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STRONGLY REGULAR FUZZY GRAPH

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Abstract: In this paper, strongly regular fuzzy graph which is analogous to the concept of strongly regular graph in crisp graph theory is introduced and examples are presented, necessary and sufficient condition for a cycle to be strongly regular fuzzy graph is provided and some properties of strongly regular fuzzy graph are studied.

Keywords: degree of a vertex, regular fuzzy graph, effective fuzzy graph, strongly regular fuzzy graph, line graph of a strongly regular fuzzy graph.

2010 Mathematics Subject Classification: 05C07, 05C72

1. INTRODUCTION:

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975. Though it is very young, it has been growing fast and has numerous applications in various fields. In this paper we introduce strongly regular fuzzy graphs and we provide a necessary and sufficient condition for a cycle to be strongly regular. Then we study about the line graph of a strongly regular fuzzy graphson a cycle.

2. PRELIMINARIES:

Throughout this paper n denotes the number of vertices of a fuzzy graph and edge between u and v is denoted as uv.

Definition2.1[3]:
Let V be a non-empty set. The triple G = (V, σ, μ) is called a fuzzy graph on V where
$\sigma: V \rightarrow [0.1]$ and $\mu: V \times V \rightarrow [0.1]$ such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $uv \in V \times V$.
The underlying crisp graph of G :(σ, μ) is denoted by G^* :(V, E)

Definition2.2[9]:
Let G = (V, σ, μ) be a fuzzy graph, Two vertices u and v in G are called adjacent if μ (uv) > 0

Definition2.3[2]:
Let G = (V, σ, μ) be a fuzzy graph. Then G is said to be effective fuzzy graph if
μ (uv) = σ (u) Λ σ (v) for all uv Є VXV

Definition2.4[3]:
A fuzzy graph G: (V, σ, μ) is called complete μ (uv) = σ (u) Λ σ (v) for all u, v Є V

Definition2.5 [4]:
The complement of a fuzzy graph G= (V, σ, μ) is a fuzzy graph $G^c = (V, \sigma^c, \mu^c)$ where
$\sigma^c(u) = \sigma(u)$for all u Є V and
μ^c (u v) = σ (u) Λ σ (v) - μ (uv) for all uvЄ V

Definition2.6 [4]:
Let G= (V, σ, μ) is a fuzzy graph with underlying graph G* (V,E)
The fuzzy line graph of G is L (G): (ω, λ) with underlying graph (Z,W)
Where Z = {S_x = {x} U {u_x,v_x}/ xЄ E, x =$u_x v_x$,u_x,v_x Є V},
W = {(S_x, S_y) / $S_x \cap S_y \neq \varphi$, x, yЄ E, x ≠y},
ω (S_x) = μ (x) ∀S_x Є Z and
λ ($S_x S_y$) = μ (x) Λ μ (y) for every ($S_x S_y$) Є W
For the sake of simplicity, the vertices of L(G) may be denoted by x instead of S_x and the edges by xy instead of $S_x S_y$.

Definition2.7 [1]:
Let G= (V, E) be a graph Then G is said to be strongly regular if it satisfies the following axioms

i) G is a k-regular.
ii) There exists λ number of common neighbours between the adjacent vertices.
iii) There exists μ number of common neighbours between the non-adjacent vertices.

Any strongly regular graph G is denoted by G: (n, k, λ, ρ).

Lemma 2.8[5]:
Let G :(σ, μ) be a fuzzy graph on G* :(V, E). If$d_G(v)$ =k ∀ v Є V then G is said to be k- regular fuzzy graph.

Lemma 2.9[5]:
Let G :(σ, μ) be a fuzzy graph on an odd cycle then G is regular iff μ is a constant function.

Lemma 2.10[5]:
Let G :(σ, μ) is a fuzzy graph on an even cycle thenG is regular iff either μ is a constant function or alternate edges have same membership values.

Definition 2.11[6]:
Let G be a fuzzy graph. The sequence $(d_1,d_2,d_3,\ldots\ldots,d_n)$ with $d_1 \geq d_2 \geq d_3 \geq \ldots\ldots\ldots \geq d_n$, where $d_1,d_2,d_3,\ldots\ldots,d_n$ are the degree of the vertices of G, is the degree sequence of a fuzzy graph G.

