NUMERICAL STUDY OF FORCED CONVECTIVE HEAT GENERATION FLOW THROUGH A PERMEABLE WALLS WITH SUCTION/INJECTION

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Abstract

In this paper, forced convective heat generation in a steady flow of an incompressible viscous fluid through a channel permeable walls was studied. The nonlinear governing equations along with boundary are converted into ordinary differential equations using appropriate similarity transformations. The nonlinear governing equations were solved numerically using weighted residual method and the residuals were minimized using the collocation method and computed using mathematica software for various values of physical parameters. The effect of various flow parameters on velocity and temperature profile are graphically discussed.

Keywords: Heat Generation, Forced Convective, Weighted Residual Method, Collocation Method.

1 Introduction

In the last several years, there has been a very rapid increase in the intensity of researches in the field of forced convectives,The increased intensity is due to the importance of convective heat transfer in science and engineering. Such flow can be found in magnetohydrodynamics(MHD) generator, heat exchanger, oil extraction, geothermal reservoirs, flow meter and mircofluid devices [1]. various categories of fluid flows and heat transfer problems for stretching surfaces have been explored in some of investigations [2]. Also, [3] stressed on the combined effect of variable viscosity and electrical conducting on hydromagnetic flow and transfer between a fixed plate and moving parallel plates was numerically analyzed. Numerical analysis of heat transfer and inherent its reversibility in compressible fluid flow through a channel partially pulled with porous medium was studies by [4]. [5] investigated the radiation effect of heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. In [6-8], it was discovered that the layer value of buoyancy parameter can be used to control the temperature and concentration boundary layer and that suction stabilizes the boundary layer. Researchers have discussed heat and mass transfer fluid under various physical situations.The combined effect of free and forced convection on MHD flow in porous channel under the uniform magnetic field was investigated and the solutions were obtained for all heat absorption condition and restricted heat generation condition[9]. The study of hydromagnetic natural convective fluid flow between vertical parallel plates with time periodic boundary conditions using Adomian decomposition method was analyzed and the results of their computation shows that an increase in the magnetic field intensity has significantly influenced the fluid flow where the effect of heat generation and suction are neglected [10]. Efficient energy utilization during the convection in fluid is one of the fundamental problem of engineering processes to improve the system, several researchers have theoretically studied heat generation in the flow systems under many physical situations, see [11-13]. However, no attempt has yet been presented for effect of variable heat generation on forced convective with surface boundary condition. Hence, in this study, the effect of heat generation on forced convective fluid flow through a channel with permeable walls was investigated and the model equations were obtained numerically using collocation method and the results were presented graphically to analyze the effect of various parameter on velocity and temperature profile.

2 PROBLEM FORMULATION

In this study, we considered a steady incompressible flow of an electrically conducting variable viscosity fluid between two fixed permeable parallel infinite plates. The flow is fully developed and the edge effects are disregarded. A constant magnetic field of strength B_o is imposed transversely in the y-direction. The applied magnetic field is assumed to be strong enough so that the induced magnetic field due to motion is weak, It is assumed that the lower permeable plate, where fluid injection occurs is convective heated, while at the upper permeable plate both fluid suction and convective heat loss takes place. Under these assumptions, the governing equation for the momentum equation and energy balance in one dimension can be written as [13,5,7]:

$$V\frac{du}{dy} = -\frac{1}{\rho}\frac{dP}{dx} + \frac{1}{\rho}\frac{d}{dy}\left(\overline{\mu}(T)\frac{du}{dy}\right) - \frac{\sigma B_0^2 u}{\rho} \tag{1}$$

$$V\frac{dT}{dy} = \alpha\frac{d^2T}{dy^2} + \frac{\overline{\mu}(T)}{\rho c_p}\left(\frac{du}{dy}\right)^2 + \frac{\sigma B_0^2 u^2}{\rho c_p} + \frac{Q_0}{\rho c_p}(T - T_f) \tag{2}$$

