A Comparison of Alternative Option Pricing Models

[1]Chukwudi A. Ugomma, [2]Felix N Nwobi and [3]Eleazar.C Nwogu

[1,2]Department of Statistics, Imo State University, Owerri, 460222, Nigeria
[3]Department of Statistics, Federal University of Technology, Owerri, Nigeria

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Abstract- In this paper we compare Weibull and Mixed-Lognormal-Weibull Option Pricing Models. The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX) which consists of 50 enlisted stocks in the clearing house as products of monthly market summary for long term options from 3rd January, 2017 to 31st December, 2019. The data were properly arranged according to 25, 27, 28, 29 and 30 maturity days. With the help R-package, we obtain the parameters of Weibull (WBS) and MLWBS and the goodness of fit test was conducted to find how best fit the two models are in Black-Scholes Option Pricing Model and the result revealed that under the null hypothesis of a good fit it is accepted (P>0.05) that WBS is a good fit in Black-Scholes option pricing model while MLWBS is a good fit only at the maturity days of 25 and 27 (shorter maturity days) Hence, we affirmed that WBS is a selective alternative option pricing model while MLWBS should only be applied when the expiration days of options are shorter.

Index Terms- Black-Scholes, Option Pricing, Weibull, Mixture Distributions, Goodness – Of – Fit Test

I. Introduction

A number of option pricing models are now available in recent times after the foundation of the method laid long time ago by Charles Castelli in 1877 where different purposes of options were discussed. The first known analytical valuation (pricing) for options was presented in 1897 by Louis Bachelier. The pitfalls in Bachelier's work were that the process produced negative security prices and the option prices of the underlying asset is greater than the prices of the underlying asset, see for example, Kalavrezos and Wennermo (2007).

Bachelier's model is based on the assumption that stock prices followed an Arithmetic Geometric Brownian Motion with zero drift as in Merton (1973) described as

$$dX = \mu_X dt + \sigma_t \lambda \sqrt{dt} \qquad (1)$$

Equation (1) describes the price movements of the underlying asset, μ is the drift term, λ_t is the Weiner process and σ_t is the volatility of the underlying asset.

More advanced option models were presented by Sprinkle (1961)) and Bones (1964) where they expressed the value of options in terms of Brownian motion. They were calculating the expected payoff of European puts and calls, but were all using different choices for the discount factor and the stocks appreciation rate. A decade later, the major development came from Fischer Black and Myron Scholes in 1973 where they introduced the popular option pricing model called Black – Scholes model which allowed investors to approximate for a European style option described as:

$$C(x,t) = xN(d_1) - xe^{-rt}N(d_2) \qquad (2)$$

$$P(x,t) = xe^{-rt}N(-d_2) - xN(-d_1) \qquad (3)$$

where C and P in equations (2) and (3) are European call and put options.

The models formed by each of the practitioners' show resemblances and each of these models had at least one random parameter. But it was the developers of the Black – Scholes model that were able to find a solution to their stochastic partial differential equation that was not reliant on any known variables. The violation of some of the assumptions of the Black and Scholes model have attracted much attention from financial practitioners, who have tried to find a thicker – tailed, more left – skewed distribution to better fit time series data. These efforts are acceptable by evidence from time series studies that stock returns are not independent and identically distributed normal, and there is also the question of whether continuous model is suitable as a model for asset prices. For this reason, many researchers over the years have taken different expressions at the Black and Scholes option pricing model by relaxing some of its assumptions by proposing alternative option pricing models in place of Black – Scholes option pricing model. Hence, in this paper, we intend to compare Weibull-Black-Scholes and Mixed-lognormal –Weibull-Black-Scholes option pricing models as alternative Option Pricing Models.

II. Alternative Option Pricing Models.

Although, the Black – Scholes option pricing model reflects the reality of the option price movements very well and it is acceptable in practice. It also has its weaknesses which inspired the development of even more complex and realistic models for option pricing. Therefore, different option pricing models have been developed as basically the extensions of the Black-Scholes model, each making different assumptions concerning the process of underlying asset price, the interest rate process and the market price of factor risks. Among the notable extensions of Black – Scholes model include stochastic volatility models, see, for example Cox and Ross (1976), Jumps and Jump Diffusion models by Merton (1976), Johnson and Shannon (1987); Hull and White (1987), Heston (1993), Trautmann and Beinert (1994), Bates (1996), Renault and Touzi (1996), Scott (1997), Carr (1998), Bates (1998), Heston and Nandi (2000), Carr, et al (2002), Baek (2006), Jang, et al (2014), Singh (2015); others include Binomial model by Carr, et al (1979), Hull (2008); Garch models by Zohra and Dash (2004), and many other alternative models such as Savickas (2002) and Ugomma and Nwobi (2022).

