Nano Generalized Pre Homeomorphisms in Nano Topological Space

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Abstract- The purpose of this paper is to define and study some properties of Nano generalized pre-homeomorphisms in Nano topological spaces.

Index Terms- Nano Topology, Ngp-closed sets, Ngp-closed map, Ngp-open map, Ngp- homeomorphisms

I. INTRODUCTION

The notion of homeomorphism plays a very important role in topology. Many researchers have generalized the notions of homeomorphism in topological spaces. Maki *et al*.[5] have introduced and investigated *g*-homeomorphism and *gc* - homeomorphism in topological spaces. Lellis Thivagar [4] introduced Nano homeomorphisms in Nano Topological Spaces. Bhuvaneswari *et al*. [2] introduced and studied some properties of Nano generalized homeomorphisms in Nano topological spaces. In this paper, a new class of homeomorphism called Nano generalized pre homeomorphism is introduced and some of its properties are discussed.

II. PRELIMINARIES

Definition 2.1. A bijective function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called a generalized homeomorphism (g-homeomorphism) [5] if *f* is both *g* - continuous and *g* - open.

Definition 2.2. A bijective function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called a *gc* - homeomorphism [5] if both *f* and f^{-1} are *gc* - irresolute functions.

Definition 2.3. [6]

Let *U* be a non-empty finite set of objects called the universe and *R* be an equivalence relation on *U* named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X\subseteq U$.

(i) The lower approximation of *X* with respect to *R* is the set of all objects, which can be for certain classified as *X* with respect to *R* and its is denoted by $L_R(X)$.
That is, $L_R(X)=\bigcup_{x\in u}\{R(x):R(x)\subseteq X\}$ where $R(x)$ denotes equivalence class determined by *x*.

(ii) The upper approximation of *X* with respect to *R* is the set of all objects, which can be possibly classified as *X* with respect to *R* and it is denoted by $U_R(X)$. That is, $U_R(X)=\bigcup_{x\in u}\{R(x):R(x)\cap X\neq\phi\}$

(iii) The boundary region of *X* with respect to *R* is the set of all objects, which can be classified neither as *X* nor as not-*X* with respect to *R* and it is denoted by $B_R(X)$. That is, $B_R(X)=U_R(X)-L_R(X)$

Property 2.4. [6]

If (U,R) is an approximation space and *X*, $Y\subseteq U$, then

(i) $L_R(X)\subseteq X\subseteq U_R(X)$

(ii) $L_R(\phi)=U_R(\phi)=\phi$ and $L_R(U)=U_R(U)=U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.5. [3] Let U be the Universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.4., $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub – collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub – collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on *U* called the Nano topology on *U* with respect to *X*.

We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets. The elements of the complement of $\tau_R(X)$ are called as Nano-closed sets.

Definition 2.6. [4] A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be Nano closed map (resp. Nano open map) if the image of every Nano closed set (resp. Nano open set) in $(U, \tau_R(X))$ is Nano closed (resp. Nano open) in $(V, \tau_{R'}(Y))$

Definition 2.7. [4] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be Nano homeomorphism if , f is 1-1 and onto, f is Nano continuous and f is Nano open.

Definition 2.8. [2] A bijection $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Nano generalized homeomorphism (shortly, Ng-homeomorphism) if f is both Ng-continuous and Ng-open.

III. PROPERTIES OF NANO GENERALIZED PRE HOMEOMORPHISM IN NANO TOPOLOGICAL SPACE

Definition 3.1. A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be Nano generalized pre-closed map (shortly- Ngp-closed map) if the image of every Nano closed set in $(U, \tau_R(X))$ is Ngp-closed in $(V, \tau_{R'}(Y))$.

Definition 3.2. A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be Ngp*-closed map if the image $f(A)$ is Ngp-closed in $(V, \tau_{R'}(Y))$ for every Ngp-closed set *A* in $(U, \tau_R(X))$.

Definition 3.3. A bijection $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is called Nano generalized pre homeomorphism (shortly, Ngp-homeomorphism) if f is both Ngp-continuous and Ngp-open.

Example 3.4. Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then $\tau_R(X)$ = { Φ, {a}, {a, b, d}, {b, d}, U }.Then $\tau_{R'}(Y)$ = { Φ, {a, b, d}, {a, b},{d}, V } with V/R'' = {{d},{c},{a, b}} and Y = {a, d}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=b$, $f(b)=a$, $f(c)=c$, $f(d)=d$. Then f is bijective, Ngp-continuous and Ngp-open. Therefore f is an Ngp-homeomorphism.

Theorem 3.5. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a one to one onto mapping. Then f is an Ngp-homeomorphism if and only if f is Ngp-closed and Ngp-continuous.

