# Optimal Linear Control of an Energy Harvesting System

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***Abstract-*** This paper presents an optimal linear control of an energy harvesting system. Energy harvesting system models are typically non linear and therefore utilize a non linear controller. In this work, an investigation on the performance of a linear controller for a nonlinear harvester model is shown. The aim is to maximize the energy produced by implementing an optimal linear control algorithm. A mathematical model of the harvester is given. All simulations were conducted in matlab and simulink. Results obtained show that a current of 0.48mA and a voltage of 4.2Volts is harnessed. This study demonstrates that for local applications where transients and sensitivity to plant parameters is not a strong requirement, a linear controller can be utilized.

***Index Terms-*** Linear, Control, Energy, Harvester

## I. Introduction

For most applications, the universal goal is power density [3]. In these instances, optimal control of the harvester is essential. Control of energy harvesting systems is an active research area and there have been myriads of recent developments in many forms and applications. One novel research was conducted by Jeff T Scruggs et al in [4]. Their report was focused on maximizing the energy harvested from a vibratory energy harvesting system using actively controlled electronics. A stationary stochastic disturbance model was studied. Their discussion was purely systematic and showed that with the use of LMI (Linear Matrix Inequality) techniques, maximum power can be harnessed from active stochastic energy harvesting systems using feedback controllers based on the requirements of the closed-loop. This work presents a linear controller designed to maximize the energy produced from a continuously rotating energy harvesting system. The objective is to investigate the performance of a linear controller for a nonlinear harvester model.

## II. PROBLEM STATEMENT

Nonlinear systems such as the harvester model presented in this paper can display a chaotic and unpredictable behavior contrasting with linear systems. However, nonlinear systems act very much linear around operating points (equilibria are operating points). Despite the fact that linearizations only give local approximations, they are remarkably useful when they work [5]. If a system can be linearized, then a linear controller can be designed. Not all nonlinear systems can be linearized. A typical example is the unicycle whose nonlinearity is so severe that it cannot be approximated around operating points. The nonlinear model of the harvester presented in this work can be linearized. Hence, a linear controller can be designed.

## III. CONTROLLER DESIGN METHODOLOGY

### 3.1 Harvester Model

The conceptual model of the harvester is presented in [2]. It consists of a direct current generator with the stator attached to the rotating source. An offset mass is coupled to the rotor of the generator at some distance away from the rotational axis. In the experimental set up, a direct current (dc) motor served as the host. In reality, the host could be a wheels or a vehicle, bicycle or any other rotational structure. When the host rotates, a rotational torque is exerted on the suspended mass. This causes the suspended mass to deflect by an angle,ϑ. When the mass deflects, gravitational torque acts on it that counteracts the torque exerted on it due to the rotating host. It is the difference between the two torques that causes power to be produced.

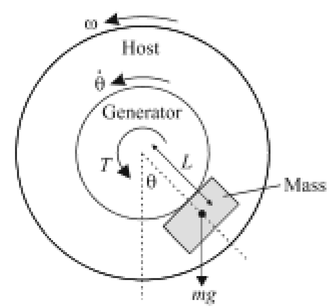


**Figure 1:** ***Rotational energy harvester*** **[2]**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{-mgl\sin x_1}{J} + \frac{k_E k_T (w_s - x_2)}{J\left(R_a + (1-\delta)^2 R_L\right)} \\ \frac{-(1-\delta)x_4}{L} + \frac{k_E (w_s - x_2)(1-\delta)^2 R_L}{L\,(R_a + (1-\delta)^2 R_L)} \\ \frac{(1-\delta)x_3}{C} - \frac{x_4}{R_L C} \end{bmatrix} \qquad (1)$$

Proof of the mathematical model of the harvester is presented in [1].

$k_E$ is the motor torque constant, L represents inductor, C is capacitor, $R_L$ is the load. m represents the mass of the offset body. $k_T$ = torque constant of the motor. The moment of inertia of the semi-circular mass is J. $w_s$ is the source rotating speed. δ is the duty cycle of the system.

