A Stochastic Model based on Order Statistics for Estimation of Expected Time to Recruitment in a two Grade Manpower System under Correlated Wastage

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***Abstract*-**In this paper, an organization with two grades subjected to exit of personnel due to the policy decisions taken by the organization is considered. As the exit of personnel is unpredictable a new univariate recruitment policy involving two thresholds one is optional and the other one mandatory is suggested to enable the organization to plan its decision on recruitment .Three mathematical models are constructed using a univariate policy based on shock model approach assuming that (i) the amount of wastage at each decision epoch are identically distributed constantly correlated and exchangeable exponential random variables(ii)the inter-decision times form an order statistics and (iii) the optional and mandatory thresholds follow various distribution. Theanalytical results are substantiated with numerical illustrations.

***Index Terms*-**Correlated wastage, Order statistics, Shock models, Univariate recruitment policy, Variance of the time to recruitment

I. Introduction

Exits of personnel which is in other words known as wastage is an important aspect in the study of manpower planning .Many models have been discussed using different kinds of wastages and thresholds following various distributions .Such models could be seen in [1],[3].In [5],for a single grade man-power system with mandatory exponential threshold for the loss of man-power, the authors have studied the system characteristics namely mean and variance of time to recruitment when the inter-decision times form a sequence of independent and identically distributed exponential random variables and the amount of wastage at each decision epoch forms a sequence of exchangeable and constantly correlated exponential random variables. In [13],the authors have extended the results in [5] for a two-grade system .In [2] ,for a single grade manpower system ,the author has considered a new recruitment policy involving two thresholds for the loss of man-power in the organization in which one is optional and the other is mandatory and obtained the mean time to recruitment under different conditions on the nature of thresholds according as the inter –decision times are exchangeable and constantly correlated exponential random variables. In [14-17], the authors have extended the results in [2] for a two-grade system according as the thresholds are exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables. In [6],for a single grade man-power system with mandatory exponential threshold for the loss of man-power the author studied system characteristics when (i) the inter-decision times form an order statistics and the loss of man-power forms a sequence of identically distributed exponential random variables. In [7-12], the authors have extended the results in [6] for a man-power system consisting of two grades, involving two thresholds by assuming different distributons for thresholds.

The objective of the present paper is to estimate the mean and variance of the time to recruitment for a two- grade system, when (i) the inter- decision times form an order statistics (ii) the loss of man-hours at each decision epoch are identically distributed constantly correlated and exchangeable exponential random variables and (iii) the thresholds for the loss of man-hours in each grade follow different distributions. This paper is organized as follows. In sections II, models I, II and III are described and analytical expressions for mean and variance of the time to recruitment are derived. In section III, the analytical results are numerically illustrated and relevant conclusions are presented.

II. RESEARCH ELLABORATIONS

MODEL DESCRIPTION AND ANALYSIS OF MODEL –I

An organization takes k decisions in $(0,\infty)$ at each decision there is a random amount of manpower wastage and is represented in terms of man hours. Wastages are linear and cumulative. Let X_i be the loss of man-hours due to the i^{th} decision epoch , i=1,2,3… with cumulative distribution function G(.) and probability density function is g(.) with mean $\frac{1}{\alpha}$ ($\alpha>0$). Let X_i's be identically distributed and constantly correlated exchangeable and exponential random variable. Let ρ be the correlation between X_i and X_j where $i\neq j$.Let the inter-decision times are independent and identically distributed exponential random variables with cumulative distribution function F(.) ,probability density function f(.).Let Y_1, Y_2 (Z_1, Z_2) be random variables denoting optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with cumulative distribution function H(.),probability density function is h(.) and mean $\frac{1}{\theta_1},\frac{1}{\theta_2},\frac{1}{\alpha_1},\frac{1}{\alpha_2}$ respectively, where $\theta_1,\theta_2,\alpha_1,\alpha_2$ are positive .It is assumed that $Y_1< Z_1$ and $Y_2< Z_2$ .Write $Y = Max (Y_1, Y_2)$ and $Z = Max (Z_1, Z_2)$ where Y (Z) is the optional (mandatory) threshold for the loss of man-hours in the organization. The loss of man-hours and the optional and mandatory thresholds are statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function L (.), probability density function l (.), mean E (T) and variance V (T). Let $F_k(.)$ be the k fold convolution of F(.). Let $l^*(.)$ and $f^*(.)$, be the Laplace transform of l(.) and f(.), respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in (0, t]. It is known from Renewal theory [4] that $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$. Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y. The Univariate recruitment policy employed in this paper is as follows: If the total loss of man-hours exceeds the optional threshold Y, the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z, the recruitment is necessary.

$$P(T>t)=\sum_{k=0}^{\infty}V_k(t)P\left(\sum_{i=1}^{k}X_i\le Y\right)+p\sum_{k=0}^{\infty}V_k(t)P\left(\sum_{i=1}^{k}X_i>Y\right)\times P\left(\sum_{i=1}^{k}X_i<Z\right) \quad (1)$$

Since X_i's are assumed to be identical constantly correlated and exchangeable exponential random variables with parameter α, cumulative distribution function of the partial sum

$S_k = X_1+X_2+\ldots\ldots+X_k$ is given by

$$G_k(y)=(1-\rho)\sum_{i=0}^{\infty}\frac{(k\rho)^i}{(1-\rho+k\rho)^{i+1}}\frac{\phi\left(k+i,\frac{y}{b}\right)}{(k+i-1)!} \quad (2)$$

where ρ is constant correlation between X_i and X_j ,$i\neq j$

$$\phi\left(k+i,\frac{y}{b}\right)=\int_0^{\frac{y}{b}}e^{-z}z^{k+i-1}dz,\; b=\alpha(1-\rho) \quad (3)$$

Since $l(t)=\frac{d}{dt}L(t), L(t)=1-P(T>t)\, and\, l^*(s)=L(l(t))$ (4)

Let U_1, U_2, U_3, ... U_k be arranged in an increasing order so that we have sequence $U_{(1)},U_{(2)},U_{(3)},\ldots\ldots U_{(k)}$ .Here $U_{(r)}$ is the r^{th} order statistics, r=1, 2,3…..k

The random variable $U_{(1)},U_{(2)},U_{(3)},\ldots\ldots U_{(k)}$ are not independent.

