_2^T X$, $\hat{B}_2 = B_2 + \gamma^{-2} YXB_2$, $L = -YC_2^T$, $F = -B_2^T X$ and

$$\left[\begin{array}{c|c} \tilde{A}_{Q_{0,i}} & \tilde{B}_{Q_{0,i}} \\ \hline \tilde{C}_{Q_{0,i}} & \tilde{D}_{Q_{0,i}} \end{array}\right] = \left[\begin{array}{c|c} R_i A_{Q_{0,i}} R_i^{-1} & R_i B_{Q_{0,i}} \\ \hline C_{Q_{0,i}} R_i^{-1} & D_{Q_{0,i}} \end{array}\right] \tag{21}$$

With $\left[\begin{array}{c|c} A_{Q_{0,i}} & B_{Q_{0,i}} \\ \hline C_{Q_{0,i}} & D_{Q_{0,i}} \end{array}\right] = \left[\begin{array}{cc|c} \hat{A}+\hat{B}_2 D_{k,i} C_2 & \hat{B}_2 C_{k,i} & \hat{B}_2 D_{k,i} - L \\ B_{k,i} C_2 & A_{k,i} & B_{k,i} \\ \hline D_{k,i} C_2 - F & C_{k,i} & D_{k,i} \end{array}\right]$ and R_i is a derived from $X_i = R_i^T R_i$ according to (17)

Proof:

According to lemma 4, the H~ performance $\|F_1(G,K)\|_\infty$ is equivalent with $\|F_1(\hat{J},K)\|_\infty$ by the controller K. That is, when the controller $K = F_1(J,Q_0)$ is chosen, we have $\|F_1(\hat{J},F_1(J,Q_0))\|_\infty = \|Q_0\|_\infty$. That is the H~ performance of the closed-loop system is equivalent with the system Q_0. Thus, we can construct the switching system Q_0 combined with $Q_{0,i}$, which has H~ performance realization with the bound γ.

If each switching system of Q_0 is chosen as

$Q_{0,i} = F_l(\hat{J},K_i)$, where $\hat{J} := \left[\begin{array}{c|cc} \hat{A} & -L & \hat{B}_2 \\ \hline -F & 0 & I \\ C_2 & I & 0 \end{array}\right]$ is shown in Fig.3, the state space realization of $Q_{0,i}$ could be obtainable as

$$Q_{0,i} = \left[\begin{array}{c|c} A_{Q_{0,i}} & B_{Q_{0,i}} \\ \hline C_{Q_{0,i}} & D_{Q_{0,i}} \end{array}\right] = \left[\begin{array}{cc|c} \hat{A}+\hat{B} D_{k,i} C_2 & \hat{B}_2 C_{k,i} & \hat{B}_2 D_{k,i} - L \\ B_{k,i} C_2 & A_{k,i} & B_{k,i} \\ \hline D_{k,i} C_2 - F & C_{k,i} & D_{k.i} \end{array}\right]$$

Since each of $K_i(s)$ is stabilizing the plant (1) with H~ performance γ_i, we have $\|Q_{0,i}\|_\infty \le \gamma_i \le \gamma$, with $\gamma := \max(\gamma_i)$. According to theorem 1, an H~ performance realization with the bound γ of $Q_{0,i}$ could be written as

$$\left[\begin{array}{c|c} \tilde{A}_{Q_{0,i}} & \tilde{B}_{Q_{0,i}} \\ \hline \tilde{C}_{Q_{0,i}} & \tilde{D}_{Q_{0,i}} \end{array}\right] = \left[\begin{array}{c|c} R_i A_{Q_{0,i}} R_i^{-1} & R_i B_{Q_{0,i}} \\ \hline C_{Q_{0,i}} R_i^{-1} & D_{Q_{0,i}} \end{array}\right],$$

Where R_i is derived from $X_i = R_i^T R_i$ with (17). Even though state space realizations of $Q_{0,i}$ may be different, transfer function expression of each system $Q_{0,i}$ is the same. Finally, when the realization of $Q_{0,i}$ is substituted into $K_i = F_l(J,Q_{0,i})$, a set of state space realizations of $K_i(s)$ could be obtainable as (19)-(20).

Remark: The theorem above gives the answer to the problem presented in section II. Each system $Q_{0,i}$ can be selected in such a way shown as Fig.4 that the resulting controller transfer function is the desired one $K_i(s)$. The only problem concerned with Q_0 is its realization, which cannot be arbitrary. Here, an H~ realization method of Q_0 is given as (21).

