

Solving Multi-objective Generalized Solid Transportation Problem by IFGP approach

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Abstract

In this paper multi objective generalized solid transportation problem (MOGSTP) has been considered. All the objective functions are to be optimized under same restrictions. Normally the objectives are conflicting in nature, otherwise by solving any one of them we can find positive optimum solution of positive problem. The problem has been solved by interactive fuzzy goal programming (IFGP) approach. This approach is to focus on minimizing the upper bound of the costs of each objective functions to obtain a compromising solution which is closed to the lower bound of the respective objective functions. A numerical example also presented.

Key words : Generalized solid transportation problem; Multi objective transportation problem; Fuzzy programming; Linear membership function.

Mathematics Subject Classification : 90B06, 90C08.

1 Introduction

Decision making is a critical process. The best decision among the several alternatives depends on the decision maker. There are several restrictions. All of them may or may not be in the same priority level, in any organisation. Decision on a single criteria may not sufficient for a practical problem rather consideration of multiple criteria makes the proposition more feasible. In general, the real life problems are modeled with multi-objectives which are measured in different scales and at the same time in conflict. The decision will be optimized (maximized or minimized) on the basis of one or more objectives under the same restrictions. These situations arise in transportation areas [15, 16, 21, 25].

There are many existing methods to solve multi object transportation problem. Some of them are

- i. Interactive method
- ii. Non interactive method
- iii. Goal programming method

iv. Fuzzy programming method

1.1 Interactive method

In this method decision maker is directly involved to find the efficient solutions and then best compromise-solution is chosen. But for large scale problem it is difficult to evaluate all the efficient solutions. Several authors studied in this method [8, 18, 20, 23].

1.2 Non interactive method

In this method, firstly the decision maker enumerates all the efficient solutions which is a long process. Then take the final decision. But limitations come from the in-experience and incomplete information of decision makers. Several authors studied in this process [9, 12, 13].

1.3 Goal programming

This technique is very useful tool to solve multi objective programming. Many researchers solve in this method [4, 7, 11, 12, 23]. But for setting of weights to different goal plays a vital role. Otherwise it gives inefficient solution.

1.4 Fuzzy programming method

In solving Multi object transportation problem, fuzzy set theory has wide applications. It is a powerful tool for incomplete preference or information of decision maker. In this method the problem is characterized by fuzzy membership functions. There are many types of such fuzzy membership functions. Several authors studied in this way [1, 2, 4, 6, 10, 17, 24].

There are many real world problems such as routing problem, machine assignment problem, inventory problem, aircraft routing problem are very close to generalized transportation problem. Generalized transportation problem has studied by several authors [3, 5, 14, 19].

In this paper, we have used interactive fuzzy goal programming (IFGP) technique to solve the general multiple objectives generalized solid transportation problem. If the decision maker is satisfied by the outcome, iteration is stopped else interactive procedure applies to find compromising solution. In section 2, we formulated the problem and some relevant definition has also been given. In section 3.1, solution procedure has been developed. Algorithm is presented in section 4. Also in Section 5, presents the numerical example to test the validity of the approaches.

2 Problem formulation of MOGSTP

Let there be l origins, m destinations and n products in a generalized solid multi objective transportation problem. x_{ijk} = the amount of k^{th} type of product transported from the i^{th} origin to the j^{th} destination, r_{ijk}^p = p^{th} type of cost involved in transporting per unit of k^{th} type product from i^{th} origin to j^{th} destination, therefore they are positive quantities by nature, $a_i =$

the number of units available at the i^{th} origin; $1 \leq i \leq l$, b_j = the number of units required at the j^{th} destination; $1 \leq j \leq m$, c_k = requirement of the number of units of k^{th} product; $1 \leq k \leq n$, And d_{ijk}^1, d_{ijk}^2 = positive constants other than unity.

The cost minimizing multi objective generalized transportation problems is:

$$\text{Min } Z^p(x) = \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}; \quad \text{for } 1 \leq p \leq P \quad (2.1)$$

Subject to constraints,

$$\sum_{j=1}^m \sum_{k=1}^n d_{ijk}^1 x_{ijk} \leq a_i; \quad 1 \leq i \leq l, \quad (2.2)$$

$$\sum_{i=1}^l \sum_{k=1}^n x_{ijk} = b_j; \quad 1 \leq j \leq m, \quad (2.3)$$

$$\sum_{i=1}^l \sum_{j=1}^m d_{ijk}^2 x_{ijk} \leq c_k; \quad 1 \leq k \leq n, \quad (2.4)$$

and $x_{ijk} \geq 0$ for $1 \leq i \leq l, 1 \leq j \leq m, 1 \leq k \leq n$,

Where at any destination j , we allocate the amount x_{ijk} , then the permissible amount from origin i is $d_{ijk}^1 x_{ijk}$ and the permissible k^{th} type of product is $d_{ijk}^2 x_{ijk}$.