For r=1,2,3…k the probability density function of $U_{(r)}$ is given by

$$f_{u(r)}(t)=r\,kc_r\,[F(t)]^{r-1}f(t)[1-F(t)]^{k-r},\; r=1,2,3.k \quad (5)$$

Case (i): The distribution of optional and mandatory thresholds follow exponential distribution

Assume $f(t)=f_{u(1)}(t)$

In this case it is known that

$$f_{u(1)}(t) = k\, f(t)(1 - F(t))^{k-1} \quad (6)$$

By hypothesis $f(t) = \lambda e^{-\lambda t}$ and $g(t) = ce^{-ct}$ (7)

Therefore from (6) and (7) we get,

$$f^{*}_{u(1)}(s) = \frac{k\lambda}{k\lambda + s} \quad (8)$$

Since

$$E(T) = -\left.\frac{d(l^{*}(s))}{ds}\right|_{s=0},\; E(T^2) = \left.\frac{d^2(l^{*}(s)}{ds^2}\right|_{s=0} \; and \;\; V(T) = E(T^2) - (E(T))^2 \quad (9)$$

For this case the first two moments of time to recruitment are found to be

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}\big((W_{1k} + W_{2k} - W_{3k}) + p(W_{4k} + W_{5k} - W_{6k}) - p(1-\rho)(W_{1k} + W_{2k} - W_{3k})(W_{4k} + W_{5k} - W_{6k})\big) \quad (10)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left(\frac{k+1}{k^2}\right)((W_{1k} + W_{2k} - W_{3k}) + p(W_{4k} + W_{5k} - W_{6k}) - p(1-\rho)(W_{1k} + W_{2k} - W_{3k})(W_{4k} + W_{5k} - W_{6k})) \quad (11)$$

Where $W_{1k} = \frac{1}{(b\theta_1 + 1)^{k-1}[(1-\rho+k\rho)(b\theta_1 + 1) - k\rho]}, W_{2k} = \frac{1}{(b\theta_2 + 1)^{k-1}[(1-\rho+k\rho)(b\theta_2 + 1) - k\rho]}$

$$W_{3k} = \frac{1}{(b(\theta_1+\theta_2)+1)^{k-1}[(1-\rho+k\rho)(b(\theta_1+\theta_2)+1) - k\rho]}, W_{4k} = \frac{1}{(b\alpha_1 + 1)^{k-1}[(1-\rho+k\rho)(b\alpha_1 + 1) - k\rho]}$$

$$W_{5k} = \frac{1}{(b\alpha_2 + 1)^{k-1}[(1-\rho+k\rho)(b\alpha_2 + 1) - k\rho]}, W_{6k} = \frac{1}{(b(\alpha_1+\alpha_2)+1)^{k-1}[(1-\rho+k\rho)(b(\alpha_1+\alpha_2)+1) - k\rho]} \quad (12)$$

The variance of time to recruitment can be calculated from (9), (10) and (11).

Assume $f(t) = f_{u(k)}(t)$

In this case it is known that

$$f_{u(k)}(t) = (F(t))^{k-1} f(t) \quad (13)$$

Therefore from (7) and (13) we get

$$f^{*}_{u(k)}(s) = \frac{k!\,\lambda^k}{(s+\lambda)(s+2\lambda)\ldots(s+k\lambda)} \quad (14)$$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty} N\big((W_{1k} + W_{2k} - W_{3k}) + p(W_{4k} + W_{5k} - W_{6k}) - p(1-\rho)(W_{1k} + W_{2k} - W_{3k})(W_{4k} + W_{5k} - W_{6k})\big) \quad (15)$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\big((2k+1)N^2 + M\big)((W_{1k} + W_{2k} - W_{3k}) + p(W_{4k} + W_{5k} - W_{6k}) - p(1-\rho)(W_{1k} + W_{2k} - W_{3k})(W_{4k} + W_{5k} - W_{6k})) \quad (16)$$

Where $N=\sum_{n=1}^{k}\frac{1}{n}, M=\sum_{n=1}^{k}\frac{1}{n^2}$

The variance of time to recruitment can be calculated from (9), (15) and (16).

Case (ii): The distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

Assume $f(t)=f_{u(1)}(t)$

Using (6),(7) and (8) the first two moments of time to recruitment are found to be

$$\begin{aligned}E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((&2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})\\ &-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}))\end{aligned} \quad (17)$$

$$\begin{aligned}E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(&2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})\\ &-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})]\end{aligned} \quad (18)$$

Where W_{jk} ,(j=1,2,3,…..6) are given by (12)

$$\begin{aligned}
&W_{7k}=\frac{1}{(b(2\theta_1)+1)^{k-1}[(1-\rho+k\rho)(b(2\theta_1)+1)-k\rho]}, && W_{8k}=\frac{1}{(b(2\theta_2)+1)^{k-1}[(1-\rho+k\rho)(b(2\theta_2)+1)-k\rho]}\\
&W_{9k}=\frac{1}{(b(2(\theta_1+\theta_2))+1)^{k-1}[(1-\rho+k\rho)(b(2(\theta_1+\theta_2))+1)-k\rho]}, && W_{10k}=\frac{1}{(b(\theta_1+2\theta_2)+1)^{k-1}[(1-\rho+k\rho)(b(\theta_1+2\theta_2)+1)-k\rho]}\\
&W_{11k}=\frac{1}{(b(\theta_2+2\theta_1)+1)^{k-1}[(1-\rho+k\rho)(b(\theta_2+2\theta_1)+1)-k\rho]}, && W_{12k}=\frac{1}{(b(2\alpha_1)+1)^{k-1}[(1-\rho+k\rho)(b(2\alpha_1)+1)-k\rho]}\\
&W_{13k}=\frac{1}{(b(2\alpha_2)+1)^{k-1}[(1-\rho+k\rho)(b(2\alpha_2)+1)-k\rho]}, && W_{14k}=\frac{1}{(b(2(\alpha_1+\alpha_2))+1)^{k-1}[(1-\rho+k\rho)(b(2(\alpha_1+\alpha_2))+1)-k\rho]}\\
&W_{15k}=\frac{1}{(b(\alpha_1+2\alpha_2)+1)^{k-1}[(1-\rho+k\rho)(b(\alpha_1+2\alpha_2)+1)-k\rho]}, && W_{16k}=\frac{1}{(b(\alpha_2+2\alpha_1)+1)^{k-1}[(1-\rho+k\rho)(b(\alpha_2+2\alpha_1)+1)-k\rho]}
\end{aligned} \quad (19)$$

The variance of time to recruitment can be calculated from (9), (17) and (18).

Assume $f(t)=f_{u(k)}(t)$

Using (7),(13) and (14) the first two moments of time to recruitment are found to be

$$\begin{aligned}E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((&2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})\\ &-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}))\end{aligned} \quad (20)$$

$$\begin{aligned}E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(&2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})\\ &-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})\\ &(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})]\end{aligned} \quad (21)$$

The variance of time to recruitment can be calculated from (9), (20) and (21).

Case (iii): The distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution with shape parameter 2.

For this case the first two moments of time to recruitment are found to be

Assume $f(t) = f_{u(1)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{1k}+W_{2k}-W_{3k})+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}) - p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})) \quad (22)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{1k}+W_{2k}-W_{3k})+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}) - p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})] \quad (23)$$

where W_{jk} ,(j=1,2,3,…..16) are given by (12) and (19)

The variance of time to recruitment can be calculated from (9), (22) and (23).

Assume $f(t) = f_{u(k)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((W_{1k}+W_{2k}-W_{3k})+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}) - p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})) \quad (24)$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{1k}+W_{2k}-W_{3k})+p(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k}) - p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(2W_{4k}+2W_{5k}-4W_{6k}-W_{12k}-W_{13k}-W_{14k}+2W_{15k}+2W_{16k})] \quad (25)$$

The variance of time to recruitment can be calculated from (9), (24) and (25).