5 NUMERICAL EXAMPLE

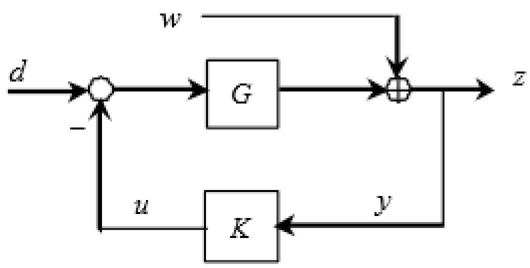

Fig. 4. Mixed sensitivity function minimization problem

We consider the practical example of Hespanha and Morse (2002) in [23] corresponding to the control of the roll angle of an aircraft. The model is defined by the transfer function

$$G(s) = \frac{-1000}{s(s+0.875)(s+50)}$$

Two controllers are considered to guarantee different H~ performance for mixed sensitivity function minimization problem, the first one is designed for good noise rejection properties, whereas the second is a little fast but sensitive to measurement noise. Furthermore, both of controllers are considering the properties of not only the output of controlled plant but also the output of the controller.

The controlled input vector is defined in Fig. 4 as follows:

$$\begin{bmatrix} z \\ u \end{bmatrix} = \begin{bmatrix} (I+GK)^{-1}G & (I+GK)^{-1} \\ K(I+GK)^{-1}G & (I+GK)^{-1}K \end{bmatrix} \begin{bmatrix} d \\ w \end{bmatrix}, \tag{22}$$

Where d is the input disturbance and w is the measurement noise. In this case, the state space presentation of the generalized plant is

$$P := \left[\begin{array}{c|cc} A & \begin{bmatrix} B & 0 \end{bmatrix} & -B \\ \hline \begin{bmatrix} C \\ 0 \end{bmatrix} & \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ C & \begin{bmatrix} 0 & 1 \end{bmatrix} & 0 \end{array}\right] \tag{23}$$

Since D_{11} of above generalized plant is not zero matrix, it does not satisfy the assumption of by the solution of traditional Riccati equations. We utilize the elimination procedure of D_{11} and scaling transformation introduced in [29]. The generalized plant satisfying all assumptions is obtained as follows.

$$\tilde{G} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] = \left[\begin{array}{c|cc} A & \begin{bmatrix} B & 0 \end{bmatrix} & -B \\ \hline p(1-\gamma^{-2})^{(1/2)}C & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \end{bmatrix} \\ (1-\gamma^{-2})^{(-1/2)}C & \begin{bmatrix} 0 & -1 \end{bmatrix} & 0 \end{array}\right] \tag{24}$$

where the weighting function ρ is chosen as a positive number. And the positive number γ is a prescribed H~ performance criterion. The H infinite controller for (23) can be obtained as

$$K = \sqrt{1-\gamma^{-2}\tilde{K}} \tag{25}$$

where $\tilde{K}$ is the H~ controller for (24), which could be solved by the traditional method of Riccati equations. It is easily found that the generalized plant above satisfies all assumptions. Here ρ is chosen equal to 100 and 300 for K_1 and K_2, respectively. This choices of ρ result in K_2 exhibiting a faster response than K_1 with performance criterions $\gamma_1 = 107.7$ and $\gamma_2 = 313.2$, respectively as

$$K_1 = \frac{-6.7e4s^2 - 3.5e6s - 7.7e6}{s^3 + 729.7s^2 + 7.3e4s + 3.1e6},$$
$$K_2 = \frac{-3.035e5s^2 - 1.585e7s - 3.362e7}{s^3 + 1208s^2 + 1.694e5s + 1.101e7}$$

In this case, to construct LFT formulation of controller for $K_i = F_l(J, Q_{0,i})$ in Fig.3, K_l is chosen to be the central solution, where symmetric nonnegative stabilizing matrices X and Y of two H~ like Riccati equations are solved as

$$X = \begin{bmatrix} 57.9 & 4.6e^3 & 1.0e^5 \\ 4.6e^3 & 4.1e^5 & 1.1e^7 \\ 1.0e^5 & 1.1e^7 & 4.6e^8 \end{bmatrix} \quad and \quad Y = \begin{bmatrix} 9.7e^{-3} & 8.1e^{-5} & -2.6e^{-5} \\ 8.1e^{-5} & 1.3e^{-4} & 3.4e^{-5} \\ -2.6e^{-5} & 3.4e^{-5} & 2.2e^{-5} \end{bmatrix}$$

We can obtain

$$J := \left[\begin{array}{ccc|cc} -108.7 & -4.6e^3 & -9.9e^4 & 6.0e^{-1} & -9.9e^{-1} \\ 1.0 & 0 & -1.0e^3 & -1.0 & -9.3e^{-3} \\ 0 & 1 & -6.2e^2 & -6.2e^{-1} & -5.7e^{-3} \\ \hline 57.8 & 4.6e^3 & 1.0e^5 & 0 & 1 \\ 0 & 0 & 1.0e^3 & 1 & 0 \end{array}\right]$$

To design the multi-controller for $K := \{K_{1,}K_2\}$, we have

$$Q_{0,i} = F_l(\hat{J}, K_i), \quad where \quad \hat{J} = \left[\begin{array}{c|cc} \hat{A} & -L & \hat{B}_2 \\ \hline -F & 0 & I \\ C_2 & I & 0 \end{array}\right]$$ is shown in Fig.4