There are some special character in generalized solid transportation problem. They are stated below:

1. The rank of the co-efficient matrix of $[x_{ijk}]$ are in general $l + m + n$ rather than $l + m + n - 2$, i.e. all the constraints are in general independent.
2. The integrability property of x_{ijk} may not be hold in generalized transportation problem.
3. The activity vector is $r_{ijk}^p = d_{ijk}^1 e_i + e_{l+j} + d_{ijk}^2 e_{l+m+k}$, whereas in classical transportation problem, it is $r_{ijk}^p = e_i + e_{l+j} + e_{l+m+k}$.
4. In generalized transportation problem, it need not be true that cells corresponding to a basic solution form a tree. Or in other words, the vectors in the loop are linearly independent. The vectors in the loop are in general independent.

Definition 2.1. Non-dominated solution: A feasible vector $x^0 \in S$ (S is the feasible region) yields a non-dominated solution, if and only if, there is no other vector $x \in S$ such that $\sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk} \leq \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}^0$ for all p and $\sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk} < \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}^0$ for some $p, p = 1, 2, \dots, P$.

Definition 2.2. Efficient solution: A point $x^0 \in S$ is efficient if there does not exist another $x \in S$ such that $\sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk} \leq \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}^0 \quad \forall p$ and $\sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk} < \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}^0$ for some p . Otherwise x^0 is an inefficient solution. For example $x^0 \in S$ is efficient if its criterion vector is not dominated by the criterion vector of another point in the feasible region S .

Definition 2.3. Compromise solution: A feasible vector $X^* \in S$ is called a compromise solution iff $X^* \in E$ and $Z(X^*) \leq \Lambda X \in SZ(X)$, Where Λ stands for "minimum and E is the set of efficient solutions.

This definition imposes two conditions on the solution for it to be a compromise solution. First, the solution should be efficient. Second, the feasible solution vector X^* should have the minimum deviation from the ideal point than any other point in S . Simply put, the compromise solution is the closest solution to the ideal one that maximizes the underlying utility function of the decision maker.

In real-world cases, knowledge of the set of efficient solutions E is not always necessary. On the other hand, the decision makers preferences are to be considered in determining the final compromise solution.

Definition 2.4. Preferred compromise solution: *If the compromise solution satisfies the decision makers preferences, then the solution is called the preferred compromise solution.*

Definition 2.5. Membership function: *A real fuzzy number $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in R$ and two functions $f(x), g(x) : R \rightarrow [0, 1]$, where $f(x)$ is non decreasing and $g(x)$ is non increasing, such that we can define membership function satisfy the following conditions*

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} f(x) & \text{for } \alpha_1 \leq x \leq \alpha_2 \\ 1 & \text{for } \alpha_2 \leq x \leq \alpha_3 \\ g(x) & \text{for } \alpha_3 \leq x \leq \alpha_4 \\ 0 & \text{otherwise} \end{cases}$$

3 Solution procedure of MOGSTP

From the proposed problem, we first solve the single objective generalized solid transportation problem for $p = 1, 2, \dots, P$ as follows. The cost minimizing objective function of the solid transportation problem is

$$\text{Min } Z^p(x) = \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n r_{ijk}^p x_{ijk}; \quad 1 \leq p \leq P \tag{3.1}$$

Subject to constraints,

$$\sum_{j=1}^m \sum_{k=1}^n d_{ijk}^1 x_{ijk} \leq a_i; \quad 1 \leq i \leq l, \tag{3.2}$$

$$\sum_{i=1}^l \sum_{k=1}^n x_{ijk} = b_j; \quad 1 \leq j \leq m, \tag{3.3}$$

$$\sum_{i=1}^l \sum_{j=1}^m d_{ijk}^2 x_{ijk} \leq c_k; \quad 1 \leq k \leq n, \tag{3.4}$$

$$\text{and } x_{ijk} \geq 0 \text{ for } 1 \leq i \leq l, 1 \leq j \leq m, 1 \leq k \leq n,$$

Now we find the lower bound L_p and upper bound U_p for the p^{th} (where $1 \leq p \leq P$) objective function Z^p and $d_p = U_p - L_p$ (where $1 \leq p \leq P$) the degradation allowance for objective p .