The boundary conditions are

$$\begin{aligned} u(0) &= 0, \qquad & u(h) &= 0, \\ -k\frac{dT}{dy}(0) &= \gamma_0(T_f - T(0)), \qquad & -k\frac{dT}{dy}(0) &= \gamma_1(T(h) - T_\infty) \end{aligned} \tag{3}$$

(x, y) is the axial and normal coordinates, u is the velocity of the fluid, p is the fluid pressure, v is the uniform suction/injection velocity at the channel walls, γ_0 is the heat transfer coefficient at the lower plate, γ_1 is the heat transfer coefficient at the upper plate, α is the thermal diffusivity, ρ is the fluid density,σ is the fluid electrical conductivity, k is the thermal conductivity coefficient, c_p is the specific heat at constant pressure, where (G) is the pressure gradient parameter, T_f is the temperature of the hot fluid at the lower permeable plate, T is the channel fluid temperature and T_∞ is the ambient temperature above the upper plate. The temperature dependent viscosity μ can be written as [14,17].

$$\overline{\mu}(T) = \mu_0 \ell^{-m(T - T_\infty)} \tag{4}$$

where m is a viscosity variation parameter and μ_0 is the fluid dynamic viscosity at the ambient temperature. Thus, the following non-dimensional quantities were introduced:

$$\begin{aligned} &G = \frac{\partial \overline{p}}{\partial x}, \quad \mu = \frac{\overline{\mu}}{\mu_0}, \quad \alpha = \frac{k}{\rho c_p}, \quad \mu(T - T_\infty) = \epsilon, \epsilon\theta = (T_f - T_\infty) \\ &Q = \frac{Q_0}{\rho c_P}(T - T_f), \quad W = \frac{u}{v}, \quad X = \frac{x}{h}, \quad \overline{P} = \frac{ph}{\mu_0 v} \\ &\theta = \frac{T - T_\infty}{T_f - T_\infty},, \quad P = \frac{\overline{P}\mu_0 v}{h}, \quad \eta = \frac{v}{h}, \quad V = \frac{\mu_0 v}{\rho} \end{aligned} \tag{5}$$

Substituting equation (5) into (1)-(4) we obtain

$$\frac{d^2w}{d\eta^2} - \epsilon\frac{d\theta}{d\eta}\frac{dw}{d\eta} - \ell^{\epsilon\theta}\left(Re\frac{dw}{d\eta} + Haw - G\right) = 0 \tag{6}$$

$$\frac{d^2\theta}{d\eta^2} - RePr\frac{d\theta}{d\eta} + EcPr\ell^{-\epsilon\theta}\left(\frac{dw}{d\eta}\right)^2 + EcPrHaw^2 + Q_0\theta = 0 \tag{7}$$

The corresponding initial and boundary conditions are:

$$w(0) = 0 \quad and \quad w(1) = 0 \tag{8}$$

$$\frac{d\theta}{d\eta}(0) = Bi_0(\theta(0) - 1) \quad and \quad \frac{d\theta}{d\eta}(0) = Bi_0(1) \tag{9}$$

$$Re = \frac{vh}{v}(Reynold - number) \quad Pr = \frac{v}{\alpha}(Prandtl - number)$$
$$Ec = \frac{v^2}{c_p}(T_f - T_\infty)(Eckert - number) \quad Q = \frac{Q_0}{\rho c_P}(T - T_f)(Heat - Generation)$$
$$Ha = \frac{\sigma B_0^2 H^2}{\mu u}(Hartmann - number) \quad \epsilon = m(T - T_\infty)(Viscosity - parameter)$$
$$Bi_0 = \frac{y_o h}{k}(Biot - number for lower - plate) \quad Bi_1 = \frac{y_1 h}{k}(Biot - number for upper - plate)$$

3 NUMERICAL COMPUTATION

Collocation Weighted Residual

The system of coupled non-linear ordinary differential equations (6) and (7) together with the boundary conditions (8) and (9) was solved numerically using Collocation Weighted Residual Method.

$$w = \sum_{K=0}^{10} a_k \eta^k \; and \; \theta = \sum_{K=0}^{10} b_k \eta^k \tag{10}$$