Definition 2.12[6] :
The set of distinct positive real numbers occurring in a degree sequence of a fuzzy graph is called its degree set

Definition2.13 [7]:
In a fuzzy graph G: (σ, μ), the degree of an edge e = uv ∈V is d(uv) = d(u) + d(v) - 2μ(uv). G is an edge regular fuzzy graph if all the edges have the same edge degree.

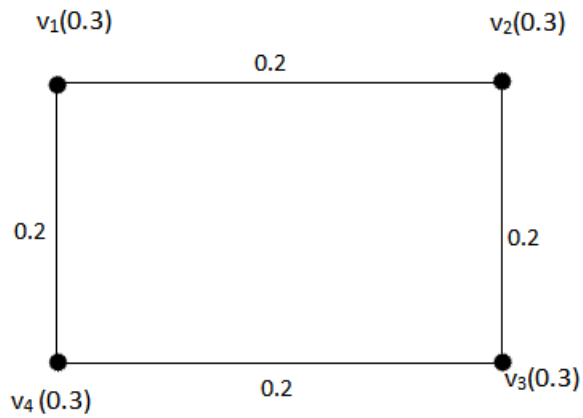
3. STRONGLY REGULAR FUZZY GRAPH

Definition 3.1:
A fuzzy graph G: (σ, μ) is said to be strongly regular if it satisfies the following axioms:-

i) G is a k-regular fuzzy graph
ii) Sum of membership values of the vertices common to the adjacent vertices λ is same for all adjacent pair of vertices,
iii) Sum of membership values of the vertices common to the non-adjacent vertices ρ is same for all non-adjacent pair of vertices

Any strongly regular fuzzy graph G is denoted by (n,k,λ,ρ) srtrongly regular fuzzy graph.

Example:



G: (4, 0.4, 0, 0.6)

Fig: 3.1

Theorem3.2

Let G be a fuzzy graph such that G* is strongly regular. Then G is strongly regular if both σ and μ are constant functions.

Proof:
Let (G*, k,λ,ρ) be a strongly regular graph.
Let σ(u) = c, a constant ∀ u ϵ V andμ (uv) = ι , a constant ∀ uv ϵ E
Now d_G(u) = $\sum_{uv\in E} \mu(uv)$

= $\sum_{uv\in E} l$

= $d_{G*}(u)l$

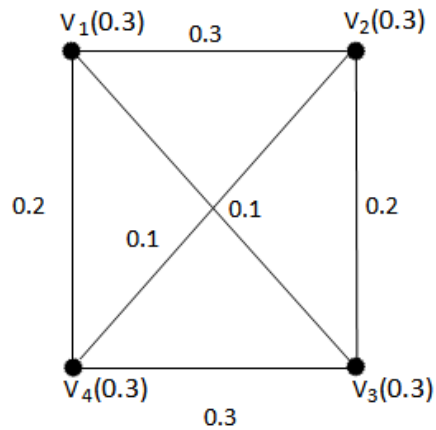
= k l.

∴ G is a k ι regular fuzzy graph.
Similarly, since $\sigma$is a constant function, the other parameters are λc andρc.
∴ G is a strongly regular with the parameters k ι, λc andρc.

Remark 3.3:

But the converse of the above theorem need not be true

In the following fig.3.2, G is (4, 0.6, 0.6, 0) strongly regular, G is (4, 3, 2, 0) strongly regular and σ is a constant but μ is not a constant function.



G: (4, 0.6, 0.6, 0)
Fig: 3. 2

Theorem 3.4:

Let G: (σ, μ) be a (n,k,λ,ρ) srtrongly regular fuzzy graph then the degree sequence of G is a constant sequence (k,k,….,k) of n elements.

Proof:

Since G is a (n,k,λ,ρ) strongly regular fuzzy graph,G is a k- regular .
Therefore all the vertices have same degree k.
Hence the degree sequence is (k,k,….,k).