Case (iv): The distributions of optional and mandatory thresholds possess SCBZ property.

Assume $f(t) = f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k}) + p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k}) - p(1-\rho)(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})) \quad (26)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k}) + p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k}) - p(1-\rho)(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})] \quad (27)$$

The variance of time to recruitment can be calculated from (26) and (27) .

where

$$W_{17k} = \frac{p_1}{(b(\delta_1+\mu_1)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_1+\mu_1)+1)-k\rho]} \quad W_{18k} = \frac{q_1}{(b(\eta_1)+1)^{k-1}[(1-\rho+k\rho)(b(\eta_1)+1)-k\rho]}$$

$$W_{19k}=\frac{p_2}{(b(\delta_2+\mu_2)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_2+\mu_2)+1)-k\rho]}$$

$$W_{20k}=\frac{p_2 p_1}{(b(\delta_1+\delta_2+\mu_1+\mu_2)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_1+\delta_2+\mu_1+\mu_2)+1)-k\rho]}$$

$$W_{21k}=\frac{p_2 q_1}{(b(\delta_2+\mu_2+\eta_1)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_2+\mu_2+\eta_1)+1)-k\rho]}\quad W_{22k}=\frac{q_2}{(b\eta_2+1)^{k-1}[(1-\rho+k\rho)(b\eta_2+1)-k\rho]}$$

$$W_{23k}=\frac{p_1 q_2}{(b(\eta_2+\delta_1+\mu_1)+1)^{k-1}[(1-\rho+k\rho)(b(\eta_2+\delta_1+\mu_1)+1)-k\rho]}$$

$$W_{24k}=\frac{q_1 q_2}{(b(\eta_1+\eta_2)+1)^{k-1}[(1-\rho+k\rho)(b(\eta_1+\eta_2)+1)-k\rho]}\quad W_{25k}=\frac{p_3}{(b(\delta_3+\mu_3)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_3+\mu_3)+1)-k\rho]}$$

$$W_{26k}=\frac{q_3}{(b\eta_3+1)^{k-1}[(1-\rho+k\rho)(b\eta_3+1)-k\rho]},\qquad W_{27k}=\frac{p_4}{(b(\delta_4+\mu_4)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_4+\mu_4)+1)-k\rho]}$$

$$W_{28k}=\frac{p_3 p_4}{(b(\delta_3+\delta_4+\mu_3+\mu_4)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_3+\delta_4+\mu_3+\mu_4)+1)-k\rho]}$$

$$W_{29k}=\frac{p_4 q_3}{(b(\delta_4+\mu_4+\eta_3)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_4+\mu_4+\eta_3)+1)-k\rho]}\quad W_{30k}=\frac{q_4}{(b\eta_4+1)^{k-1}[(1-\rho+k\rho)(b\eta_4+1)-k\rho]}$$

$$W_{31k}=\frac{p_3 q_4}{(b(\delta_3+\eta_4+\mu_3)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_3+\eta_4+\mu_3)+1)-k\rho]},\quad W_{32k}=\frac{q_3 q_4}{(b(\eta_3+\eta_4)+1)^{k-1}[(1-\rho+k\rho)(b(\eta_3+\eta_4)+1)-k\rho]}$$

(28)

If $f(t)=f_{u(k)}(t)$

The first two moments of time to recruitment are found to be

$$\begin{aligned}E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((&W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})\\&+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})\\&-p(1-\rho)(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})(W_{25k}+W_{26k}+W_{27k}-\\&W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k}))\end{aligned}\quad(29)$$

$$\begin{aligned}E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[&(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})\\&+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})\\&-p(1-\rho)(W_{17k}+W_{18k}+W_{19k}-W_{20k}-W_{21k}+W_{22k}-W_{23k}-W_{24k})\\&(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})]\end{aligned}\quad(30)$$

where W_{jk} ,(j=17,18,…..32) are given by (28).The variance of time to recruitment can be calculated from (29) and (30) .

Case (v): The distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{1k}+W_{2k}-W_{3k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})) \quad (31)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{1k}+W_{2k}-W_{3k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})] \quad (32)$$

where W_{jk} ,(j=1,2,3,25,….,32) are given by (12) and (28).The variance of time to recruitment can be calculated from (31) and (32) .

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((W_{1k}+W_{2k}-W_{3k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})) \quad (33)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{1k}+W_{2k}-W_{3k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})] \quad (34)$$

The variance of time to recruitment can be calculated from (33) and (34) .

Case (vi): The distributions of optional thresholds follow extended exponential distribution with shape parameter 2 and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})) \quad (35)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})+p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})-p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k})(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})] \quad (36)$$

The variance of time to recruitment can be calculated from (35) and (36)

where W_{jk} ,(j=1,2,3,7,8,9,10,11,25,…..,32) are given by (12),(19) and (28)

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N\Big((2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k}) \\ +p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k}) \\ -p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k}) \\ (W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})\Big) \qquad (37)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\Big((2k+1)N^2+M\Big)\Big[(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k}) \\ +p(W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k}) \\ -p(1-\rho)(2W_{1k}+2W_{2k}-4W_{3k}-W_{7k}-W_{8k}-W_{9k}+2W_{10k}+2W_{11k}) \\ (W_{25k}+W_{26k}+W_{27k}-W_{28k}-W_{29k}+W_{30k}-W_{31k}-W_{32k})\Big] \qquad (38)$$

The variance of time to recruitment can be calculated from (37) and (38).

MODEL DESCRIPTION AND ANALYSIS OF MODEL-II

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as Y=min (Y_1, Y_2) and Z=min (Z_1, Z_2). All the other assumptions and notations are as in model-I.

Case (i): The distribution of optional and mandatory thresholds follow exponential distribution

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

Proceeding as in model-I, it can be shown for the present model that

$$E(T)=\frac{1-\rho}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}\Big(W_{3k}+pW_{6k}-p(1-\rho)W_{3k}W_{6k}\Big) \qquad (39)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}\Big(W_{3k}+pW_{6k}-p(1-\rho)W_{3k}W_{6k}\Big) \qquad (40)$$

where W_{ak} ,(a=3,6) are given by (12) .

Since $V(T)=E(T^2)-(E(T))^2$ the variance of time to recruitment can be calculated from (39) and (40)

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{1-\rho}{\mu}\sum_{k=0}^{\infty}N\Big(W_{3k}+pW_{6k}-p(1-\rho)W_{3k}W_{6k}\Big) \qquad (41)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\Big((2k+1)N^2+M\Big)\Big(W_{3k}+pW_{6k}-p(1-\rho)W_{3k}W_{6k}\Big) \qquad (42)$$

Since $V(T)=E(T^2)-(E(T))^2$ the variance of time to recruitment can be calculated from (41) and (42)

Case (ii): The distribution of optional and mandatory thresholds follow extended exponential distribution

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}\big((4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big) \quad (43)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}\big[(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big] \quad (44)$$

The Variance of time to recruitment can be calculated from (43) and (44).