3.1 IFGP approach for solving MOGSTP

We now define the membership function of general multiple objective solid transportation problem as:

$$\mu_p = \begin{cases} 1 & \text{for } Z^p \leq L_p \\ \frac{U_p - Z^p}{U_p - L_p} & \text{for } L_p < Z^p < U_p \\ 0 & \text{for } Z^p \geq U_p \end{cases}$$

By using an auxiliary variable λ the above fuzzy model can be converted into the following crisp model as.

$$\text{Maximize } \lambda \tag{3.5}$$

Subject to constraints

$$Z^p + \lambda(U_p - L_p) \leq U_p; \quad 1 \leq p \leq P \tag{3.6}$$

$$\text{And } \sum_{j=1}^m \sum_{k=1}^n d_{ijk}^1 x_{ijk} \leq a_i; \quad 1 \leq i \leq l, \tag{3.7}$$

$$\sum_{i=1}^l \sum_{k=1}^n x_{ijk} = b_j; \quad 1 \leq j \leq m, \tag{3.8}$$

$$\sum_{i=1}^l \sum_{j=1}^m d_{ijk}^2 x_{ijk} \leq c_k; \quad 1 \leq k \leq n, \tag{3.9}$$

$$\text{and } x_{ijk} \geq 0 \quad \text{for } 1 \leq i \leq l, 1 \leq j \leq m, 1 \leq k \leq n,$$

4 Algorithm

The solution procedure can be presented in the following steps

Step 1: Solve each of the $Z^p(x)$ for $1 \leq p \leq P$ as a single objective solid transportation problem.

Step 2: The solutions identify all the solution X^p ; $1 \leq p \leq P$ if X^p for $1 \leq p \leq P$ are same then any one of X^p ($1 \leq p \leq P$) be the optimal solution and go to step 8.

Step 3: Evaluate best lower bound L_p ; ($1 \leq p \leq P$) and worst upper bound U_p ; ($1 \leq p \leq P$).

Step 4: Define fuzzy membership function

$$\mu_p = \begin{cases} 1 & \text{for } Z^p \leq L_p \\ \frac{U_p - Z^p}{U_p - L_p} & \text{for } L_p < Z^p < U_p \\ 0 & \text{for } Z^p \geq U_p \end{cases}$$

Step 5: Convert into crisp linear programming problem.

$$\text{Max } \lambda \tag{4.1}$$

Such that

$$Z^p + \lambda(U_p - L_p) \leq U_p; \quad 1 \leq p \leq P \tag{4.2}$$

$$\text{And} \quad \sum_{j=1}^m \sum_{k=1}^n d_{ijk}^1 x_{ijk} \leq a_i; \quad 1 \leq i \leq l, \tag{4.3}$$

$$\sum_{i=1}^l \sum_{k=1}^n x_{ijk} = b_j; \quad 1 \leq j \leq m, \tag{4.4}$$

$$\sum_{i=1}^l \sum_{j=1}^m d_{ijk}^2 x_{ijk} \leq c_k; \quad 1 \leq k \leq n, \tag{4.5}$$

and $x_{ijk} \geq 0$ for $1 \leq i \leq l, 1 \leq j \leq m, 1 \leq k \leq n,$

And let the solution is X^*

Step 6: If the decision maker is satisfies with this solution then stop and this is the solution X^* of multiple objective transportation problem. Otherwise go to step 7.

Step 7: Set $U_p = Z^p(X^*)$ and go to step 3 until decision maker satisfies or the problem terminates.

Step 8: Stop.

5 Numerical Example

Let us consider the following example to establish our method of approach.

$$\text{Min } Z^1 = 0.5x_{111} + 0.4x_{112} + 0.8x_{121} + 0.5x_{122} + 0.25x_{131} + 0.2x_{132} + 0.3x_{211} + 0.5x_{212} + 0.5x_{221} + 0.2x_{222} + 0.7x_{231} + 0.1x_{232};$$

$$\text{Min } Z^2 = 0.8x_{111} + 0.4x_{112} + 0.2x_{121} + 0.25x_{122} + 0.25x_{131} + 0.8x_{132} + 0.1x_{211} + 0.1x_{212} + 0.1x_{221} + 0.6x_{222} + 0.8x_{231} + 0.8x_{232};$$

$$\text{Min } Z^3 = 0.9x_{111} + 0.2x_{112} + 0.5x_{121} + 0.5x_{122} + 0.8x_{131} + 0.7x_{132} + 0.9x_{211} + 0.5x_{212} + 0.2x_{221} + 0.5x_{222} + 0.9x_{231} + 0.8x_{232};$$

$$\text{Min } Z^4 = 0.1x_{111} + 0.4x_{112} + 0.1x_{121} + 0.5x_{122} + 0.9x_{131} + 0.5x_{132} + 0.6x_{211} + 0.2x_{212} + 0.8x_{221} + 0.8x_{222} + 0.5x_{231} + 0.1x_{232};$$

