Mathematical and psychological perception of emotions

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Abstract : In this article, the author begins by recalling the hyperbolic partial differential equations in order to introduce the wave equation. If in the article [2], a uni-dimensional solution has been detailed, then, in this article, the author is interested in the 3-dimensional solution. Finally, the article presents the pattern of the birth of emotions in the human brain from stimuli. The article [5] is concluded with a question: can we learn emotions differently? In this article, we put the question to a psychologist. Here is his testimony. She answers three questions. First, she explains the electrical and hormonal phases leading to emotional reactions. Then, she discusses the learning of emotions from emotions. And she concludes with the importance of emotional learning through experience emphasizing the presence of an adult.

Key words : Maxwell equation, Wave equation, electromagnetic fields, psychology, emotional education.

1 The wave equation

This paragraph presents the hyperbolic partial differential equations. Recall that a general form of second-order one-variable PDEs is written in the equality 1 :

$$A\frac{\partial^2 u}{\partial t^2}+B\frac{\partial^2 u}{\partial t\partial x}+C\frac{\partial^2 u}{\partial x^2}+D\frac{\partial u}{\partial t}+E\frac{\partial u}{\partial x}+F\,u \quad = \quad 0 \tag{1}$$

The hyperbolic equations are defined as follows :

Definition 1 *If $B^2-4AC>0$, the equation is called **hyperbolic**.*

Example 1 *In dimension 1, the wave equation, is*

$$\frac{\partial^2 u}{\partial t^2}(t,x)-\frac{\partial^2 u}{\partial x^2}(t,x) \quad = \quad 0 \tag{2}$$

In this example, A =1, B =0 and $C=-1$, thus,

$$B^2-4AC=4>0$$

This equation has been studied in the article [2].

In $\mathbb{R}^3$, we denote the matrix $A=(a_{i,j})_{1\leq i,j\leq 3}$ and the vector $B=(b_i)_{1\leq i\leq 3}$. A general form of the PDEs of the second order with several variables is described in the equality 3, which we recall :

$$\sum_{i,j=1}^{n} a_{ij}(\mathbf{x})\frac{\partial^2 f}{\partial x_i\partial x_j}+\sum_{i=1}^{n} b_i(\mathbf{x})\frac{\partial f}{\partial x_i}+c(\mathbf{x})f \quad = \quad h(\mathbf{x}),\ \mathbf{x}\in U\subset\mathbb{R}^n \tag{3}$$

Proposition 1 *The PDE 3 is said to be **hyperbolic** at a given point x of the open U if the symmetric square matrix*

$$A(\mathbf{x}) \quad = \quad (a_{ij})_{1\leq i,j\leq n} \tag{4}$$

second-order coefficients admit non-zero eigenvalues which are all of the same sign except one of opposite sign.

Example 2 *Let $u : \mathbb{R} \times \mathbb{R}^3 \rightarrow \mathbb{R}$, the wave equation becomes :*

$$\frac{\partial^2 u}{\partial t^2}(t,x,y,z) - \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)(t,x,y,z) = 0 \tag{5}$$

is a hyperbolic equation. Indeed, as in the previous example, this equation contains three spatial variables x,y,z and a temporal variable t so

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{6}$$

The matrix A has four non-zero eigenvalues. Three (spatial) values are negative and the fourth (temporal) is positive. This evolution equation models propagation phenomena such as the description of the vibratory behavior of a body.

The d'Alembert equation or wave equation is a partial differential equation in physics that governs the propagation of a wave in a particular context. This is a fundamental equation in physics, which correctly describes the behavior of many everyday waves, such as sound or light.

Definition 2 *The partial differential equation is called the wave equation*

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u \tag{7}$$

where u is a function defined on $\mathbb{R}^n \times \mathbb{R}$, the first n coordinates being the space coordinates and the last the time. The constant c is the propagation speed of the wave or a celerity.

