ISSN 2250-3153

Simulation and Evaluation of Shrinkage Techniques for Image De-noising under Additive White Gaussian Noise (AWGN)

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Abstract- Image denoising is a process to restore a worth image from degraded image by choosing an appropriate filter but choosing an appropriate filter still is a challenge. Recently, wavelet transform has attracted more and more interest in image de-noising because the wavelet transform of an image produces a non-redundant image representation, which provides better spatial and spectral localization of image formation. The wavelet transform can be interpreted as image decomposition in a set of independent, spatially oriented frequency channels and each cannel can be treated as a single element for removal of noise using an appropriate shrinkage threshold and at the end all treated channels recombined to get denoised image. We discuss in this paper various shrinkage techniques used in wavelet transform for image denoising in respect of test image size, orthogonal wavelets and elapsed time under Additive White Gaussian Noise.

Index Terms- Additive White Gaussian Noise (AWGN), Image Denoising, Peak Signal to Noise Ratio (PSNR), Shrinkage Techniques, and Wavelet Transform.

I. Introduction

The image denoising [1, 2] of an image naturally corrupted by noise is a classical problem in the field of image processing. In image processing, noise is undesired information that degrades the quality of image. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can degrade the data of interest. Image denoising involves removing the effect of noise and preserves the detail to produce a high quality image. There are various methods to restore an image from noisy distortions but selecting the appropriate method plays a major role in getting the desired image. The denoising methods tend to be problem specific. For example, a method that is used to denoise satellite images may not be suitable for denoising medical images. This paper covers the wavelet transform shrinkage techniques for image denoising and their detail comparative study. The performance of each technique is compared by computing Mean Square Error (MSE) and Peak Signal to Noise Ratio (PSNR) [3, 4] besides the visual interpretation.

II. Wavelet Transform Shrinkage Techniques

Donoho and Johnstone [5] pioneered the work on filtering of additive Gaussian noise using wavelet thresholding. From their properties and behavior, wavelets play a major role in image denoising. The term wavelet thresholding is explained as decomposition of the image into wavelet coefficients, comparing the detail coefficients with a given threshold value, and shrinking these coefficients close to zero to take away the effect of noise in the image. The image is reconstructed from the modified coefficients. This process is also known as the inverse discrete wavelet transform. During thresholding, a wavelet coefficient is compared with a given threshold and is set to zero if its magnitude is less than the threshold; otherwise, it is retained or modified depending on the threshold rule.

Thresholding methods use a threshold and determine the clean wavelet coefficients based on this threshold.

A. Hard Thresholding

If the absolute value of a coefficient is less than a threshold, then it is assumed to be 0, otherwise it is unchanged. Mathematically it is given by

$$f_h(x) = x, \quad if\ x \geq \lambda$$
$$= 0, \quad otherwise$$

The hard thresholding [6] function chooses all wavelet coefficients that are greater than the given threshold λ and sets the others to zero. The threshold λ is chosen according to the signal energy and the noise variance (σ^2).

B. Soft Thresholding

The soft thresholding [6] function has a somewhat different rule from the hard thresholding function. It shrinks the wavelet coefficients by λ towards zero,

Hard thresholding is discontinuous. To overcome this, Donoho introduced the Soft Thresholding method. If the absolute value of a coefficient is less than the threshold λ, then it is assumed to be 0, otherwise its value is shrunk by λ. Mathematically, it is given by

$$f(x) = x - \lambda, \quad if\ x \geq \lambda$$
$$= 0, \quad if\ x < \lambda$$
$$= x + \lambda, \quad if\ x \leq -\lambda$$

The soft-thresholding rule is chosen over hard-thresholding, because soft-thresholding method yields more visually pleasant images over hard thresholding.

C. Visu Shrink

Visu Shrink [6] was introduced by Donoho. It uses a threshold value T that is proportional to the standard deviation of the noise. It follows the hard thresholding rule. It is also referred to as universal threshold and is defined as

$$T = \sigma\sqrt{2\log(M)} \quad (1)$$

where σ^2 is the noise variance and M is the number of pixels. This asymptotically yields a MSE estimate as *M* tends to infinity. As *M* increases, we get bigger and bigger threshold, which tends to over smoothen the image. Visu Shrink follows the global thresholding [6] scheme where there is a single value of threshold applied globally to all the wavelet coefficients [1].