Which can be interpreted as

$$w(\eta) = a_0 + a_1\eta + \eta^2 a_2 + \eta^3 a_3 + a_4\eta^4 + a_5\eta^5 + a_6\eta^6 + a_7\eta^7 + a_8\eta^8 + a_9\eta^9 + a_{10}\eta^{10} \tag{11}$$

$$\theta(\eta) = b_0 + b_1\eta + \eta^2 b_2 + b_3\eta^3 + b_4\eta^4 + b_5\eta^5 + b_6\eta^6 + b_7\eta^7 + b_8\eta^8 + b_9\eta^9 + b_{10}\eta^{10} \tag{12}$$

Substituting equation (6) into (10), we obtain

$$\begin{aligned} w(\eta) &= 2a_2 + 6a_3\eta + 12a_4\eta^2 + 20a_5\eta^3 + 30a_6\eta^4 + 42a_7\eta^5 + 56a_8\eta^6 + 72a_9\eta^7 + 90a_{10}\eta^8 - \\ &\ell^{\left(b_0+b_1\eta+\eta^2 b_2+b_3\eta^3+b_4\eta^4+b_5\eta^5+b_6\eta^6+b_7\eta^7+b_8\eta^8+b_9\eta^9+b_{10}\eta^10\right)}(a_0 + a_1 + a_1\eta + 2a_2\eta + a_2\eta^2 \\ &+ 3a_3\eta^2 + a_3\eta^3 + 4a_4\eta^3 + a_4\eta^4 + 5a_5\eta^4 + a_5\eta^5 + 6a_6\eta^5 + a_6\eta^6 + 7a_7\eta^6 + a_7\eta^7 + 8\eta^7 a_8 + \eta^8 a_8 + \\ &9\eta^8 + \eta^9 a_9 + 10\eta^9 a_{10} + \eta^{10} a_{10}) - (a_1 + 2a_2\eta + 3a_3\eta^2 + 4a_4\eta^3 + 5a_5\eta^4 + 6a_6\eta^5 + 7a_7\eta^6 + 8a_8\eta^7 + \\ &9a_9\eta^8 + 10a_{10}\eta^9)(b_1 + 2b_2\eta + 3b_3\eta^2 + 4b_4\eta^3 + 5b_5\eta^4 + 6b_6\eta^5 + 7b_7\eta^6 + 8b_8\eta^7 + 9b_9\eta^8 + 10b_{10}\eta^9) \end{aligned} \tag{13}$$

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Substituting equation (7) into (11), we have

$$\theta(\eta) = \left(2b_2 + 6\eta b_3 + 12\eta^2 b_4 + 20\eta^3 b_5 + 30\eta^4 b_6 + 42\eta^5 b_7 + 56\eta^6 b_8 + 72\eta^7 b_9 + 90\eta^8 b_{10}\right) -$$
$$\left(b_1 + 2b_2\eta + 3b_3\eta^2 4b_4\eta^3 + 5b_5\eta^4 + 6b_6\eta^5 + 7b_7\eta^6 + 8b_8\eta^7 + 8b_8\eta^7 + 9b_9\eta^8 + 10b_{10}\eta^9\right) +$$
$$\ell^{\left(b_0+b_1\eta+\eta^2 b_2+b_3\eta^3+b_4\eta^4+b_5\eta^5+b_6\eta^6+b_7\eta^7+b_8\eta^8+b_9\eta^9+b_{10}\eta^{10}\right)}$$
$$\left(a_1 + 2a_2\eta + 3a_3\eta^2 + 4a_4\eta^3 + 5a_5\eta^4 + 6a_6\eta^5 + 7a_7\eta^6 + 8a_8\eta^7 + 9a_9\eta^8 + 10a_{10}\eta^9\right)^2 +$$
$$\left(a_1\eta + \eta^2 a_2 + \eta^3 a_3 + a_4\eta^4 + a_5\eta^5 + a_6\eta^6 + a_7\eta^7 + a_8\eta^8 + a_9\eta^9 + a_{10}\eta^{10}\right)^2 +$$
$$(b_0 + b_1\eta + \eta^2 b_2 + b_3\eta^3 + b_4\eta^4 + b_5\eta^5 + b_6\eta^6 + b_7\eta^7 + b_8\eta^8 + b_9\eta^9 + b_{10}\eta^{10}) \quad (14)$$