Example 3

1. *The speed of sound $c = 343\ m.s^{-1}$.*
2. *Since 1983, the International System of Units (SI) has fixed the speed of light in a vacuum at exactly $c = 299\,792\,458\ m.s^{-1}$.*
3. *To the author's knowledge, the celerity of brain waves has never been measured. In most of the works carried out, the celerity of brain waves is replaced by the celerity of electromagnetic waves in a vacuum. This value is calculated from the permittivity of the vaccum and the permeability of the vaccum. See equation (4) in the article [2].*

2 Solving the wave equation in dimension 3

Theorem 4 *Let be the parallelogram $R =]0,a[\times]0,b[\times]0,c[$ and let be the following problem :*

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} & ,t > 0 \ et\ (x,y,z) \in R \\ u(x,y,z,0) = f(x,y,z) & (x,y,z) \in R \\ \frac{\partial u}{\partial t}(x,y,z,0) = g(x,y,z) & (x,y,z) \in R \\ u = 0 & \text{on the faces of } R \end{cases} \tag{8}$$

It is assumed that the functions f and g are zero on the faces of R. A solution of the system 8 is :

$$u(x,y,z,t) = \sum_{l,m,n \in \mathbb{N}^*} \left(K_{l,m,n} cos(\delta t) + L_{l,m,n} sin(\delta t)\right) sin\left(\frac{l\pi}{a}x\right) sin\left(\frac{m\pi}{b}y\right) sin\left(\frac{n\pi}{c}z\right) \tag{9}$$

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with Fourier coefficients :

$$\left\{\begin{array}{lcl} K_{l,m,n} & = & \frac{8}{abc}\int_0^a\int_0^b\int_0^c f(x,y,z,)\ sin\left(\frac{l\pi}{a}x\right)\ sin\left(\frac{m\pi}{b}y\right)\ sin\left(\frac{n\pi}{c}z\right)dxdydz \\ L_{l,m,n} & = & \frac{8}{abc\delta}\int_0^a\int_0^b\int_0^c g(x,y,z,)\ sin\left(\frac{l\pi}{a}x\right)\ sin\left(\frac{m\pi}{b}y\right)\ sin\left(\frac{n\pi}{c}z\right)dxdydz \end{array}\right. \quad (10)$$

où $\delta = \pi\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}$.

Demonstration 5 (Theorem 4) *The demonstration is carried out by the method of the separation of the variables. We are looking for a solution in the form*

$$u(x,y,z,t) = X(x)Y(y)Z(z)T(t) \quad (11)$$

the equation will therefore be written

$$XYZT'' = X''YZT + XY''ZT + XYZ''T \quad (12)$$

Dividing the two members by $XYZT \neq 0$ *we get*

$$\frac{T''}{T} = \frac{X''}{X}+\frac{Y''}{Y}+\frac{Z''}{Z} \quad (13)$$

Since the variables are independent, there is a real constant k such that

$$\frac{T''}{T} = k \quad (14)$$

Therefore,

$$k = \frac{X''}{X}+\frac{Y''}{Y}+\frac{Z''}{Z} \quad (15)$$

and for the same reason, there are constants $\alpha,\beta,\gamma \in \mathbb{R}$

$$\left\{\begin{array}{lcl} \frac{X''}{X} & = & \alpha \\ \frac{Y''}{Y} & = & \beta \\ \frac{Z''}{Z} & = & \gamma \end{array}\right. \quad (16)$$

Let's solve the equation by X

$$\frac{X''}{X} = \alpha \quad \Rightarrow \quad X'' - \alpha X = 0 \quad (17)$$

Its characteristic equation is$r^2 = \alpha$.