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N\big((4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big) \quad (45)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\big((2k+1)N^2+M\big)\big[(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big] \quad (46)$$

where W_{jk} ,(j=3,6,9,10,11,14,15,16)are given by (12) and (19).The variance of time to recruitment can be calculated from (45) and (46)

Case (iii): The distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution with shape parameter 2.

Assume $f(t)=f_{u(1)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}\big(W_{3k}+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(W_{3k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big) \quad (47)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}\big[(W_{3k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(W_{3k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big] \quad (48)$$

where W_{jk} ,(j=3,6,14,15,16) are given by (12) and (19).The variance of time to recruitment can be calculated from (47) and (48)

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N\big(W_{3k}+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(W_{3k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big) \quad (49)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\big((2k+1)N^2+M\big)\big[(W_{3k})+p(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})$$
$$-p(1-\rho)(W_{3k})(4W_{6k}+W_{14k}-2W_{15k}-2W_{16k})\big] \quad (50)$$

The Variance of time to recruitment can be calculated from (49) and (50).

Case (iv): The distributions of optional and the mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{20k}+W_{21k}+W_{23k}+W_{24k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{20k}+W_{21k}+W_{23k}+W_{24k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})) \qquad (51)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{20k}+W_{21k}+W_{23k}+W_{24k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{20k}+W_{21k}+W_{23k}+W_{24k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \qquad (52)$$

where W_{jk} ,(j=20,21,23,24,28,29,31,32) are given by (28).The variance of time to recruitment can be calculated from (51) and (52)

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((W_{20k}+W_{21k}+W_{23k}+W_{24k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{20k}+W_{21k}+W_{23k}+W_{24k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})) \qquad (53)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{20k}+W_{21k}+W_{23k}+W_{24k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{20k}+W_{21k}+W_{23k}+W_{24k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \qquad (54)$$

The variance of time to recruitment can be calculated from (53) and (54)

Case (v): The distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}\left((W_{3k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})-p(1-\rho)(W_{3k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})\right) \qquad (55)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{3k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{3k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \qquad (56)$$

where W_{jk} ,(j=3,28,29,31,32) are given by (12) and (28).The variance of time to recruitment can be calculated from (55) and (56)

Assume $f(t)=f_{u(k)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N\left((W_{3k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})-p(1-\rho)(W_{3k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})\right) \qquad (57)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{3k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})$$
$$-p(1-\rho)(W_{3k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \qquad (58)$$

The variance of time to recruitment can be calculated from (57) and (58).

Case (vi): The distributions of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t) = f_{u(k)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})- p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})) \quad (59)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})- p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \quad (60)$$

where W_{jk} ,(j=3,9,10,11,28,29,31,32) are given by (12) ,(19) and (28).The variance of time to recruitment can be calculated from (59) and (60)

Assume $f(t) = f_{u(k)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})- p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(W_{28k}+W_{29k}+W_{31k}+W_{32k})) \quad (61)$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})+p(W_{28k}+W_{29k}+W_{31k}+W_{32k})- p(1-\rho)(4W_{3k}+W_{9k}-2W_{10k}-2W_{11k})(W_{28k}-W_{29k}+W_{31k}+W_{32k})] \quad (62)$$

The variance of time to recruitment can be calculated from (61) and (62)

MODEL DESCRIPTION AND ANALYSIS OF MODEL-III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y=Y_1+Y_2$ and $Z=Z_1+Z_2$ .All the other assumptions and notations are as in model-I. Proceeding as in model-I, it can be shown for the present model that

Case (i): The distributions of optional and mandatory thresholds follow exponential distribution.

Assume $f(t) = f_{u(1)}(t)$

For this case the first two moments of time to recruitment are found to be

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{33k}-W_{34k})+p(W_{35k}-W_{36k})-p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})) \quad (63)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{33k}-W_{34k})+p(W_{35k}-W_{36k})-p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})] \quad (64)$$

where $W_{33k} = \frac{\theta_1}{(\theta_1-\theta_2)(b\theta_2+1)^{k-1}[(1-\rho+k\rho)(b\theta_2+1)-k\rho]}, W_{34k} = \frac{\theta_2}{(\theta_1-\theta_2)(b\theta_1+1)^{k-1}[(1-\rho+k\rho)(b\theta_1+1)-k\rho]}$

$$W_{35k} = \frac{\alpha_1}{(\alpha_1-\alpha_2)(b\alpha_2+1)^{k-1}[(1-\rho+k\rho)(b\alpha_2+1)-k\rho]}, W_{36k} = \frac{\alpha_2}{(\alpha_1-\alpha_2)(b\alpha_1+1)^{k-1}[(1-\rho+k\rho)(b\alpha_1+1)-k\rho]} \quad (65)$$

The variance of time to recruitment can be calculated from (63) and (64).

Assume $f(t) = f_{u(k)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty} N((W_{33k} - W_{34k}) + p(W_{35k} - W_{36k}) - p(1-\rho)(W_{33k} - W_{34k})(W_{35k} - W_{36k})) \quad (66)$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}((2k+1)N^2 + M)[(W_{33k} - W_{34k}) + p(W_{35k} - W_{36k}) - p(1-\rho)(W_{33k} - W_{34k})(W_{35k} - W_{36k})] \quad (67)$$

The variance of time to recruitment can be calculated from (66) and (67).

Case (ii): If the distributions of optional and mandatory thresholds follow extended exponential distribution with shape parameter 2.

Assume $f(t) = f_{u(1)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{37k} + W_{38k} - W_{39k} - W_{40k}) + p(W_{41k} + W_{42k} - W_{43k} - W_{44k}) - p(1-\rho)(W_{37k} + W_{38k} - W_{39k} - W_{40k})(W_{41k} + W_{42k} - W_{43k} - W_{44k})) \quad (68)$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{37k} + W_{38k} - W_{39k} - W_{40k}) + p(W_{41k} + W_{42k} - W_{43k} - W_{44k}) - p(1-\rho)(W_{37k} + W_{38k} - W_{39k} - W_{40k})(W_{41k} + W_{42k} - W_{43k} - W_{44k})] \quad (69)$$

where $W_{37k} = \dfrac{4\theta_2^2}{(\theta_1 - \theta_2)(\theta_1 - 2\theta_2)(b\theta_1 + 1)^{k-1}[(1-\rho+k\rho)(b\theta_1 + 1) - k\rho]}$

$$W_{38k} = \frac{4\theta_1^2}{(\theta_1 - \theta_2)(2\theta_1 - \theta_2)(b\theta_2 + 1)^{k-1}[(1-\rho+k\rho)(b\theta_2 + 1) - k\rho]}$$

$$W_{39k} = \frac{\theta_2^2}{(\theta_1 - \theta_2)(2\theta_1 - \theta_2)(b(2\theta_1) + 1)^{k-1}[(1-\rho+k\rho)(b(2\theta_1) + 1) - k\rho]}$$

