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A Set with Special Arrangement and Semi-Group on a New System Called the Stacked System

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Abstract- This paper put a new mathematical system (stacked system) interested in arranging elements in a manner simple analytical set (according to the importance of the element in the set) and to prove that this system is a semi-group and prove some properties of semi-groups in this system.

Index Terms- Stacked system , Stacked set , Stacked semi-group , the order element on stacked system, The level stacked system , The path on stacked –system .

I. Introduction

There is no doubt that the progress of science and technology has led to the development of tools serve this development, and creating a language to explain and translate this harmony.

We are in a world filled with many of the complex relationships between the elements, and these overlap in the relationships between them confirms that we are still in the way of expanding the simple concept of sets.

Each set has a simple arrangement, whether from the largest to the smallest, or vice versa, or the order of elements according to their role in influencing their sets, and out of this sets.

But there is no study speaks clearly about the order of the elements affected each other within the same group. Simple concept of the set could not explain some of the relationships between the elements of this set .

Every element of this set has its own environment, making the bloc with elements of the same set concept is a little .

Suppose that there is a factory produces three varieties of products and each product has three sizes of packaging. This means that the set of products (S) of this factory has nine elements (three rows and three columns).

S =

	Products		
packaging	x_{11}	x_{12}	x_{13}
	x_{21}	x_{22}	x_{23}
	x_{31}	x_{32}	x_{33}

Table 1

Suppose that the purchasing power of each row (the product) and each column (the package) was distributed as in the table 2

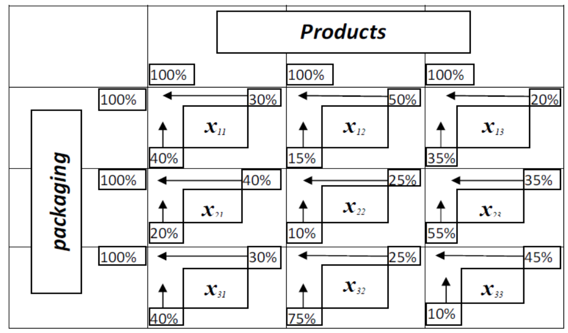

Table 2

Now, What is the most sales of products? And what is next?. And the next? .....

It is noted that these questions are designed to explain a particular order of these elements in this set is not like we know each arrangement. It can determine the order of any element calculates its relationship with the four elements that affect it in rank within the set and that affect it (two elements in the same row and two in the same column).

In general this study delve into the depths of the relations between the element in the same set .

The conversion of these sets into sets arranged a special arrangement that takes into account the relationship between each constituent element in the same group .Since the set theory assumes that the order of sets is a binary operations, and make this a semi-group (S , <) and (S , >) , this study is transforming this new arrangement to the semi-group system .

II. PRELIMINARIES

2.1 Definition[7]

A set X is a collection of elements (numbers, points, herring, etc.); we write $x \in X$, to denote x belonging to X. Two sets X and Y are defined to be equal, denoted by X = Y, if they are comprised of exactly the same elements: for every element x, we have $x \in X$ if , and only if $x \in Y$.

2.2 Definition[7]

A subset of a set X is a set S each of whose elements also belongs to X: If $s \in S$, then $s \in X$. We denote S being a subset of X by $S \subseteq X$.

2.3 Definition[6]

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If X and Y are subsets of a set Z, then their intersection is the set, $X \cap Y = \{ z \in Z : z \in X \text{ and } z \in Y \}$.

More generally, if $\{A_i : i \in I\}$ is any, possibly infinite, family of subsets of a set Z, then their intersection is : $\cap_{i \in I} A_i = \{ z \in Z : z \in A_i \text{ for all } i \in I \}$. It is clear that $X \cap Y \subseteq X$ and $X \cap Y \subseteq Y$. In fact, the intersection is the largest such subset: if $S \subseteq X$ and $S \subseteq Y$, then $S \subseteq X \cap Y$. Similarly,
$\cap_{i \in I} A_i \subseteq A_j$ for all $j \in I$. $X \cap Y$.

2.4 Definition[6]

If X and Y are subsets of a set Z, then their union is the set , $X \cup Y = \{ z \in Z : z \in X \text{ or } z \in Y \}$.[2]

More generally, if $\{ A_i : i \in I \}$ is any, possibly infinite, family of subsets of a set Z, then their union is

$\cup_{i \in I} A_i = \{ z \in Z : z \in A_i \text{ for some } i \in I \}$. It is clear that $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$. In fact, the union is the smallest such subset: if $X \subseteq S$ and $Y \subseteq S$, then $X \cup Y \subseteq S$. Similarly, $A_j \subseteq \cup_{i \in I} A_i$ for all $j \in I$.

2.5 Definition[6]

If X and Y are sets, then their difference is the set , $X - Y = \{ x \in X : x \notin Y \}$. The difference Y - X has a similar definition and, of course, Y - X and X – Y need not be equal. In particular, if X is a subset of a set Z, then its complement in Z is the set , $X^C = Z - X = \{ z \in Z : z \notin X \}$.

2.6 Definition[3]

Two sets are said to be disjoint if their intersection is empty; that is, if they have no elements in common. A collection $\{A_i : i \in I\}$ of sets is said to be disjointed if A_i and A_j are disjoint for all i and j in I with $i \neq j$.

2.7 Definition[3]

The symmetric difference of A and B, $A \triangle B$ is defined by $A \triangle B = A \setminus B \cup B \setminus A$.

2.8 Property[4]

All A,B,C ∈ V satisfy the following laws.

Commutativity: $A \cup B = B \cup A$, $A \cap B = B \cap A$;

Associativity: $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$;

Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;

De Morgan Laws: $A - (B \cap C) = (A - B) \cup (A - C)$, $A - (B \cup C) = (A - B) \cap (A - C)$.

2.9 Definition[7]

Observe that if X and Y are finite sets, say, $|X| = m$ and $|Y| = n$ (we denote the number of elements in a finite set X by $|X|$), then $|X \times Y| = mn$.

2.10 Definition[4]

Given a set X, one could collect all the subsets of X to form a new set. This procedure is called the power set operation.

1.4.11 Axiom[4]

(Power Set). For every set x, there exists a (unique) set, called the power set of x, whose elements are exactly subsets of x. This set is denoted by P(x). Then : $\emptyset \in P(x)$ for every set $x \in V$. Also $x \in P(x)$. If x = {a, b, c, d} then P(x) = {Ø, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c},{a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}}.

2.11 Axiom[4]

(Comprehension, defining subsets). Given a property p(y) of sets, for any set A, there exists a (unique) set B such that $x \in B$ if and only if $x \in A$ and p(x) holds.

2.12 Convention[4]

The notation $\{x \in A \mid p(x)\}$ stands for the set of all $x \in A$ which satisfy p(x).

2.13 Definition[7]

If X and Y are (not necessarily distinct) sets, then their Cartesian product $X \times Y$ is the set of all ordered pairs (x, y), where $x \in X$ and $y \in Y$. The plane is $R \times R$. The only thing we need to know about ordered pairs is that $(x, y) = (x^*, y^*)$ if and only if $x = x^*$ and $y = y^*$.

2.13 Definition[2]

If A is a non-void set, a non-void subset $R \subseteq A \times A$ is called a relation on A. If $(a , b) \in R$ we say that a is related to b, and we write this fact by the expression a ~ b.

2.14 Definition[2]

the several properties which a relation may possess.

1) If $a \in A$, then a ~ a. (reflexive)
2) If a ~ b; then b ~ a. (symmetric)
2$'$) If a ~ b and b ~ a, then a = b. (anti-symmetric)
3) If a ~ b and b ~ c; then a ~ c. (transitive).

2.15 Definition[2]

A relation which satisfies 1), 2$'$), and 3) is called a partial ordering. In this case we write a ~ b as $a \leq b$. Then:

1) If $a \in A$, then $a \leq a$.
2$'$) If $a \leq b$ and $b \leq a$, then a = b.
3) If $a \leq b$ and $b \leq c$, then $a \leq c$.

2.16 Definition[2]

A linear ordering is a partial ordering with the additional property that, if $a, b \in A$, then $a \leq b$ or $b \leq a$.

2.17 Definition[9]

The mathematical systems is a set of interacting or interdependent components forming an integrated whole or a set of elements (often called 'components') and relationships which are different from relationships of the set or its elements to other elements or sets .

2.18 Definition[9]

A binary operation on a set S is a mapping of the Cartesian product $S \times S$ into S.

2.19 Definition[9]

Let S be a set with a binary operation, written multiplicatively. An identity element of S is an element e of S such that $ex = x = xe$ for all $x \in S$.

2.20 Definition[9]

A binary operation on a set S (written multiplicatively) is associative when (xy) z = x (yz) for all x, y, z ∈ S .

2.21 Definition[9]

A binary operation on a set S (written multiplicatively) is commutative when x y = yx for all x, y ∈ S .

2.22 Definition[5]

Let S be a set and $\sigma : S \times S \to S$ a binary operation that maps each ordered pair (x , y) of S to an element σ(x , y) of S. The pair (S, σ) (or just S, if there is no fear of confusion) is called a groupoid .

2.23 Definition[5]

A groupoid (S, *) we shall mean a non-empty set S on which a binary operation * is defined. That is to say, we have a mapping $* : S \times S \to S$. We shall say that (S, *) is a semigroup if * is associative, i.e. if $(\forall\ x, y, z \in S)\ ((x, y)*, z)* = (x, (y, z)*)*$.

2.24 Definition[9]

A semigroup is an ordered pair of a set S , the underlying set of the semigroup, and one associative binary operation on S . A semigroup with an identity element is a monoid. A semigroup or monoid is commutative when its operation is commutative.

2.25 Definition[9]

Let S be a semigroup (written multiplicatively). Let a ∈ S and let $n \geq 1$ be an integer ($n \geq 0$ if an identity element exists). The nth power a^n of a is the product $x_1 x_2 \cdots x_n$ in that $x_1 = x_2 = \cdots = x_n = a$.

2.26 Definition[5]

S is a finite semigroup if it has only a finitely many elements.

2.27 Definition[5]

A commutative semigroup is a semigroup S with property : $(\forall x, y \in S)\ (xy = yx)$.

2.28 Definition[5]

If there exists an element 1 of S such that $(\forall x \in S)\ x1 = 1x = x$ we say that 1 is an identity (element) of S and that S is a semigroup with identity.

2.29 Definition[5]

If S has no identity element then it is very easy to adjoin an extra element 1 to S to form a monoid. We define 1S = S1 = S for all s in S , and 1 1 = 1, and it is a routine matter to check that $S \cup \{ 1 \}$ becomes a monoid

2.30 Definition[5]

If there exists an element 0 of S such that $(\forall x \in S)\ x0 = 0x = 0$ we say that 0 is a zero element of S and that S is a semigroup with zero.

2.31 Definition[5]

Let $a \neq \emptyset$ be a (nonempty) subset of a semigroup (S , σ). We say that (A ,σ) is a subsemigroup of S, denoted by $A \leq S$, if A is closed under the product of S: $\forall x , y \in A: x\, \sigma\, y \in A$, that is, $A \leq S \Leftrightarrow A^2 \subseteq A$.

2.32 Definition[8]

A non-empty subset A of a semigroup S is called a left ideal if $SA \subseteq A$, a right ideal if $AS \subseteq A$, and a (two-sided) ideal if it is both a left and a right ideal.

2.33 Definition[1]

A subsemigroup I of a semigroup S is a called an interior ideal of S if $SIS \subseteq I$.

2.34 Definition[1]

A subsemigroup I of a semigroup S is a called a bi-ideal of S if $ISI \subseteq I$.

2.35 Definition[1]

We call a non-empty subset L of S which is a left ideal of S to be prime if for any two ideal A and B of S such that $AB \subseteq L$, it implies that $A \subseteq L$ or $B \subseteq L$.

III. STACKED SET ,SYSTEMS AND SEMI-GROUPS

3.1 Stacked Set And Systems

3.1.1 Definition

let T_α be a finite set , where T_α be a stacked set if and only if $a_\alpha \in T_\alpha$, $\alpha \in N/0$, α is the number of methods stacking elements in the set, and it is called paths ($P_1, P_2, \ldots, P_\alpha$).

Ex: (T_2 is times stacking two paths (columns and rows)).

(T_2) is in terms of operations between the elements in the form like a matrix $(n \times n) \cong T_2$, that mean we have a stacked in two paths , any element that has a link package, with two classes of elements of T_2 :

	C_1	C_2	C_3	C_4	C_5	….	C_n
R_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	….	a_{1n}
R_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	….	a_{2n}
R_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	….	a_{3n}
R_4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	….	a_{4n}
R_5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	….	a_{5n}
….	….	….	….	….	….	….	….
R_n	a_{n1}	a_{n2}	a_{n3}	a_{n4}	a_{n5}	….	a_{nn}

Table 3

Elements in this case are in the case of stack in the form of packages each of which has to do with some of the elements , So we cannot call them matrix, and the matrices known . just we say it is in stack . This stacking is possible to have a regular or irregular .

The question here is how to distinguish the rows of columns, meaning that when we say that this path of rows or columns?.

3.1.2 Definition

If T_2 is stacked set then T_2 has two paths , C_i and R_i ($1 \leq i \leq n$)such that $C_1 \cap C_2 \cap \ldots \cap C_n = \emptyset$, $R_1 \cap R_2 \cap \ldots \cap R_n = \emptyset$, and $C_\rho \cap R_\sigma = \{ a_{\rho\sigma} \mid a_{\rho\sigma} \in T_2 \}$.

Then

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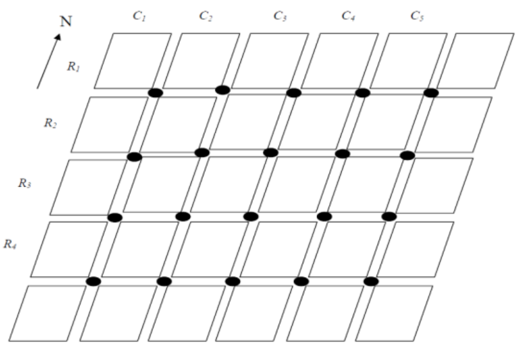
$\bigcap_{i\in n} C_i = \bigcap_{i\in n} R_i = \emptyset$,
and
$C_\rho \cap R_\sigma = \{ a_{\rho\sigma} \mid a_{\rho\sigma} \in T_2 \}$.

So it could be likened to the idea of the order of the set (T), as the city square in shape, with parallel paths from north to south and another parallel from east to west, and must be no intersections between the two types of paths .