Using the boundary conditions,

$$w(0) = 0 \Rightarrow a_0 = 0 \quad (15)$$

$$w(0) = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} = 0 \quad (16)$$

$$\theta(0) = -b_0 + b_1 + 1 = 0 \quad (17)$$

$$\theta(1) = b_0 + 2b_1 + 3b_2 + 4b_3 + 5b_4 + 6b_5 + 7b_6 + 8b_7 + 9b_8 + 10b_9 + 11b_{10} = 0 \quad (18)$$

4 RESULTS AND DISCUSSION

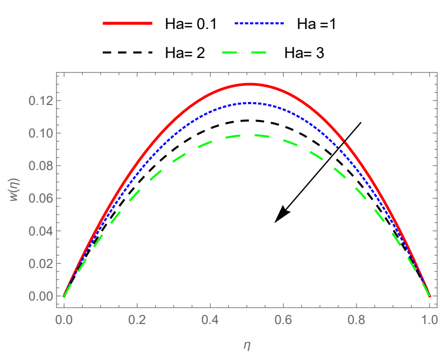

Fig. 1: velocity profile for different values of Ha when $Re = 0.1$, Pr=0.7, $Ec = 0.1$, $\epsilon = 0.1$, $Q = 0.1$, $Bi_0 = 0.1$, $Bi_1 = 0.1$

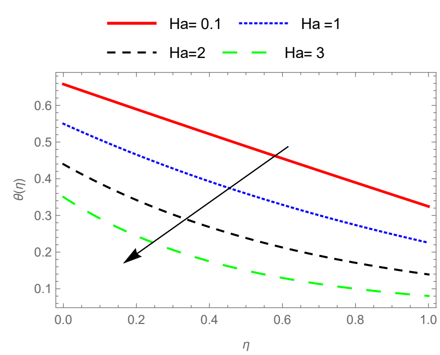

Fig. 2: Temperature profile for different values of Ha when $Re = 0.1$, $Pr = 0.7$, $Ec = 0.1$, $\epsilon = 0.1$, $Q = 0.1$, $Bi_0 = 0.1$, $Bi_1 = 0.1$

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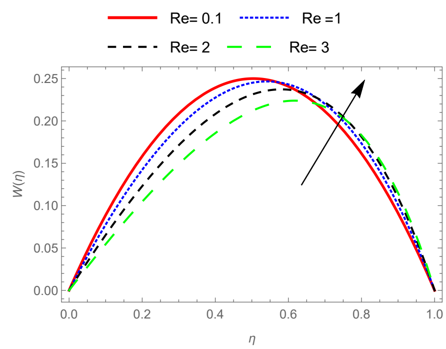

Fig. 3: velocity profile for different values of Re when $Ha = 1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Bi_0 = 0.1, Bi_0 = 0.1$

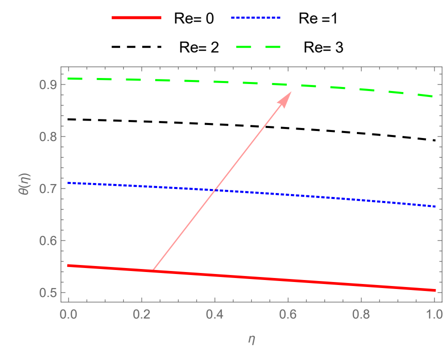

Fig. 4: Temperature profile for different values of Re when $Ha = 0.1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Bi_0 = 0.1, Bi_0 = 0.1$

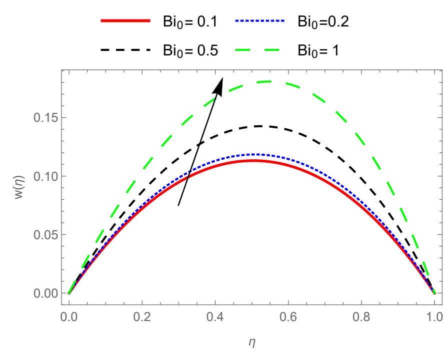

Fig. 5: velocity profile for different values of Bi_0 when $Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Ha = 0.1, Bi_0 = 0.1, Re = 1$