2.2 *The Black-Scholes Option Pricing Model under Weibull Distribution*

The call price can be expressed as the Weibull Call Option pricing in our case given by

$$C_{web} = \mathrm{e}^{-rT} \max \int_0^\infty (\mathrm{X}_t - K) f(X_t)\, dX_t \tag{4}$$

where $f(*)$ is a density function of lognormal distribution in Black -Scholes of 1976, X_t is the underlying price and K is the underlying price of the option.

Substituting equation (4) into Weibull pdf gives the Weibull call option as

$$f(x_t) = \mathrm{C}_{web} = \mathrm{e}^{-rT} Max \int_K^\infty (\mathrm{X}_t - K)\left(\frac{\beta}{\alpha}\right)\left(\frac{X_t}{\alpha}\right)\exp\left(-\frac{X_t}{\alpha}\right)^\beta d\mathrm{X}_t \tag{5}$$

Equation (5) can also be written as

$$f(\mathrm{X}_t) = \mathrm{e}^{-rT} \int_K^\infty \mathrm{X}_t \exp\left\{-\left(\frac{\mathrm{X}_t}{\alpha}\right)^\beta \frac{\beta X_t^{\beta-1}}{\alpha^\beta}\right\} d\mathrm{X}_t - K\exp\left\{-\frac{\mathrm{X}_t}{\alpha}\right\}^\beta \tag{6}$$

Further simplification of (6) yields

$$\int_K^\infty \mathrm{X}_t \exp\left\{-\left(\frac{\mathrm{X}_t}{\alpha}\right)^\beta \frac{\beta X_t^{\beta-1}}{\alpha^\beta}\right\} d\mathrm{X}_t = \frac{\beta}{\alpha^\beta}\int_K^\infty \mathrm{X}_t^\beta \exp\left\{-\left(\frac{X_t}{\alpha}\right)^\beta\right\} d\mathrm{X}_t$$

$$= \frac{\beta}{\alpha^\beta}\int_K^\infty \left(\frac{X_t}{\alpha}\right)^\beta \exp\left\{-\left(\frac{X_t}{\alpha}\right)^\beta\right\} \frac{\alpha}{\beta}\left(\frac{X_t}{\alpha}\right)^{\beta\left(\frac{1}{\beta-1}\right)} d\mathrm{X}_t \tag{7}$$

Let $\mathrm{y} = \left(\frac{\mathrm{X}_t}{\alpha}\right)^\beta$, $\mathrm{Z} = \alpha \mathrm{y}^{\frac{1}{\beta}}$, and $\mathrm{dz} = \frac{\alpha}{\beta}\mathrm{y}^{\frac{1}{\beta}-1}$ in (7)

Then we have

$$\frac{\beta}{\alpha}\int_{\left(\frac{K}{\alpha}\right)^\beta}^\infty \alpha^\beta y(\exp(-y))\frac{\alpha}{\beta} y^{\frac{1}{\beta}-1} dy \tag{8}$$

$$\alpha\int_0^\infty y^{\frac{1}{\beta}}(\exp(-y))dy - \alpha\int_0^{\left(\frac{K}{\alpha}\right)^\beta} y^{\frac{1}{\beta}}(\exp(-y))dy$$

$$= \alpha\Gamma\left(1+\frac{1}{\beta}\right) - \alpha\Gamma\left(1+\frac{1}{\beta}\right)\frac{\int_0^{\left(\frac{K}{\alpha}\right)^{\beta}} y^{\frac{1}{\beta}}\left(\exp(-y)\right)dy}{\Gamma\left(1+\frac{1}{\beta}\right)} \tag{9}$$

$$\Gamma_t(*) = \int_0^t y^{\frac{1}{\beta}-1}\frac{\exp(-y)\,dy}{\Gamma\left(1+\frac{1}{\beta}\right)}$$

where

Substituting equation gamma function into (10) and discounting it as the risk free rate yields

$$C_{web} = e^{-rT}\left[X_0\left(1-\frac{\Gamma y+\left(1+\frac{1}{\beta}\right)}{\Gamma\left(1+\frac{1}{\beta}\right)}\right)-K\exp(-y)\right] \tag{11}$$

