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Estimation of Variance of Time to Recruitment for a Single Grade Manpower System Under Two Sources of Depletion Using Univariate Policy of Recruitment

Amudha. T [*], Srinivasan. A [**]

[*] M.Phil., Scholar, PG and Research Department of Mathematics, Bishop Heber College, Trichy-17, (T.N), India

[**] Professor Emeritus, PG and Research Department of Mathematics, Bishop Heber College, Trichy-17, (T.N), India

Abstract- In this paper a marketing organization consisting of one grade with policy and transfer decisions forming two sources of depletion is considered. The problem of time to recruitment for this organization is analyzed when the breakdown threshold for the cumulative loss of manpower, a level of maximum allowable manpower depletion, have single component as the exponential threshold. A mathematical model is constructed and using univariate policy of recruitment based on the shock model approach, analytical results are obtained for the variance of time to recruitment by assuming that the loss in man power, inter-policy and inter-transfer decision times form three different sequences of independent and non-identically distributed exponential random variables.

Keywords- Single grade, Inter-policy and inter-transfer decision times, Two sources of depletion, Univariate policy, Shock model approach, Variance of time to recruitment.

I. INTRODUCTION

Attrition, which leads to depletion of manpower, is a common phenomenon in any marketing organization whenever this organization announces policy decisions regarding sales target, revision of wages, incentives and perquisites. The depletion may also be due to the transfer of personnel to sister organizations. Thus these are two sources of depletion, one due to policy decisions and the other due to the transfer decisions. In the two sources of depletion, the policy decisions are recurring and the transfer decisions are non-recurrent and hence it would not be realistic if we combine the policy and transfer decisions to form a single source of depletion. These depletions produce loss in manpower, which adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to the cost of recruitment. As the loss of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. An univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: *"Recruitment is made whenever the cumulative loss of man hours exceeds a breakdown threshold".*

Several researchers have studied the problem of time to recruitment for a single grade manpower system using shock model approach. In [1],[3],[4],[5],[7],[8],[9],[10],[11],[12],[13],[14],[15],[16] and [17], the authors have studied the problem of time to recruitment for a single grade manpower system with single source for depletion, using different recruitment policies under different conditions on the loss of manpower, inter-policy decisions and threshold for the loss of manpower. In [2], the authors have studied the problem for a single grade manpower system with two sources for depletion and obtained the variance of time to recruitment when the loss of manpower, inter-policy decision times, inter-transfer decision times and the single mandatory threshold for the loss of manpower are independent and identically distributed exponential random variables, using the univariate CUM policy of recruitment. No attempt has been made so far in the context of studying the problem of time to recruitment for a single grade manpower system under two sources of depletion of manpower when the loss in manpower for each decision, inter-policy decision times and the inter-transfer decision times form separately a sequence of independent and non-identically distributed random variables. In this paper, an attempt has been made to study the work of [2] under the above cited non-identical setup for an organization, using univariate CUM policy of recruitment.

II. MODEL DESCRIPTION

Consider an organization with single grade subject to loss in manpower due to two sources of depletions namely policy and transfer decisions. It is assumed that the loss of manpower is linear and cumulative. For $i = 1,2,3,...$ ($j = 1,2,3,...$), let X_{iP} (X_{jT}) be independent and non-identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) due to i^{th} policy decision (j^{th} transfer decision) with distribution $G_{iP}(\cdot)$ ($G_{jT}(\cdot)$), probability density function $g_{iP}(\cdot)$

($g_{jT}(\cdot)$) and mean $\frac{1}{\alpha_{iP}}$ ($\frac{1}{\alpha_{jT}}$). Here α_{iP} and α_{jT} are positive. Let $\overline{X}_{mP}$ ($\overline{X}_{nT}$) be the cumulative loss of man hours in the first m-policy decisions (n-transfer decisions) with cumulative distribution $\overline{G}_{mP}(\cdot)$ ($\overline{G}_{nT}(\cdot)$) and probability density function $\overline{g}_{mP}(\cdot)$ ($\overline{g}_{nT}(\cdot)$). Let U_{iP} (U_{jT}), the time between $(i-1)^{th}$ and i^{th} policy decisions ($(j-1)^{th}$ and j^{th} transfer decisions) be independent and non-identically distributed exponential random variables with cumulative distribution $F_P(\cdot)$ ($F_T(\cdot)$), probability density function $f_P(\cdot)$ ($f_T(\cdot)$) and mean $\frac{1}{\lambda_{iP}}$ ($\frac{1}{\lambda_{jT}}$). Here λ_{iP} and λ_{jT} are positive. Let $\overline{U}_{mP}$ ($\overline{U}_{nT}$) be the waiting time upto m-policy decisions (n-transfer decisions) with cumulative distribution $\overline{F}_{mP}(\cdot)$ ($\overline{F}_{nT}(\cdot)$), probability density function $\overline{f}_{mP}(\cdot)$ ($\overline{f}_{nT}(\cdot)$). Let Y be an exponential random variable representing the breakdown threshold for the organization with mean $\frac{1}{\theta}$, $\theta > 0$. Let $N_P(t)$ ($N_T(t)$) be the number of policy decisions (transfer decisions) taken in $(0,t]$. Let W be the time to recruitment for the organization with cumulative distribution $L(\cdot)$, probability density function $l(\cdot)$, Laplace transform $l^*(\cdot)$, mean $E[W]$ and variance $V(W)$. It is assumed that the loss of manhour process, the processes of inter-policy decision times, inter-transfer decision times and the breakdown threshold are statistically independent. The univariate CUM policy of recruitment employed in this model is stated as follows: "***Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold*** Y **".**

