Innovative Technique for Analysis of Retaining Wall – using Dimensional Analysis

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Abstract- Analysis of retaining wall is one of the major part in Geotechnical Engineering. This can be done by various methods such as Rankine's Theory, Coulomb's Theory, Culmann's Graphical Method which depends on various assumptions . To overcome this, various approaches has been adopted, in this paper we have tried to formulate the same with the help of Dimension Analysis method by which results can be obtained without any assumptions and an universal formula can be developed. We have used the modified Buckingham-pi theorem for this analysis. The results of dimensional analysis so obtained are then compared with the actual test results. The comparison shows the validity of this method.

Index Terms- Retaining wall; Active Earth Pressure; Dimensional Analysis; Trial and error.

I. Introduction

Every Engineer seeks to have a simple, fast and reliable solution to his problems. For this he took help of various analysis tools. There are various methods of analysis such as Limit Equilibrium Analysis, Finite Element Analysis etc. which involves simple rules and assumptions to more complex techniques. Dimensional analysis is a mathematical technique which makes use of study of dimensions as an aid to the solution of several engineering problems. The various physical quantities used by engineers to describe the given phenomenon can be described by a set of quantities which are in a sense independent of each other. These quantities are known as fundamental or primary quantities. The expression for a derived quantity in terms of the primary quantities is called the dimension of the physical quantity

II. Materials and Methods

2.1 Experimental Study On Earth Pressure Of Retaining Wall By Field Tests by Minaro Matsuo,Satosuo Kenmochi [3]

Earth pressure acting on retaining wall has been theoretically & experimentally studied by many researchers but the large ranged measured result are comparatively low. In this paper Influence of displacement of wall on magnitude of earth pressure are shown.

Retaining wall structure

Height 10 meter Made up of concrete

Backfill Silty Sand & Slag (Produced in Iron manufacturing process)

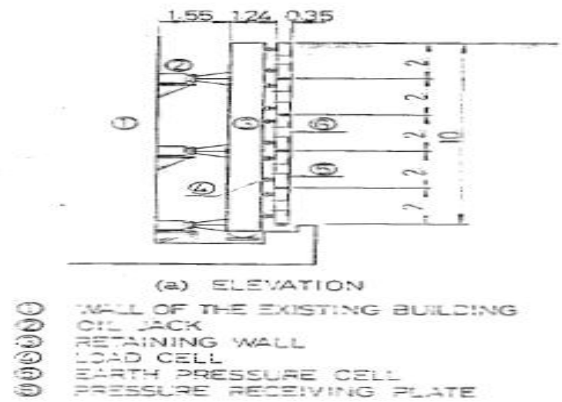
Duration One Year (Including preparation of the test)

Procedure in General

When Earth Pressure becomes stable at rest wall is moved outside to Active State and after completion of the same wall has returned back to original position to measure Passive Pressure.

Experimental Set up

Wall is made up of concrete of 10m height. Lowest end is supported by Hinge (Provides not only inclined displacement but also horizontal displacement)Pressure receiving plates – 2m in each unit length Load Cells – Four to each pressure receiving plate. Two at the lowest end (Measure wall friction)Oil Jacks – Between Retaining wall & wall of existing building.



Pre-Experimental Procedure

Retaining wall set up with supports by Oil Jacks between existing wall & test wall. Place the lowest pressure receiving plate and fill the backfill up to height of 2mContinue the process up to 10m height.

Unit Weight

Silty Sand 1.9 t/m3 Slag (A) 2.0 t/m3 Slag(B) 1.6 t/m3

Influence of Underground water

Sand mat is placed between original ground & backfill Water always pumped out

Experimental Procedure

At Rest Firstly wall is kept at rest condition – 2 Months

Active State Tilting deformation is given – Twenty Days (Displacement is measured by Dial gauge) Passive State Wall has been bring to original position. This is actually not perfect passive state but gives the Rough information. Beyond this practically not possible due anticipated danger

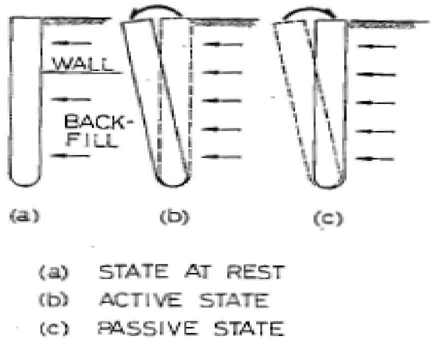


Fig. 6. Illustrations of deformation of retaining wall

Relationship between Earth Pressure & Time.