Theorem 3.5:

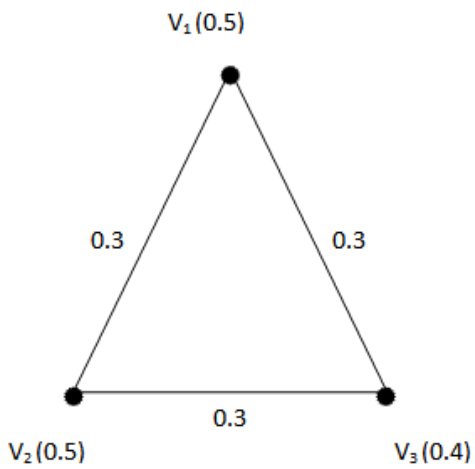
Let G: (σ, μ) be a (n,k,λ,ρ) srtrongly regular fuzzy graph then the degree set of G is the singleton set {k}.

Proof:

Since G is - k regular, all the vertices have same degree k .
Therefore the degree set of G is {k}.

Remark 3.5:

The converse of theorem 3.4 and theorem 3.5 need not be true. For example consider the fuzzy graph G in the fig 3.3 has the constant degree sequence (0.6, 0.6, 0.6) , degree set is {0.6} which is not strongly regular.



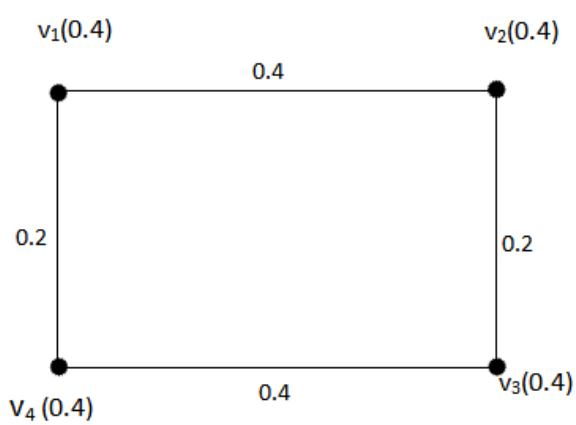
G: (σ, μ)

Fig.3.3

Remark 3.6:

If G is strongly regular then its edge degree sequence need not be a constant sequence .

This can be verified by the strongly regular fuzzy graph G in the following fig.3.4.



G: (4, 0.6, 0.8, 0.8)

Fig: 3. 2

Edge degree sequence of G is {0.8, 0.4}

Theorem 3.7:

Let G: (σ, μ) be a strongly regular fuzzy graph such that μ is a constant function then the edge degree sequence of G is a constant sequence and the edge degree set is the singleton set.

Proof:

Since G is strongly regular, G is k-regular . Given

$\Rightarrow\ d_G(u) = k\ \forall\ u \in V$

Therefore $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$

$= k + k - 2\mu(uv)$

$= 2k - 2c\ \ \forall\ uv \in E.$

Hence the edge degree sequence of G is a constant sequence and its edge degree set {2k-2c}.

4. STRONGLY REGULAR FUZZY GRAPH ON CYCLES

Theorem 4.1:

Let G be a fuzzy graph such that G* is an odd cycle. Then G is strongly regular iff both μ and σ are constant functions and $n \leq 5$.

Proof:

Let G be a strongly regular fuzzy graph. Since G* is an odd cycle, $n \geq 3$.
Suppose that G* is a cycle $v_1e_1v_2e_2v_3e_3\ldots\ldots\ldots\ldots v_ne_nv_1$, where $n \geq 6$.
Consider the non-adjacent vertices v_1 and v_3. Then v_2 is the only vertex common to v_1 and v_3. Therefore the sum of membership values of the vertices to the non-adjacent vertices v_1 and v_3 is $\sigma(v_2)$.
Since $n \geq 6$, no vertex is common to the non-adjacent vertices v_1 and v_4. Therefore the sum of membership values of the vertices common to the non-adjacent vertices v_1 and v_4 is 0. Therefore G is not a strongly regular, which is a contradiction. Hence G is an odd cycle with $n \leq 5$.
Suppose μ is not a constant function. Then G cannot be a regular fuzzy graph by lemma 2.8.
To prove that σ is a constant function.