D. Bayes Shrink

Bayes Shrink [7] was proposed by Chang, Yu and Vetterli [8]. It uses soft thresholding and is subband dependent, which means that thresholding is done at each band of resolution in the wavelet decomposition. It is smoothness adaptive. In particular, it is assumed that, for the various subbands and decomposition levels, the wavelet coefficients of the original image follow approximately a Generalized Gaussian Distribution (GGD) [4]. The Bayes threshold 'T' is defined as

$$T = \frac{\hat{\sigma}_n^{\,2}}{\hat{\sigma}_x} \quad (2)$$

if $\hat{\sigma}_x$ is non-zero.

Otherwise, it is set to some predetermined maximum value.

$$\hat{\sigma}_x = \sqrt{max(\hat{\sigma}_y^2 - \hat{\sigma}^{\,2}, 0)} \quad (3)$$

$\hat{\sigma}_y$ is calculated as,

$$\hat{\sigma}_y^{\,2} = \frac{1}{n^2}\sum_{i,j=1}^{n} Y_{ij}^{\,2}\,; \qquad Y_{ij} \in subband\ (HH_1) \quad (4)$$

The noise variance $\hat{\sigma}_n$ is estimated from the HH band as,

$$\hat{\sigma}_n = \frac{Median\ (|Y_{ij}|)}{0.6745} \quad (5)$$

D. Neigh Shrink

In NeighShrink [9] method, the wavelet coefficients are threshold according to the magnitude of the squared sum of all the wavelet coefficients i.e. the local energy within the neighborhood window.

Let d(i,j) denote the wavelet coefficients of interest and B(i,j) is a neighborhood window around d(i,j). Also let S2=Σd2(i,j) over the window B(i,j). Then the wavelet coefficient to be threshold, is shrinked according to the formulae,

$$d(i,j)= d(i,j)^* B(i,j) \quad (6)$$

where the shrinkage factor can be defined as B(i,j) = (1-T2/ S2(i,j))$_+$, and the sign + at the end of the formulae means to keep the positive value while set it to zero when it is negative.

E. Gaussian Noise

Gaussian noise is evenly distributed over the signal [10]. This means that eachpixel in the noisy image is the sum of the true pixel value and a random Gaussiandistributed noise value. As the name indicates, this type of noise has a Gaussiandistribution, which has a bell shaped probability distribution function given by,

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(g-m)^2}/2\sigma^2 \quad (7)$$

where *g* represents the gray level, *m* is the mean or average of the function, and σ is the standard deviation of the noise.

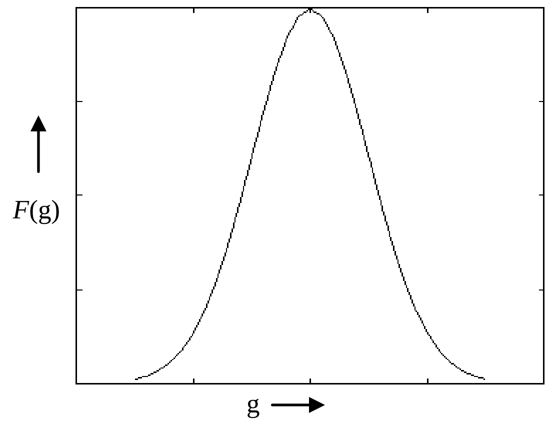

Fig.1Gaussian distribution

III. PERFORMANCE MEASURE

To measure the performance of the various shrinkage techniques for image denoising, a test image is added with some known Gaussian noise. This would then be given as input to the denoising algorithm, which produces an image close to the original test image as per shrinkage techniques deployed. The performance of each shrinkage algorithm is compared by computing MSE and PSNR, besides the visual interpretation.

MSE and PSNR are the two main parameters for comparison of shrinkage denoising techniques. In statistics, the mean squared error of an estimator is the difference between an estimator and the true value of the quantity being estimated i.e. difference between the original image and the denoised image.