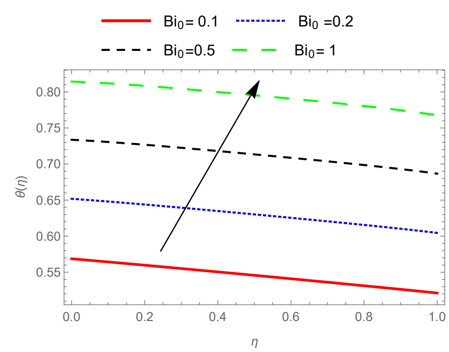

Fig. 6: Temperature profile for different values of Bi_0 when $Ha = 0.1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Bi_0 = 0.1, Re = 0.1$

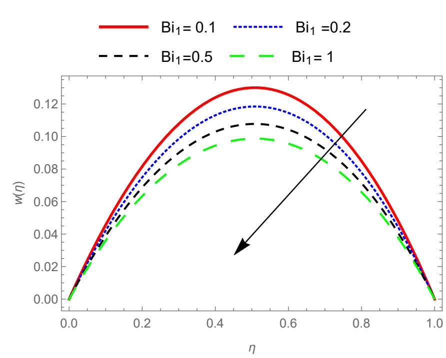

Fig. 7: velocity profile for different values of Bi_1 when $Re = 1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Bi_0 = 0.1, Ha = 0.1$

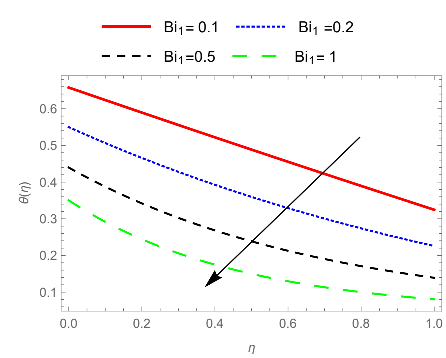

Fig. 8: Temperature profile for different values of Bi_1 when $Ha = 0.1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Q = 0.1, Bi_0 = 0.1, Ha = 0.1$

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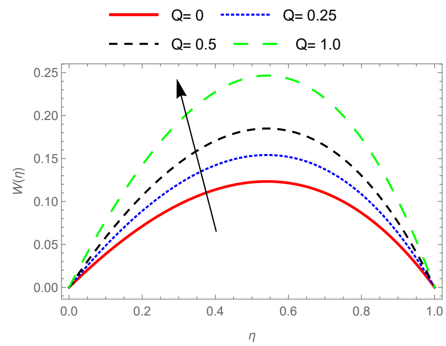

Fig. 9: velocity profile for different values of Q = when $Re = 1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1$, $Bi_1 = 0.1, Bi_0 = 0.1, Ha = 0.1$

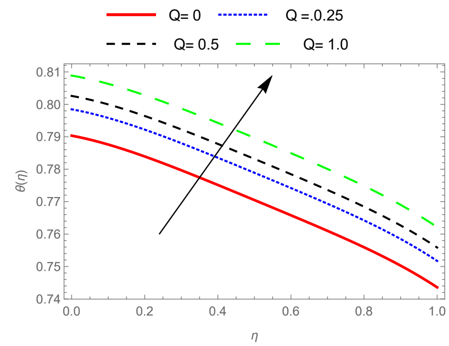

Fig. 10:Temperature profile for different values of Q = when $Ha = 0.1, Pr = 0.7, Ec = 0.1, \epsilon = 0.1, Bi_1 = 0.1, Bi_0 = 0.1, Ha = 0.1$