$$W_{40k} = \frac{\theta_1^2}{(\theta_1 - \theta_2)(\theta_1 - 2\theta_2)(b(2\theta_2) + 1)^{k-1}[(1-\rho+k\rho)(b(2\theta_2) + 1) - k\rho]}$$

$$W_{41k} = \frac{4\alpha_2^2}{(\alpha_1 - \alpha_2)(\alpha_1 - 2\alpha_2)(b\alpha_1 + 1)^{k-1}[(1-\rho+k\rho)(b\alpha_1 + 1) - k\rho]}$$

$$W_{42k} = \frac{4\alpha_1^2}{(\alpha_1 - \alpha_2)(2\alpha_1 - \alpha_2)(b\alpha_2 + 1)^{k-1}[(1-\rho+k\rho)(b\alpha_2 + 1) - k\rho]}$$

$$W_{43k} = \frac{\alpha_2^2}{(\alpha_1 - \alpha_2)(2\alpha_1 - \alpha_2)(b(2\alpha_1) + 1)^{k-1}[(1-\rho+k\rho)(b(2\alpha_1) + 1) - k\rho]}$$

$$W_{44k} = \frac{\alpha_1^2}{(\alpha_1 - \alpha_2)(\alpha_1 - 2\alpha_2)(b(2\alpha_2) + 1)^{k-1}[(1-\rho+k\rho)(b(2\alpha_2) + 1) - k\rho]} \quad (70)$$

The variance of time to recruitment can be calculated from (68) and (69).

Assume $f(t) = f_{u(k)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty} N((W_{37k}+W_{38k}-W_{39k}-W_{40k})+p(W_{41k}+W_{42k}-W_{43k} \quad (71)$$
$$-W_{44k})-p(1-\rho)(W_{37k}+W_{38k}-W_{39k}-W_{40k})(W_{41k}+W_{42k}-W_{43k}-W_{44k}))$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{37k}+W_{38k}-W_{39k}-W_{40k})+p(W_{41k}+W_{42k}-W_{43k}-W_{44k})$$
$$-p(1-\rho)(W_{37k}+W_{38k}-W_{39k}-W_{40k})(W_{41k}+W_{42k}-W_{43k}-W_{44k})] \quad (72)$$

The variance of time to recruitment can be calculated from (71) and (72).

Case (iii): If the distributions of optional thresholds follow exponential distribution and mandatory thresholds follow extended exponential distribution.

Assume $f(t) = f_{u(1)}(t)$

$$E(T^2) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{33k}-W_{34k})+p(W_{41k}+W_{42k}-W_{43k}-W_{44k})- \quad (73)$$
$$p(1-\rho)(W_{33k}-W_{34k})(W_{41k}+W_{42k}-W_{43k}-W_{44k}))$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}[(W_{33k}-W_{34k})+p(W_{41k}+W_{42k}-W_{43k}-W_{44k}) \quad (74)$$
$$-p(1-\rho)(W_{33k}-W_{34k}(W_{41k}+W_{42k}-W_{43k}-W_{44k})]$$

where W_{jk} ,(j=33,34,35,36,41,42,43,44) are given by(65) and (70). The variance of time to recruitment can be calculated from (73) and (74).

Assume $f(t) = f_{u(k)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty} N((W_{33k}-W_{34k})+p(W_{41k}+W_{42k}-W_{43k}-W_{44k}) \quad (75)$$
$$-p(1-\rho)(W_{33k}-W_{34k})(W_{41k}+W_{42k}-W_{43k}-W_{44k}))$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)[(W_{33k}-W_{34k})+p(W_{41k}+W_{42k}-W_{43k}$$
$$-W_{44k})-p(1-\rho)(W_{33k}-W_{34k}(W_{41k}+W_{42k}-W_{43k}-W_{44k})] \quad (76)$$

The variance of time to recruitment can be calculated from (75) and (76)

Case (iv): If the distributions of optional and mandatory thresholds possess SCBZ property.

Assume $f(t) = f_{u(1)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{45k}-W_{46k}+W_{47k}-W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})+p(W_{53k}-W_{54k}+$$
$$W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{45k}-W_{46k}+W_{47k}-W_{48k} \quad (77)$$
$$+W_{49k}-W_{50k}+W_{51k}-W_{52k})(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k}))$$

$$E(T^2) = \frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}(W_{45k}-W_{46k}+W_{47k}-W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})+p(W_{53k}$$
$$-W_{54k}+W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{45k}-W_{46k}+W_{47k}$$
$$-W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k}$$
$$-W_{58k}+W_{59k}-W_{60k})$$

(78)where

$$W_{45k}=\frac{(\delta_1+\mu_1)p_1p_2}{(\delta_1-\delta_2+\mu_1-\mu_2)(b(\delta_2+\mu_2)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_2+\mu_2)+1)-k\rho]}$$

$$W_{46k}=\frac{(\delta_2+\mu_2)p_1p_2}{(\delta_1-\delta_2+\mu_1-\mu_2)(b(\delta_1+\mu_1)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_1+\mu_1)+1)-k\rho]}$$

$$W_{47k}=\frac{\eta_1p_2q_1}{(\eta_1-\delta_2-\mu_2)(b(\delta_2+\mu_2)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_2+\mu_2)+1)-k\rho]}$$

$$W_{48k}=\frac{(\delta_2+\mu_2)p_2q_1}{(\eta_1-\delta_2-\mu_2)(b\eta_1+1)^{k-1}[(1-\rho+k\rho)(b\eta_1+1)-k\rho]}$$

$$W_{49k}=\frac{(\delta_1+\mu_1)p_1q_2}{(\delta_1+\mu_1-\eta_2)(b\eta_2+1)^{k-1}[(1-\rho+k\rho)(b\eta_2+1)-k\rho]}$$

$$W_{50k}=\frac{\eta_2p_1q_2}{(\delta_1+\mu_1-\eta_2)(b(\delta_1+\mu_1)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_1+\mu_1)+1)-k\rho]}$$

$$W_{51k}=\frac{\eta_1q_1q_2}{(\eta_1-\eta_2)(b\eta_2+1)^{k-1}[(1-\rho+k\rho)(b\eta_2+1)-k\rho]},\qquad W_{52k}=\frac{\eta_2q_1q_2}{(\eta_1-\eta_2)(b\eta_1+1)^{k-1}[(1-\rho+k\rho)(b\eta_1+1)-k\rho]}$$

$$W_{53k}=\frac{(\delta_3+\mu_3)p_3p_4}{(\delta_3-\delta_4+\mu_3-\mu_4)(b(\delta_4+\mu_4)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_4+\mu_4)+1)-k\rho]}$$

$$W_{54k}=\frac{(\delta_4+\mu_4)p_3p_4}{(\delta_3-\delta_4+\mu_3-\mu_4)(b(\delta_3+\mu_3)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_3+\mu_3)+1)-k\rho]}$$

$$W_{55k}=\frac{(\delta_3+\mu_3)p_3q_4}{(\delta_3+\mu_3-\eta_4)(b\eta_4+1)^{k-1}[(1-\rho+k\rho)(b\eta_4+1)-k\rho]}$$