Figures 1

3.1.3 Definition

Any set $A \notin \emptyset$ has at least one stack or stack is T_1 , and the set $\emptyset$ is T_0 .

From this concept we define paths inside the sets that define it, or some type of disassembly in the definition of the set and of the elements inside the sets .

3.2 Paths in stacked set

3.2.1 Definition

P_ρ is path on T_α if $\forall\, a \in T_\alpha \Leftrightarrow a \in P_\rho$, $1 \le \rho \le \alpha$,and T_α has α paths $\alpha \in N$.

3.2.2 Definition

If $T_{(\alpha\,,\,n)}$ is stacked set then it has α paths [$P_1 , P_2 , \ldots , P_\alpha$], and any path contains n sub-paths . such that $P_\rho = P_{\rho 1} \cup P_{\rho 2} \cup \ldots \cup P_{\rho n}$, $\rho \in \{ 1 , 2 , \ldots , \alpha \}$, and $P_{\rho\gamma} \cap P_{\rho\delta} = \emptyset$. ($\gamma , \delta \in \{ 1, 2, \ldots , n\}$).

3.2.3 Definition

Any path P_ρ in T_α , contains a disjoint sub-paths $P_{\rho\sigma}$ .and any sub-path $P_{\rho\sigma}$ (if $\alpha \ge 3$)contains a disjoint sub- sub-paths $P_{\rho\sigma\beta}$. Then, $P_{11} \cap P_{12} \cap \ldots \cap P_{1n} = \emptyset$, $P_{21} \cap P_{22} \cap \ldots \cap P_{2n} = \emptyset$, …. , $P_{\alpha 1} \cap P_{\alpha 2} \cap \ldots \cap P_{\alpha n} = \emptyset$.

3.2.4 Definition

If $T_{(\alpha\,,n\,,\,m)}$ is stacked set then it has α paths [$P_1 , P_2 , \ldots , P_\alpha$], and any path contains n sub-paths . such that $P_\rho = P_{\rho 1} \cup P_{\rho 2} \cup \ldots \cup P_{\rho n}$, $\rho \in \{ 1 , 2 , \ldots , \alpha \}$, and any sub-path has m sub-sub-paths $P_{\rho\sigma} = P_{\rho\sigma 1} \cup P_{\rho\sigma 2} \cup \ldots \cup P_{\rho\sigma m}$. ($\rho ,\sigma , m , n \in N/0$).

3.2.5 Theorem

the total elements in the stacked set T_α is $\wp(T_\alpha) = \wp(P_\rho)$.

proof :

From definition of the paths, P_ρ is path on T_α if $\forall\, a \in T_\alpha \Leftrightarrow a \in P_\rho$, $1 \le \rho \le \alpha$. then $\wp(T_\alpha) = \wp(P_\rho)$.

3.2.6 Theorem

the total elements in the stacked set $T_{(\alpha\,,n\,)}$ is $\wp(T_{(\alpha\,,n\,)}) \le m\, n$, that's where the total elements in the sub-path $P_{\rho\,\sigma}$ is $\wp(P_{\rho\,\sigma}) \le m$. ($\rho ,\sigma , m , n \in N$) .

proof :

Let $T_{(\alpha\,,n\,)}$ is stacked set ,then from definition above , it has α paths [$P_1 , P_2 , \ldots , P_\alpha$], and any sub-path has n elements . such that $\wp(T_{(\alpha\,,n\,)}) = \wp(P_\rho)$, $P_\rho = P_{\rho 1} \cup P_{\rho 2} \cup \ldots \cup P_{\rho n}$, $\rho \in \{ 1 , 2 , \ldots , \alpha \}$, and $P_{\rho\gamma} \cap P_{\rho\delta} = \emptyset$. ($\gamma , \delta \in \{ 1, 2, \ldots , n\}$), then $\wp(T_{(\alpha\,,n\,)}) = \wp(P_\rho) = \sum_{i=1}^{n} \wp_i(P_{\rho\gamma})$. And when ; $\wp(P_{\rho\,\sigma}) \le m$. Then ; $\wp(T_{(\alpha\,,n\,)}) = \wp(P_\rho) \le m\, n$.

3.2.7 Example

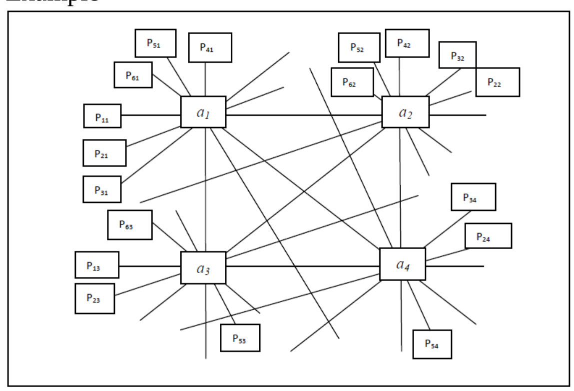


Figure 2 $T_{(6,4)}$

$P_1 = P_{11} \cup P_{12} \cup P_{13} \cup P_{14} = \{ a_1 , a_2 \} \cup \emptyset \cup \{ a_3 , a_4 \} \cup \emptyset = T_{(6,4)}$.
$P_2 = P_{21} \cup P_{22} \cup P_{23} \cup P_{24} = \{ a_1 \} \cup \{ a_2 \} \cup \{ a_3 \} \cup \{ a_4 \} = T_{(6,4)}$.
$P_3 = P_{31} \cup P_{32} \cup P_{33} \cup P_{34} = \{ a_1 \} \cup \{ a_2 , a_3 \} \cup \emptyset \cup \{ a_4 \} = T_{(6,4)}$.
$P_4 = P_{41} \cup P_{42} \cup P_{43} \cup P_{44} = \{ a_1 , a_3 \} \cup \{ a_2 , a_4 \} \cup \emptyset \cup \emptyset = T_{(6,4)}$.
$P_5 = P_{51} \cup P_{52} \cup P_{53} \cup P_{54} = \{ a_1 \} \cup \{ a_2 \} \cup \{ a_3 \} \cup \{ a_4 \} = T_{(6,4)}$.
$P_6 = P_{61} \cup P_{62} \cup P_{63} \cup P_{64} = \{ a_1 , a_4 \} \cup \{ a_2 \} \cup \{ a_4 \} \cup \emptyset = T_{(6,4)}$.

3.2.8 Example

The stacked set $T_{(4,5)} = P_{1(5)} \cup P_{2(5)} \cup P_{3(5)} \cup P_{4(5)}$, so :
$T_{(4,5)} = [P_{1,1} \cup P_{1,2} \cup P_{1,3} \cup P_{1,4} \cup P_{1,5} \cup P_{2,1} \cup \ldots P_{2,5} \cup P_{3,1} \cup \ldots P_{3,5} \cup P_{4,1} \cup \ldots P_{4,5} \cup P_{5,1} \cup \ldots P_{5,5}]$,
We note in this example also
$P_{1,1} \cap P_{1,2} = \emptyset$.

4.4 The stacked-systems

4.4.1 Definition

The system (T_α , τ) called stacked - system if and only if $a_\gamma\ \tau\ b_\beta = \min_0 (a_\gamma , b_\beta)$, and the system looking for (zero convergence) , and The system (T_α , Ʈ) called stacked - system if and only if a_γ Ʈ $b_\beta = \max_0 (a_\gamma , b_\beta)$, and the system looking for (zero spacing) . a_γ and $b_\beta \in T_\alpha$.

4.4.2 Definition

The system (T_α , τ) called stacked - system if and only if a_γ τ $b_\beta = \min_t(a_\gamma , b_\beta)$, and the system looking for (convergence of t) , and The system (T_α , Ʈ) called stacked - system if and only if a_γ Ʈ $b_\beta = \max_t(a_\gamma , b_\beta)$, and the system looking for (spacing of t) . a_γ and $b_\beta \in T_\alpha$, $t \in \mathcal{R}$.

4.4.3 Definition

The order element on stacked system T_α ,where the system looking for zero convergence or zero spacing, is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ , ($O_0(a_\gamma)$) :

$$O_0(a_\gamma) = \lceil a_\gamma \rfloor_0 = \left[\frac{|a_{\gamma 1}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \frac{|a_{\gamma 2}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} + \ldots + \frac{|a_{\gamma \alpha}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha = \left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i}|}{|\sum_{i=1}^{\alpha} a_{\gamma i}|} \right] / \alpha .$$

4.4.4 Definition

The order element on stacked system T_α ,where the system looking for convergence of t (or spacing of t), is amount contributes to this element in the system, and this estimate is calculated relationship of this element in every path that contains this element, then the element order of a_γ ,($O_t(a_\gamma)$) :

$$O_t(a_\gamma) = \lceil a_\gamma \rfloor_t = \left[\frac{|a_{\gamma 1} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \frac{|a_{\gamma 2} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} + \ldots + \frac{|a_{\gamma \alpha} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha = \left[\sum_{i=1}^{\alpha} \frac{|a_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |a_{\gamma i} - t|} \right] / \alpha .$$

4.4.5 Definition

The order stacked set T_α in zero convergence system is set $O_0(T_\alpha) = \{x_1, x_2 , \ldots , x_n\}$ if $\lceil x_1 \rfloor_0 < \lceil x_2 \rfloor_0 < \ldots < \lceil x_n \rfloor_0$ .And the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1} , \ldots , x_1\}$ if $\lceil x_1 \rfloor_0 > \lceil x_2 \rfloor_0 > \ldots > \lceil x_n \rfloor_0$.

The order stacked set T_α , where the system looking for convergence of t (or spacing of t), is set $O_0(T_\alpha) = \{x_1, x_2 , \ldots , x_n\}$ if $\lceil x_1 \rfloor_t < \lceil x_2 \rfloor_t < \ldots < \lceil x_n \rfloor_t$ .And the order stacked set T_α in zero spacing system is set $O_0[T_\alpha] = \{x_n, x_{n-1} , \ldots , x_1\}$ if $\lceil x_1 \rfloor_t > \lceil x_2 \rfloor_t > \ldots > \lceil x_n \rfloor_t$.

4.4.6 Definition

- If (T_α , τ) or (T_α , Ʈ) is stacked – system then $\forall a_\alpha , b_\beta \in T_\alpha$:

$\mathrm{Max}_t (a_\alpha , b_\beta) = a_\alpha$ Ʈ b_β , and $\mathrm{Min}_t (a_\alpha , b_\beta) = a_\alpha$ τ b_β

- If (T_α , τ) or (T_α , Ʈ) is stacked – system then $\forall a_\alpha , b_\beta \in T_\alpha$:

$$\max_t (a_\alpha , b_\beta) = \begin{cases} a_\alpha : if \lceil a_\alpha \rfloor_t > \lceil b_\beta \rfloor_t . \\ b_\beta : if \lceil b_\beta \rfloor_t > \lceil a_\alpha \rfloor_t . \end{cases}$$

$$\min_t (a_\alpha , b_\beta) = \begin{cases} a_\alpha : if \lceil a_\alpha \rfloor_t < \lceil b_\beta \rfloor_t . \\ b_\beta : if \lceil b_\beta \rfloor_t < \lceil a_\alpha \rfloor_t . \end{cases}$$

- If $a_\alpha = t$,(in $\lceil a_\alpha \rfloor_t$) then we suppose that $| a_\alpha - t | = \Delta t$, and where $\sum_{i}^{\alpha} | a_i - t |$, $\alpha \in \{1 , ..i\}$ then, we compensate $| a_\alpha - t | = 0$.

- If $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil a_\alpha \rfloor_t \neq \lceil b_\beta \rfloor_t$ the system is type-1 .

4.4.7 theorem

If the system is type-1 , and $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$, then a = b . a , b is element in this system .

proof:

from the definition above , if $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$ (one order element in two different places)so we have many type of this system , and if $\lceil a_\alpha \rfloor_t \neq \lceil b_\beta \rfloor_t$ the system is type-1 . so if the system is type-1 , and $\lceil a_\alpha \rfloor_t = \lceil b_\beta \rfloor_t$, then a = b (one order element one place).

4.4.8 Definition

The level stacked -system (lT_α) is defined as if the system looking for convergence of t , then $lT_\alpha = \{ 1 , 2 , \ldots , n \}$, $n \in N/0$. And if the system looking for spacing of t ,then $lT_\alpha = \{ n , n-1 , \ldots , 1 \}$, $n \in N/0$. Any element in T_α has only one element corresponding in lT_α , and in the same staking .

The lT_α helps us to calculate the $\min_t$ and the $\max_t$ in any a stacked-system. We arrange the elements of the smallest to the largest, or the opposite is true, according to the following concept $L : T_\alpha \to lT_\alpha$, $l : T_\alpha \to lT_\alpha$

$l (a_\eta) = 1$ if $\lceil a_\eta \rfloor_t$ is smallest, if the system looking for convergence of t , $L (a_\eta) = n$ if $\lceil a_\eta \rfloor_t$ is largest , if the system looking for spacing of t .

3.3 The stacked-semigroups

3.3.1 Definition

The minimum or maximum cost element is an element that can be an alternative for all elements with it in the same row and column , and symbolized by : $\min_t\{cost (T_\alpha)\}$, or $\max_t\{cost (T_\alpha)\}$.

3.3.2 Theorem

1- If Ʈ is an operation on the system (T_α , Ʈ) in type -1 system , then a Ʈ b = Ʈ (a , b) = $\max_t (a, b) = a \vee b$. $a, b \in T_\alpha$.
2- If τ is an operation on the system (T_α , τ) in type -1 system , then $a \tau b = \tau (a, b) = \min_t(a, b) = a \vee b$. $a, b \in T_\alpha$.

proof :

1- let Ʈ is an operation on the system (T_α , Ʈ), $a , b \in T_\alpha$, so a Ʈ $b = \max_t(a, b)$, then $\max_t(a, b) = a$, (if $\lceil a \rfloor_t > \lceil b \rfloor_t$) ,or $\max_t(a, b) = b$ if $\lceil b \rfloor_t > \lceil a \rfloor_t$,and the system is type-1 then ($\lceil a \rfloor_t \neq \lceil b \rfloor_t$) so $\max_t(a, b)$ = a or b mean $a \vee b$.
2- let τ is an operation on the system (T_α , τ), $a , b \in T_\alpha$, so $a \tau b = \min_t(a, b)$, then $\min_t(a, b) = a$, (if $\lceil a \rfloor_t < \lceil b \rfloor_t$) ,or $\min_t(a, b) = b$ if $\lceil b \rfloor_t < \lceil a \rfloor_t$,and the system is type-1 then ($\lceil a \rfloor_t \neq \lceil b \rfloor_t$) so $\min_t(a, b)$ = a or b mean $a \vee b$.