Proof . Let f be an Ngp-homeomorphism. Then f is Ngp-continuous. Let B be an arbitrary Nano closed set in $(U,\tau_R(X))$. Then B is Ngp-closed set in $(U,\tau_R(X))$. Then $U-B$ is Ngp-open .Since f is Ngp-open, $f(U-B)$ is Ngp-open in $(V,\tau_{R'}(Y))$. That is $V-f(B)$ is Ngp-open in $(V,\tau_{R'}(Y))$. Therefore $f(B)$ is Ngp-closed in $(V,\tau_{R'}(Y))$.Thus the image of every Nano closed set in $(U,\tau_R(X))$ is Ngp-closed in $(V,\tau_{R'}(Y))$

Conversely, let f be Ngp-closed and Ngp-continuous. Let B be a Nano open set in $(U,\tau_R(X))$. Then $U-B$ is Nano closed in $(U,\tau_R(X))$. Since f is Ngp-closed, $f(U-B)=V-f(B)$ is Ngp-closed in *V*. Therefore $f(B)$ is Ngp-open in $(V,\tau_{R'}(Y))$. Thus f is Ngp-open and hence f is an Ngp- homeomorphism.

Theorem 3.6. Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a bijective Ngp-continuous map. Then the following are equivalent.

(i) f is an Ngp-open map,
(ii) f is an Ngp-homeomorphism,
(iii) f is an Ngp-closed map.

Proof . (i) $\Rightarrow$ (ii) By the given hypothesis, the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is bijective, Ngp-continuous and Ngp-open. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-homeomorphism.

(ii) $\Rightarrow$ (iii) Let A be the Nano closed set in $(U,\tau_R(X))$. Then A^C is Nano open in $(U,\tau_R(X))$. By assumption, $f(A^C)$ is Ngp-open in $(V,\tau_{R'}(Y))$, i.e., $f(A^C)=(f(A))^C$ is Ngp-open in $(V,\tau_{R'}(Y))$ and hence $f(A)$ is Ngp-closed in $(V,\tau_{R'}(Y))$ for every Nano closed set A in $(U,\tau_R(X))$. Hence the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-closed map.

(iii) $\Rightarrow$ (i) Let F be a Nano open set in $(U,\tau_R(X))$. Then F^C is Nano closed set in $(U,\tau_R(X))$. By the given hypothesis, $f(F^C)$ is Ngp-closed in $(V,\tau_{R'}(Y))$. But, $f(F^C)=(f(F))^C$ is Ngp-closed , i.e., $f(F)$ is Ngp-open in $(V,\tau_{R'}(Y))$ for every Nano open set F in $(U,\tau_R(X))$. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-open map.

Theorem 3.7. Every Nano homeomorphism is Ngp-homeomorphism but not conversely.

Proof : If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Nano homeomorphism, by definition, f is bijective, Nano continuous and Nano open. Then $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngp-continuous and Ngp-open respectively. Hence the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is an Ngp-homeomorphism. Therefore, every Nano homeomorphism is Ngp-homeomorphism.

The map f in Example 3.4 is an Ngp-homeomorphism but not a Nano homeomorphism because it is not Nano continuous and Nano open map.

Theorem 3.8. Every Ng- homeomorphism is Ngp-homeomorphism but not conversely.

Proof : Since every Ng-continuous map is Ngp-continuous and every Ng-open map is Ngp-open, the theorem follows.

Example 3.9. Let $\tau_R(X)$ = {Φ, {a}, {b, c}, U} with U/R = {{a},{b ,c}} and X ={a, c } $\tau_{R'}(Y)$ = {Φ, {a, c}, V} with V/R' = {{a, c}, {b}} and Y = {a, c}. Define $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a)=c$, $f(b)=a$ and $f(c)=b$ which is an Ngp-homeomorphism. But this is not Ng-homeomorphism because for the Nano open set {b, c} in $(U,\tau_R(X))$, $f(b,c)=\{a,b\}$ is not Ng-open in $(V,\tau_{R'}(Y))$.

Remark 3.10. The composition of two Ngp-homeomorphisms need not be an Ngp-homeomorphism as seen from the following example.

Example 3.11. Let U = {a, b, c} = V = W. Then $\tau_R(X)$ = {Φ, {b}, {a, c}, U} with U/R = {{b}, {a, c}} and X = {b, c}. Let $\tau_{R'}(Y)$ = {Φ, {a}, {b, c}, V} with V/R' = {{a}, {b, c}} and Y = {a, c}. Let $\tau_{R''}(z)$ = {Φ, {a, c}, W} with W/R'' = {{a}, {b, c}} and Z = {a} Define $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ as $f(a)=a$, $f(b)=c$ and $f(c)=b$. Define $g:(V,\tau_{R'}(Y)) \to (W,\tau_{R''}(Z))$ as $g(a)=c$, $g(b)=a$ and $g(c)=b$. Then f and g are Ngp-homeomorphisms, but their composition $(g \circ f):(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is not an Ngp- homeomorphisms because for the Nano open set {b} in $(U,\tau_R(X))$, $(g \circ f)$ ({b}) = {b} which is not Ngp-open set in $(W,\tau_{R''}(Z))$. Therefore $(g \circ f)$ is not an Ngp-open map and $(g \circ f)$ is not an Ngp-homeomorphism.

We next introduce a new class of maps called Ngp*-homeomorphisms which forms a subclass of Ngp-homeomorphisms. This class of maps is closed under composition of maps.

Definition 3.12. A bijection $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is said to be Ngp*-homeomorphism if both f and f^{-1} are Ngp-irresolute.