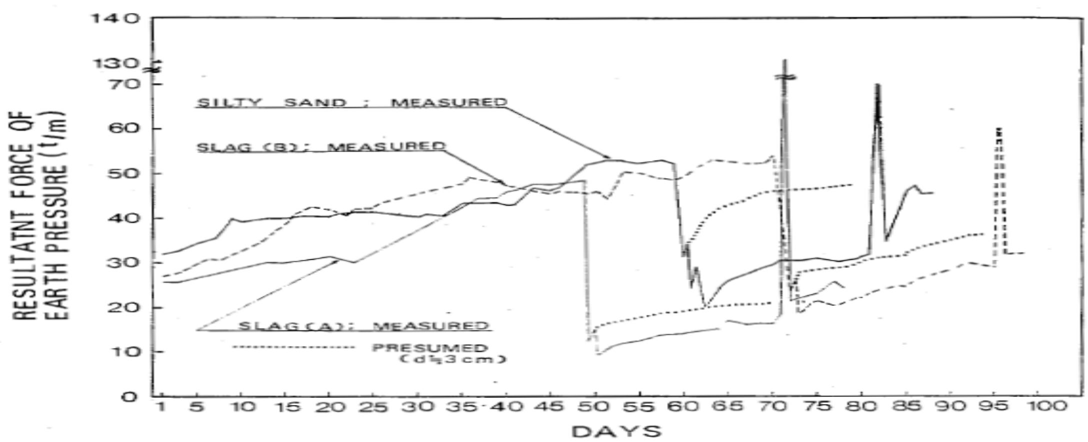


Fig. 7. Change of resultant force of earth pressure

Resultant force = Horizontal Earth pressure + Frictional Force.
Horizontal Earth pressure = Sum of values measured by twenty load cells
Frictional Force = Two load cells at the lowest end of pressure receiving plate

Distribution Of Earth Pressure in Vertical Direction

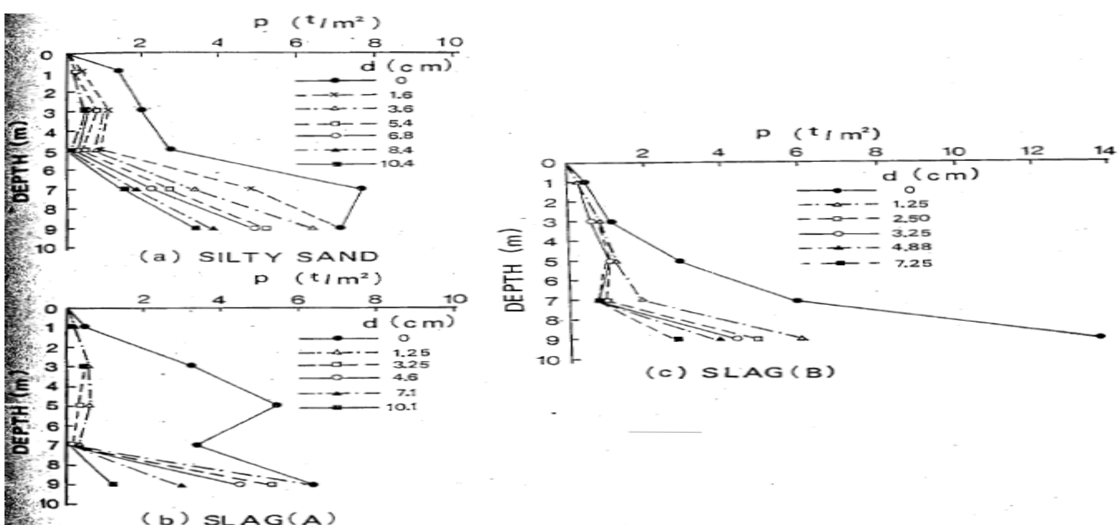


Fig. 8. Distribution of earth pressure in the vertical direction

Coefficient Of Earth Pressure, Angle Of Wall Friction, Point Of Application Of Resultant Force.