Multiplying e^{-rT} to both sides of (11), we obtain the Weibull Call Option as

$$C_{web} = \mathrm{X}_0\Phi(d_1) - \mathrm{K}e^{-rT}\Phi(d_2) \tag{12}$$

where $\Phi(*)$ is the standardized normal value obtained from normal distribution table.

$$= \mathrm{X}_0 = E(X) = \left(1+\frac{1}{\beta}\right), \quad d_1 = \left[1-\frac{\Gamma y\left(1+\frac{1}{\beta}\right)}{\Gamma\left(1+\frac{1}{\beta}\right)}\right] \text{ and } \quad d_2 = \exp\left\{-\left(\frac{X_t}{\alpha}\right)^{\beta}\right\}$$

2.1.3 ***The Mixed–Lognormal-Weibull-Black-Scholes Option Pricing Model Option (MLWBS).***

Let the call price of MLWBS be

$$C_t(X_t, K) = e^{-rT}\int_0^{\infty}(X_t - K)f(X_t)\,dX_t \tag{21}$$

This implies that the mixture distribution for the Black-Scholes Option Pricing Model is expressed as

$$f_{\theta_i}^{mix}(x_t,\theta_1,\theta_2) = wf_1(x_t,\theta_1)+(1-w)f_2(x_t,\theta_2)$$
$$= wf_1^{\log}(x_t,\theta_1)+(1-w)f_2^{web}(x_t,\theta_2)$$
$$= e^{-rT}\left[\int_0^{\infty} wf_1^{\log}(x_t,\theta_1)+(1-w)f_2^{web}(x_t,\theta_2)\right. \tag{22}$$

Substituting the Black-Scholes Models for both lognormal and Weibull in (21), we obtain

$$e^{-rT}\left[w\left(\frac{X_0\Phi\ln\left(\frac{X_0}{K}\right)-\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-Ke^{-rT}\ln\left(\frac{X_0}{K}\right)-\left(r-\frac{\sigma^2}{2}\right)T\right)+(1-w)X_0\left(\alpha\Gamma\left(1+\frac{1}{\beta}\right)-\Gamma_y\left(\frac{1+\frac{1}{\beta}}{\Gamma\left(1+\frac{1}{\beta}\right)}\right)\right)-Ke^{-rT}\exp(-y)\right] \tag{3.94}$$

Through simplification and collecting like terms together, we obtain

$$=\left[w(1-w)X_0\Phi d_1 - w(1-w)Ke^{-rT}\Phi d_2\right]$$

Hence,

$$\therefore C_{tMix} = w(1-w)\left[X_0\Phi d_1 - Ke^{-rT}\Phi d_2\right] \tag{23}$$

III. DATA AND METHOD OF ANALYSIS

3.1 *Data Description*

The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX). The sample consists of fifty (50) enlisted stocks in the clearing house as products of monthly market summary for long term options which consists of the period of January, 3rd 2017 to December, 31st 2019 when there are no significant structural changes among the products. For each transaction, our sample contains the following information: the opening and closing dates of the options, option prices comprising opening and closing prices otherwise referred in our case as the underlying and strike prices respectively. The final sample consists of 50 stocks for the period of 36 months (720 trading days). The maturity period of the options was gotten from the difference between the opening date and closing date of the options over the trading days. The data for the analysis were arranged in accordance to the maturity days of 25, 27, 28, 29 and 30 days. The data were actually obtained at http//www.asx.com/au/product/equity_options/options_statistics.htm.

3.2 *Testing Methodology*

The technique adopted for this study will estimate the absolute returns of the underlying price and the volatility from annualized standard deviation (implied volatility) using log – difference of option prices that equates to theoretical option pricing models.
The data in each of the maturity days (expiration time) were tested in accordance with 252 trading days.