Let original image is $f(x,y)$ of size $m \times n$ and $\hat{f}(x,y)$ is estimated image after restoration then MSE can be defined as-

$$MSE = \frac{1}{m \times n}\sum_{x=1}^{m}\sum_{y=1}^{n}\left[f(x,y) - \hat{f}(x,y)\right]^2 \quad (8)$$

PSNR is the peak signal to noise ratio, here signal is the original image and noise is error introduced by restoration. PSNR is the most commonly used parameter to measure the quality of reconstruction image with respect to the original image. A higher PSNR would normally indicate that the reconstruction is of higher quality. PSNR is usually expressed in terms of the logarithmic decibel scale (dB).

$$PSNR = 10\ log_{10}\left(\frac{255^2}{MSE}\right) \quad (9)$$

Using (8), we get

$$PSNR = 10\ log_{10}\left(\frac{255^2}{\frac{1}{m \times n}\sum_{x=1}^{m}\sum_{y=1}^{n}\left[f(x,y) - \hat{f}(x,y)\right]^2}\right) \quad (10)$$

Here $f(x,y)$ is the original image of size $m \times n$ and $\hat{f}(x,y)$ is the reconstructed image after restoration. In the expression of

PSNR, the value of the numerator is 255^2; it is the square of the maximum possible pixel value of the image $f(x,y)$. When $f(x,y)$ is the 8-bit gray scale image, the maximum possible pixel value of the Image $f(x,y)$ is 255.

Elapsed Time is the amount of time which MATLAB takes to complete all operations of that particular denoising algorithms and displays the time in seconds.

IV. Simulated Results and Discussion

The wavelet transforms shrinkage methods: Visu Shrink, Bayes Shrink, Neigh Shrink, Bi Shrink are simulated in MATLAB. The test images: Lena, Barbara and Boats of sizes 512×512 and House and Peppers of sizes 256×256 are corrupted with AWGN of standard deviation sigma at different intervals i.e. sigma (σ) = 10, 20, 30, 40.The wavelets used are orthogonal wavelets i.e. coiflets, symlets, daubechies and haar.

Table-1(a) Performance under different sigma values of AWGN

IMAGE	Visu Shrink (Haar)		Bayes Shrink (Haar)	
LENA (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	30.6532	0.045933	29.8091	0.045855
σ = 20	26.4793	0.045079	26.4884	0.044968
σ = 30	23.6048	0.045430	23.7881	0.045166
σ = 40	21.3985	0.044816	21.6207	0.046123
BARBARA (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	27.0321	0.044753	25.1309	0.045171
σ = 20	24.0440	0.044499	23.5684	0.045761
σ = 30	22.0047	0.043889	21.9787	0.045201
σ = 40	20.3194	0.045542	20.4340	0.045865
BOATS (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	29.4420	0.045532	28.0915	0.046286
σ = 20	25.7227	0.044900	25.3604	0.045003
σ = 30	23.1188	0.046145	23.1338	0.044599
σ = 40	21.0749	0.045351	21.2112	0.045101
HOUSE (256 x 256)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	30.8739	0.010850	29.3920	0.009800
σ = 20	26.2293	0.009497	26.1226	0.010248
σ = 30	23.3050	0.009539	23.5412	0.009696
σ = 40	21.1075	0.009540	21.4390	0.009506
PEPPERS (256 x 256)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	29.3540	0.009837	27.1206	0.010131
σ = 20	25.6211	0.009505	23.9953	0.010256
σ = 30	22.8838	0.009683	22.1074	0.009936
σ = 40	20.7348	0.009626	20.4682	0.009536

Table-1(b) Performance under different sigma values of AWGN

IMAGE	Neigh Shrink (Haar)		Bi Shrink (Haar)	
LENA (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	33.3147	5.800438	32.7335	0.047611
σ = 20	29.7907	5.676601	29.0275	0.047452
σ = 30	27.9265	5.704856	26.8363	0.061307
σ = 40	26.6950	5.607180	25.2176	0.045770
BARBARA (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	31.5857	5.922542	30.3805	0.048614
σ = 20	27.4689	5.792790	26.0484	0.047630
σ = 30	25.3808	5.723764	24.0034	0.047934
σ = 40	24.0829	5.702113	22.6938	0.048180
BOATS (512 x 512)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	31.9014	5.855568	31.3918	0.048494
σ = 20	28.2986	5.706110	27.7366	0.047814
σ = 30	26.4013	5.687567	25.6460	0.045968
σ = 40	25.2007	5.663731	24.1534	0.045849
HOUSE (256 x 256)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	33.5719	1.486125	33.0123	0.010874
σ = 20	29.9426	1.444289	29.1771	0.011689
σ = 30	27.9873	1.476113	26.7781	0.010377
σ = 40	26.6621	1.419338	24.9968	0.011341
PEPPERS (256 x 256)	**PSNR**	**Elapsed time (Sec.)**	**PSNR**	**Elapsed time (Sec.)**
σ = 10	32.0660	1.488034	31.3745	0.011141
σ = 20	27.9119	1.461632	27.5441	0.011296
σ = 30	25.7133	1.426746	25.2915	0.011394
σ = 40	24.1687	1.418810	23.6835	0.011752