When n = 3, each vertex is common to the other two adjacent vertices, therefore, since G is strongly regular, all the three vertices must have the membership value. Hence σ is a constant function.
When n = 5, each vertex is the only vertex common to its neighbours which are non-adjacent. Since G is strongly regular, all the five vertices must have the same membership value. So σ is a constant function.
Conversely, if μ is a constant function then G is regular by the lemma 2.8,
Let μ (uv) = k be a constant and σ(u) = c be a constant$\forall$u $\in$V.
When n = 3, Sum of membership values of the vertices common to a pair of adjacent vertices is c. The other parameter is 0. So G is (3, 2k, c, 0) strongly regular fuzzy graph.
Similarly, When n=5 G is (5, 2k, 0, c) strongly regular fuzzy graph.

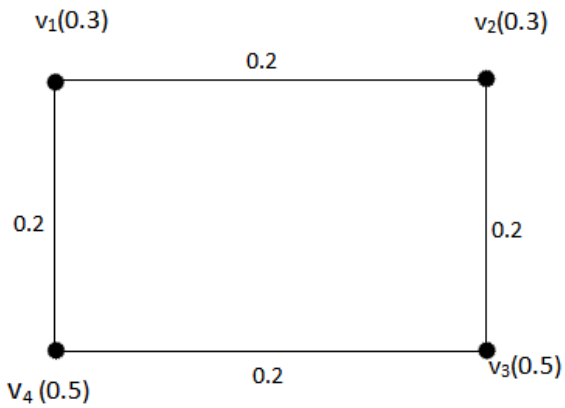
Theorem 4.2:
Let G be a fuzzy graph such that G* is a cycle with n = 4. Then G is strongly regular iff either μ is a constant function or alternate edges have same membership values and sum of membership values of the diagonally opposite vertices are equal.

Proof:
Suppose that G is a strongly regular fuzzy graph. Then by the lemma 2.9,either μ is a constant function (or) alternate edges have same membership value. Since G is a cycle with n = 4, the sum of membership values of the diagonally opposite vertices is the sum of membership values of the vertices common to a pair of non-adjacent vertices, which are equal.

Conversely, since μ is a constant function or alternate edges have same membership values, G is a regular fuzzy graph. The vertices common to a pair of non-adjacent vertices are diagonally opposite. Therefore by our assumption, sum of membership values of the vertices common to a pair of non-adjacent vertices is the same. Also no vertices common to any pair of adjacent vertices. Hence G is strongly regular.

Remark 4.3:-

Every strongly regular fuzzy graph on a cycle need not be an effective fuzzy graph and vice versa.
It can be illustrated by the following example.Fig.4.1 is strongly regular but not effective and fig.4.2 is effective but not strongly regular.



G: (4, 0.4, 0, 0.8)

Fig: 4.1

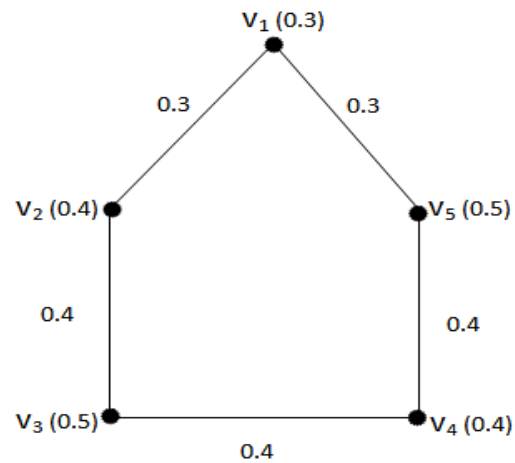


Fig: 4.2

Theorem 4.4:
Let G :(σ, μ) be an effective fuzzy graph on an odd cycle with n ≤ 5. Then G is strongly regular iff σ is a constant function.

Proof:
Let G :(σ, μ) be an effective and strongly regular fuzzy graph on an odd cycle.
Then by theorem 4.1, σ is a constant function.
Conversely let G be an effective fuzzy graph with σ as a constant function and n ≤ 5.
Let σ(u) = C ∀ u Є V.
Since G is effective, μ (uv) = σ(u) Λ σ(u) ∀ u v Є E.
= C∀ u v Є E.
Hence σ and μ are constant functions in G.
Therefore G is strongly regular fuzzy graph by the theorem 4.1.

Theorem 4.5:
Let G :(σ, μ) be an effective fuzzy graph on cycle with n ≤ 5 such that σ and μ are constant functions. Then the degree sequence and edge degree sequence of G are constant sequences.
Proof:
Let G :(σ , μ) be an effective fuzzy graph on a cycle C:v_1 ,v_2 ,v_3,……v_n ,v_1 with n ≤ 5 such that σ and μ are constant functions.