III. MAIN RESULTS

$$P(W>t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left\{\begin{array}{l}\text{Probability that there are exactly } m \text{ policy decisions and } n \text{ transfer decisions}\\ \text{in } (0,t] \text{ and the cumulative loss of manhours due to } m \text{ policy decisions and}\\ n \text{ transfer decisions does not exceed the threshold } Y\end{array}\right\}$$

By using laws of probability and from renewal theory,

$$P(W>t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} P\left(\overline{X}_{mP}+\overline{X}_{nT}\le Y\right)P\left(N_P(t)=m\right)P\left(N_T(t)=n\right) \tag{1}$$

Again by law of total probability

$$P\left(\overline{X}_{mP}+\overline{X}_{nT}\le Y\right) = \left(\overline{g}_{mP}\right)^*(\theta)\cdot\left(\overline{g}_{nT}\right)^*(\theta) \tag{2}$$

Therefore from (1) and (2), we get

$$P(W>t) = \left\{\sum_{m=0}^{\infty}P\left(N_P(t)=m\right)\left(\overline{g}_{mP}\right)^*(\theta)\right\}\left\{\sum_{n=0}^{\infty}P\left(N_T(t)=n\right)\left(\overline{g}_{nT}\right)^*(\theta)\right\} \tag{3}$$

Since $N_P(t)$ ($N_T(t)$) is a renewal process, it is known from renewal theory [6] that

$$P\left(N_P(t)=m\right)=\overline{F}_{mP}(t)-\overline{F}_{m+1,P}(t) \quad and \quad P\left(N_T(t)=n\right)=\overline{F}_{nT}(t)-\overline{F}_{n+1,\mathrm{T}}(t) \tag{4}$$

From (3) and (4)

$$P(W>t)=\left\{\sum_{m=0}^{\infty}\left[\overline{F}_{mP}(t)-\overline{F}_{m+1,P}(t)\right]\left(\overline{g}_{mP}\right)^*(\theta)\right\}\left\{\sum_{n=0}^{\infty}\left[\overline{F}_{nT}(t)-\overline{F}_{n+1,T}(t)\right]\left(\overline{g}_{nT}\right)^*(\theta)\right\}$$

$$=\left\{\sum_{m=0}^{\infty}\left[\sum_{l_1=1}^{m}a_{l_1P}\left(1-e^{-\lambda_{l_1P}t}\right)-\sum_{l_1=1}^{m+1}a_{l_1P}\left(1-e^{-\lambda_{l_1P}t}\right)\right]\left(\overline{g}_{mP}\right)^*(\theta)\right\}$$

$$\left\{\sum_{n=0}^{\infty}\left[\sum_{l_2=1}^{n}a_{l_2T}\left(1-e^{-\lambda_{jT}t}\right)-\sum_{l_2=1}^{n+1}a_{l_2T}\left(1-e^{-\lambda_{l_2T}t}\right)\right]\left(\overline{g}_{nT}\right)^*(\theta)\right\}$$

$$P(W>t)=\left\{\sum_{m=0}^{\infty}\left[-a_{m+1,P}\left(1-e^{-\lambda_{m+1,P}t}\right)\right]\left(\bar{g}_{mP}\right)^{*}(\theta)\right\}\left\{\sum_{n=0}^{\infty}\left[-a_{n+1,T}\left(1-e^{-\lambda_{n+1,T}t}\right)\right]\left(\bar{g}_{nT}\right)^{*}(\theta)\right\}$$

$$i.e.\ P(W>t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}a_{m+1,P}\ a_{n+1,T}\left[1-e^{-\lambda_{m+1,P}t}-e^{-\lambda_{n+1,T}t}+e^{-\left(\lambda_{m+1,P}+\lambda_{n+1,T}\right)t}\right]\left(\bar{g}_{mP}\right)^{*}(\theta).\left(\bar{g}_{nT}\right)^{*}(\theta) \tag{5}$$