$$W_{56k}=\frac{\eta_4p_3q_4}{(\delta_3+\mu_3-\eta_4)(b(\delta_3+\mu_3)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_3+\mu_3)+1)-k\rho]}$$

$$W_{57k}=\frac{\eta_3p_4q_3}{(\eta_3-\delta_4-\mu_4)(b(\delta_4+\mu_4)+1)^{k-1}[(1-\rho+k\rho)(b(\delta_4+\mu_4)+1)-k\rho]}$$

$$W_{58k}=\frac{(\delta_4+\mu_4)p_4q_3}{(\eta_3-\delta_4-\mu_4)(b\eta_3+1)^{k-1}[(1-\rho+k\rho)(b\eta_3+1)-k\rho]}$$

$$W_{59k}=\frac{\eta_3q_3q_4}{(\eta_3-\eta_4)(b\eta_4+1)^{k-1}[(1-\rho+k\rho)(b\eta_4+1)-k\rho]},\qquad W_{60k}=\frac{\eta_4q_3q_4}{(\eta_3-\eta_4)(b\eta_3+1)^{k-1}[(1-\rho+k\rho)(b\eta_3+1)-k\rho]} \quad (79)$$

The Variance of time to recruitment can be calculated from (77) and (78)

If $f(t)=f_{u(1)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((W_{45k}-W_{46k}+W_{47k}-W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})+p(W_{53k}-W_{54k}+ W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{45k}-W_{46k}+W_{47k}-W_{48k} +W_{49k}-W_{50k}+W_{51k}-W_{52k})(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (80)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)(W_{45k}-W_{46k}+W_{47k}-W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})+p(W_{53k} -W_{54k}+W_{55k}-W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{45k}-W_{46k}+W_{47k} -W_{48k}+W_{49k}-W_{50k}+W_{51k}-W_{52k})(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k}) \quad (81)$$

The variance of time to recruitment can be calculated from (80) and (81).

Case (v): The distributions of optional thresholds follow exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{33k}-W_{34k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{33k}-W_{34k})(W_{53k}-W_{54k}+W_{55k}-W_{56k} +W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (82)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}((W_{33k}-W_{34k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{33k}-W_{34k})(W_{53k}-W_{54k}+W_{55k}-W_{56k} +W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (83)$$

where W_{jk} ,(j=33,34,53,54,55,56,57,58,59,60) are given by(65) and (79). The variance of time to recruitment can be calculated from (82) and (83).

Assume $f(t)=f_{u(k)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}N((W_{33k}-W_{34k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{33k}-W_{34k})(W_{53k}-W_{54k}+W_{55k}-W_{56k} +W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (84)$$

$$E(T^2)=\frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\left((2k+1)N^2+M\right)((W_{33k}-W_{34k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{33k}-W_{34k})(W_{53k}-W_{54k}+W_{55k}-W_{56k} +W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (85)$$

The variance of time to recruitment can be calculated from (84) and (85).

Case (vi): If the distributions of optional thresholds follow extended exponential distribution and the distribution of mandatory thresholds possess SCBZ property.

Assume $f(t)=f_{u(1)}(t)$

$$E(T)=\frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty}\frac{1}{k}((W_{37k}+W_{38k}-W_{39k}-W_{40k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{37k}+W_{38k}-W_{39k}-W_{40k})(W_{53k}-W_{54k}+W_{55k} -W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (86)$$

$$E(T^2)=\frac{2(1-\rho)}{\mu^2}\sum_{k=0}^{\infty}\frac{(k+1)}{k^2}((W_{37k}+W_{38k}-W_{39k}-W_{40k})+p(W_{53k}-W_{54k}+W_{55k}-W_{56k}+W_{57k} -W_{58k}+W_{59k}-W_{60k})-p(1-\rho)(W_{37k}+W_{38k}-W_{39k}-W_{40k})(W_{53k}-W_{54k}+W_{55k} -W_{56k}+W_{57k}-W_{58k}+W_{59k}-W_{60k})) \quad (87)$$

where W_{jk} ,(j=37,38,39,40,53,54,55,56,57,58,59,60) are given by(70) and (79). The variance of time to recruitment can be calculated from (86) and (87).

Assume $f(t) = f_{u(k)}(t)$

$$E(T) = \frac{(1-\rho)}{\mu}\sum_{k=0}^{\infty} N((W_{37k} + W_{38k} - W_{39k} - W_{40k}) + p(W_{53k} - W_{54k} + W_{55k} - W_{56k} + W_{57k} - W_{58k} + W_{59k} - W_{60k}) - p(1-\rho)(W_{37k} + W_{38k} - W_{39k} - W_{40k})(W_{53k} - W_{54k} + W_{55k} - W_{56k} + W_{57k} - W_{58k} + W_{59k} - W_{60k})) \quad (88)$$

$$E(T^2) = \frac{(1-\rho)}{\mu^2}\sum_{k=0}^{\infty} \left((2k+1)N^2 + M\right)((W_{37k} + W_{38k} - W_{39k} - W_{40k}) + p(W_{53k} - W_{54k} + W_{55k} - W_{56k} + W_{57k} - W_{58k} + W_{59k} - W_{60k}) - p(1-\rho)(W_{37k} + W_{38k} - W_{39k} - W_{40k})(W_{53k} - W_{54k} + W_{55k} - W_{56k} + W_{57k} - W_{58k} + W_{59k} - W_{60k})) \quad (89)$$

The variance of time to recruitment can be calculated from (88) and (89).

III. NUMERICAL ILLUSTRATIONS

The mean and variance of the time to recruitment for the above models are given in the following tables for the cases (i),(ii),(iii),(iv),(v),(vi) respectively by keeping $\theta_1 = 0.4, \theta_2 = 0.9, \alpha_1 = 0.6, \alpha_2 = 0.7, \alpha = 1, p = 0.2, b = 2, \mu = 1$ fixed and varying ρ and keeping ρ fixed and varying k.