The all wavelet transform shrinkage methods are applied on all five test images with the combination of four wavelets coif5, sym8, db10 and harr for image denoising using Gaussion noise sigma variation from 10 to 40. The simulated results with haar wavelet are tabulated as given above Table - 1(a) and 1(b) and the following graphs (in Fig 2 to Fig 5) represent the comparative views of each shrinkage method with four wavelets coif5, sym8, db10 and harr applied to all test images i.e. Lena, Barbara and Boats of sizes 512×512 and House and Peppers of sizes 256×256.

The elapsed time is taken as the measure of execution time in seconds of each shrinkage denoising algorithm to determine the faster method of image denoising. Elapsed time is proportional to the resolution of the denoised image. The best performance Neigh Shrink method with coif5 wavelet is visually interpreted in Fig 6.

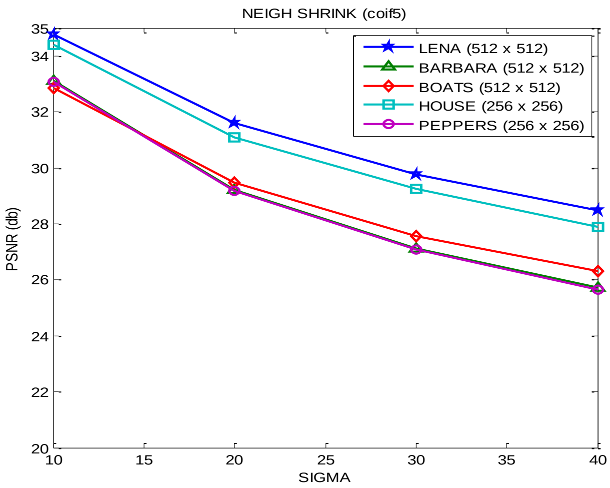

Fig.2(a) Performance Neigh Shrink using coif5 wavelet

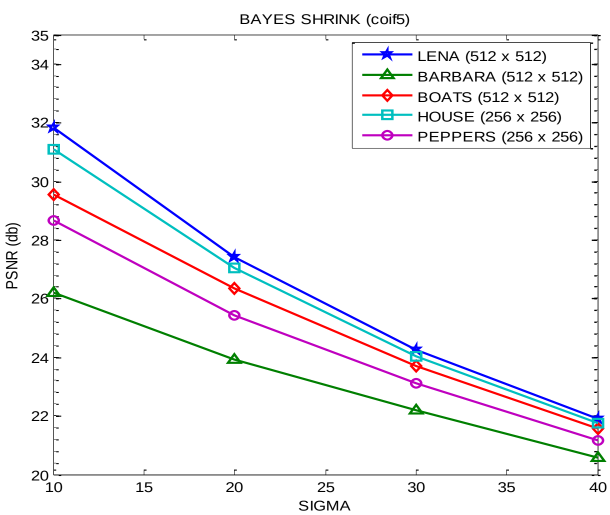

Fig.2(b) Performance Bayes Shrink using coif5 wavelet

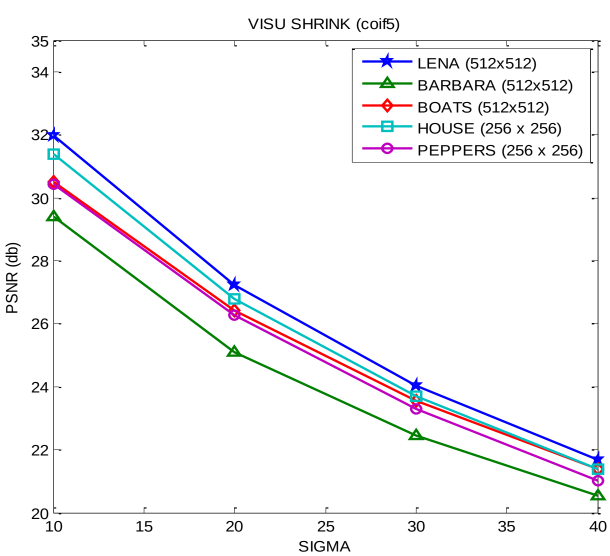

Fig.2(c) Performance Visu Shrink using coif5 wavelet

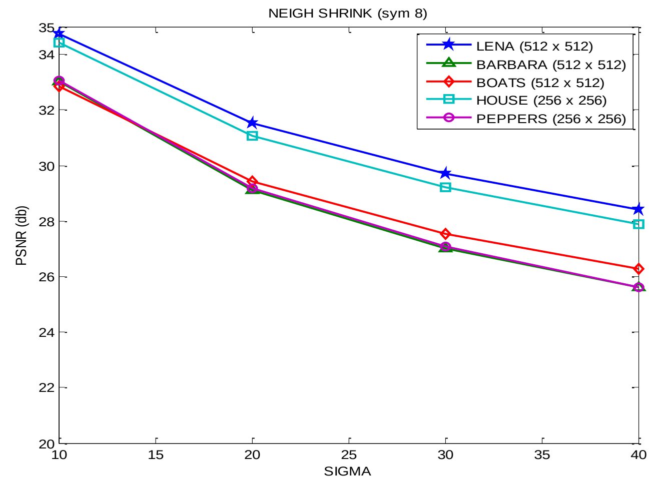

Fig.3(a) Performance Neigh Shrink using sym8 wavelet

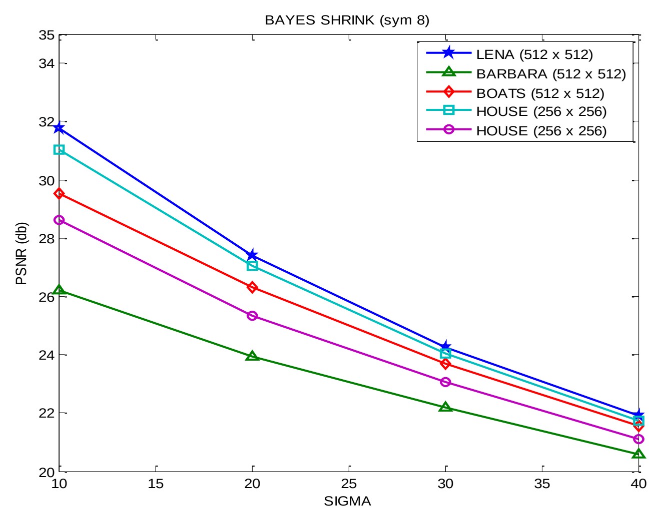

Fig.3(b) Performance Bayes Shrink using sym8 wavelet

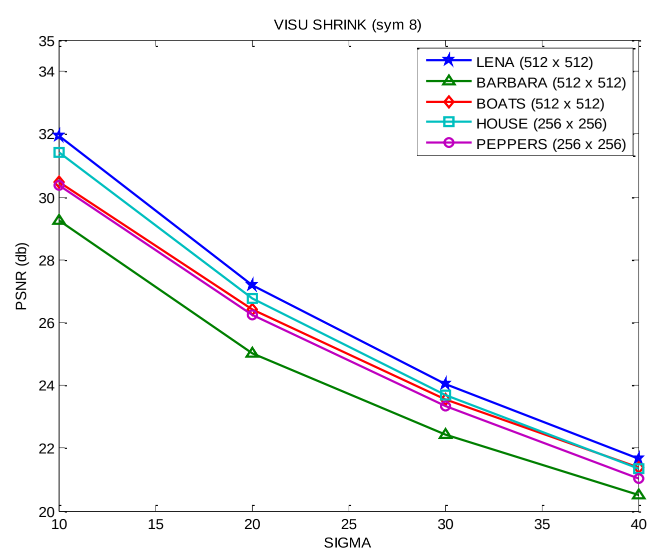

Fig.3(c) Performance Visu Shrink using sym8 wavelet

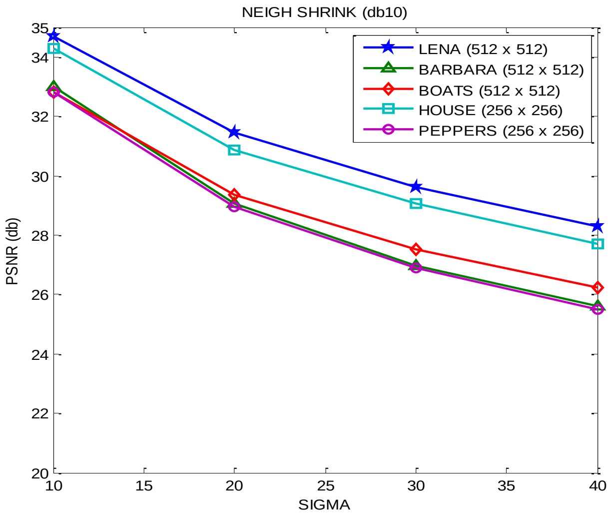