We denote the family of all Ngp-homeomorphism (resp. Ngp*-homeomorphism) of a Nano topological space $(U,\tau_R(X))$ onto itself by $Ngp-h(U,\tau_R(X))$ (resp. $Ngp*-h(U,\tau_R(X))$

Theorem 3.13. Every Ngp*-homeomorphism is Ngp-homeomorphism.
i.e., For any space $(U,\tau_R(X))$, $Ngp*-h(U,\tau_R(X)) \subseteq Ngp-h(U,\tau_R(X))$.

Proof . Since every Ngp-irresolute function is Ngp-continuous and every Ngp*-open map is Ngp-open, the theorem follows.
Remark 3.14. The converse of the above theorem is not true as shown from the following example.

Example 3.15. Let U = {a, b, c} = V = W. Then $\tau_R(X)$ = {Φ, {b}, {a, c}, U} with U/R = {{b}, {a, c}} and X = {b, c}. Let $\tau_{R'}(Y)$ = {Φ, {a}, {b, c}, V} with V/R' = {{a}, {b, c}} and Y = {a, c}. Let $\tau_{R''}(z)$ = {Φ, {a, c}, W} with W/R'' = {{a}, {b, c}} and Z = {a} Define $g:(V,\tau_{R'}(Y)) \to (W,\tau_{R''}(Z))$ as $g(a)=c$, $g(b)=a$ and $g(c)=b$. Then g is an Ngp-homeomorphism but not Ngp*-homeomorphism since for the Ngp-closed set {a, b} in $(V,\tau_{R'}(Y))$, $(g^{-1})^{-1}(\{a,b\})$ = $g(a,b)=\{a,c\}$ is not Ngp-closed in $(W,\tau_{R''}(Z))$. Therefore g^{-1} is not Ngp-irresolute and therefore g is not Ngp*-homeomorphism.

***Example* 3.16.** Ngp*-homeomorphism $\not\longrightarrow$ Nano homeomorphism

Let U = {a, b, c} = V = W. Then $\tau_R(X)$ = {Φ, {b}, {a, c}, U} with U/R = {{b}, {a, c}} and X = {b, c}. Let $\tau_{R'}(Y)$ = {Φ, {a}, {b, c}, V} with V/R' = {{a}, {b, c}} and Y = {a, c}. Let $\tau_{R''}(z)$ = {Φ, {a, c}, W} with W/R'' = {{a}, {b, c}} and Z = {a}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=a$, $f(b)=c$ and $f(c)=b$. Then f is Ngp*-homeomorphism but not Nano homeomorphism because it is not Nano continuous.

***Example* 3.17.** Ng-homeomorphism $\not\longrightarrow$ Ngp*-homeomorphism

Let $\tau_R(X)$ = {Φ, {a}, {b, c}, U} and $\tau_{R'}(Y)$ = {Φ, {a}, V}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=b$, $f(b)=a$ and $f(c)=c$ .Then f is Ng -homeomorphism. Then for any Ngp-closed set {b} in $(U,\tau_R(X))$, $(f^{-1})^{-1}(\{b\})$ = $f(b)=a$ is not Ngp-closed in $(V,\tau_{R'}(Y))$. Therefore f^{-1} is not Ngp-irresolute and therefore f is not Ngp*-homeomorphism.

***Remark* 3.18.** We obtain the following implications from the above discussions.

Figure 3.19.

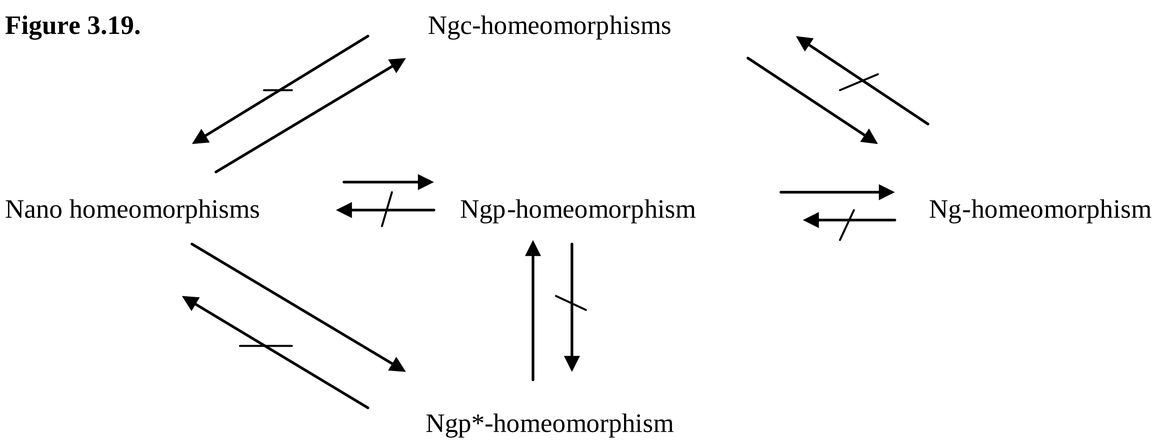

Figure 3.19 Implications of the Results

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