3.4 The stacked-semigroup

3.4.1 Definition

A binary operation on a stacked set T_α is a mapping of the Cartesian product
$T_\alpha \times T_\alpha$ into T_α .

3.4.2 Definition

The staked-groupoid is stacked-system , with binary operation .

3.4.3 Theorem

The stacked-system (T_α , τ) and (T_α , Ʈ) (in definition 3.1) is a groupoid and called a stacked-groupoid .

Proof :

1- Let $a , b \in T_\alpha$, from definition 6.4.6 , $a \tau b = \min_t (a, b)$ if the system looking for convergence of t , from that $a \tau b = a \vee b \in T_\alpha$,($a \vee b$ = a or b) , then certainly $\forall a , b \in T_\alpha$, $a \tau b \in T_\alpha$, that mean $\tau : T_\alpha \times T_\alpha \rightarrow T_\alpha$, and from definitions 7.2.1 , and 7.2.2 , the relation τ is a binary operation and the stacked-system is stacked-groupoid.
2- Let $a , b \in T_\alpha$, from definition 6.4.6 , a Ʈ $b = \max_t (a, b)$ if the system looking for spacing of t, from that a Ʈ $b = a \vee b \in T_\alpha$,($a \vee b$ = a or b) , then certainly $\forall a , b \in T_\alpha$, a Ʈ $b \in T_\alpha$, that mean Ʈ : $T_\alpha \times T_\alpha \rightarrow T_\alpha$, and from definitions 7.2.1 , and 7.2.2 , the relation Ʈ is a binary operation and the stacked-system is stacked-groupoid.

3.4.4 Theorem

If the systems (T_α , τ) and (T_α , Ʈ) are a stacked-systems (type – 1) then τ and Ʈ are associative .

proof :

- Let $a , b , c \in T_\alpha$,(($a \tau b$) τ c) = (($a \vee_t b$) τ c) =(($a \vee_t b$) $\vee$ c)=($a \vee_t$ ($b \vee_t c$)) =($a \vee_t$ ($b \tau c$)) =($a \tau$ ($b \tau c$)) , then τ is associative relation .
- Let $a , b , c \in T_\alpha$,((a Ʈ b) Ʈ c) = (($a \vee_t b$) Ʈ c) =(($a \vee_t b$) $\vee$ c)=($a \vee_t$ ($b \vee_t c$)) =($a \vee_t$ (b Ʈ c)) =(a Ʈ (b Ʈ c)) , then Ʈ is associative relation.

3.4.5 Definition

A stacked-semigroup is a stacked-system T_α , with associative binary operation .

3.4.6 Theorem

(i) If the systems (T_α , τ) is a stacked-system (type – 1), then (T_α , τ) is a semigroup and called a stacked-semigroup .
(ii) If the systems (T_α , Ʈ)is a stacked-system (type – 1), then (T_α , Ʈ) is a semigroup and called a stacked-semigroup

proof :

(i) Let $a , b \in T_\alpha$, from definition 6.4.6 , $a \tau b = \min_t (a, b)$ if the system looking for convergence of t , from that $a \tau b = a \vee b \in T_\alpha$,($a \vee b$ = a or b) , then certainly $\forall a , b \in T_\alpha$, $a \tau b \in T_\alpha$, that mean $\tau : T_\alpha \times T_\alpha \rightarrow T_\alpha$, and from definitions 7.2.1 , and 7.2.2 , the relation τ is a binary operation .

Let $a , b , c \in T_\alpha$,(($a \tau b$) τ c) = (($a \vee_t b$) τ c) =(($a \vee_t b$) $\vee$ c) = ($a \vee_t$ ($b \vee_t c$)) =($a \vee_t$ ($b \tau c$)) =($a \tau$ ($b \tau c$)) , then τ is associative relation . then from definition 7.2.5 , the stacked-systems (T_α , τ) is a semigroup and called a stacked-semigroup .

(ii) Let $a , b \in T_\alpha$, from definition 6.4.6 , a Ʈ b = $\max_t (a, b)$ if the system looking for spacing of t, from that a Ʈ $b = a \vee b \in T_\alpha$,($a \vee b$ = a or b) , then certainly $\forall a , b \in T_\alpha$, a Ʈ $b \in T_\alpha$, that mean Ʈ : $T_\alpha \times T_\alpha \rightarrow T_\alpha$, and from definitions 7.2.1 , and 7.2.2 , the relation Ʈ is a binary operation.

Let $a , b , c \in T_\alpha$,((a Ʈ b) Ʈ c) = (($a \vee_t b$) Ʈ c) =(($a \vee_t b$) $\vee$ c)=($a \vee_t$ ($b \vee_t c$)) =($a \vee_t$ (b Ʈ c)) =(a Ʈ (b Ʈ c)) , then Ʈ is associative relation . then from definition 7.2.5 , the stacked-systems (T_α , Ʈ) is a semigroup and called a stacked-semigroup .

3.4.7 Theorem

(i) If the semigroup (T_α , τ)is a stacked-semigroup (type – 1), then (T_α , τ) is a commutative semigroup.
(ii) If the semigroup (T_α , Ʈ)is a stacked-semigroup (type – 1), then (T_α , Ʈ) is a commutative semigroup.

proof :

(i) Let $x , y \in T_\alpha$ then $x \tau y = x \vee_t y = y \vee_t x = y \tau x \in T_\alpha \Rightarrow x \tau y = y \tau x \Rightarrow$ (T_α , τ) is a commutative semigroup.
(ii) Let $x , y \in T_\alpha$ then x Ʈ y = $x \vee_t y = y \vee_t x$ = y Ʈ x $\in T_\alpha \Rightarrow$ x Ʈ y = y Ʈ x $\Rightarrow$ (T_α , Ʈ) is a commutative semigroup.

3.4.8 Theorem

(i) If the semigroup (T_α , τ)is a stacked-semigroup , then (T_α , τ) is a finite semigroup.
(ii) If the semigroup (T_α , Ʈ)is a stacked-semigroup , then (T_α , Ʈ) is a finite semigroup.

proof :

(i) From definition 2.6 , any set is finite semigroup if it has only a finitely many elements . from definition 3.1 , T_α is a stacked finite set ,then (T_α , τ) is a finite semigroup .

(ii) And so (T_α , Ʈ) is a finite semigroup .

3.4.9 Theorem

The stacked-semigroups (T_α , τ) and (T_α , Ʈ) have an identity element ,

proof :

let (T_α , τ) is a stacked-system so (T_α , τ) is a semigroup , so it is looking for convergence of t , and let $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$ so T_α is finite set , then $x_a \tau x_b = \min_t (x_a , x_b)$, $\forall x_a , x_b \in T_\alpha$, there is exist $x_e \in T_\alpha$, and it is the last element we looking for it in the system such that $\forall x \in T_\alpha$, $x \tau x_e = x_e \tau x = x$, then x_e is the identity element , and so if the system looking for spacing of t (T_α , Ʈ), then $T_\alpha = \{ x_n , x_{n-1} , \dots , x_1 \}$, and $\forall x \in T_\alpha$, x Ʈ $x_e = x_e$ Ʈ x = x , then x_e is the identity element .

3.4.10 Theorem

The stacked-semigroups (T_α , τ) and (T_α , Ʈ) have zero element.

Proof :

Let (T_α , τ) is a stacked-system so (T_α , τ) is a semigroup with an associative binary composition in T_α . It is looking for convergence of t , and let $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$ then $x_a \tau x_b = \min_t(x_a , x_b)$, $\forall x_a , x_b \in T_\alpha$, there is exist $x_{zero} \in T_\alpha$ such that $x_1 \tau x_2 \tau \dots \tau x_n = \min_t(x_1 , x_2 , \dots , x_n) = x_{zero}$, that mean x_e is the nearest convergence of t element we looking for it , so $\forall x \in T_\alpha$, $x_{zero} \tau x = x \tau x_{zero} = x_{zero}$ .then x_{zero} is the zero element . If the system looking for spacing of t (T_α , Ʈ), then the zero element is the furthest spacing of t element.

3.4.11 Lemma

i. In the stacked-semigroup (T_α , Ʈ) the identity element $x_e = l(1)$, such that $T_\alpha = \{ x_n , x_{n-1} , \dots , x_1 \}$, $n \in N/0$.

ii. In the stacked-semigroup (T_α , τ) the identity element $x_e = l(n)$, such that $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$, $n \in N/0$.

proof :

From the theorem 4.7, any stacked-semigroup has an identity element ,it is the last element of the order set T_α , so if the system looking for convergence of t ,then $x_e = x_n$, and it is) corresponds the element n in $l(T_\alpha)$, so $x_e = l(n)$. if the system looking for spacing of t, the element $x_e = x_1$ in (T_α , Ʈ) corresponds the element 1in $l(T_\alpha)$, then $x_e = l(1)$.

3.4.12 Lemma

i. In the stacked-semigroup (T_α , Ʈ) the zero element $x_{zero} = l(n)$ if $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$.

ii. In the stacked-semigroup (T_α , τ) the zero element $x_{zero} = l(1)$

Proof :

from the theorem 4.8, any stacked-semigroup has a zero element ,it is the first element of the order set T_α , so if the system looking for convergence of t ,then $x_{zero} = x_1$, and it is corresponds the element 1 in $l(T_\alpha)$, so $x_{zero} = l(1)$. if the system looking for spacing of t, the element $x_{zero} = x_n$ in (T_α , Ʈ) corresponds the element n in $l(T_\alpha)$, then $x_{zero} = l(n)$.

3.4.13 Theorem

If (T_α , τ) and (T_α , Ʈ) are a stacked-semigroups, then $[x_\alpha] = \{ x_\alpha \}$, $\forall x_\alpha \in (T_\alpha , \tau)$, or $\forall x_\alpha \in$ (T_α , Ʈ) .

Proof :

i. let (T_α , τ) is a stacked-semigroup $\Longrightarrow T_\alpha$ is finite set $\Longrightarrow |T_\alpha| = n$, $n \in N$, then $\forall x \in T_\alpha$: $x^1 = x$, $x^2 = x \tau x = x$, $x^3 = x \tau x \tau x = x$, so $x^n = x \tau x \dots \tau x$ (to repeat it n times) = $x \Longrightarrow [x_\alpha] = \{ x_\alpha \}$.

ii. let (T_α , Ʈ) is a stacked-semigroup $\Longrightarrow T_\alpha$ is finite set $\Longrightarrow |T_\alpha| = n$, $n \in N/0$, then $\forall x \in T_\alpha$: $x^1 = x$, $x^2 = x$ Ʈ x = x , $x^3 = x$ Ʈ x Ʈ x = x , so $x^n = x$ Ʈ x .... Ʈ x (to repeat it n times) = $x \Longrightarrow [x_\alpha] = \{ x_\alpha \}$.

3.4.14 Definition

i. Let $A_\alpha \subseteq T_\alpha$ be a (nonempty) subset of a stacked-semigroup (T_α , τ). We saying that (A_α , τ) is a stacked- subsemigroup of (T_α , τ) denoted by $A \le T$, if A_α is closed under the product of T_α : $\forall x , y \in A_\tau$; $x \tau y \in A_\tau$.

ii. Let $B_\alpha \subseteq T_\alpha$ be a (nonempty) subset of a stacked-semigroup (T_α , Ʈ). We saying that (B_α , Ʈ) is a stacked- subsemigroup of (T_α , Ʈ), denoted by $B_\alpha \le T_\alpha$, if B_α is closed under the product of T_α : $\forall x , y \in B_\alpha$; x Ʈ $y \in B_\alpha$.

3.4.15 Theorem

If (T_α , τ) and (T_α , Ʈ), be stacked-semigroups, then any element in the power set of the set T_α with τ or Ʈ is a stacked-subsemigroup.

proof :

1- let (T_α , τ) be a stacked-semigroup, then $\forall x \in T_\alpha$, $x^2 = x \tau x = x \vee x = x \in (\{ x \} , \tau) \subseteq (T , \tau)$, $\forall x , y \in T_\alpha$: $x \tau y = (x \vee_t y) \in (\{ x , y \} , \tau) \subseteq (T_\alpha , \tau)$ so $\forall x_1, x_2 \dots x_n \in T_\alpha$: $x_1 \tau x_2 \tau \dots \tau x_n = (x_1 \vee_t x_2 \vee_t \dots \vee_t x_n) \in (\{ x_1 , x_2 , \dots x_n \} , \tau) \subseteq (T_\alpha , \tau)$. So any subset in T_α with τ is staked - subsemigroup on T_α , but the set of all subset in T_α is power set of T_α , so any element in the power set of the set T_α with τ is a stacked-subsemigroup.

2- let (T_α , Ʈ) be a stacked-semigroup, then $\forall x \in T_\alpha$, x^2 = x Ʈ x = $x \vee x = x \in$ ({ x } , Ʈ) ⊆ (T , Ʈ), $\forall x , y \in T_\alpha$: x Ʈ y = $(x \vee_t y) \in$ ({ x , y } , Ʈ) ⊆ (T_α , Ʈ) so $\forall x_1, x_2 \dots x_n \in T_\alpha$: x_1 Ʈ x_2 Ʈ … Ʈ $x_n = (x_1 \vee_t x_2 \vee_t \dots \vee_t x_n) \in$ ({ $x_1 , x_2 , \dots x_n$ } , Ʈ) ⊆ (T_α , Ʈ) . So any subset in T_α with Ʈ is staked - subsemigroup on T_α , but the set of all subset in T_α is power set of T_α , so any element in the power set of the set T_α with Ʈ is a stacked-subsemigroup

3.4.16 Example

If there are three distribution centers, consumer products of the type (A, B, C), where they are transported to sales centers (X, Y, Z) at a cost, as in the following table:

	X	Y	Z
A	1	3	2
B	4	5	1
C	7	3	1

Table 4

So there is a process of transferring between (A, B, C) and (X, Y, Z).
Transportation between A and X cost 1, so
cost (A,X) = 1 ≡ 1_{11}
cost (A,Y) = 3 ≡ 3_{12}
cost (A,Z) = 2 ≡ 2_{13}
cost (B,X) = 4 ≡ 4_{21}
cost (B,Y) = 5 ≡ 5_{22}
cost (B,Z) = 1 ≡ 1_{23}
cost (C,X) = 7 ≡ 7_{31}
cost (C,Y) = 3 ≡ 3_{32}
cost (C,Z) = 1 ≡ 1_{33} .
And $T_{2,3}$ (Non-Order)= { 1_{11} , 3_{12} , 2_{13} , 4_{21} , 5_{22} , 1_{23} , 7_{31} , 3_{32} , 1_{33} } .

or $T_{2,3}$ =

1	3	2
4	5	1
7	3	1

Table 5

If we were looking for a less expensive transfer this means the search for convergence of elements from zero
Then :

1- $\lceil 1_{11} \rfloor_0$ =[(1/ (1+3+2))+(1/(1+4+7))]/ 2 = 0.125 .
$\lceil 3_{12} \rfloor_0$ = 0.38636 .
$\lceil 2_{13} \rfloor_0$= 0.41667 .
$\lceil 4_{21} \rfloor_0$ = 0.36667 .
$\lceil 5_{22} \rfloor_0$ = 0.47727 .
$\lceil 1_{23} \rfloor_0$ = 0.175 .
$\lceil 7_{31} \rfloor_0$ = 0.60985 .
$\lceil 3_{32} \rfloor_0$ = 0.27273 .
$\lceil 1_{33} \rfloor_0$ = 0.17045 .