Table I: **(Effect of ρ on the performance measures E (T) and V (T))**

Model –I	Order	ρ	-0.6	-0.3	0	0.3	0.6	0.9
Case(i)	I	E(T)	1.2293	1.0997	0.9502	0.7706	0.5333	0.1730
		V(T)	2.9201	2.7987	2.6016	2.2828	1.7312	0.6326
	K	E(T)	2.4668	2.0765	1.6764	1.2666	0.8122	0.2421
		V(T)	15.2161	12.8380	10.3242	7.7625	4.9615	1.4875
Case(ii)	I	E(T)	1.4700	1.3563	1.2021	1.0010	0.7164	0.2443
		V(T)	3.0851	3.0415	2.9315	2.6913	2.1696	0.8703
	K	E(T)	3.0972	2.7354	2.2766	1.7602	1.1552	0.3557
		V(T)	17.9588	16.1373	13.6020	10.6345	7.0742	2.2388
Case(iii)	I	E(T)	1.2481	1.1218	0.9734	0.7935	0.5532	0.1818
		V(T)	2.0343	2.8219	2.6350	2.3266	1.7818	0.6625
	K	E(T)	2.5240	2.1417	1.7380	1.3190	0.8504	0.2558
		V(T)	15.5339	13.2402	10.7208	8.1047	5.2109	1.5771
Case(iv)	I	E(T)	1.3298	1.2128	1.0690	0.8876	0.6354	0.2201
		V(T)	2.9973	2.9143	2.7631	2.4927	1.9753	0.7885
	K	E(T)	2.7484	2.3910	1.9905	1.5494	1.0282	0.3238
		V(T)	16.6397	14.6517	12.3026	9.6547	6.4700	2.0738

Case(v)	I	E(T)	1.2424	1.1178	0.9729	0.7968	0.5603	0.1887
		V(T)	2.9281	2.8147	2.6297	2.3273	1.7945	0.6843
	K	E(T)	2.5134	2.1399	1.7514	1.3437	0.8784	0.2710
		V(T)	15.5111	13.2830	10.8884	8.3641	5.4823	1.7065
Case(vi)	I	E(T)	1.4669	1.3541	1.2025	1.0045	0.7229	0.2509
		V(T)	3.0833	3.0385	2.9286	2.6917	2.1791	0.8903
	K	E(T)	3.0908	2.7353	2.2889	1.7827	1.1814	0.3704
		V(T)	17.9507	16.1729	13.7321	10.8482	7.3135	2.3627

Table II: (Effect of ρ on the performance measures E (T) and V (T))

Model-II	Order	ρ	-0.6	-0.3	0	0.3	0.6	0.9
Case(i)	I	E(T)	0.5711	0.4629	0.3721	0.2784	0.1730	0.0479
		V(T)	1.8004	1.5465	1.2930	1.0016	0.6449	0.1854
	K	E(T)	0.9693	0.6757	0.5020	0.3551	0.2112	0.0564
		V(T)	6.5763	4.0283	2.8000	1.8966	1.0957	0.2873
Case(ii)	I	E(T)	0.8957	0.7510	0.6147	0.4702	0.3009	0.0865
		V(T)	2.4940	2.2503	1.9570	1.5834	1.0739	0.3295
	K	E(T)	1.6107	1.2041	0.9021	0.6440	0.3889	0.1063
		V(T)	10.3108	7.2549	5.1643	3.5577	2.0995	0.5686
Case(iii)	I	E(T)	0.6165	0.5047	0.4085	0.3082	0.1938	0.0545
		V(T)	1.9073	1.6567	1.3986	1.0963	0.7167	0.2103
	K	E(T)	1.0660	0.7578	0.5651	0.4016	0.2407	0.0650
		V(T)	7.2267	4.5946	3.2123	2.1849	1.2697	0.3363
Case(iv)	I	E(T)	0.6910	0.5741	0.4688	0.3570	0.2272	0.0650
		V(T)	2.0741	1.8299	1.5646	1.2438	0.8278	0.2491
	K	E(T)	1.2261	0.9022	0.6809	0.4871	0.2940	0.0802
		V(T)	8.2897	5.6193	4.0239	2.7691	1.6232	0.4346
Case(v)	I	E(T)	0.5772	0.4692	0.3781	0.2837	0.1771	0.0494
		V(T)	1.8139	1.5622	1.3094	1.0177	0.6584	0.1908

	K	E(T)	0.9852	0.6916	0.5156	0.3657	0.2183	0.0586
		V(T)	6.7021	4.1599	2.9099	1.9786	1.1469	0.3022
Case(vi)	I	E(T)	0.8667	0.7236	0.5903	0.4494	0.2868	0.0815
		V(T)	2.4435	2.1926	1.8964	1.5241	1.0248	0.3109
	K	E(T)	1.5463	1.1493	0.8598	0.6123	0.3683	0.1001
		V(T)	9.9375	6.9236	4.9224	3.3845	1.9901	0.5356

Table III: (Effect of ρ on the performance measures E (T) and V (T))

Model-III	Order	ρ	-0.6	-0.3	0	0.3	0.6	0.9
Case(i)	I	E(T)	1.3685	1.2519	1.1037	0.9144	0.6506	0.2204
		V(T)	3.0255	2.9537	2.8123	2.5455	2.0167	0.7910
	K	E(T)	2.8443	2.4822	2.0585	1.5885	1.0404	0.3195
		V(T)	17.0227	15.0497	12.5813	9.7670	6.4425	2.0138
Case(ii)	I	E(T)	1.6135	1.5308	1.3980	1.2018	0.8961	0.3267
		V(T)	3.1074	3.1054	3.0700	2.9324	2.5156	1.1282
	K	E(T)	3.5392	3.2642	2.8356	2.2726	1.5449	0.4988
		V(T)	18.7988	17.5483	15.5019	12.7368	8.9781	3.0864
Case(iii)	I	E(T)	1.3861	1.2750	1.1308	0.9437	0.6787	0.2348
		V(T)	3.0320	2.9681	2.8392	2.5891	2.0784	0.8380
	K	E(T)	2.9044	2.5594	2.1420	1.6669	1.1023	0.3438
		V(T)	17.3444	15.5087	13.1147	10.2895	6.8639	2.1819
Case(iv)	I	E(T)	1.4487	1.3487	1.2138	1.0309	0.7602	0.2767
		V(T)	3.0608	3.0170	2.9191	2.7103	2.2437	0.9692
	K	E(T)	3.0872	2.7765	2.3808	1.9019	1.2959	0.4230
		V(T)	18.1695	16.6387	14.5233	11.8205	8.2300	2.7768
Case(v)	I	E(T)	1.3640	1.2512	1.1077	0.9233	0.6640	0.2318
		V(T)	3.0192	2.9474	2.8094	2.5514	2.0400	0.8263
		E(T)	2.8434	2.4985	2.0926	1.6345	1.0876	0.3439

	K	V(T)	17.0772	15.226	12.8982	10.1832	6.8602	2.2148
Case(vi)	I	E(T)	1.6032	1.5187	1.3850	1.1890	0.8854	0.3240
		V(T)	3.1076	3.1026	3.0615	2.9161	2.4924	1.1181
	K	E(T)	3.5066	3.2294	2.8046	2.2509	1.5345	0.4991
		V(T)	9.7757	9.2822	8.3433	7.0743	5.3046	2.0446

Table IV: (Effect of k on the performance measures E (T) and V (T))