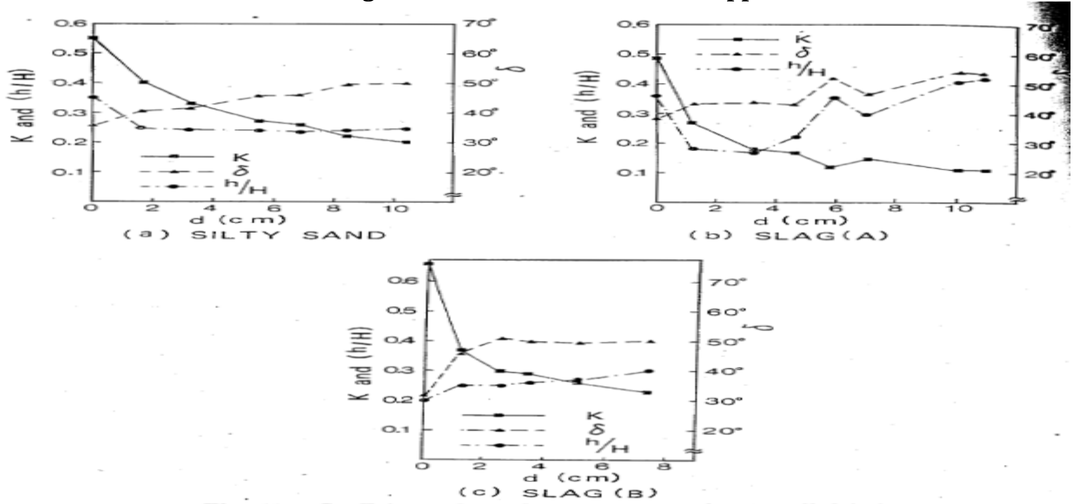


Fig. 11. Coefficient of earth pressure, angle of wall friction and application point of resultant force

2.2 *Formulation of Dimensional Analysis Equation*

Dimensional analysis looks to reduce to a minimum the dimension space in which the behaviour of problem can be studied by joining and systematically positioning the assumed governing variables (V) = (V_1, V_2, V_3... V_n), consisting total of 'm' independent primary dimensions (D) = (D_1, D_2, D_3...D_m), into N = (n - m) dimensionless groups, that are (π_1, π_2, π_3…π_n), with N being less than V [1].

Thus, using modified Buckingham – pi theorem [1] we have V = { Pa, γ, z, k, Φ, δ, d }hence, n = 7. Writing the dimensions of the variables, V= { $M^1 L^{-1} T^{-2}$, $M^1 L^{-2}T^{-2}$, L^1, O, O, O, L^1 }, m = 3.

As a requirement the condition of 'Dmin' which looks to reduce the total number of primary dimensions in set {V} is fulfilled by minimizing the value of m, therefore, assuming the S = [$M^1L^{-1}T^{-2}$], V= [S^1, S^1L^{-1},L^1,0,0,0,L^1]; hence, m = 2; the number of repeating variables forming a set Q. Thus, the number of dimensional pi-groups and the number of isolated variable in N = (n-m) = 7 – 3 = 5.

R is a set o variables in V that dimensions totally different from one another,

R= { p, γ, z }. Dimensionless gropes are formed by the combination of the repeating (Q) and isolated (NOTQ) variable sets; Since Q is to be selected from R,

Q = { Pa, γ } .... (1)

NOTQ = { z, k, Φ, δ, d } .... (2)

Therefore the dimensionless groups are,

π_1= { Pa, γ, z } .... (3i)

Or

π_2= { Pa, γ, k } .... (3ii)

Or

π_3= { Pa, γ,Φ } .... (3iii)

Or

π_4= { Pa, γ, δ } .... (3iv)

Or

π_5= { Pa, γ, d } .... (3v)

Now, representing

π_1= { Pa, γ, z } .... (4)

In dimensional form eq. (4) can be stated as follows:

$[M^o L^o T^o]= [M^1 L^{-1} T^{-2}]^a \times [M^1 L^{-2}T^{-2}]^b \times [L^1]^c$

Comparing the indices of left hand side and the right hand side; for

M: 0 = a +b
L: 0 = -a-2b+c
T: 0 = -2a - 2b

Hence, a =1 then b = -1, c = -1

Substituting these values in eq. (4)

$\pi_1 = \{ Pa\ \gamma^{-1}\ z^{-1} \} = Pa / (\gamma z)$ .... (5)

Similarly, by solving the eq. for π_2, π_3, π_4 and π_5 we find that

$\pi_2 = k$ .... (6)

$\pi_3 = \Phi$ .... (7)

$\pi_4 = \delta$ .... (8)

$\pi_5 = Pa / (\gamma d)$ .... (9)

We consider $\pi_1 = \psi (\pi_2, \pi_3, \pi_4, \pi_5)$ and by substituting from eqs. (5),(6),(7),(8)and (9), we have

$Pa / (\gamma z) = \psi [k, \Phi, \delta, Pa / (\gamma d)]$ .... (10)

To find exact nature of (ψ) a power product relationship of the dimensionless groups is used as follows;