2- So : $\forall a_\alpha , b_\beta \in T_{2,3} \Rightarrow \lceil a_\alpha \rfloor_0 \neq \lceil b_\beta \rfloor_0 \Rightarrow (T_{2,3} , \tau)$ is type – 1 .

3- $lT_{3,2}$ (0) = { 1_{11} , 1_{33} , 1_{23} , 3_{32} , 4_{21} , 3_{12} , 2_{13} , 5_{22} , 7_{31} }.

or $lT_{3,2}$ (0) =

1	6	7
5	8	3
9	4	2

Table 6

4- | $lT_{3,2}$ | = n = 9 .
5- Min_0 (4_{21} , 3_{12}) = 4_{21} τ 3_{12} = 4_{21} .
6- Min_0(1_{11} , 7_{31}) = 1_{11} τ 7_{31} = 1_{11} .
7- X_{zero} = x_1 = 1_{11} , x_e = x_9 = 7_{31} .

3.5 stacked- ideal

3.5.1 remark

The stacked-semigroup T_α means (T_α , τ) and (T_α , Ʈ).

3.5.2 Definition

A non-empty subset I_α of a stacked-semigroup (T_α , τ) or (T_α , Ʈ) is called a stacked- ideal if $T_\alpha I_\alpha \subseteq I_\alpha$, and $I_\alpha T_\alpha \subseteq I_\alpha$.

3.5.3 Theorem

Let I_α is a stacked-ideal of the stacked-semigroup (T_α , τ) , | I_α | = r, T_α = {x_1 , x_2 ,… ,x_r , … x_n }, and the level stacked -semigroup(lT_α) = {1,2,…,r,…n}, then I_α = { x_1 , x_2 , … , x_r }, and lI_α (x)∊ {1,2,…,r} .

Proof :

- Let $I_{\alpha(r)}$ is a stacked-ideal of the stacked-semigroup (T_α , τ) , | $I_{\alpha(r)}$ | = r, T_α = {x_1 , x_2 ,… ,x_r , … x_n }, so the level stacked -semigroup(lT_α) = {1,2,…,r,…n} ,∀ x ∈ $I_{\alpha(r)}$ ⟹ x τ T_α ∈ $I_{\alpha(r)}$ ⟹ If 1 ≤ s ≤ r then x_s ∈ $I_{\alpha(r)}$ ⟹ $I_{\alpha(r)}$ = { x_1 , x_2 , … , x_r }.
- Next , lT_α (x_1) = 1 , lT_α (x_2)= 2, … , lT_α (x_r) = r ⟹ x_1 = img(1) , x_2 = img(2) , … , x_r = img(r)⟹ ∀ x,y∊ lT_α , x ≤ y , img(x) τ img(y) = img(x), and 1 ≤ 2 ≤ … ≤ r ,then ∀ t ∊ lT_α , t ≤ r ⟹img(t) τ img(r) = img(r) τ img(t) = img(t) , but T_α I_α ⊆ $I_{\alpha(r)}$, and $I_\alpha T_\alpha \subseteq I_\alpha$, so img(t)∊ $I_{\alpha(r)}$ ⟹ ∀ t ≤ r , and | $I_{\alpha(r)}$ | = r then t ∊ $I_{\alpha(r)}$ ⟹ $I_{\alpha(r)}$ = { img(1) , img(2) , … img(t) , … , img(r) } ⟹ the level-stacked-ideal (lI_α) = { 1 , 2 , … , r } ⟹ ∀ x∊ $I_{\alpha(r)}$ then l(x)∊ {1,2,…,r} .

3.5.4 Theorem

Let $I_{\alpha(r)}$ is a stacked-ideal of the stacked-semigroup (T_α , Ʈ) , | $I_{\alpha(r)}$| = n – r + 1 , T_α = {x_1 , x_2 ,… ,x_r , … x_n }, and the level stacked -semigroup(lT_α) = {1,2,…,r,…n}, then $I_{\alpha(r)}$ = { x_r , x_{r+1} , x_{r+2} , … , x_n }, and l $I_{\alpha(r)}$ (x)∊ {1 , 2 ,…, n – r + 1 } .

Proof :

- Let $I_{\alpha(r)}$ is a stacked-ideal of the stacked-semigroup (T_α , Ʈ), | $I_{\alpha(r)}$| = n – r + 1, T_α = {x_1 , x_2 ,… ,x_r , … x_n }, so the level stacked -semigroup(lT_α) = {1,2,…,r,…n} ,from the definition (4.11) ,∀ x ∈ $I_{\alpha(r)}$ ⟹ x Ʈ T_α ∈ $I_{\alpha(r)}$. If r ≤ s ≤ n then x_s ∈ $I_{\alpha(r)}$ ⟹ $I_{\alpha(r)}$ = { x_r , x_{r+1} , x_{r+2} , … , x_n }.
- Next , lT_α (x_r) = 1 , lT_α (x_{r+1})= 2, … , lT_α (x_n) = n ⟹ x_r = img(1) , x_{r+1} = img(2) , … , x_n = img(n – r + 1)⟹ ∀ x,y∊ lT_α , x ≤ y , img(x) τ img(y) = img(x), and ,(n ≥ r and r , n ∈ N/0), so 1 ≤ 2 ≤ … ≤ n – r + 1 then ∀ t ∊ lT_α , t ≤ n – r + 1 ⟹ img(t) τ img(n – r + 1) = img(n – r + 1) τ img(t) = img(n – r + 1) , but T_α $I_{\alpha(r)}$, $I_\alpha T_\alpha \subseteq$ $I_{\alpha(r)}$

so img(t)$\in I_{\alpha(r)} \Rightarrow r \le t \le n - r + 1$, and $| I_{\alpha(r)} | = n - r + 1$, then $t \in I_{\alpha(r)} \Rightarrow I_{\alpha(r)} = \{ img(1) , img(2) , \dots img(t) , \dots , img(n - r + 1) \} \Rightarrow$ the level-stacked-ideal $(lI_{\alpha}) = \{ 1 , 2 , \dots , n - r + 1 \} \Rightarrow \forall x \in I_{\alpha(r)}$ then $l(x) \in \{1,2,\dots, n - r + 1\}$.

3.5.5 Theorem

Any a stacked-ideal I of the stacked-semigroup T_{α} be an interior ideal .

Proof :

1- Let I is a stacked-ideal of stacked-semigroup (T_{α} , τ) $\Rightarrow T_{\alpha} I , IT_{\alpha} \subseteq I$, then $\forall s \in I , \forall x,y \in T_{\alpha} , x \tau s = s \tau x = (x \vee s) \in I$ and $x \tau s \tau y = y \tau s \tau x = ((x \vee s) \vee y) \in I$, then $T_{\alpha} IT_{\alpha} \subseteq I$ so I is an interior ideal , and any a stacked-ideal I of the stacked-semigroup be an interior ideal .
2- Let I is a stacked-ideal of stacked-semigroup (T_{α} , Ʈ) $\Rightarrow T_{\alpha} I , IT_{\alpha} \subseteq I$, then $\forall s \in I , \forall x,y \in T_{\alpha}$, x Ʈ s = s Ʈ x = $(x \vee s) \in I$ and x Ʈ s Ʈ y = y Ʈ s Ʈ x = $((x \vee s) \vee y) \in I$, then $T_{\alpha} IT_{\alpha} \subseteq I$ so I is an interior ideal , and any a stacked-ideal I of the stacked-semigroup be an interior ideal .

3.5.6 Theorem

Any a stacked-ideal I of the stacked-semigroup T_{α} be bi-ideal .

Proof :

1- Let I is a stacked-ideal of stacked-semigroup (T_{α} , τ) $\Rightarrow T_{\alpha} I , IT_{\alpha} \subseteq I$, then $\forall s , t \in I , \forall x \in T_{\alpha} , x \tau s = s \tau x = (s \vee x) \in I$ and $(s \tau x) \tau t = t \tau (x \tau s) = ((s \vee x) \vee t) = s \vee x \vee t \in I$, then $IT_{\alpha} I \subseteq I$ so I is bi-ideal , and any a stacked-ideal I of the stacked-semigroup T_{α} be bi-ideal .
2- Let I is a stacked-ideal of stacked-semigroup (T_{α} , Ʈ) $\Rightarrow T_{\alpha} I , IT_{\alpha} \subseteq I$, then $\forall s , t \in I , \forall x \in T_{\alpha}$, x Ʈ s = s Ʈ x = $(s \vee x) \in I$ and $(s \tau x)$ Ʈ t = t Ʈ (x Ʈ s) = $((s \vee x) \vee t) = s \vee x \vee t \in I$, then $IT_{\alpha} I \subseteq I$ so I is bi-ideal , and any a stacked-ideal I of the stacked-semigroup T_{α} be bi-ideal .

3.5.7 Theorem

If I is a stacked-ideal of stacked-semigroup T_{α} , then I be not completely prime.

Proof:

Let I is a stacked-ideal of stacked-semigroup T_{α} , then $\forall x , y \in T_{\alpha}$, $x \tau y = x \vee y$(or x Ʈ y = $x \vee y$) . if $x \vee y \in I$, there are three cases, $x \in I$ or $y \in I$ or $x , y \in I$ but the definition of the prime has just two cases , $x \in I$ or $y \in I$ so any a stacked-ideal I of a stacked-semigroup T_{α} be not completely prime .

3.5.8 Theorem

If I is a stacked-ideal of stacked-semigroup T_{α} , then I is completely semiprime.

Proof :

Let I is a stacked-ideal of stacked-semigroup T_{α} , then $\forall x \in I : x^2 = x \tau x = x \in I$ (or x Ʈ x = $x \in I$) , so from the definition of the completely semiprime, $x^2 \in I$ implies that $x \in I$. So any a stacked-ideal I of stacked-semigroup T_{α} is completely semi prime.

3.5.9 lemma

If I is a stacked-ideal of T_{α} , then I is a stacked-subsemigroup of T_{α} .

Proof:

If $IT_{\alpha} \subseteq I$ or $T_{\alpha} I \subseteq I$, then certainly $II \subseteq I$, since $I \subseteq T_{\alpha}$.

3.5.10 Definition

The stacked-ideal I of T_{α} are T_{α} itself and{0} . A stacked-ideal I of T_{α} such that $\{0\} \subset I \subset T_{\alpha}$ (strictly) is called proper-stacked.

3.5.11 Theorem

If any an ideal-stacked I of a stacked-semigroup T_{α} with {0}, then $I = T_{\alpha}$.

Proof :

1- Let $I_{\alpha(r)}$ is a stacked-ideal of the stacked-semigroup (T_{α} , τ) , $| I_r | = r$, and the level stacked -semigroup(lT_{α}) = $\{1,2,\dots,r,\dots n\}$, then $l(x) \in \{1,2,\dots,r\}, \forall x \in I_{\alpha(r)}$, and $lT_{\alpha} (n) = 0 \Rightarrow img(0) = n$, but the stacked-ideal I with { 0 } , and from the definition to the zero element of the stacked-semigroups l(0) = n , since $r = n \Rightarrow I = T_{\alpha}$.
2- Let $I_{\alpha(r)}$ is a stacked-ideal of the stacked-semigroup (T_{α} , Ʈ) , $| I_r | = n - r + 1$, and the level stacked -semigroup(lT_{α}) = $\{1,2,\dots,r,\dots n\}$, then $l(x) \in \{r ,r+1 ,\dots,n \}_{T\alpha} \equiv \{1,2,\dots, n - r + 1\}_I , \forall x \in I_{\alpha(r)}$, and $lT_{\alpha} (1) = 0 \Rightarrow img(0) = 1$, but the stacked-ideal I with { 0 } , and from the definition to the zero element of the stacked-semigroups (T_{α} , Ʈ) ,l(0) = 1 , since $n - r + 1 = 1 \Rightarrow n - r = 0 \Rightarrow n = r \Rightarrow I = T_{\alpha}$.

3.5.12 Theorem

The a stacked-semigroup T_{α} be the proper-stacked .

Proof :

From the Definition 3.5.10 and theorem 3.5.11 it is proofed

3.5.13 Theorem

Let I_{γ} , I_{β} are a stacked-ideals of a stacked-semigroup (T_{α} , τ), $I_{\gamma} \cap I_{\beta} \neq T_{\alpha}$, and $|I_{\gamma}| \le |I_{\beta}|$ then $I_{\gamma} \cap I_{\beta} = I_{\gamma}$, and $I_{\gamma} \cup I_{\beta} = I_{\beta}$.

Proof :

Let $I_{\gamma} = \{ x_1 , x_2 , \dots , x_r \}$, $I_{\beta} = \{ x_1 , x_2 , \dots ,x_s \}$, $\Rightarrow lT_{\alpha} (I_{\gamma}) = \{ 1 , 2 , \dots , r \}$,$lT_{\alpha} (I_{\beta}) = \{ 1 , 2 , \dots , s \}$, and if $|I_{\gamma}| \le |I_{\beta}|$, then $r \le s \Rightarrow lT_{\alpha} (I_{\gamma}) \subseteq l(I_{\beta})$, but $lT_{\alpha} (I_{\gamma}) \subseteq lT_{\alpha}$ and $lT_{\alpha} (I_{\beta}) \subseteq lT_{\alpha} \Rightarrow I_{\gamma} \subseteq I_{\beta} \subseteq T_{\alpha} \Rightarrow I_{\gamma} \cap I_{\beta} = I_{\alpha}$, and $I_{\gamma} \cup I_{\beta} = I_{\beta}$.