- *If* $\alpha \geq 0$, *since* $X(0) = X(a) = 0$ *(because X is zero on the faces of R) then* $X \equiv 0$.
- *If* $\alpha < 0$ *then* $r = \pm i\sqrt{-\alpha}$ *where* $i^2 = -1$ *and we have*

$$X(x) = C_1\ cos\left(x\sqrt{-\alpha}\right) + C_2\ sin\left(x\sqrt{-\alpha}\right)$$

The boundary conditions $X(0) = X(a) = 0$ *give*

$$\left\{\begin{array}{lcl} C_1 & = & 0 \\ C_2\ sin\left(a\sqrt{-\alpha}\right) & = & 0 \end{array}\right. \quad (18)$$

Since X is not identically zero, we have

$$sin\left(a\sqrt{-\alpha}\right) = 0 \quad (19)$$

Which implies that

$$a\sqrt{-\alpha} = l\pi \quad (20)$$

with $l \in \mathbb{N}^*$, *hence*

$$\alpha = \frac{l^2\pi^2}{a^2}, \quad l \in \mathbb{N}^* \quad (21)$$

And thus we obtain the family of solutions of the equation in X;

$$X_l(x) = A_l sin\left(\frac{l\pi}{a}x\right) \quad , l \in \mathbb{N}^*,\ A_l \in \mathbb{R} \quad (22)$$

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Let's solve the equation by Y

In the same way, we have the family of solutions of the equation in Y:

$$Y_m(y) = B_m sin\left(\frac{m\pi}{b}y\right),\ m \in \mathbb{N}^*,\ B_m \in \mathbb{R} \tag{23}$$

with

$$\beta = \frac{m^2\pi^2}{b^2} \tag{24}$$

Let's solve the equation by Z

Also, by a similar reasoning, the family of solutions of the equation in Z is :

$$Z_n(z) = C_n sin\left(\frac{n\pi}{c}x\right), \quad n \in \mathbb{N}^*,\ C_n \in \mathbb{R} \tag{25}$$

with

$$\gamma = \frac{n^2\pi^2}{c^2} \tag{26}$$

Let's solve the equation by T

Solving the equation 14, in T :

$$T'' = kT \tag{27}$$

On the one hand, by hypothesis, we have

$$k = \alpha + \beta + \gamma \tag{28}$$

so

$$k < 0 \tag{29}$$

D'autre part,

$$\alpha + \beta + \gamma = \frac{l^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{c^2} \tag{30}$$

Hence

$$k = \frac{l^2\pi^2}{a^2} + \frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{c^2} \tag{31}$$

Then let's set $k = -\delta^2$ *with*

$$\delta = \pi\sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2} + \frac{n^2}{c^2}} \tag{32}$$

The equation becomes

$$T'' + \delta^2 T = 0 \tag{33}$$

It admits for family of solutions

$$T_{l,m,n}(t) = D_{l,m,n} cos\left(\delta t\sqrt{-\alpha}\right) + E_{l,m,n} sin\left(\delta t\sqrt{-\alpha}\right) \tag{34}$$

with $D_{l,m,n}, E_{l,m,n} \in \mathbb{R}$.

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Form of the solution u

Finally, by the principle of superposition we write

$$u(x,y,z,t) = \sum_{l,m,n\in\mathbb{N}^*} X_l(x)\,Y_m(y)\,Z_n(z)\,T_{l,m,n}(t) \tag{35}$$

In other words,

$$u(x,y,z,t) = \sum_{l,m,n\in\mathbb{N}^*} \left(K_{l,m,n}cos(\delta t) + L_{l,m,n}sin(\delta t)\right) sin\left(\frac{l\pi}{a}x\right) sin\left(\frac{m\pi}{b}y\right) sin\left(\frac{n\pi}{c}z\right) \tag{36}$$

It remains only to determine the coefficients $K_{l,m,n}$ and $L_{l,m,n}$ from the initial conditions

$$\begin{cases} u(x,y,z,0) = f(x,y,z) & , (x,y,z) \in R \\ \frac{\partial u}{\partial t}(x,y,z,0) = g(x,y,z) & , (x,y,z) \in R \end{cases} \tag{37}$$

we have

$$f(x,y,z) = \sum_{l,m,n\in\mathbb{N}^*} K_{l,m,n}\, sin\left(\frac{l\pi}{a}x\right)\, sin\left(\frac{m\pi}{b}y\right)\, sin\left(\frac{n\pi}{c}z\right) \tag{38}$$

and

$$g(x,y,z) = \sum_{l,m,n\in\mathbb{N}^*} \delta\, L_{l,m,n}\, sin\left(\frac{l\pi}{a}x\right)\, sin\left(\frac{m\pi}{b}y\right)\, sin\left(\frac{n\pi}{c}z\right) \tag{39}$$