Subjectto

$$\begin{aligned} 0.4x_{111} + 0.6x_{112} + 0.5x_{121} + 0.4x_{122} + 0.2x_{131} + 0.4x_{132} &\leq 230; \\ 0.25x_{211} + 0.6x_{212} + 0.2x_{221} + 0.4x_{222} + 0.4x_{231} + 0.5x_{232} &\leq 650; \\ x_{111} + x_{112} + x_{211} + x_{212} &= 200; \\ x_{121} + x_{122} + x_{221} + x_{222} &= 600; \\ x_{131} + x_{132} + x_{231} + x_{232} &= 300; \\ 0.2x_{111} + 0.7x_{121} + 0.5x_{131} + 0.2x_{211} + 0.5x_{221} + 0.3x_{231} &\leq 300; \\ 0.5x_{112} + 0.6x_{122} + 0.3x_{132} + 0.4x_{212} + 0.5x_{222} + 0.7x_{232} &\leq 1010; \end{aligned}$$

Table 1:

	Z^1	Z^2	Z^3	Z^4
X^1	210	620	720	630
X^2	475	200	550	700
X^3	438	380	373	700
X^4	519	435	640	238
L_p =lower bound	210	200	373	238
U_p =upper bound	519	620	720	700
$d_p = U_p - L_p$	309	420	347	462

The corresponding objective functions values are

The membership functions of the objective functions based on membership function (MF) formula are as follows:

$$\text{Max } \lambda$$

Subject to

$$0.5x_{111} + 0.4x_{112} + 0.8x_{121} + 0.5x_{122} + 0.25x_{131} + 0.2x_{132} + 0.3x_{211} + 0.5x_{212} + 0.5x_{221} + 0.2x_{222} + 0.7x_{231} + 0.1x_{232} + \lambda 309 \leq 519;$$

$$0.8x_{111} + 0.4x_{112} + 0.2x_{121} + 0.25x_{122} + 0.25x_{131} + 0.8x_{132} + 0.1x_{211} + 0.1x_{212} + 0.1x_{221} + 0.6x_{222} + 0.8x_{231} + 0.8x_{232} + \lambda 420 \leq 620;$$

$$0.9x_{111} + 0.2x_{112} + 0.5x_{121} + 0.5x_{122} + 0.8x_{131} + 0.7x_{132} + 0.9x_{211} + 0.5x_{212} + 0.2x_{221} + 0.5x_{222} + 0.9x_{231} + 0.8x_{232} + \lambda 347 \leq 720;$$

$$0.1x_{111} + 0.4x_{112} + 0.1x_{121} + 0.5x_{122} + 0.9x_{131} + 0.5x_{132} + 0.6x_{211} + 0.2x_{212} + 0.8x_{221} + 0.8x_{222} + 0.5x_{231} + 0.1x_{232} + \lambda 462 \leq 700;$$

$$0.4x_{111} + 0.6x_{112} + 0.5x_{121} + 0.4x_{122} + 0.2x_{131} + 0.4x_{132} \leq 230;$$

$$0.25x_{211} + 0.6x_{212} + 0.2x_{221} + 0.4x_{222} + 0.4x_{231} + 0.5x_{232} \leq 650;$$

$$x_{111} + x_{112} + x_{211} + x_{212} = 200;$$

$$x_{121} + x_{122} + x_{221} + x_{222} = 600;$$

$$x_{131} + x_{132} + x_{231} + x_{232} = 300;$$

$$0.2x_{111} + 0.7x_{121} + 0.5x_{131} + 0.2x_{211} + 0.5x_{221} + 0.3x_{231} \leq 300;$$

$$0.5x_{112} + 0.6x_{122} + 0.3x_{132} + 0.4x_{212} + 0.5x_{222} + 0.7x_{232} \leq 1010;$$

Using Lingo 15 optimizing software,

We get, $\lambda = 0.4474208$,

$$Z^1 = 381, Z^2 = 432, Z^3 = 565, Z^4 = 493.$$

If the decision maker needs more improvement, then, further iteration may be performed.

6 Conclusion

Nothing new will be discussed if the constraints

$$\sum_{i=1}^m \sum_{k=1}^q x_{ijk} = b_j; \quad 1 \leq j \leq n,$$

is changed into

$$\sum_{i=1}^m \sum_{k=1}^q d_{ijk}^3 x_{ijk} = b_j; \quad 1 \leq j \leq n,$$

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