3.5.14 Theorem

Let I_φ , I_δ are a stacked-ideals of a stacked-semigroup (T_α , Ʈ) ,$I_\varphi \cap I_\delta \neq T_\alpha$, and $|I_\varphi| \leq |I_\delta|$ then $I_\varphi \cap I_\delta = I_\gamma$, and $I_\varphi \cup I_\delta = I_\delta$.

Proof :

Let $I_\varphi = \{ x_r , x_{r+1} , \dots , x_n \}$, $I_\delta = \{ x_s , x_{s+1} , \dots , x_n \}$, $\Longrightarrow$ $lT_\alpha (I_\varphi) = \{ 1 , 2 , \dots , n - r + 1 \}$,$lT_\alpha (I_\delta) = \{ 1 , 2 , \dots , n - s + 1 \}$, and if $|I_\varphi| \leq |I_\delta|$, then $n - r + 1 \leq n - s + 1 \Longrightarrow r \geq s$, when $I_\varphi = \{ x_n , x_{n-1} , \dots , x_r \}$, and $I_\delta = \{ x_n , x_{n-1} , \dots , x_s \}$,so $lT_\alpha (I_\varphi) \subseteq l(I_\delta)$, but $lT_\alpha (I_\varphi) \subseteq lT_\alpha$ and $lT_\alpha (I_\delta) \subseteq lT_\alpha \Longrightarrow I_\varphi \subseteq I_\delta \subseteq T_\alpha \Longrightarrow I_\varphi \cap I_\delta = I_\varphi$, and $I_\varphi \cup I_\delta = I_\delta$.

3.5.15 Lemma

If I and J are a stacked-ideals of a stacked- semigroup T_α with $I \cap J \neq T$ then $I \cap J$, and $I \cup J$ is a stacked-ideals.

Proof :

I , J are a stacked-ideals of a stacked-semigroup T_α , and $|I| \leq |J|$ or $|J| \leq |I|$, then from the theorem 4.16 and theorem 4.16 , $I \cap J = I$ and $I \cup J = J$ that if and $|I| \leq |J|$ or $I \cap J = J$, and $I \cup J = I$ that if $|J| \leq |I|$. In both cases, true , $I \cap J$, and $I \cup J$ is a stacked-ideals.

3.5.16 Definition

A stacked-ideal I of a stacked-semigroup T_α is said to be minimal-stacked , if for all stacked-ideals J of T_α , $J \subseteq I$ implies that J = I.

3.5.17 Theorem

The only minimal-stacked of a stacked-semigroup T_α is I = { e } .

Proof :

let I is a stacked-ideal of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$ and $I = \{ x_1 \}$, so there is no stacked-ideal $J : J \subseteq I$, just only I = J , and a stacked-ideal I of a stacked-semigroup T_α is said to be minimal-stacked , if for all stacked-ideals J of T_α , $I \subseteq J$ implies that I = J, then the only minimal-stacked of a stacked-semigroup T_α is $I = \{ x_1 \} = \{e\}$.

3.5.18 Lemma

If I is a minimal-stacked ideal, and J is any stacked-ideal of T_α , then $I \subseteq J$

Proof :

The only minimal-stacked of a stacked-semigroup T_α is $I = \{ e \} = \{ x_1 \}$, J is any stacked-ideal of T_α so $\{ x_1 \} \subseteq J \subseteq T_\alpha$,then $I \subseteq J$.

3.5.19 Theorem

Let x_r is element of a stacked-semigroup (T_α , τ) , then $(x_rT_\alpha , \tau) =(T_\alpha x_r ,\tau)$ is a stacked-ideal of T_α , and $(x_rT_\alpha , \tau) =(T_\alpha x_r ,\tau)= I_{\alpha (r)}$

Proof :

Let the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$, hence $x_rT_\alpha = x_r \tau [x_1 , x_2 , \dots , x_r , \dots , x_n] = T_\alpha x_r =\{ x_r \tau x_1 , x_r \tau x_2 , \dots , x_r \tau x_r , x_r \tau x_{r+1} , \dots , x_r \tau x_n \} = [x_1 , x_2 , \dots , x_r , x_r \dots , x_r] = \{ x_1 , x_2 , \dots , x_r \}$, then $[\{ x_1 , x_2 , \dots , x_r \} , \tau] =(I_{\alpha (r)} , \tau)$ is stacked-ideal $\Longrightarrow (x_rT_\alpha , \tau) =(T_\alpha x_r ,\tau) = I_{\alpha (r)}$ and it is stacked-ideal of (T_α , τ) .

3.5.20 Theorem

Let x_s is element of a stacked-semigroup (T_α , Ʈ), then (x_sT_α , Ʈ) =($T_\alpha x_s$, Ʈ) is a stacked-ideal of (T_α , Ʈ) , and (x_sT_α , Ʈ) =($T_\alpha x_s$, Ʈ)= $I_{\alpha (s)}$.

Proof :

Let the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_s , x_{s+1} , \dots , x_n \}$, hence $x_sT_\alpha = x_s$ Ʈ $[x_1 , x_2 , \dots , x_s , x_{s+1} , \dots , x_n] = T_\alpha x_s$ ={ x_s Ʈ x_1 , x_s Ʈ x_2 , … , x_s Ʈ x_s , x_s Ʈ x_{s+1} , … , x_s Ʈ x_n } = $[x_s , x_s , \dots , x_s , x_{s+1} \dots , x_n] = \{ x_s , x_{s+1} , \dots , x_n \}$, then [$\{ x_s , x_{s+1} , \dots , x_n \}$, Ʈ] = ($I_{\alpha (s)}$, Ʈ) is stacked-ideal $\Longrightarrow$ (x_rT_α , Ʈ) =($T_\alpha x_r$, Ʈ)= $I_{\alpha (s)}$, and it is stacked-ideal of (T_α , Ʈ).

3.5.21 Theorem

Let x_rT_α is stacked-ideal of the stacked-semigroup(T_α , τ) then $[x_rT_\alpha]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \}$,

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$, hence $x_rT_\alpha = [x_1 , x_2 , \dots , x_r]$, so $[x_rT_\alpha]^c = T_\alpha - x_rT_\alpha = \{ x_{r+1} , x_{r+2} , \dots x_n \}$.

3.5.22 Theorem

Let x_rT_α is stacked-ideal of the stacked-semigroup(T_α , Ʈ) then $[x_rT_\alpha]^c = \{ x_1 , x_2 , \dots x_{r-1} \}$,

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_{r-1} , x_r , x_{r+1} , \dots , x_n \}$, hence $x_rT_\alpha = [x_r , x_{r+1} , \dots , x_n]$, so $[x_rT_\alpha]^c = T_\alpha - x_rT_\alpha = \{ x_1 , x_2 , \dots x_{r-1} \}$.

3.5.23 Theorem

Let $T_\alpha = \{ x_1 , x_2 , \dots , x_{r-1} , x_r , x_{r+1} , \dots , x_n \}$, is stacked-set , then :

1- $\{[(x_rT_\alpha , \tau)]^c \}$ = { ($x_{r+1} T_\alpha$, Ʈ)} .
2- {[(x_rT_α , Ʈ)]^c } = $\{ (x_{r-1} T_\alpha , \tau)\}$.

Proof:

Let $T_\alpha = \{ x_1 , x_2 , \dots , x_{r-1} , x_r , x_{r+1} , \dots , x_n \}$,

1- From theorem above $[(x_rT_\alpha , \tau)]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \}$, and from theorem above {[(x_rT_α , Ʈ)]} = $\{ x_r , x_{r+1} , \dots , x_n \}$ $\Longrightarrow$ {[($x_{r+1}T_\alpha$, Ʈ)]} = $\{ x_{r+1} , x_{r+2} , \dots , x_n \}$, so $\{[(x_rT_\alpha , \tau)]^c \}$ = { ($x_{r+1} T_\alpha$, Ʈ)} .
2- From theorem above {[(x_rT_α , Ʈ)]^c } = $\{ x_1 , x_2 , \dots x_{r-1} \}$, and from theorem above $\{[(x_rT_\alpha , \tau)]\} = \{ x_1 , x_2 , \dots , x_r \} \Longrightarrow \{ (x_{r-1} T_\alpha , \tau)\} = \{ x_1 , x_2 , \dots x_{r-1} \}$, so {[(x_rT_α , Ʈ)]^c } = $\{ (x_{r-1} T_\alpha , \tau)\}$

3.5.24 Theorem

Let $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$ is stacked-set , and $I_{\alpha (r)}$ is the stacked-ideal of the stacked-semigroup (T_α , τ) then$[I_{\alpha (r)}]^c$ is not , stacked-ideal of (T_α , τ) , and it is not stacked-subsemigroup in (T_α , τ) .

Proof

let $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$ is a stacked-semigroup $I_{\alpha (r)} =\{ x_1 , x_2 , \dots , x_r \}$ is stacked-ideal in T_α , then $[I_{\alpha (r)}]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \} \Longrightarrow \{ x_1 \}= \{ 0_{T_\alpha} \} \nsubseteq [I_{\alpha (r)}]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \} \Longrightarrow x_1 \tau [I_{\alpha(r)}]^c = 0_{T_\alpha} \tau [I_{\alpha(r)}]^c = 0_{T_\alpha} \notin [I_{\alpha(r)}]^c$ then from the definition of the ideal and the stacked-subsemigroups, $[I_{\alpha(r)}]^c$ is not stacked-ideal in T_α , and it is not subsemigroup in T_α .

3.5.25 Theorem

Let $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$ is stacked-set , and $I_{\alpha (r)}$ is the stacked-ideal of the stacked-semigroup (T_α , Ʈ) then$[I_{\alpha (r)}]^c$ is not , stacked-ideal of (T_α , Ʈ) , and it is not stacked-subsemigroup in (T_α , Ʈ) .

Proof

let $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_n \}$ is a stacked-semigroup $I_{\alpha (r)} =\{ x_r , x_{1+1} , \dots , x_n \}$ is stacked-ideal in T_α , then $[I_{\alpha (r)}]^c = \{ x_1 , x_2 , \dots x_{r-1} \}$ Ʈ $\{ x_n \}= \{ 0_{T_\alpha} \}$ Ʈ $[I_{\alpha (r)}]^c = \{ x_1 , x_2 , \dots , x_{r-1} \}$ Ʈ x_n Ʈ $[I_{\alpha (r)}]^c = 0_{T_\alpha}$ Ʈ $[I_{\alpha (r)}]^c = 0_{T_\alpha}$ Ʈ $[I_{\alpha (r)}]^c$ then from the definition of the ideal and the stacked-subsemigroups, $[I_{\alpha (r)}]^c$ is not stacked-ideal in T_α , and it is not subsemigroup in T_α

3.5.26 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup(T_α , τ) = [{ $x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n$ } , τ] , and $r \le s \le n$, then $x_rT_\alpha \cup x_sT_\alpha = x_sT_\alpha$

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n \}$, and $r \le s \le n$,hence $x_rT_\alpha = \{x_1 , x_2 , \dots , x_r \}$, $x_sT_\alpha = \{x_1 , x_2 , \dots , x_r , \dots x_s \}$, then $x_rT_\alpha \cup x_sT_\alpha = \{x_1 , x_2 , \dots , x_r \}\cup \{x_1 , x_2 , \dots , x_r , \dots x_s \} = \{x_1 , x_2 , \dots , x_r , \dots x_s \} = x_sT_\alpha$.

3.5.27 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup(T_α , Ʈ) = [{ $x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n$ } , Ʈ] , and $r \le s \le n$, then $x_rT_\alpha \cup x_sT_\alpha = x_rT_\alpha$

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n \}$, and $r \le s \le n$,hence $x_rT_\alpha = \{x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \}$, $x_sT_\alpha = \{ x_s , x_{s+1} , \dots , x_n \}$, then $x_rT_\alpha \cup x_sT_\alpha = \{ x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \} \cup \{ x_s , x_{s+1} , \dots , x_n \} = \{ x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \} = x_rT_\alpha$.

3.5.28 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup(T_α , τ) = [{ $x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n$ } , τ] , and $r \le s \le n$, then $x_rT_\alpha \cap x_sT_\alpha = x_rT_\alpha$

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n \}$, and $r \le s \le n$,hence $x_rT_\alpha = \{x_1 , x_2 , \dots , x_r \}$, $x_sT_\alpha = \{x_1 , x_2 , \dots , x_r , \dots x_s \}$, then $x_rT_\alpha \cap x_sT_\alpha = \{x_1 , x_2 , \dots , x_r \}\cap \{x_1 , x_2 , \dots , x_r , \dots x_s \} = \{x_1 , x_2 , \dots , x_r , \dots x_s \} = x_rT_\alpha$.

3.5.29 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup(T_α , Ʈ) = [{ $x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n$ } , Ʈ] , and $r \le s \le n$, then $x_rT_\alpha \cap x_sT_\alpha = x_sT_\alpha$

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots , x_s ,\dots , x_n \}$, and $r \le s \le n$,hence $x_rT_\alpha = \{x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \}$, $x_sT_\alpha = \{ x_s , x_{s+1} , \dots , x_n \}$, then $x_rT_\alpha \cap x_sT_\alpha = \{ x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \} \cap \{ x_s , x_{s+1} , \dots , x_n \} = \{ x_r , x_{r+1} , \dots , x_s ,x_{s+1} ,\dots , x_n \} = x_sT_\alpha$.

3.5.30 Theorem

Let (T_α , τ) is stacked-semigroup ,$T_\alpha = \{ x_1 , x_2 , \dots, x_n \}$, then $x_1T_\alpha =\{ x_1\}$, $x_2T_\alpha = \{ x_1 , x_2 \}$,……, $x_rT_\alpha = \{ x_1 , x_2 , \dots, x_r \}$, $x_nT_\alpha = T_\alpha$.

Proof:

From the theorems it is easy to proof it : $x_1T_\alpha = I_{\alpha (1)}$, $x_2T_\alpha =I_{\alpha (2)}$, … , $x_rT_\alpha = I_{\alpha (r)}$, …. , $x_nT_\alpha = I_{\alpha (n)} = T_\alpha$.

3.5.31 Theorem

Let (T_α , Ʈ) is stacked-semigroup ,$T_\alpha = \{ x_1 , x_2 , \dots, x_n \}$, then $x_nT_\alpha =\{ x_n\}$, $x_{n-1}T_\alpha = \{ x_{n-1} , x_n \}$,……, $x_rT_\alpha = \{ x_r , x_{r+1} , \dots, x_n \}$, $x_1T_\alpha = T_\alpha$.