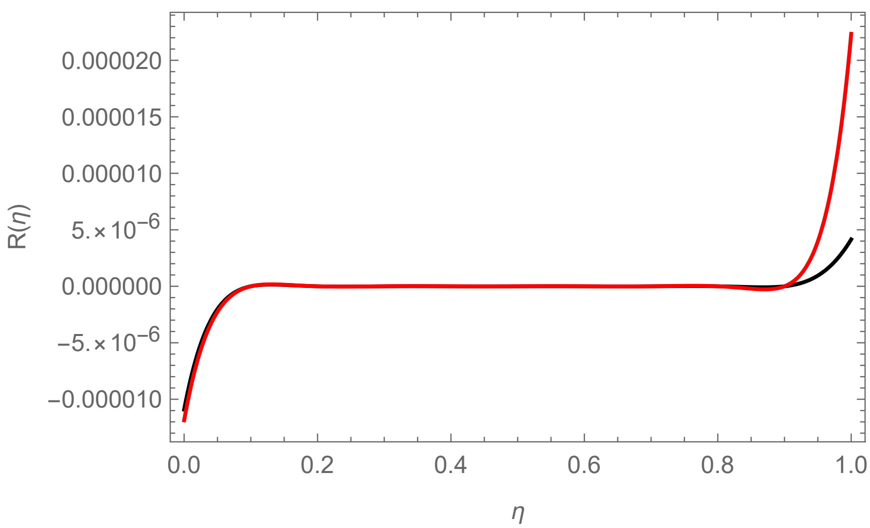

Fig 11: Minimized residual error $(R(\eta))$

Fig. (1) shows the effect of Hartmann number (Ha) on velocity profile and we observed that decrease in Hartmann decreases the velocity profile and it serves as electromagnetic force to the viscous force. The presence of Lorentz force also act as resistance to flow. **Fig. (2)** shows the effect of magnetic (Ha) on the flow fluid, Increase in values of (Ha) decreases the Temperature profile, then the fluid suction and convection losses despite the presence of Lorentz heating which serves as additional heat source to the flow.**Fig. (3)** shows the response of the fluid velocity to variation in suction Reynolds number, the fluid velocity decreases and skewed towards the upper plate as Reynolds number (Re) increases due to increase in injection at the lower plate and injection and suction at the upper plate.**Fig. (4)** shows that the suction Reynolds number (Re) increases with increases number then

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suction Reynolds number (Re) increasing the fluid viscosity becomes lighter and viscous heating increase due to increase convective heating at the lower plate increase leading to a rise in fluid in the temperature. **Fig. (5) and (7)** shows that the effect of (Bi_0) rise in the fluid temperature is observed with increasing convective heating at the lower plate, (Bi_1) in the fluid temperature decreases due to increase in convective heat loss at the upper plate.**Figs. (6) and (8)** graphically shows that the (Bi_0) increases with increase in connectivity heating at lower plate and (Bi_1) decreases with increase in convection cooling at the upper plate, this is expected since the fluid become lighter and flow faster with increasing temperature due to convective heating. **Figs. (9) and (10)** shows the heat generation due to the viscous heating increasing with the parameter values of heat generation and it is observed that fluid velocity and temperature profiles increase with increasing value of heat generation (Q), hence produces an increase in the heat transfer and flow faster.**Fig. (11)** shows the graph of the residual functions $R(\eta)$ and it was observed that the residuals are minimized in the domain (0 *to* 1)

5 CONCLUSION

This study investigated the effects of heat generation on variable viscosity channel flow with suction/injection together with convective heating/cooling at the walls have been investigated. The nonlinear model problem is tackled numerically using (WRM) collocation method. Based on the results presented above, the following conclusions are deduced.

The impact of different parameter such as effect of different parameter such as Hartman number (Ha), Biot number lower plate (Bi_0), Biot number upper plate (Bi_1), Reynold number (Re), Heat Generation (Q) are discussed graphically.

- An increase in (Q),(Pr),(Bi_0),(Ec),(ϵ) and increases the velocity profiles, while an increase in (Ha) and (Bi_1)decreases the velocity profile.
- An increase in (Q),(Pr),(Bi_0),(Re),(Ec),(ϵ) and increases the temperature profiles, while an increase in (Ha) and (Bi_1) decreases the temperature profile.

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