$\pi_1 = \beta_1 (\pi_2^{\beta 2}, \pi_3^{\beta 3}, \pi_4^{\beta 4}, \pi_5^{\beta 5})$ .... (11)

To solve eq. (11), a logarithmic form of this eq. is used
$\log(\pi_1) = \log(\beta1) + \beta2 \log(\pi_2) + \beta3 \log(\Pi3) + \beta4 \log(\Pi4) + \beta5 \log(\Pi5)$
$\log(Pa/(\gamma z)) = (\log\beta_1) + \beta_2 \log(k) + \beta_3 \log(\Phi) + \beta4 \log(\delta) + \beta5 \log(p/(\gamma d))$ .... (12)

Now π_1 is treated as the dependent variable and the value of constants (β_1, β_2, β_3) can be found out by performing a multiple regression analysis of eqs. (11) and (12) or by the trial-and-error procedure of eq. (12).

Now using the Properties of Backfill material with reference to the Experiment performed on site and using Rankines formula to find the Active Pressure at mid height of wall .We will use this as a control point to formulate the D.A for Slag B as Backfill material we will get

$Pa = k \gamma z = 0.1325 * 1.6 * 5 = 1.06$ t/m2, And taking the values of $\Phi = 50°$ $\delta = \Phi/2 = 25°$ d = 0.0325m

By using the test results obtained from the site test, and substituting the values of the test results in eq. (12) - that is, using this point as a control point to formulate the dimensional analysis, we get
$-2.021 = \log(\beta1) - 2.0212\,\beta2 + 3.912\,\beta3 + 3.212\beta4 + 3.015\beta5$ .... (13)

Table 1 explains the trial and error procedure with values of constants for eq.(13), in the first trial, β_1, β_2, β_3 and β_4 are assumed and the corresponding $\beta5$ value is back calculated by using eq.(13) for values π_1, π_2, π_3, π_4 and π_5 of the first set of test results, which are use as a control point. Then the determined β values are submitted, along with the values of π_1, π_2, π_3, π_4 and π_5 for the second set of test results, as a check point in eq.(11). Hence the control point is used to prepare the dimensional analysis equation and the checkpoint is used to authenticate it.

Table 1. Trial and Error Procedure with Values of Constants for Equation (13)

Trial No.	β_1	β_2	β_3	$\beta4$	$\beta5$	**Results**	**Remark**
1	1	1	1	1	-2.3627	0.1349	NOT VALID
2	1	1	1	0.9	-2.2562	0.13475	NOT VALID
3	1	1	0.6	0.6	-1.4176	0.1339	NOT VALID
4	1	1	0.5	0.5	-1.1813	0.1337	NOT VALID
5	1	1	0.4	0.4	-0.945	0.1334	NOT VALID
I. **6**	1	1	0.3	0.3	-0.70878	0.1332	NOT VALID
II. 7	1	1	0.2	0.2	-0.4725	0.1329	NOT VALID
III. 8	1	1	0.1	0.1	-0.2362	0.1328	NOT VALID
IV. 9	1	1	0.09	0.09	-0.2125	0.1326	**VALID**

Thus the trial 9, as shown in Table 5, resulted in a valid equation and correct set of values are
$\beta_1=1$, $\beta_2=1$, $\beta_3=0.09$, $\beta_4=0.09$, $\beta_5=-0.2125$

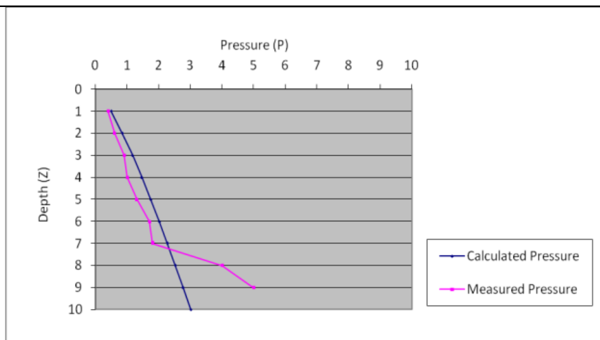
By substituting the preceding β values values in Eq.(9) and solving, we get the universal formula
$$\mathbf{Pa = \{ k\ z\ \gamma^{1.2125}\ (\Phi\ \delta)^{.09}\ d^{\ 0.2125} \}} \quad .... (14)$$

III. Results and Discussion

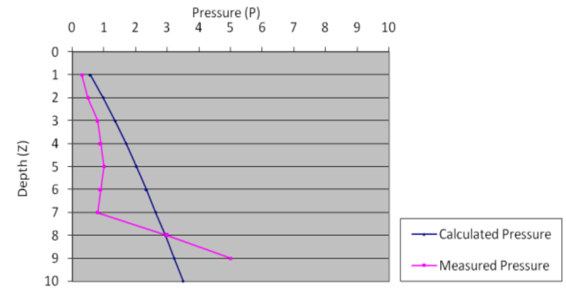
3.1 *Comparison of Experimental Results and Dimensional Analysis*

To verify the proposed method we have used 5 test results. Below graphs shows the results of the tests conducted at site and the results obtained by the proposed method. The results shows that the prediction capability of this new method is very good.