Proof:

From the theorems it is easy to proof it : $x_nT_\alpha = I_{\alpha (n)}$, $x_{n-1}T_\alpha =I_{\alpha (n-1)}$, … , $x_rT_\alpha = I_{\alpha (r)}$, …. , $x_1T_\alpha = I_{\alpha (1)} = T_\alpha$.

3.5.32 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup (T_α , τ) = $\{ x_1 , x_2 , \dots , x_r , \dots, x_n \}$, then $[x_rT_\alpha]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \}$ is a new stacked-semigroup .

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots, x_n \}$, hence $[x_rT_\alpha]^c = \{ x_{r+1} , x_{r+2} , \dots x_n \}$.

(1) $\forall\, x_t , x_s \in T_\alpha$, $n \ge t \ge r$, and $n \ge s \ge r \Longrightarrow x_t , x_s \in [x_rT_\alpha]^c$, hence $x_t\, \tau^1\, x_s =(x_t \vee_t x_s) \in [x_rT_\alpha]^c$, then τ^1 is a binary operation , so : $[x_rT_\alpha]^c \times [x_rT_\alpha]^c \rightarrow [x_rT_\alpha]^c$.

(2) $\forall\, x_m \in [x_rT_\alpha]^c$, $x_m\, \tau^1\, (x_t\, \tau^1\, x_s) = x_m \vee_t (x_t \vee_t x_s) =(x_m \vee_t x_t) \vee_t x_s =(x_m\, \tau^1\, x_t)\, \tau^1\, x_s \Longrightarrow ([x_rT_\alpha]^c , \tau^1)$ is associative system , from (1) and (2) $[x_rT_\alpha]^c =(\{ x_{r+1} , x_{r+2} , \dots x_n \} , \tau^1)$ is a new stacked-semigroup .

3.5.33 Theorem

Let the stacked-ideal x_rT_α of the stacked-semigroup (T_α , Ʈ) = $\{ x_1 , x_2 , \dots , x_r , \dots, x_n \}$, then $[x_rT_\alpha]^c = \{ x_1 , x_2 , \dots x_{r-1} \}$ is a new stacked-semigroup .

Proof :

Let the stacked-ideal x_rT_α of the stacked-semigroup $T_\alpha = \{ x_1 , x_2 , \dots , x_r , \dots, x_n \}$, hence $[x_rT_\alpha]^c = \{ x_1 , x_2 , \dots, x_{r-1} \}$.

(1) $\forall x_t , x_s \in T_\alpha$, $r > t \geq 1$, and $r > s \geq 1 \Longrightarrow x_t , x_s \in [x_r T_\alpha]^c$, hence x_t $Ʈ^1$ $x_s = (x_t \vee_t x_s) \in [x_r T_\alpha]^c$, then $Ʈ^1$ is a binary operation , so : $[x_r T_\alpha]^c \times [x_r T_\alpha]^c \rightarrow [x_r T_\alpha]^c$.
(2) $\forall x_m \in [x_r T_\alpha]^c$, x_m $Ʈ^1$ $(x_t$ $Ʈ^1$ $x_s) = x_m \vee_t (x_t \vee_t x_s) = (x_m \vee_t x_t) \vee_t x_s = (x_m$ $Ʈ^1$ $x_t)$ $Ʈ^1$ x_s Ʈ ($[x_r T_\alpha]^c$, $Ʈ^1$) is associative system , from (1) and (2) $[x_r T_\alpha]^c = (\{ x_{r+1} , x_{r+2} , \dots x_n \} , Ʈ^1)$ is a new stacked-semigroup .

3.5.34 Theorem

Let $x_s T_\alpha$ and $x_t T_\alpha$ are tow stacked-ideal of stacked-semigroup (T_α , τ) , if $s \geq t$ then $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha \cap [x_t T_\alpha]^c$, and it is not stacked-ideal on (T_α , τ) , but it is a new stacked-semigroup.

Proof :

1- Let $x_s T_\alpha$ and $x_t T_\alpha$ are tow stacked-ideal of stacked-semigroup (T_α , τ), if $s \geq t$ then $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha - x_t T_\alpha = [[\{ x_1 , x_2 , \dots , x_t , \dots , x_s \} - \{ x_1 , x_2 , \dots , x_t \}] , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau]$ (i). And hence $x_t T_\alpha = \{ x_1 , x_2 , \dots , x_t \}$ so $[x_t T_\alpha]^c = \{ x_{t+1} , \dots , x_s , \dots , x_n \}$, then $x_s T_\alpha \cap [x_t T_\alpha]^c = [[\{ x_1 , x_2 , \dots , x_t , x_{t+1} , \dots , x_s \} \cap \{ x_{t+1} , \dots , x_s , \dots , x_n \}] , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau]$ (ii) . from (i) and (ii) $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha \cap [x_t T_\alpha]^c$

2- Let $x_m \in (T_\alpha , \tau)$, and $x_m \notin [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau]$ when $m \leq t < s$, then $x_m \tau \{ x_{t+1} , x_{t+2} , \dots , x_s \} \notin [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau]$, and from definition of the stacked-ideals $[\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau]$is not stacked-ideal of stacked-semigroup (T_α , τ).

3- $\forall x_a , x_b \in [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1]$, $x_a \tau^1 x_b = (x_a \vee_t x_b) \in [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1]$, then τ^1 is a binary operation , so : $[\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1] \times [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1] \rightarrow [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1]$ (i). $\forall x_a , x_b , x_c \in \{ x_{t+1} , x_{t+2} , \dots , x_s \}$, $x_a \tau^1 (x_b \tau^1 x_c) = x_a \vee_t (x_b \vee_t x_c) = (x_a \vee_t x_b) \vee_t x_c = (x_a \tau^1 x_b) \tau^1 x_c \Longrightarrow (\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1)$ is associative system(ii). From (i) and (ii) $[\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1]$ is a new stacked-semigroup .

3.5.35 Theorem

Let $x_s T_\alpha$ and $x_t T_\alpha$ are tow stacked-ideal of stacked-semigroup (T_α , Ʈ) , if $s \leq t$ then $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha \cap [x_t T_\alpha]^c$, and it is not stacked-ideal on (T_α , Ʈ) , but it is a new stacked-semigroup.

Proof :

1- Let $x_s T_\alpha$ and $x_t T_\alpha$ are tow stacked-ideal of stacked-semigroup (T_α , Ʈ), if $s \leq t$ then $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha - x_t T_\alpha = [[\{ x_n , x_{n-1} , \dots , x_t , x_{t-1} , \dots , x_s \} - \{ x_n , x_{n-1} , \dots , x_t \}]$, Ʈ] = $[\{ x_{t-1} , x_{t-2} , \dots , x_s \}$, Ʈ] (i). And hence $x_t T_\alpha = \{ x_n , x_{n-1} , \dots , x_t \}$ so $[x_t T_\alpha]^c = \{ x_{t-1} , \dots , x_s , \dots , x_n \}$, then $x_s T_\alpha \cap [x_t T_\alpha]^c = [[\{ x_n , x_{n-1} , \dots , x_t , x_{t-1} , \dots , x_s \} - \{ x_{t-1} , x_{t-2} , \dots , x_s , \dots , x_n \}]$, Ʈ] = $[\{ x_{t-1} , x_{t-2} , \dots , x_s \}$, Ʈ] (ii) . from (i) and (ii) $x_s T_\alpha \setminus x_t T_\alpha = x_s T_\alpha \cap [x_t T_\alpha]^c$

2- Let $x_m \in$ (T_α , Ʈ), and $x_m \notin [\{ x_{t-1} , x_{t-2} , \dots , x_s \}$, Ʈ] when $m \geq t > s$, then x_m Ʈ $\{ x_{t-1} , x_{t-2} , \dots , x_s \} \notin [\{ x_{t-1} , x_{t-2} , \dots , x_s \}$, Ʈ] , and from definition of the stacked-ideals $[\{ x_{t+1} , x_{t+2} , \dots , x_s \}$, Ʈ]is not stacked-ideal of stacked-semigroup (T_α , Ʈ).

3- $\forall x_a , x_b \in [\{ x_{t-1} , x_{t-2} , \dots , x_s \} , Ʈ^1]$, x_a $Ʈ^1$ $x_b = (x_a \vee_t x_b) \in [\{ x_{t-1} , x_{t-2} , \dots , x_s \} , Ʈ^1]$, then $Ʈ^1$ is a binary operation , so : $[\{ x_{t+1} , x_{t+2} , \dots , x_s \} , Ʈ^1] \times [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , Ʈ^1] \rightarrow [\{ x_{t+1} , x_{t+2} , \dots , x_s \} , Ʈ^1]$ (i). $\forall x_a , x_b , x_c \in \{ x_{t+1} , x_{t+2} , \dots , x_s \}$, x_a $Ʈ^1$ $(x_b$ $Ʈ^1$ $x_c) = x_a \vee_t (x_b \vee_t x_c) = (x_a \vee_t x_b) \vee_t x_c = (x_a$ $Ʈ^1$ $x_b)$ $Ʈ^1$ x_c Ʈ $(\{ x_{t+1} , x_{t+2} , \dots , x_s \} , \tau^1)$ is associative system(ii). From the definition of the semigroups , (i) and (ii) $[\{ x_{t+1} , x_{t+2} , \dots , x_s \} , Ʈ^1]$ is a new stacked-semigroup .

3.5.36 Theorem

Let $x_r T_\alpha$ is stacked-ideal of stacked-semigroup(T_α , τ) then$[[x_r T_\alpha]^c]^c = x_r T_\alpha$.

Proof :

Let $x_r T_\alpha$ is stacked-ideal of stacked-semigroup(T_α , τ)= $[\{ x_1 , x_2 , \dots , x_r , x_{r+1} , \dots , x_n \} , \tau]$,then $x_r T_\alpha = [\{ x_1 , x_2 , \dots , x_r \} , \tau] \Longrightarrow [x_r T_\alpha]^c = [\{ x_{r+1} , x_{r+2} , \dots , x_n \} , \tau] \Longrightarrow [[x_r T_\alpha]^c]^c = [\{ x_1 , x_2 , \dots , x_r \} , \tau] \Longrightarrow [[x_r T_\alpha]^c]^c = x_r T_\alpha$.

3.5.37 Theorem

Let $x_r T_\alpha$ is stacked-ideal of stacked-semigroup(T_α , Ʈ) then$[[x_r T_\alpha]^c]^c = x_r T_\alpha$.

Proof :

Let $x_r T_\alpha$ is stacked-ideal of stacked-semigroup(T_α , Ʈ)= $[\{ x_1 , x_2 , \dots , x_{r-1} , x_r , x_{r+1} , \dots , x_n \}$, Ʈ] ,then $x_r T_\alpha = [\{ x_r , x_{r+1} , \dots , x_n \}$, Ʈ] $\Longrightarrow [x_r T_\alpha]^c = [\{ x_1 , x_2 , \dots , x_{r-1} \}$, Ʈ]$\Longrightarrow [[x_r T_\alpha]^c]^c = [\{ x_r , x_{r+1} , \dots , x_n \}$, Ʈ] $\Longrightarrow [[x_r T_\alpha]^c]^c = x_r T_\alpha$.

3.5.38 Theorem

Let $x_r T_\alpha$, $x_s T_\alpha$ and $x_t T_v$ are stacked-ideal of stacked-semigroup (T_α , τ) , if $s \geq t$, then $[x_s T_\alpha \setminus x_t T_\alpha] \cap x_r T_\alpha = [x_s T_\alpha \cap x_r T_\alpha] \setminus [x_t T_\alpha \cap x_r T_\alpha]$.

Proof :

Let $x_r T_\alpha$, $x_s T_\alpha$ and $x_t T_\alpha$ are stacked-ideal of stacked-semigroup (T_α , τ), if $s \geq t$ then we have three conditions , (1) $r \leq t$, (2) $t < r < s$, and (3) $r \geq s$.

(1) If $r \leq t$, then $x_r T_\alpha = [\{ x_1 , x_2 , \dots , x_r \} , \tau]$, $x_s T_\alpha = [\{ x_1 , x_2 , \dots ,x_r , \dots , x_t , \dots , x_s \}, \tau]$, and $x_t T_\alpha = [\{ x_1 , x_2 , \dots , x_r , \dots , x_t \}$. So $[x_s T_\alpha \setminus x_t T_\alpha] \cap x_r T_\alpha = [x_s T_\alpha - x_t T_\alpha] \cap x_r T_\alpha = [[\{ x_1 , x_2 , \dots ,x_r , \dots , x_t , \dots , x_s \} - \{ x_1 , x_2 , \dots , x_r , \dots , x_t \}] , \tau] \cap [\{ x_1 , x_2 , \dots , x_r \} , \tau] = \emptyset$ (i).

And $[x_s T_\alpha \cap x_r T_\alpha] \setminus [x_t T_\alpha \cap x_r T_\alpha] = [[\{ x_1 , x_2 , \dots ,x_r , \dots , x_t , \dots , x_s \} \cap \{ x_1 , x_2 , \dots , x_r\}] , \tau]- [[\{ x_1 , x_2 , \dots ,x_r , \dots , x_t \} \cap \{ x_1 , x_2 , \dots , x_r \}]= [\{ x_1 , x_2 , \dots , x_r\} , \tau] - [\{ x_1 , x_2 , \dots , x_r\} , \tau] = \emptyset$ (ii).

From (i) and (ii): $[x_s T_\alpha \setminus x_t T_\alpha] \cap x_r T_\alpha = [x_s T_\alpha \cap x_r T_\alpha] \setminus [x_t T_\alpha \cap x_r T_\alpha]$.

(2) If $s < r < t$, then $x_r T_\alpha = [\{ x_1 , x_2 , \dots , x_t , \dots , x_r \} , \tau]$, $x_s T_\alpha =[\{ x_1 , x_2 , \dots ,x_t , \dots , x_r , \dots , x_s \}, \tau]$, and $x_t T_\alpha = [\{ x_1 , x_2 , \dots , x_t \},\tau]$. So $[x_s T_\alpha \setminus x_t T_\alpha] \cap x_r T_\alpha =[x_s T_\alpha - x_t T_\alpha] \cap x_r T_\alpha = [[\{ x_1 , x_2 , \dots ,x_t , \dots , x_r , \dots ,$

$x_s \} - \{ x_1 , x_2 , \dots , x_t \}] , \tau] \cap [\{ x_1 , x_2 , \dots , x_t , \dots , x_r\} , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_r\} , \tau]$ (i) .