We recognize the Fourier coefficients :

$$K_{l,m,n} = \frac{8}{abc}\int_0^a\int_0^b\int_0^c f(x,y,z,)\, sin\left(\frac{l\pi}{a}x\right)\, sin\left(\frac{m\pi}{b}y\right)\, sin\left(\frac{n\pi}{c}z\right) dxdydz \tag{40}$$

and

$$L_{l,m,n} = \frac{8}{abc\delta}\int_0^a\int_0^b\int_0^c g(x,y,z,)\, sin\left(\frac{l\pi}{a}x\right)\, sin\left(\frac{m\pi}{b}y\right)\, sin\left(\frac{n\pi}{c}z\right) dxdydz \tag{41}$$

with $\delta = \pi\sqrt{\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}}$.

3 The cycle of emotions

The study of emotions is of increasing interest to scientists. Known to be the object of media, psychological, social and human studies, emotions are now of interest to applied mathematics and engineering scientists, see article [1] and the thesis [6]. Indeed, emotions being electro-chemical reactions originating in the human brain, it may be possible to decipher them in a brain recording. The cerebral electrical signal is studied thanks to electroencephalograms, for example. An example of a systemic study of decryption devices is presented in the article [5]. Figure 1 illustrates a device that can be used for learning emotions from early childhood. But what about the psychological phenomenon?

Figure 1: The photo illustrates an imagined device that emits waves simmulating emotions [5].

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3.1 From a psychological point of view

A description of the emotional cycle is described in the diagram 2. Several stimuli can awaken the human senses. They are called emotional stimuli. These data are sent to the amygdala to be processed through different organs such as the thalamus or the cerebral cortex or brainstem nuclei. The hippocampus sends the context of these emotional stimuli to the amygdala. The amygdala processes this information and decides the biological reaction to this emotional stimuli. This is how the subjective experience of emotion and the expression of emotion are developed in the unconscious. The expression of emotion consists of emotional behaviors, autonomic responses, and endocrine responses.

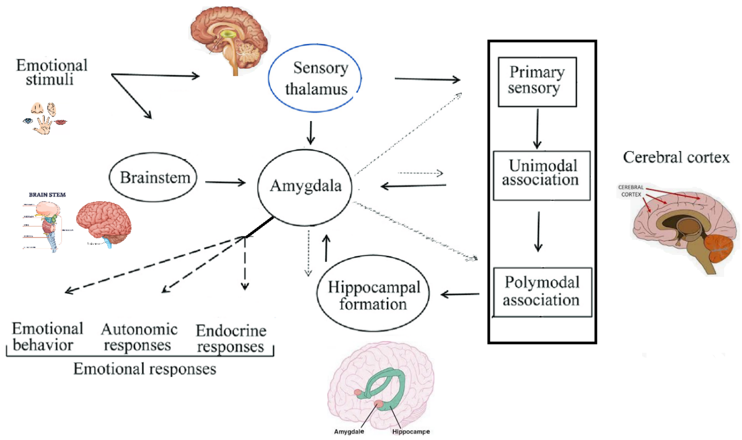

Figure 2: This diagram is based on the document [11].

In the cortex, there are three brain areas specialized in the processing of sensory data: the auditory cortex in the temporal lobe, the visual cortex located in the occipital lobe and the somatosensory cortex in the parietal lobe.

3.2 The explanations of a psychiatrist

We asked the following questions to a psychologist :

1. How to understand emotions from a psychological point of view?
2. How is the learning of emotions with children done?
3. In some countries, schools have a learning program based on facial recognition of basic emotions: anger, joy, sadness, fear, surprise and disgust. What do you think about it?