3.2.1 *Computation of the Annualize (Implied Volatility)*

In order to get the implied volatility of the models, we first estimate the historic volatility (standard deviation) of option prices using opening and closed prices as underlying and strike prices respectively.

Let $X_i = ABS \ln\left(\frac{X_t}{X_{t-1}}\right)$, X_t is the underlying option price at time t.

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\sigma_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

So that the implied volatility is obtained by

$$\sigma_{im} = \sqrt{\frac{T}{n} Var(X)} \qquad (24)$$

where T is 252 trading days per annum and n is number of stocks.
and the rate of return is estimated by

$$r = \frac{1}{T}\ln\left(\frac{K}{X_0}\right) \qquad (25)$$

3.3 *Goodness – of – Fit – Test of the Option Pricing Models*

In this study, we wish to test the hypothesis whether the option pricing models are good fit for pricing options against that it does not.

We shall reject the null hypothesis that the models are not good fit if $\chi_c^2 > \chi_\alpha^2$, otherwise accept the null hypothesis.

The test statistic approximately follows the Chi – Square distribution with $k-1$ degrees of freedom at 5% level of significance given by

$$\chi_c^2 = \sum_{j=1}^{N} \frac{\left(P_j - P_j\left(\hat{\sigma}\right)\right)^2}{P_j\left(\hat{\sigma}\right)} \qquad (26)$$

where

P_j is the observed option price of the jth category and $P_j\left(\hat{\sigma}\right)$ is the expected (predicted) option pricing models of the jth category and n is the sample size.

IV. Results and Discussions

4.1 *Descriptive Statistics of ASX Data*

The output in Table 1 is a descriptive statistics of the absolute returns of ASX data for the period under study

Table 1: Summary Statistics of Absolute Returns of ASX Original Data

Maturity Days	Sample Size	Sample Mean	Sample Standard Dev	Skewness	Kurtosis	Implied Volatility	Rate of Return (r)
25	99	1.1983	1.0483	1.5414	6.0265	1.6726	-0.01
27	199	1.2150	1.0801	1.5255	5.8071	1.2154	-0.02
28	399	1.2408	1.0780	1.4158	5.2195	0.8564	-0.01
29	449	1.2593	1.1117	1.4866	5.6435	0.8329	-0.03
30	499	1.2466	1.0890	1.4330	5.3350	0.7740	0.01

4.2 *Goodness – of – Fit – Test of the Option Pricing Models*

H_0 : The models are good fit for pricing options

H_1 : The models are not good fit for pricing options

Model	Maturity Days	Sample Size	χ^2	df	P-value	Decision
WBS	25	99	9603	9506	0.2401	Accept
	27	199	38400	38208	0.2433	Accept
	28	399	149600	149226	0.2466	Accept
	29	449	187650	187233	0.2476	Accept
	30	499	222055	221610	0.2518	Accept
MLWBS	25	99	9408	9312	0.2401	Accept
	27	199	38400	38016	0.0822	Accept
	28	399	149600	148478	0.0199	Reject
	29	449	187650	185565	0.0003	Reject
	30	499	222055	220720	0.0224	Reject

From the result displayed in Table 2, we observed that the null hypothesis of a good fit is accepted (P>0.05) for Weibull model at all the maturity days, while MLWBS is a good fit only for options with shorter maturity days (25 and 27 days). We, therefore, conclude that Weibull Distribution is a good fit in Black- Scholes Option Pricing Model when the option price has while MLWBS is not useful when options have longer expiration days.

V. CONCLUSION

In this work, we compared the Weibull and Mixed-Lognormal-Weibull Black-Scholes option pricing model on the effect of Weibull distribution in Black-Scholes option pricing model and we observe that Weibull distribution is a better fit in Black-Scholes option pricing model in line with Savickas (2002) and Nwobi and Ugomma (2021), and that the mixture distribution (MLWBS) is a good fit only for options with shorter maturity (expiation) days.

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AUTHORS

First Author – Chukwudi A. Ugomma, Department of Statistics, Imo State University, Owerri, 460222, Nigeria
Second Author – Felix N Nwobi, Department of Statistics, Imo State University, Owerri, 460222, Nigeria
Third Author – Eleazar.C Nwogu, Department of Statistics, Federal University of Technology, Owerri, Nigeria