And $[x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]= [[\{ x_1 , x_2 , \dots ,x_t , \dots , x_r , \dots , x_s \} \cap \{ x_1 , x_2 , \dots , x_t , \dots , x_r\}] , \tau]- [[\{ x_1 , x_2 , \dots , x_t \} \cap \{ x_1 , x_2 , \dots , x_t , \dots , x_r\}]= [\{ x_1 , x_2 , \dots , x_t , \dots , x_r\}, \tau] - [\{ x_1 , x_2 , \dots , x_t\} , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_r\} , \tau]$ (ii) .

From (i) and (ii): $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

(3) If $r \geq s$, , then $x_rT_\alpha = [\{ x_1 , x_2 , \dots , x_t , \dots , x_s , \dots , x_r \} , \tau]$, $x_sT_\alpha =[\{ x_1 , x_2 , \dots ,x_t , \dots , x_s \}, \tau]$, and $x_tT_\alpha = [\{ x_1 , x_2 , \dots , x_t \}$. So $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha =[x_sT_\alpha - x_tT_\alpha] \cap x_rT_\alpha = [[\{ x_1 , x_2 , \dots ,x_t , \dots , x_s \} - \{ x_1 , x_2 , \dots , x_t \}] , \tau] \cap [\{ x_1 , x_2 , \dots , x_t , \dots , x_s , \dots , x_r\} , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_s\} , \tau]$ (i) .

And $[x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]= [[\{ x_1 , x_2 , \dots ,x_t , \dots , x_s \} \cap \{ x_1 , x_2 , \dots , x_t , \dots , x_s , \dots , x_r\}] , \tau]- [[\{ x_1 , x_2 , \dots , x_t \} \cap \{ x_1 , x_2 , \dots , x_t , \dots , x_s , \dots , x_r\}]= [\{ x_1 , x_2 , \dots , x_t , \dots , x_s\} \tau] - [\{ x_1 , x_2 , \dots , x_t\} , \tau] = [\{ x_{t+1} , x_{t+2} , \dots , x_s\} , \tau]$ (ii) .

From (i) and (ii): $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

3.5.39 Theorem

Let x_rT_α , x_sT_α and x_tT_α are stacked-ideal of stacked-semigroup (T_α , Ţ) , if $s \leq t$, then $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

Proof :

Let x_rT_α , x_sT_α and x_tT_α are stacked-ideal of stacked-semigroup (T_α , Ţ), if $s \leq t$ then we have three conditions , (1) $r \geq t$, (2) $t > r > s$, and (3) $r \leq s$.

(1) If $r \geq t$, then $x_rT_\alpha = [\{ x_n , x_{n-1} , \dots , x_r \} ,$ Ţ $]$, $x_sT_\alpha =[\{ x_n , x_{n-1} ,\dots ,x_r , \dots , x_t , \dots , x_s \},$ Ţ $]$, and $x_tT_\alpha = [\{ x_n , x_{n-1} , \dots , x_r , \dots , x_t \},$ Ţ$]$. So $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha =[x_sT_\alpha - x_tT_\alpha] \cap x_rT_\alpha = [[\{ x_n , x_{n-1} , \dots ,x_r , \dots , x_t , \dots , x_s \} - \{ x_n , x_{n-1} , \dots , x_r , \dots , x_t \}] ,$ Ţ $] \cap [\{ x_n , x_{n-1} , \dots , x_r \} ,$ Ţ $] = \emptyset$ (i).

And $[x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]= [[\{ x_n , x_{n-1} , \dots ,x_r , \dots , x_t , \dots , x_s \} \cap \{ x_n , x_{n-1} , \dots , x_r\}] ,$ Ţ $]- [[\{ x_n , x_{n-1} , \dots ,x_r , \dots , x_t \} \cap \{ x_n , x_{n-1} , \dots , x_r \}],$ Ţ $] = [\{ x_n , x_{n-1} , \dots , x_r\} ,$ Ţ $] - [\{ x_n , x_{n-1} , \dots , x_r\} ,$ Ţ $] = \emptyset$ (ii).

From (i) and (ii): $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

(2) If $s \leq r \leq t$, then $x_rT_\alpha = [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_r \} ,$ Ţ $]$, $x_sT_\alpha =[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_r , \dots , x_s \},$ Ţ $]$, and $x_tT_\alpha = [\{ x_n , x_{n-1} , \dots , x_t \},$ Ţ$]$. So $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha =[x_sT_\alpha - x_tT_\alpha] \cap x_rT_\alpha = [[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_r , \dots , x_s \} - \{ x_n , x_{2n-1} , \dots , x_t \}] ,$ Ţ $] \cap [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_r\} ,$ Ţ$] = [\{ x_{t+1} , x_{t+2} , \dots , x_r\} ,$ Ţ $]$ (i) .

And $[x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]= [[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_r , \dots , x_s \} \cap \{ x_n , x_{n-1} , \dots , x_t , \dots , x_r\}] ,$ Ţ $]- [[\{ x_n , x_{n-1} , \dots , x_t \} \cap \{ x_n , x_{n-1} , \dots , x_t , \dots , x_r\},$ Ţ$]= [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_r\},$ Ţ $] - [\{ x_n , x_{n-1} , \dots , x_t\} ,$ Ţ $] = [\{ x_{t+1} , x_{t+2} , \dots , x_r\} ,$ Ţ $]$ (ii) .

From (i) and (ii): $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

(3) If $r \leq s$, , then $x_rT_\alpha = [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_s , \dots , x_r \} ,$ Ţ $]$, $x_sT_\alpha =[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_s \},$ Ţ $]$, and $x_tT_\alpha = [\{ x_n , x_{n-1} , \dots , x_t \},$ Ţ$]$. So $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha =[x_sT_\alpha - x_tT_\alpha] \cap x_rT_\alpha = [[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_s \} - \{ x_n , x_{n-1} , \dots , x_t \}] ,$ Ţ $] \cap [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_s , \dots , x_r\} ,$ Ţ $] = [\{ x_{t+1} , x_{t+2} , \dots , x_s\} ,$ Ţ $]$ (i) .

And $[x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]= [[\{ x_n , x_{n-1} , \dots ,x_t , \dots , x_s \} \cap \{ x_n , x_{n-1} , \dots , x_t , \dots , x_s , \dots , x_r\}] ,$ Ţ $]- [[\{ x_n , x_{n-1} , \dots , x_t \} \cap \{ x_n , x_{n-1} , \dots , x_t , \dots , x_s , \dots , x_r\} ,$ Ţ$]= [\{ x_n , x_{n-1} , \dots , x_t , \dots , x_s\} ,$ Ţ $] - [\{ x_n , x_{n-1} , \dots , x_t\} ,$ Ţ $] = [\{ x_{t+1} , x_{t+2} , \dots , x_s\} ,$ Ţ $]$ (ii) .

From (i) and (ii): $[x_sT_\alpha \setminus x_tT_\alpha] \cap x_rT_\alpha = [x_sT_\alpha \cap x_rT_\alpha] \setminus [x_tT_\alpha \cap x_rT_\alpha]$.

3.5.40 Theorem

Let x_sT_α and x_tT_α are tow stacked-ideal of stacked-semigroup(T_α , τ) ,and $s \geq t$, then:
$[x_sT_\alpha \cap x_tT_\alpha]^c = [x_sT_\alpha]^c \cup [x_tT_\alpha]^c$.

Proof :

Let x_sT_α and x_tT_α are stacked-ideal of stacked-semigroup (T_α , τ), if $s \geq t$,then $x_sT_\alpha =[\{ x_1 , x_2 , \dots , x_t , \dots , x_s \}, \tau]$, and $x_tT_\alpha = [\{ x_1 , x_2 , \dots , x_t \} , \tau]$. And $[x_sT_\alpha \cap x_tT_\alpha]=[[\{ x_1 , x_2 , \dots , x_t , \dots , x_s \}, \tau] \cap [\{ x_1 , x_2 , \dots , x_t \}, \tau]] = [\{ x_1 , x_2 ,\dots , x_t \}, \tau]$, so $[x_sT_\alpha \cap x_tT_\alpha]^c = [\{ x_{t+1} , x_{t+2} , \dots , x_s , \dots , x_n \}, \tau]$, it is a new stacked-semigroup (i) . Then $[x_sT_\alpha]^c = [\{x_{s+1} , x_{s+2} , \dots , x_n \}, \tau]$, $[x_tT_\alpha]^c= [\{ x_{t+1} , x_{t+2} , \dots , x_s , \dots , x_n \}, \tau]$, so $[x_sT_\alpha]^c \cup [x_tT_\alpha]^c= [\{ x_{t+1} , x_{t+2} , \dots , x_s , \dots , x_n \}, \tau]$, it is a new stacked-semigroup (ii), from (i) and (ii) $[x_sT_\alpha \cap x_tT_\alpha]^c = [x_sT_\alpha]^c \cup [x_tT_\alpha]^c$.

3.5.41 Theorem

Let x_sT_α and x_tT_α are tow stacked-ideal of stacked-semigroup(T_α , Ţ) ,and $t \geq s$, then:
$[x_sT_\alpha \cap x_tT_\alpha]^c = [x_sT_\alpha]^c \cup [x_tT_\alpha]^c$.

Proof :

Let x_sT_v and x_tT_α are stacked-ideal of stacked-semigroup (T_α , Ţ), if $t \geq s$,then $x_sT_\alpha =[\{ x_n , x_{n-1} , \dots , x_t , \dots , x_s \},$ Ţ$]$, and $x_tT_\alpha = [\{ x_n , x_{n-1} , \dots , x_t \} ,$ Ţ $]$. And $[x_sT_\alpha \cap x_tT_\alpha]=[[\{ x_n , x_{n-1} , \dots , x_t , \dots , x_s \},$ Ţ$] \cap [\{ x_n , x_{n-1} , \dots , x_t \},$ Ţ $]] = [\{ x_n , x_{n-1} ,\dots , x_t \},$ Ţ $]$, so $[x_sT_\alpha \cap x_tT_\alpha]^c = [\{ x_{t-1} , x_{t-2} , \dots , x_s , \dots , x_1 \},$ Ţ $]$, it is a new stacked-semigroup (i) . And $[x_sT_\alpha]^c = [\{x_{s-1} , x_{s-2} , \dots , x_1 \},$ Ţ $]$, $[x_tT_\alpha]^c= [\{ x_{t-1} , x_{t-2} , \dots , x_s , \dots , x_1 \},$ Ţ $]$, so $[x_sT_\alpha]^c \cup [x_tT_\alpha]^c= [\{ x_{t-1} , x_{t-2} , \dots , x_s , \dots , x_1 \},$ Ţ $]$, it is a new stacked-semigroup (ii), from (i) and (ii) $[x_sT_\alpha \cap x_tT_\alpha]^c = [x_sT_\alpha]^c \cup [x_tT_\alpha]^c$.

3.5.42 Theorem

Let x_sT_α and x_tT_α are tow stacked-ideal of stacked-semigroup (T_α , τ) , and $s \geq t$, then :
$[x_sT_\alpha \cup x_tT_\alpha]^c = [x_sT_\alpha]^c \cap [x_tT_\alpha]^c$.

Proof :

Let x_sT_α and x_tT_α are stacked-ideal of stacked-semigroup (T_α , τ), if $s \geq t$,then $x_sT_\alpha =[\{ x_1 , x_2 , \dots , x_t , \dots , x_s \}, \tau]$, and $x_tT_\alpha = [\{ x_1 , x_2 , \dots , x_t \} , \tau]$. And $[x_sT_\alpha \cup x_tT_\alpha]=[[\{ x_1 , x_2 , \dots , x_t ,$

$\dots , x_s \}, \tau] \cup [\{ x_1 , x_2 , \dots , x_t \}, \tau]] = [\{ x_1 , x_2 ,\dots , x_t ,\dots , x_s \}, \tau]$, so $[x_sT_\alpha \cup x_tT_\alpha]^c = [\{ x_{s+1} , x_{s+2} , \dots , x_n \}, \tau]$, it is a new stacked-semigroup (i) . And $[x_sT_\alpha]^c = [\{x_{s+1} , x_{s+2} , \dots , x_n \}, \tau]$, $[x_tT_\alpha]^c= [\{ x_{t+1} , x_{t+2} , \dots , x_s , \dots , x_n \}, \tau]$, so $[x_sT_\alpha]^c \cap [x_tT_\alpha]^c= [\{ x_{s+1} , x_{s+2} , \dots , x_n \}, \tau]$, it is a new stacked-semigroup (ii), from (i) and (ii) $[x_sT_\alpha \cup x_tT_\alpha]^c = [x_sT_\alpha]^c \cap [x_tT_\alpha]^c$.

3.5.43 Theorem

Let x_sT_α and x_tT_α are tow stacked-ideal of stacked-semigroup (T_α , Ʈ) , and $s \geq t$, then :
$[x_sT_\alpha \cup x_tT_\alpha]^c = [x_sT_\alpha]^c \cap [x_tT_\alpha]^c$.

Proof :

Let x_sT_α and x_tT_α are stacked-ideal of stacked-semigroup (T_α , Ʈ), if $t \geq s$,then x_sT_α =[{ $x_n , x_{n-1} , \dots , x_t , \dots , x_s$ }, Ʈ], and x_tT_α = [{ $x_n , x_{n-1} , \dots , x_t$ } , Ʈ] . And [$x_sT_\alpha \cup x_tT_\alpha$]=[[{ $x_n , x_{n-1} , \dots , x_t , \dots , x_s$ }, Ʈ] ∪ [{ $x_n , x_{n-1} , \dots , x_t$ }, Ʈ]] = [{ $x_n , x_{n-1} ,\dots , x_t ,\dots , x_s$ }, Ʈ], so $[x_sT_\alpha \cup x_tT_\alpha]^c$ = [{ $x_{s-1} , x_{s-2} , \dots , x_1$ }, Ʈ], it is a new stacked-semigroup (i) . And $[x_sT_\alpha]^c$ = [{$x_{s-1} , x_{s-2} , \dots , x_1$ }, Ʈ], [$x_tT_\alpha]^c$= [{ $x_{t-1} , x_{t-2} , \dots , x_s , \dots , x_1$ }, Ʈ], so $[x_sT_\alpha]^c \cap [x_tT_\alpha]^c$= [{ $x_{s-1} , x_{s-2} , \dots , x_1$ }, Ʈ], it is a new stacked-semigroup (ii), from (i) and (ii) $[x_sT_\alpha \cup x_tT_\alpha]^c = [x_sT_\alpha]^c \cap [x_tT_\alpha]^c$.