The answers were as follows :

1. The emotions are solicited by external stimuli via the five senses. Through a network of neurons, the brain triggers hormones such as adrenaline, sterolanine or endorphin. These substances are distributed throughout the body thanks to the blood circuit and generate the emotion felt in the body.

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2. The learning of emotions with young children (4 to 6 years old) is done indirectly during his experiences. However, the child is not always able to evaluate situations. Therefore, it is important that an adult explains it to him. This learning can begin by pronouncing this emotion in a sentence. The child then discovers that his psychological state has a name. Then, it is important to explain to the child the consequences of an emotion on relationships with others. For example, the way of expressing one anger can hurt another. It is therefore important that an adult intervenes either to explain or to punish or to reward. An apprenticeship must be concluded by a return to a reassuring situation for the child because he records what he has experienced and uses it as a reference in his future experiences.

3. It is difficult to carry out emotional learning in schools because teachers are most of the time busy with a scientific program. It is preferable that this takes place elsewhere in local social institutions at the neighborhood level. Regarding the facial recognition of emotions, there are studies on the facial impact of the maternal face on the baby. This is part of the psychology of the form, the theory of Gestalt or gestaltism.

4 Conclusion

The applications related to the processing of brain signals have evolved a lot as it is said in the article [1]. From a mathematical point of view, this brain activity is modeled by Maxwell's equations. In the article [3], These equations are reformulated in the form of wave equation. The resolution of this equation in one dimension figures in the article [2]. This article presents a resolution of this equation in 3 dimension. Indeed, in reality, brain waves propagate in a three-dimensional space. The hypothesis of a decryption of brain waves into emotions, and the possibility of the manufacture of new learning devices have been described in the article [5]. This article is concluded by a testimony of a psychologist explaining the procedure of emotional reactions from stimuli received by the five senses to the reactions felt in the body.

References

[1] Annabi, Y. (2019). *Multidisciplinary Researches using the Magneto-Electroencephalography*, International Journal of Emerging Technology and Advanced Engineering, Volume 9, Issue 7. https://ijetae.com/Volume9Issue7.html

[2] Annabi, Y. (2023). *Mathematical and Electronic Perception of Electromagnetism*, International Journal of Innovation in Science and Mathematics, Volume 11, Issue 2. http://ijism.org/administrator/components/com_jresearch/files/publications/IJISM_994_FINAL.pdf

[3] Annabi, Y. (2022). *Maxwell's Equations, Emotions and Beta Wave*, International Journal of Innovation in Science and Mathematics, Volume 10, Issue 6. http://ijism.org/administrator/components/com_jresearch/files/publications/IJISM_984_FINAL.pdf

[4] Annabi, Y. (2023). *Introduction to partial differential equations*, European University Edition. ISBN : 978-620-3-44714-9.

[5] Annabi, Y. (2023). *Systemic study of devices processing electromagnetic waves. Application to brain waves and emotions*, International Journal of Innovative Science and Research Technology, Volume 8, Issue 6.

http://dx.doi.org/10.29322/IJSRP.13.06.2023.p13854

[6] Grandchamp, R. (2012). *Electroencephalography and Brain-Machine Interfaces: new methods for studying mental states*. Thesis, University of Toulouse.

[7] Goleman, D. (1995). *Emotional intelligence*. Edition Déjà Lu.

[8] Dautray, R. ,Lions J. L.(1985), *Mathematical analysis and numerical calculation for science and technology, Volume 3*, Masson Edition.

[9] Brezis, H. (1983). *Functional analysis, Theory and applications*. Applied Mathematics Collection for the Master's degree, Masson Edition.

[10] Ekman, P. (1994). *All Emotions Are Basic*. In Ekman, P. Davidson, R. (Eds.), The Nature of Emotion : Fundamental Questions (pp. 15-19). New York : Oxford University Press.

[11] Vergnes J. (2018). *Emotions in the kindergarten classroom*. Education. URL : dumas.ccsd.cnrs.fr/dumas-02137774

http://dx.doi.org/10.29322/IJSRP.13.06.2023.p13854