If n = 3 (or) 5 then by theorem 4.4, G is strongly regular .Hence the degree sequence is a Constant sequence.

If n =4, then by theorem 4.2, G is strongly regular .Hence the degree sequence is a Constant sequence.

Since G is a cycle, the degree of each edge in G is the sum of the membership values of two edges incident on it .Here μ is a constant function, therefore the edge degree sequence of G is a constant sequence.

Theorem 4.6:
Let G :(σ, μ) be an effective fuzzy graph on a cycle such that σ is a constant function. Then the degree sequence and edge degree sequence of G are constant sequences. Hence the degree set and edge degree set of G are singleton sets.

Proof:

Let G:(σ , μ) be an effective fuzzy graph on a cycle G*:v_1 ,v_2 ,v_3,……v_n ,v_1 such that σ is a constant function. Then μ is also a constant function.

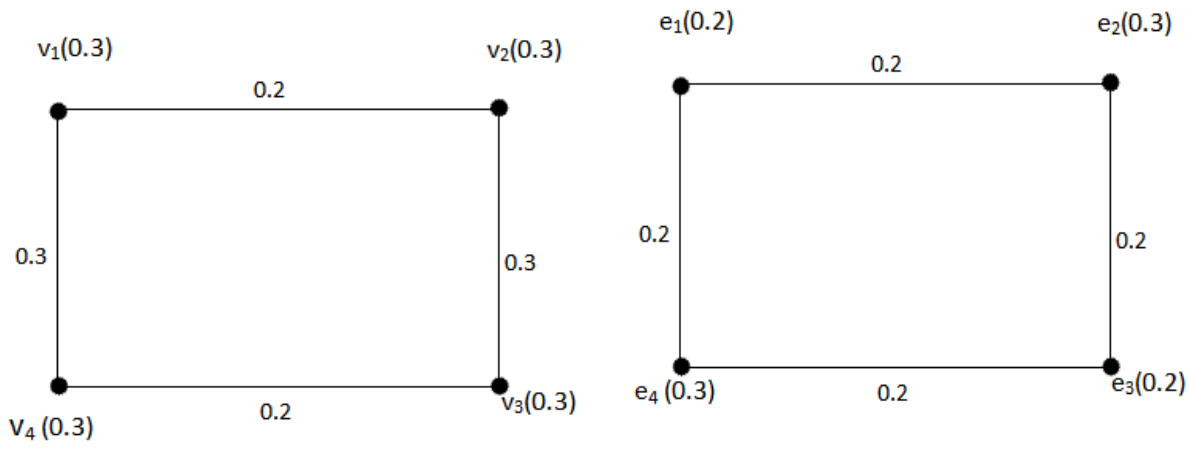
Since G* is a cycle, the degree of each vertex in G is the sum of the membership values of two edges incident on it and the degree of each edge in G is the sum of the membership values of two edges adjacent to it. Therefore if μ is a constant function of constant value k, then G is a 2k-regular and 2k-edge regular fuzzy graph. Therefore the degree sequence and the edge degree sequence of G are constant sequences.

Corollary 4.6:
Let G :(σ, μ) be an effective fuzzy graph on a cycle such that σ is a constant function. Then the degree set and edge degree set of G are singleton sets.

5. LINE GRAPH OF STRONGLY REGULAR FUZZY GRAPH ON YCLES

Remark5.1:
If G is a strongly regular fuzzy graph on a cycle then L (G) need not be strongly regular.