MODEL-I	Order	ρ=0.9	Case-i	Case-ii	Case-iii	Case-iv	Case-v	Case-vi
k=1	I	E(T)	0.1453	0.2002	0.1522	0.1796	0.1561	0.2040
		V(T)	0.5601	0.7608	0.5855	0.6860	0.6002	0.7743
	K	E(T)	0.1453	0.2002	0.1522	0.1796	0.1561	0.2040
		V(T)	0.5601	0.7608	0.5855	0.6880	0.6002	0.7743
k=2	I	E(T)	0.1675	0.2350	0.1759	0.2112	0.1818	0.2406
		V(T)	0.6199	0.8499	0.6489	0.7685	0.6685	0.8679
	k	E(T)	0.2120	0.3045	0.2234	0.2744	0.2331	0.3138
		V(T)	1.0918	1.5768	1.1521	1.4332	1.2115	1.6328
k=3	I	E(T)	0.1730	0.2443	0.1818	0.2201	0.1887	0.2509
		V(T)	0.6326	0.8703	0.6625	0.7885	0.6843	0.8903
	k	E(T)	0.2421	0.3557	0.2558	0.3238	0.2710	0.3704
		V(T)	1.4875	2.2388	1.5771	2.0738	1.7065	2.3627

Table V: (Effect of k on the performance measures E (T) and V (T))

MODEL-II	Order	ρ=0.9	Case-i	Case-ii	Case-iii	Case-iv	Case-v	Case-vi
k=1	I	E(T)	0.0442	0.0779	0.0499	0.0586	0.0453	0.0735
		V(T)	0.1747	0.3057	0.1972	0.2308	0.1793	0.2724
	k	E(T)	0.0442	0.0779	0.0499	0.0586	0.0453	0.0735
		V(T)	0.1747	0.3057	0.1972	0.0753	0.1793	0.2885
		E(T)	0.0475	0.0854	0.0540	0.0641	0.0489	0.0804

k=2	I	V(T)	0.1843	0.3268	0.2089	0.2468	0.1896	0.2922
	k	E(T)	0.0541	0.1003	0.0620	0.2308	0.0560	0.0943
		V(T)	0.2564	0.4877	0.2965	0.3677	0.2667	0.4589
k=3	I	E(T)	0.0479	0.0865	0.0545	0.0650	0.0494	0.0815
		V(T)	0.1854	0.3295	0.2103	0.2491	0.1908	0.3109
	k	E(T)	0.0564	0.1063	0.0650	0.0802	0.0586	0.1001
		V(T)	0.2873	0.5686	0.3363	0.4346	0.3022	0.5356

Table VI: (Effect of k on the performance measures E (T) and V (T))

MODEL-III	Order	ρ=0.9	Case-i	Case-ii	Case-iii	Case-iv	Case-v	Case-vi
k=1	I	E(T)	0.1812	0.2599	0.1918	0.2202	0.1880	0.2564
		V(T)	0.6920	0.9721	0.7305	0.8322	0.7165	0.9598
	K	E(T)	0.1812	0.2599	0.1918	0.2202	0.1880	0.2554
		V(T)	1.4970	0.9721	0.7305	0.8322	0.7165	0.9598
k=2	I	E(T)	0.2121	0.3113	0.2255	0.2633	0.2220	0.3081
		V(T)	0.7725	1.0969	0.8176	0.9408	0.8047	1.0857
	K	E(T)	0.2739	0.4141	0.2930	0.3496	0.2900	0.4114
		V(T)	1.4220	2.1529	1.5246	1.8373	1.5183	2.1481
k=3	I	E(T)	0.2204	0.3267	0.2348	0.2767	0.2318	0.3240
		V(T)	0.7910	1.1282	0.8380	0.9692	0.8263	1.1181
	K	E(T)	0.3195	0.4988	0.3438	0.4230	0.3439	0.4991
		V(T)	2.0138	3.0864	2.1819	2.7768	2.2148	2.0446

IV.FINDINGS

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment for all the models are reported below. It is observed that

(i)When correlation coefficient increases, the mean and variance of time to recruitment decreases for both first and k th order statistics.

(ii) If k, the number of decision epochs in (0, t], increases, the mean and variance of time to recruitment increases for both first and k th order statistics.

REFERENCES

[1] D.J .Bartholomew, and A.F. Forbes, Statistical Techniques for Manpower Planning, John Wiley and Sons (1979).

[2] J.B.Esther Clara, Contributions to the study on some stochastic models in manpower planning, 2012,Ph.D., thesis Bharathidasan University

[3] R.C. Grinold, andK.J. Marshall, Man Power Planning, North Holland, NewYork (1977).

[4]J.Medhi , Stochastic Processes, Wiley Eastern, New Delhi.

[5] A.Muthaiyan ,R.Sathiyamoorthi,A stochastic model for estimation of expected time to recruitment under correlated wastage ,Ultra sci.phys.sci.21,no.2 (M) (2009),pp,433-438,

[6]A.Muthaiyan, A. Sulaiman, and R. Sathiyamoorthi, A stochastic model basedon order statistics for estimation of expected time to recruitment, Acta Ciencia Indica, Vol 5, No.2, (2009), pp, 501-508.

[7] J. Sridharan,P. Saranya., and A.Srinivasan, A Stochastic model based on order statistics for estimation of expected time to recruitment in a two-grade man-power system with different types of thresholds, International Journal of Mathematical Sciences and Engineering Applications , Vol 6,No.5, (2012),pp,1-10.

[8] J. Sridharan,P. Saranya., and A.Srinivasan, A Stochastic model based on order statistics for estimation of expected time to recruitment in a two-grade man-power system using a univariate recruitment policy involving geometric threshold, Antarctica journal of Mathematics , Vol 10,No.1, (2013),pp ,11-19.

[9]J. Sridharan,P. Saranya., and A.Srinivasan,Variance of time to recruitment in a two-grade man-power system with extended exponential thresholds using Order Statistics for inter-decision times, Archimedes journal of Mathematics,Vol 3,No.1,(2013),pp,19-30.

[10] J. Sridharan,P. Saranya., and A.Srinivasan, Variance of time to recruitment in a two- grade manpower system with exponential and extended exponential thresholds using order statistics for inter-decision times, Bessel Journal of Mathematics ,Vol 3,No.1,(2013),pp,29-38.

[11]J. Sridharan,P. Saranya., and A.Srinivasan, Expected time to recruitment in an organization with two-grades involving two thresholds, Bulletin of Mathematical Sciences and Applications, Vol 1,No.2,(2012),pp,53-69.

[12]J. Sridharan,P. Saranya., and A.Srinivasan, A Stochastic model for estimation of expected time to recruitment in a two grade manpower system under correlatedwastage, accepted for publication in Archimedes journal of Mathematics.

[13] A.Srinivasan, Vasudevan.V, Variance of the time to recruitment in an organization with two grades, Recent Research in Science and technology, Vol 3,No.1, (2011),pp, 128-131.

[14]A. Srinivasan, Vasudevan.V, A Stochastic model for the expected time to recruitment in a two graded manpower system .Antarctica Journal of Mathematics, Vol 8,No.3, (2011), pp,241-248.

[15]A. Srinivasan, Vasudevan.V, A manpower model for a two-grade system with univariate policy of recruitment ,International review of pure and Applied Mathematics ,Vol 7,No.1, (2011) ,pp,79-88.

[16] A. Srinivasan, Vasudevan.V, A Stochastic model for the expected time to recruitment in a two graded manpower system with two discrete thresholds ,International Journal of Mathematical Analysis and Applications,Vol 6,No.1,(2011),pp,119-126.

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