3.6 mapping on stacked-semigroups

3.6.1 Definition

Let T_α and T_η be two stacked-semigroups. A mapping $\upsilon : T_\alpha \to T_\eta$ is a stacked-homomorphism, if

$\forall\, x , y \in T_\alpha : \upsilon(x\, \tau\, y) = \upsilon(x)\, \tau_1\, \upsilon(y) \in T_\eta$.
Or
$\forall\, x , y \in T_\alpha$: φ(x Ʈ y) = φ(x) $Ʈ_1$ φ(y) ∈ T_η .

3.6.2 Theorem

Let (T_α , τ) is a stacked-semigroup, then (lT_α , τ) , and (lT_α , Ʈ) are stacked-semigroups .

Proof :

Let (T_α , τ) is a stacked-semigroup , then $T_\alpha = \{ x_1 , x_2 , \dots , x_n \}$,and $lT_\alpha = \{ 1 , 2 , \dots , n \} = \{ l(x_1) , l(x_2) , \dots , l(x_n) \}$ .And (T_α , Ʈ) is a stacked-semigroup , then $T_\alpha = \{ x_n , x_{n-1} , \dots , x_1 \}$,and $lT_\alpha = \{ n , n-1 , \dots , 1 \} = \{ l(x_n) , l(x_{n-1}) , \dots , l(x_1) \}$. From definition of the level-staked , we have a corresponding between the elements (T_α and lT_α) and they have a same staking , Then (lT_α , τ) and (lT_α , Ʈ) are stacked-semigroups .

3.6.3 Theorem

Let (T_α , Ʈ) is a stacked-semigroups , and (lT_α , $Ʈ_1$) is the level stacked-semigroups . A mapping $L : T_\alpha \to lT_\alpha$ is a stacked-homomorphism .

Proof:

Let $x , y \in T_\alpha$ and ⌈ x ⌋ > ⌈ y ⌋, and ⌈ x_γ ⌋ = $\left[\sum_{i=1}^{\alpha} \frac{|x_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |x_{\gamma i} - t|}\right] / \alpha$., $\alpha \in N/0$. L[$\max_t$(x , y)] = L(x Ʈ y) = L(x $\vee_t$ y) = L(x) = max [L(x) , L(y)]= L(x) $Ʈ_1$ L(y), hence L(x Ʈ y)= L(x) $Ʈ_1$ L(y) . ($Ʈ_1$[a , b] = max[a , b]) , then A mapping $L : T_\alpha \to lT_\alpha$ is a stacked-homomorphism .

3.6.4 Theorem

Let (T_α , τ) is a stacked-semigroups , and (lT_α , τ_1) is the level stacked-semigroups . A mapping $l : T_\alpha \to lT_\alpha$ is a stacked-homomorphism .

Proof:

Let $x , y \in T_\alpha$ and ⌈ x ⌋ < ⌈ y ⌋ , and ⌈ x_γ ⌋ = $\left[\sum_{i=1}^{\alpha} \frac{|x_{\gamma i} - t|}{\sum_{i=1}^{\alpha} |x_{\gamma i} - t|}\right] / \alpha$., $\alpha \in N/0$.
l[$\min_t$(x , y)] = $l\,(x\, \tau\, y) = l\,(x \vee_t y) = l\,(x)$ = min [l (x) , l (y)]= $l\,(x)\, \tau_1\, l\,(y)$, hence $l\,(x\, \tau\, y)= l\,(x)\, \tau_1\, l\,(y)$. ($\tau_1$ [a , b] = min[a , b]), then A mapping $l : T_\alpha \to lT_\alpha$ is a stacked-homomorphism .

3.6.5 Definition

The stacked-homomorphism is an embedding or a monomorphism , denoted $\alpha : T_\alpha \to T_{\alpha 1}$, if it is injective, that is, if $\beta(x) = \beta(y)$ implies x = y.

3.6.6 Theorem

The stacked-homomorphism is denoted $\upsilon : T_\alpha \to lT_\alpha$ or $\varphi : T_\alpha \to lT_\alpha$ an embedding or a monomorphism if it is type-1 .

Proof :

From the theorem 4.4: if the system is type-1 , and ⌈ a_α ⌋ = ⌈ b_β ⌋ , then a = b . and from definition above , The stacked-homomorphism is denoted $l : T_\alpha \to lT_\alpha$ or $L : T_\alpha \to lT_\alpha$ is injective , so it is an embedding or a monomorphism .

3.6.7 Definition

The stacked-homomorphism is an epimorphism , denoted $\beta : T_\alpha \to T_{\alpha 1}$, if it is surjective , that is, if for all $y \in T_{\alpha 1}$,there exists $x \in T_\alpha$ with α(x) = y.

3.6.8 Theorem

The stacked-homomorphism denoted $l : T_\alpha \to lT_\alpha$,or $L : T_\alpha \to lT_\alpha$ is an epimorphism .

Proof :

Let T_α is a stacked-semigroup ⟹ (T_α , τ) = { $x_1 , x_2 , \dots , x_n$ } or (T_α , Ʈ) = { $x_n , x_{n-1} , \dots , x_1$ } ,and $|T_\alpha| = n$, then the level-stacked semigroup(lT_α ,τ) = { $l\,(x_1) , l(x_2) , \dots , l(x_n)$ } or (lT_α , Ʈ) = { $L(x_n) , L(x_{n-1}) , \dots , L(x_1)$ } and $|lT_\alpha| = n$.
and when $l : T_\alpha \to lT_\alpha$ or $L : T_\alpha \to lT_\alpha$ so :

$x_1 \to l(x_1) = 1$ $\qquad$ $x_n \to L(x_n) = n$
$x_2 \to l(x_2) = 2$ $\qquad$ $x_{n-1} \to L(x_{n-1}) = n-1$

$\vdots$ $\vdots$
$x_n \rightarrow l(x_n) = n$ $x_1 \rightarrow L(x_1) = 1$

then for all $y \in lT_\alpha$,there exists $x \in T_\alpha$ with $\alpha(x) = y$, so the stacked-homomorphism is surjective, and it is an epimorphism .

3.6.9 Definition

The stacked-homomorphism is an isomorphism , denoted $\alpha : T_\alpha \rightarrow T_{\alpha 1}$, if it is both an embedding and an epimorphism .

3.6.10 Theorem

The stacked-homomorphism denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an isomorphism .

Proof :

From theorem 5.2 , the stacked-homomorphism is denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an embedding or a monomorphism, and from theorem 5.3 , the stacked-homomorphism denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an epimorphism , then The stacked-homomorphism is an isomorphism.

3.6.11 Definition

The stacked-homomorphism, denoted $\alpha : T_\alpha \rightarrow T_{\alpha 1}$ is an endomorphism, if $T_\alpha = T_{\alpha 1}$.

3.6.12 Theorem

The stacked-homomorphism, denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an endomorphism if $T_\alpha = \{ 1 , 2 , \dots , n \}$.

Proof :

Hence $lT_\alpha = \{ \varphi(x_1) , \varphi(x_2) , \dots , \varphi(x_n) \} = \{ 1 , 2 , \dots , n \}$, or $lT_\alpha = \{ \upsilon(x_1) , \upsilon(x_2) , \dots , \upsilon(x_n) \} = \{ 1 , 2 , \dots , n \}$ and if $T_\alpha = \{ 1 , 2 , \dots , n \}$, then $T_\alpha = lT_\alpha$, so it is an endomorphism .

3.6.13 Definition

The stacked-homomorphism, denoted $\alpha : T_\alpha \rightarrow T_{\alpha 1}$ is an automorphism , if it is both an isomorphism and an endomorphism .

3.6.14 Theorem

The stacked-homomorphism, denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an automorphism, if $T_\alpha = \{ 1 , 2 , \dots , n \}$

Proof :

Hence $lT_\alpha = \{ \varphi(x_1) , \varphi(x_2) , \dots , \varphi(x_n) \} = \{ 1 , 2 , \dots , n \}$, or $lT_\alpha = \{ \upsilon(x_1) , \upsilon(x_2) , \dots , \upsilon(x_n) \} = \{ 1 , 2 , \dots , n \}$ and if $T_\alpha = \{ 1 , 2 , \dots , n \}$, then $T_\alpha = lT_\alpha$, so it is an endomorphism (i)
From theorem 5.2 , the stacked-homomorphism is denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an embedding or a monomorphism, and from theorem 5.3 , the stacked-homomorphism denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an epimorphism , then The stacked-homomorphism is an isomorphism(ii)
Then from (i) and (ii), the stacked-homomorphism, denoted $\upsilon : T_\alpha \rightarrow lT_\alpha$ or $\varphi : T_\alpha \rightarrow lT_\alpha$ is an automorphism .

3.6.15 Theorem

Let(T_α , τ) = [{ $x_1 , x_2 , \dots , x_n$ } , τ] is stacked-semigroup , $lT_\alpha (x) = \upsilon(x)$, and $lT_\alpha = \{ \upsilon(x_1) , \upsilon(x_2) , \dots , \upsilon(x_n) \}$, then $\upsilon = \upsilon^2 = \upsilon^3 = \dots$.

Proof :

Let(T_α , τ) = [{ $x_1 , x_2 , \dots , x_n$ } , τ] is stacked-semigroup , $lT_\alpha (x) = \upsilon(x)$, and $lT_\alpha = \{ \upsilon(x_1) , \upsilon(x_2) , \dots , \upsilon(x_n) \}$. Hence : $\upsilon(x_1) = 1$, $\upsilon(x_2) = 2$, $\upsilon(x_3) = 3 , \dots , \upsilon(x_n) = n$.
Then $\upsilon^2(x_1) = \upsilon(\upsilon(x_1)) = \upsilon(1) = 1$, so :

$$\upsilon = \begin{pmatrix} x_1 \, x_2 \, x_3 \dots x_n \\ 1 \;\; 2 \;\; 3 \dots n \end{pmatrix}$$

$$\upsilon^2 = \begin{pmatrix} 1 \;\; 2 \;\; 3 \dots n \\ 1 \;\; 2 \;\; 3 \dots n \end{pmatrix}$$

$$\upsilon^3 = \begin{pmatrix} 1 \;\; 2 \;\; 3 \dots n \\ 1 \;\; 2 \;\; 3 \dots n \end{pmatrix}$$

$$\upsilon^\infty = \begin{pmatrix} 1 \;\; 2 \;\; 3 \dots n \\ 1 \;\; 2 \;\; 3 \dots n \end{pmatrix}$$

Then $\upsilon = \upsilon^2 = \upsilon^3 = \dots$.

3.6.16 Theorem

Let(T_α , τ) = [{ $x_n , x_{n-1} , \dots , x_1$ } , τ] is stacked-semigroup , $lT_\alpha (x) = \varphi(x)$, and $lT_\alpha = \{ \varphi(x_1) , \varphi(x_2) , \dots , \varphi(x_n) \}$, then $\varphi = \varphi^2 = \varphi^3 = \dots$.

Proof :

Let(T_α , τ) = [{ $x_n , x_{n-1} , \dots , x_1$ } , τ] is stacked-semigroup , $lT_\alpha (x) = \varphi(x)$, and $lT_\alpha = \{ \varphi(x_n) , \varphi(x_{n-1}) , \dots , \varphi(x_1) \}$. Hence : $\varphi(x_n) = n$, $\varphi(x_{n-1}) = n-1$, $\varphi(x_{n-2}) = n-2 , \dots , \varphi(x_1) = 1$.
Then $\varphi^2(x_n) = \varphi(\varphi(x_n)) = \varphi(n) = n$, so :

$$\varphi = \begin{pmatrix} x_n & x_{n-1} & x_{n-2} & \dots & x_1 \\ n & n-1 & n-2 & \dots & 1 \end{pmatrix}$$

$$\varphi^2 = \begin{pmatrix} n & n-1 & n-2 & \dots & 1 \\ n & n-1 & n-2 & \dots & 1 \end{pmatrix}$$

$$\varphi^3 = \begin{pmatrix} n & n-1 & n-2 & \dots & 1 \\ n & n-1 & n-2 & \dots & 1 \end{pmatrix}$$

$$\varphi^\infty = \begin{pmatrix} n & n-1 & n-2 & \dots & 1 \\ n & n-1 & n-2 & \dots & 1 \end{pmatrix}$$

Then $\varphi = \varphi^2 = \varphi^3 = \dots$.

3.6.17 Definition

let(T_1 , τ_1) , (T_2 , τ_2) are stacked-subsemigroup of the stacked-semigroup (T , τ), then $\{ \tau_1 , \tau_2 \} \subseteq \tau$

3.6.18 Definition

let(T_1 , τ_1) , (T_2 , τ_2) are stacked-semigroup , if $T_1 \cap T_2 = T$, then they are tow disjoint-stacked-semigroup . If $T_1 \cap T_2 \neq T$, then they are tow undisjoint-stacked-semigroup .

3.6.19 Definition

Let $T_1 = \{ x_1 , x_2 , \dots , x_n \}$ and $T_2 = \{ y_1 , y_2 , \dots , y_m \}$ are stacked-semigroup , then we defined the multiplication between T_1 and T_2 by $T_1 \bullet T_2 = T_1 T_2 = T$. Then T is a new system with a new order set .

And $| T | = | T_1 | | T_2 |$.

Then $T_1 T_2 = T = \{ x_1 y_1 , x_1 y_2 ,\dots x_1 y_m , x_2 y_1 , x_2 y_2 ,\dots , x_2 y_m ,\dots , x_n y_m\}$

3.6.20 Theorem

Let $T_1 = \{ x_1 , x_2 , \dots , x_n \}$ and $T_2 = \{ y_1 , y_2 , \dots , y_m \}$ are stacked-semigroup , then $T_1 T_2 \neq T_2 T_1$.

Proof :

From the definition above let $T_1 = \{ x_1 , x_2 , \dots , x_n \}$ and $T_2 = \{ y_1 , y_2 , \dots , y_m \}$, $T_1 T_2 = \{ x_1 y_1 , x_1 y_2 ,\dots x_1 y_m , x_2 y_1 , x_2 y_2 ,\dots , x_2 y_m ,\dots , x_n y_m\}$,

$T_1 T_2 = \{ y_1 x_1 , y_1 x_2 ,\dots y_1 x_n , y_2 x_1 , y_2 x_2 ,\dots , y_2 x_n ,\dots , y_m x_n\}$.

IV. CONCLUSION

Thus can be considered the stacked system is semi-group , and can be applied to data that need to be addressed in the same way, and will follow this paper sheets in the same subject as applications addressing some of the issues in mathematics relevant to this data taken in the form of stacked system .

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