The corresponding transfer matrices $\{Q_{0,1}, Q_{0,2}\}$ are then computed using (21). Since K_1 is chosen to be the central solution, we have $Q_{0,1} = 0$. The state space realization of $Q_{0,2}$ can be obtainable as

ISSN 2250-3153

$$Q_{0,2}=\left[\begin{array}{cccccc|c} -51.2 & -68.6 & -5.4e^{-2} & -57.5 & -4.6e^{3} & -9.9e^{4} & 0.6 \\ 1.5 & 4.3 & 9.3e^{2} & -5.4e^{-1} & -4.3e^{1} & -9.3e^{2} & -1.0 \\ 3.3e^{-3} & 2.7 & 5.7e^{2} & -3.3e^{-1} & -2.6e^{1} & -5.7e^{2} & -0.6 \\ 0 & 0 & -6.0e^{2} & -1.1e^{2} & -4.6e^{3} & -9.9e^{4} & 0.6 \\ 0 & 0 & 1.0e^{3} & 1.0 & 0 & -1.0e^{3} & -1.0 \\ 0 & 0 & 6.2e^{2} & 0 & 1.0 & -6.2e^{2} & -0.6 \\ \hline -57.9 & -4.6e^{3} & -1.0e^{5} & 57.9 & 4.6e^{3} & 1.0e^{5} & 0 \end{array}\right]$$

The fact that $Q_{0,1}=0$ is a consequence of having used $K=K_1$. We then pick a minimal realization $\{\tilde{A}_{Q_{0,2}},\tilde{B}_{Q_{0,2}},\tilde{C}_{Q_{0,2}}\}$ for $Q_{0,2}$ and the trivial realization $\{\tilde{A}_{Q_{0,2}},0,\tilde{C}_{Q_{0,2}}\}$ for $Q_{0,1}$. Since both realizations share the same stable $\tilde{A}_{Q_2}$ matrix. As mentioned before, it would have been possible to choose realization for $Q_{0,1}$ and $Q_{0,2}$ with this property even if $Q_{0,1}$ is nontrivial. The desired controller realizations are then given by (19)–(20). This guarantees H~ performance: $\gamma:=\max(\gamma_1,\gamma_2)=352.5$ of the switched closed-loop system. When external disturbance $d(t)$ is injected into the system for $t\in[18,40]$ as Fig.5,

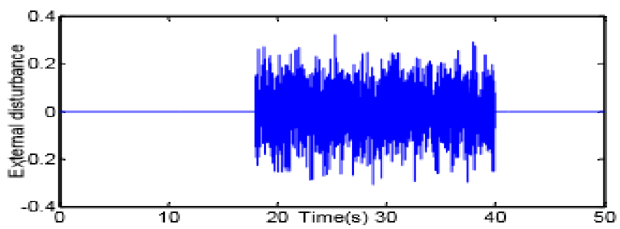

Fig.5. External disturbance $d(t)$

The outputs of controller plant and the controller are illustrated as Fig.6 and Fig.7, respectively.

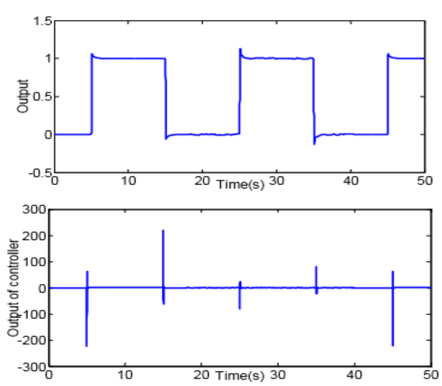

Fig.6. The switching control with H∞ performance

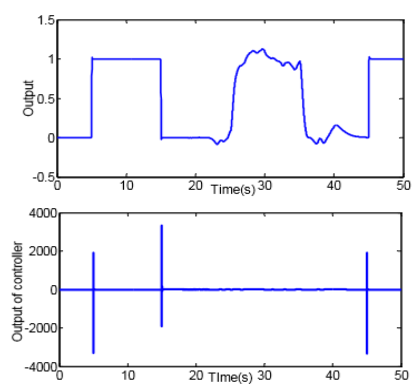

Fig.7. The switching strategy between stabilizing controllers

The main objective in [23] was to guarantee stability when switching between stabilizing controllers regardless the switching signal.

The paper presented here is to guarantee H~performance of the closed-loop system, when switching between H~performance controllers regardless the switching signal. Since the mixed sensitivity function minimization problem considered here guarantees the properties of not only the output of controlled plant but also the output of the controller. Comparing Fig. 6 and Fig. 7 we notice that the magnitude of the control signal in our method is much lower than that proposed in [23]. This makes clear the validity of the proposed technique.

6 CONCLUSIONS

To deal with the control of complex systems where conflicting requirements make a single LTI controller unsuitable, the paper provides a switching strategy between H~performance controllers of mixed sensitivity control problem for a specific LTI plant. It is shown that it is possible to find proper realizations for the given family of H~performance controllers so that the closed-loop system remains the H~ control performance, no matter how we switch among the controller.

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