Since $l(t)=\frac{d}{dt}\left[1-P(W>t)\right]$, from (5) $l^{*}(s)$ is found to be

$$l^{*}(s)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left(a_{m+1,P}a_{n+1,T}\left[\frac{\lambda_{m+1,P}+\lambda_{n+1,T}}{\lambda_{m+1,P}+\lambda_{n+1,T}+s}-\frac{\lambda_{m+1,P}}{\lambda_{m+1,P}+s}-\frac{\lambda_{n+1,T}}{\lambda_{n+1,T}+s}\right]\left(\bar{g}_{mP}\right)^{*}(\theta).\left(\bar{g}_{nT}\right)^{*}(\theta)\right) \tag{6}$$

Mean and variance of time to recruitment can be computed from (6) and from the result

$$E[W]=-\left[\frac{d}{ds}\left\{l^{*}(s)\right\}\right]_{s=0},\quad E\left[W^{2}\right]=\left[\frac{d^{2}}{ds^{2}}\left\{l^{*}(s)\right\}\right]_{s=0}\ \text{and}\ Var(W)=E\left[W^{2}\right]-\left(E[W]\right)^{2} \tag{7}$$

From (6), (7) and on simplification, we get

$$E[W]=\sum_{m=0}^{\infty}a_{m+1,P}\left\{\sum_{n=0}^{\infty}\left(a_{n+1,T}\left[\frac{1}{\lambda_{m+1,P}+\lambda_{n+1,T}}-\frac{1}{\lambda_{m+1,P}}-\frac{1}{\lambda_{n+1,T}}\right]\left(\bar{g}_{nT}\right)^{*}(\theta)\right)\right\}\left(\bar{g}_{mP}\right)^{*}(\theta) \tag{8}$$

$$E\left[W^{2}\right]=2\sum_{m=0}^{\infty}\left(a_{m+1,P}\left\{\sum_{n=0}^{\infty}\left(a_{n+1,T}\left[\frac{1}{\left(\lambda_{m+1,P}+\lambda_{n+1,T}\right)^{2}}-\frac{1}{\lambda_{m+1,P}^{2}}-\frac{1}{\lambda_{n+1,T}^{2}}\right]\left(\bar{g}_{nT}\right)^{*}(\theta)\right)\right\}\left(\bar{g}_{mP}\right)^{*}(\theta)\right) \tag{9}$$

and

$$\begin{aligned}Var(W)&=2\sum_{m=0}^{\infty}\left(a_{m+1,P}\left\{\sum_{n=0}^{\infty}\left(a_{n+1,T}\left[\frac{1}{\left(\lambda_{m+1,P}+\lambda_{n+1,T}\right)^{2}}-\frac{1}{\lambda_{m+1,P}^{2}}-\frac{1}{\lambda_{n+1,T}^{2}}\right]\left(\bar{g}_{nT}\right)^{*}(\theta)\right)\right\}\left(\bar{g}_{mP}\right)^{*}(\theta)\right)\\&\quad-\left(\sum_{m=0}^{\infty}\left[a_{m+1,P}\left\{\sum_{n=0}^{\infty}\left(a_{n+1,T}\left[\frac{1}{\lambda_{m+1,P}+\lambda_{n+1,T}}-\frac{1}{\lambda_{m+1,P}}-\frac{1}{\lambda_{n+1,T}}\right]\left(\bar{g}_{nT}\right)^{*}(\theta)\right)\right\}\left(\bar{g}_{mP}\right)^{*}(\theta)\right]\right)^{2}\end{aligned} \tag{10}$$

where from [6],

$$\left(\bar{g}_{mP}\right)^{*}(\theta)=\sum_{l_1=1}^{m}b_{l_1P}\frac{\alpha_{l_1P}}{\theta+\alpha_{l_1P}},\quad b_{l_1P}=\prod_{r_1=1}^{m}\frac{\alpha_{r_1P}}{\alpha_{r_1P}-\alpha_{l_1P}},\quad r_1\neq l_1 \tag{11}$$

and

$$\left(\bar{g}_{nT}\right)^{*}(\theta)=\sum_{l_2=1}^{n}b_{l_2T}\frac{\alpha_{l_2T}}{\theta+\alpha_{l_2T}},\quad b_{l_2T}=\prod_{r_2=1}^{n}\frac{\alpha_{r_2T}}{\alpha_{r_2T}-\alpha_{l_2T}},\quad r_2\neq l_2 \tag{12}$$

NOTE:

1. Suppose the loss of man hours due to policy decisions are independent and identically distributed exponential random variables with mean $\frac{1}{\alpha_P}$ and the loss of man hours due to transfer decisions are independent and identically distributed exponential random variables with mean $\frac{1}{\alpha_T}$. Then from equations (8) and (10) we find that

$$E[W] = \sum_{m=0}^{\infty} a_{m+1,P} \left\{ \sum_{n=0}^{\infty} a_{n+1,\mathrm{T}} \left[\frac{1}{\lambda_{m+1,P} + \lambda_{n+1,T}} - \frac{1}{\lambda_{m+1,P}} - \frac{1}{\lambda_{n+1,T}} \right] \left(\frac{\alpha_T}{\alpha_T + \theta} \right)^n \right\} \left(\frac{\alpha_P}{\alpha_P + \theta} \right)^m$$

and

$$Var(W) = 2\left(\sum_{m=0}^{\infty} a_{m+1,P} \left\{ \sum_{n=0}^{\infty} a_{n+1,\mathrm{T}} \left[\frac{1}{(\lambda_{m+1,P} + \lambda_{n+1,T})^2} - \frac{1}{\lambda_{m+1,P}^2} - \frac{1}{\lambda_{n+1,T}^2} \right] \left(\frac{\alpha_T}{\alpha_T + \theta} \right)^n \right\} \left(\frac{\alpha_P}{\alpha_P + \theta} \right)^m \right)$$
$$- \left(\sum_{m=0}^{\infty} a_{m+1,P} \left\{ \sum_{n=0}^{\infty} a_{n+1,\mathrm{T}} \left[\frac{1}{\lambda_{m+1,P} + \lambda_{n+1,T}} - \frac{1}{\lambda_{m+1,P}} - \frac{1}{\lambda_{n+1,T}} \right] \left(\frac{\alpha_T}{\alpha_T + \theta} \right)^n \right\} \left(\frac{\alpha_P}{\alpha_P + \theta} \right)^m \right)^2$$

2. Suppose the inter-policy decision times and inter-transfer decisions are independent and identically distributed exponential random variables with mean $\frac{1}{\lambda_P}$ and $\frac{1}{\lambda_T}$ respectively, Then from equations (8) and (10) we find that

$$E[W] = \frac{1}{\lambda_P + \lambda_T} \sum_{m=0}^{\infty} \frac{\lambda_P{}^m}{m!} \left\{ \sum_{n=0}^{\infty} \frac{\lambda_T{}^n}{n!} \left[\frac{(m+n)!}{(\lambda_P + \lambda_T)^{m+n}} \right] (\bar{g}_{nT})^* (\theta) \right\} (\bar{g}_{mP})^* (\theta)$$

and

$$Var(W) = \frac{2}{(\lambda_P + \lambda_T)^2} \left(\sum_{m=0}^{\infty} \frac{\lambda_P{}^m}{m!} \left\{ \sum_{n=0}^{\infty} \frac{\lambda_T{}^n}{n!} \left[\frac{(m+n+1)!}{(\lambda_P + \lambda_T)^{m+n}} \right] (\bar{g}_{nT})^* (\theta) \right\} (\bar{g}_{mP})^* (\theta) \right)$$
$$- \frac{1}{(\lambda_P + \lambda_T)^2} \left[\sum_{m=0}^{\infty} \frac{\lambda_P{}^m}{m!} \left\{ \sum_{n=0}^{\infty} \frac{\lambda_T{}^n}{n!} \left[\frac{(m+n)!}{(\lambda_P + \lambda_T)^{m+n}} \right] (\bar{g}_{nT})^* (\theta) \right\} (\bar{g}_{mP})^* (\theta) \right]^2$$

where $(\bar{g}_{mP})^* (\theta)$ and $(\bar{g}_{nT})^* (\theta)$ are given by (11) and (12)

3. The analytical result for variance of time to recruitment in [2], when the loss of man hours, inter-policy decision times and inter-transfer decision times are independent and identically distributed exponential random variables can be deduced from our results by taking

$$\alpha_{iP} = \alpha_P,\ \alpha_{jT} = \alpha_{\mathrm{T}},\ \lambda_{iP} = \lambda_P,\ \lambda_{jT} = \lambda_T\ .$$

IV. Conclusion

The manpower planning model developed in this paper is more general compare to earlier work in this direction and it can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels in the context of attrition. There is a scope for studying the applicability of the designed model using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this paper. The findings given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain. The present work can be studied for a two grade manpower system.

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AUTHORS

First Author – T. Amudha, M.Phil., Scholar, PG and Research Department of Mathematics, Bishop Heber College, Trichy-17, (T.N), India, amudhat14@gmail.com

Second Author – Dr.A.Srinivasan, Professor Emeritus, PG and Research Department of Mathematics, Bishop Heber College, Trichy-17, (T.N), India, mathsrinivas@yahoo.com.