This can be seen from the following figure 5.1

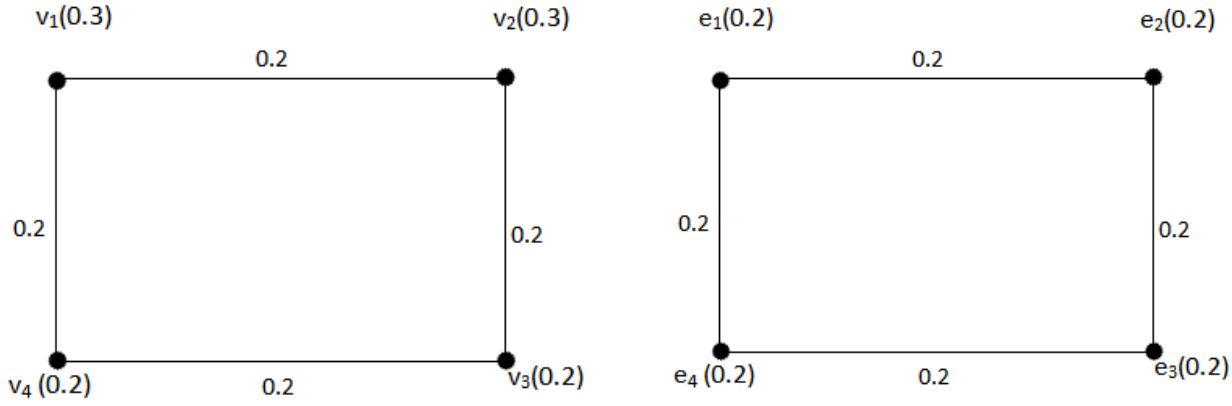


G: (4,0.5,0,0.6) L(G)

Fig: 5.1

Remark5.2:
If L (G) is a strongly regular fuzzy graph on a cycle then G need not be strongly regular.

This can be seen from the following figure 5.2



G L(G): (4, 0.4, 0, 0.4)

Fig: 5.2

Theorem 5.3:

If G: (σ, μ) is a strongly regular fuzzy graph on a cycle G*, then L^m (G) is a strongly regular fuzzy graph for every positive integer m.

Proof:

Let G: (σ, μ) be a strongly regular fuzzy graph on a cycle with n vertices.. Then by the theorem 4.1, $3 \leq n \leq 5$ and σ and μ are constant functions.

Let μ (uv) = k, a constant, ∀uv ∈E and σ(u) = c, a constant, ∀u ∈V.

Then in L(G), L^2(G), …, L^n(G), …, the vertex membership functions and the edge membership functions are all constant functions of same constant value c

Also if G* is a cycle with n vertices, L^m(G*) is also a cycle with n vertices for every positive integer m.

Since G* has n, $3 \leq n \leq 5$, vertices, L^m(G*) also has n, $3 \leq n \leq 5$, vertices for every positive integer m.

Hence L^m (G) is a strongly regular fuzzy graph for every positive integer m.

6. CONCLUSION

In this paper we have defined strongly regular fuzzy graph and we have found the necessary and sufficient condition for a cycle to be strongly regular. Also we have derived the necessary and sufficient condition for line graph of a strongly regular fuzzy graph to be strongly regular and discussed about degree sequence, edge degree sequence of strongly regular fuzzy graphs.

7. REFERENCES

[1] X. Hubat, Strongly regular graphs, Discrete Math. 13 (1975) 357-381.

[2].A.Nagoorgani and K.Radha, Conjuction of two Fuzzy graphs, International Review of Fuzzy Mathematics, Vol.3 ((no.1) 2008), 61-71.

[3].A.Nagoorgani and K.Radha, Regular property of fuzzy graphs, Bulletin of Pure and applied Sciences, Vol.27E ((no.2) 2008), 411-419.

[4]. A.Nagoorgani and K.Radha,Some sequences in Fuzzy graphs, east journal of Applied Mathematics, Vol.31 ((no.3) 2008), 321-335.

[5].A.Nagoorgani and K.Radha, On Regular fuzzy graphs, Journal of physical sciences 12 (2008), 33-40.

[6] K. Radha and A.Rosemine , "Degree sequence of a fuzzy graph", Journal Academic Research Journal, ISSN 0973-0303,635-638.

[7] K. Radha and A.Rosemine , "Edge Degree sequence of a fuzzy graph",International Journal Advancesin Mathematics and Statistics, Vol.2, Issue 1(2017), 13-22.

[8].M.S.Sunitha and A.V.Kumar,Complements of Fuzzy graphs, Indian Journal of pure and Applied Math.,vol.33,9(2002),1451-1460.

[9].R.Sattanathan and S.Lavanya,Complementary Fuzzy graphs and Fuzzy chromatic number,International journal of algorithms, computing and Mathematics, vol.2.no.3 (2009), 21-24.