Fig.4(a) Performance Neigh Shrink using db10 wavelet

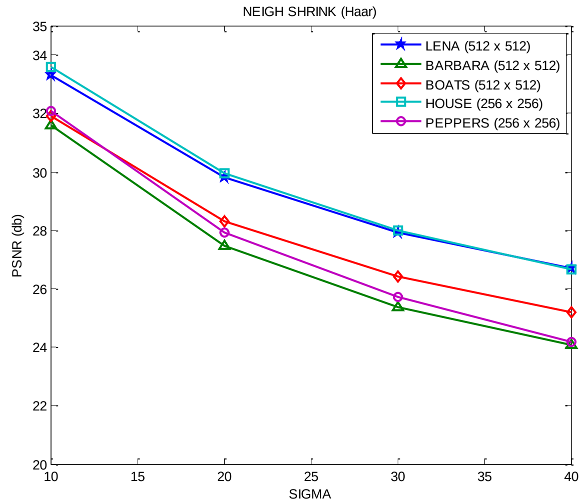

Fig.5(a) Performance Neigh Shrink using haar wavelet

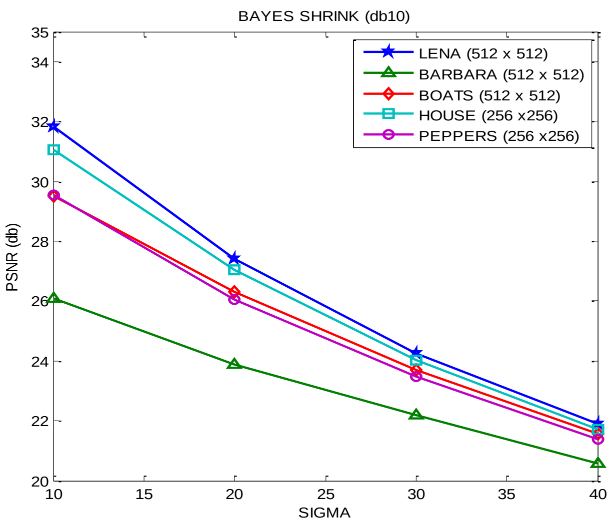

Fig.4(b) Performance Bayes Shrink using db10 wavelet

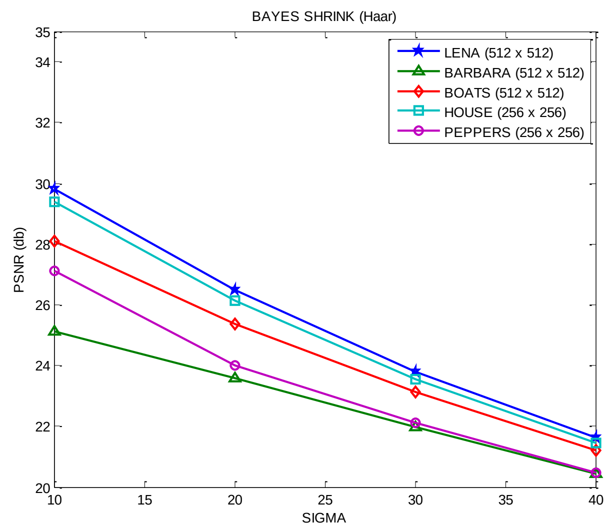

Fig.5(b) Performance Bayes Shrink using haar wavelet

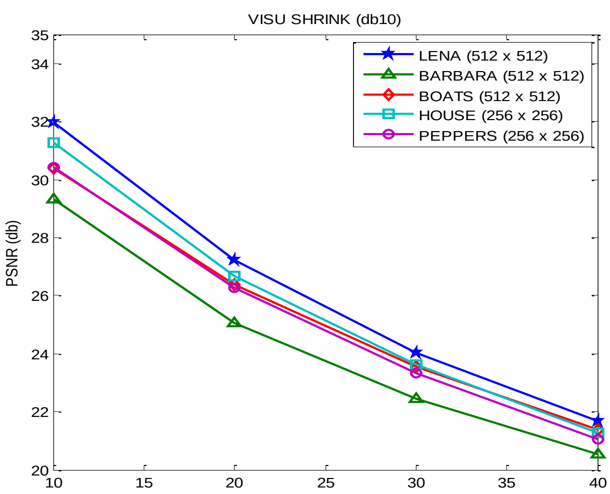

Fig.4(c) Performance Visu Shrink using db10 wavelet

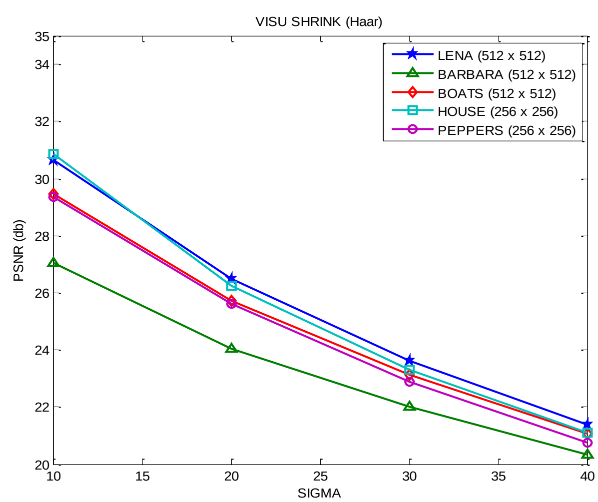

Fig.5(c) Performance Visu Shrink using haar wavelet

Fig.6(a) Original Test Image LENA (512 x 512)

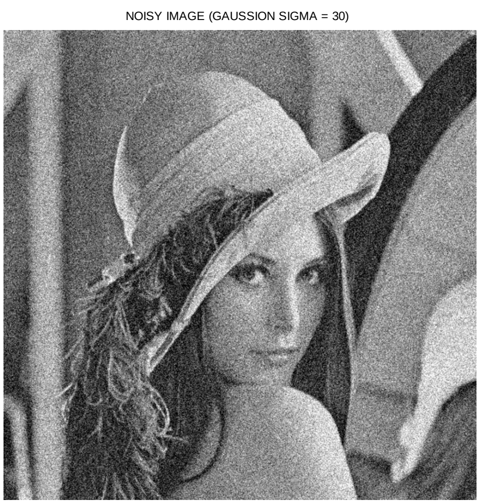

Fig.6(b) Noisy Image with Gaussion Noise (σ = 30)

Fig.6(c) Denoised Image with Neigh Shrink method using coif5 wavelet at Gaussion Noise (σ = 30)

V. Conclusion

The PSNR and elapsed time (execution time) are taken as performance measures. The PSNR values and elapsed time of the different filters for various test images are tabulated in table 1 and different shrinkage techniques for orthogonal wavelets are graphed in Fig 2 to Fig 5.These simulated result shows that the Neigh Shrink method with coif5 wavelet gives best performance (high PSNR) at the cost of more execution time (elapsed time) among all used shrinkage methods for image denoising in respect of different test images and their size. The performance of wavelets coif5 and sym8, haar and db1 are almost same in terms of PSNR values. The filter with less elapsed time is usually implemented in real-time applications.

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