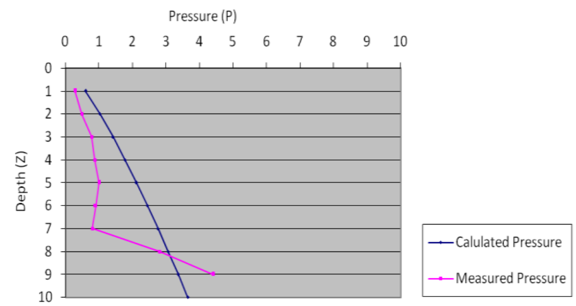
Comparison Active Pressure For Slag B
$d = 0.0125$, $k = 0.36$, $\delta = 45°$, $\Phi = 50°$, $\gamma = 1.6$



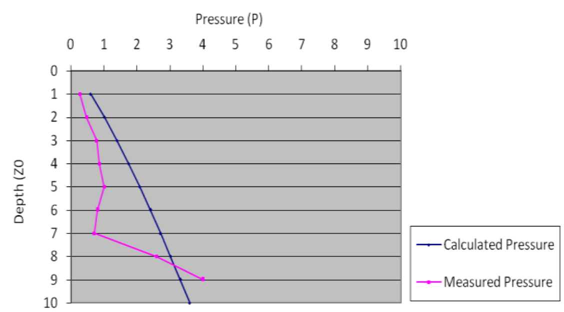
Comparison Active Pressure For Slag B
d = 0.025, k = 0.30, δ = 51°, Φ = 50°, γ = 1.6



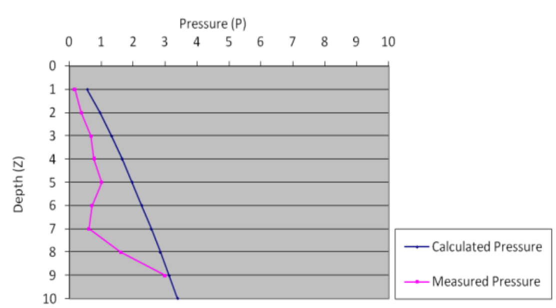
Comparison Active Pressure For Slag B
d = 0.0325, k = 0.3, δ = 50°, Φ = 50°, γ = 1.6



Comparison Active Pressure For Slag B
d = 0.0488, k = 0.27, δ = 49°, Φ = 50°, γ = 1.6



Comparison Active Pressure For Slag B
d = 0.0725, k = 0.23, δ = 50°, Φ = 50°, γ = 1.6



Graphical Comparison between Experimental and Analytical Results

IV. Conclusion

- The It is possible to formulate a DA for any type of Geotechnical problem , only thing that various parameters involved in the problem should be known along with there interrelationship with each other.
- Various assumptions required in the conventional analysis are not required for the D.A.
- From any point called as Control Point either obtained by traditional analysis or an experimental tests; the predictions of the other points with different soil conditions, displacements could be possible.
- The results obtained for Active conditions are quantitatively conservative as well as qualitatively conversant with the lab. Test and also field test.

- D.A.is simpler method than any other traditional method to take the wall displacement into consideration.
- D.A. is simpler and faster method as compared to any other rigorous methods of analysis such as Finite Element Analysis, Limit Equilibrium Methods, etc. and have potential to replace them.

Notations:

The following symbols are used in this paper:

Pa = Active Earth pressure
γ = Soil Density
z = Depth of backfill
k = Coefficient of Earth Pressure
Φ = Angle of Internal Friction
Δ = Angle of wall Friction
D = Wall displacement
V = Set of independent and dependent parameters involved in the phenomenon; and an Unknown function
M = Mass dimension
L = Length dimension
T = Time dimension
D = Set of independent primary dimensions
*D*min = Minimum number of independent and nonzero elements
n = Number of variables or parameters
m = Number of primary dimensions in *D*
N = (n−m) = Number of dimensionless groups
Q = Set of repeating variables
R = Set of variables in *V* that have dimensions totally distinct from one another

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