### 3.2 State Variables

ϑ = x1
wr (rotor speed) = x2
IL (inductor current) = x3
Vc (capacitor voltage) = x4.

### 3.3 Conditions of Optimality

The current drawn, $I = \frac{mgl\sin\vartheta}{K_T} = \frac{kE(ws-wr)}{(RL+(1-\delta)^2RL)}$ (2)

It is easy to see that maximum current is drawn when wr = 0

$$\Rightarrow I(max) = \frac{kE(ws)}{(RL+(1-\delta)^2RL)} \quad (3)$$

Also, when $\vartheta = 90^0$, $I(max) = \frac{mgl}{kT}$ (4)

wr = 0 and $\vartheta = 90^0$ forms the two conditions of optimality and will be utilized in the controller design.

### 3.4 Constant Controller Design

This controller ensures that $x1 = \frac{\pi}{2}$ *and* $x2 = 0$. Where $x1$ *and* $x2$ represents the angular position and angular velocity of the offset mass respectively. From equation (1), it can be observed that the only term that can be controlled and varied is the duty cycle, $\delta$. This term will be used as the constant controller. If $x1 \rightarrow \pi\ 2$ *and* $x2 \rightarrow 0 \Rightarrow (\dot{x}1, \dot{x}2) = (0, 0)$.

Equating $\dot{x}_1$ *and* $\dot{x}_2$ *in* equation (1) to 0, and substituting $x1 = \frac{\pi}{2}$ and $x2 = 0$.

$$\frac{-mgl\sin x1}{J} + \frac{kEkT(ws-x2)}{J(Ra+(1-\delta)^2RL)} = 0 \quad (5)$$

Substituting $x1 = \frac{\pi}{2}$ and $x2 = 0$, $\frac{mgl}{J} = \frac{kEkT(ws)}{J(Ra+(1-\delta)^2RL)}$

$$(1-\delta)^2 = \frac{kEkT(ws)}{RLmgl} - \frac{Ra}{RL}$$

$$\Rightarrow \delta = 1 \mp \sqrt{\left(\frac{kEkT(ws)}{RLmgl} - \frac{Ra}{RL}\right)} \quad (6)$$

Since $\delta$ represents the duty cycle, it is expected that $(t)\ \epsilon\ [0,1]$ for all $t \geq 0$

$$\therefore \boldsymbol{\delta} = \mathbf{1} - \sqrt{(\boldsymbol{kEkT}(\boldsymbol{ws})\ \boldsymbol{RLmgl} - \boldsymbol{Ra}\ \boldsymbol{RL})} \quad (7)$$

Equation (7) is the constant controller. Since it is required for $(t)\ \epsilon\ [0,1]$ *for all* $t \geq 0$,

$$\Rightarrow 1 \geq (ws)\ RLmgl - Ra\ RL \geq 0 \text{ for all } t \geq 0$$

### 3.5 Linear Controller Design by Pole Placement

From standard controller design techniques, to design a linear controller for the harvester, the nonlinear model is linearized. Nonlinear systems act linear around operating points. Consider the harvester model in equation (1). Matlab script was used to perform the linearization and compute a state feedback controller. Since rank (ctrb(A,B)) = 4, the system is controllable. Therefore, a suitable control law can be designed. Poles of the closed loop system are chosen to have strictly negative real parts with a good balance between response speed and amount of control. Poles of the resulting closed loop system were placed at $2*Re(\lambda(A))$.

The script returns a controller, K = [0.0000 -0.7248 0.0680 0.0554].

Linear control law is given as

$$\boldsymbol{u} = \boldsymbol{K1x1} + \boldsymbol{K2x2} + \boldsymbol{K3x3} + \boldsymbol{K4x4} \quad (8)$$

Where $K1 = 0$ , $K2 = -0.7248$, $K3 = 0.0680$ and $K4 = 0.0554$

### 3.6 Proposed Linear Control Implementation

The controller is implemented in both state and output feedback modes. In state feedback, the states of the harvester are assumed known and available for feedback. For output feedback, the controller is fed with estimates of the states. The schematic diagrams for each are shown below.

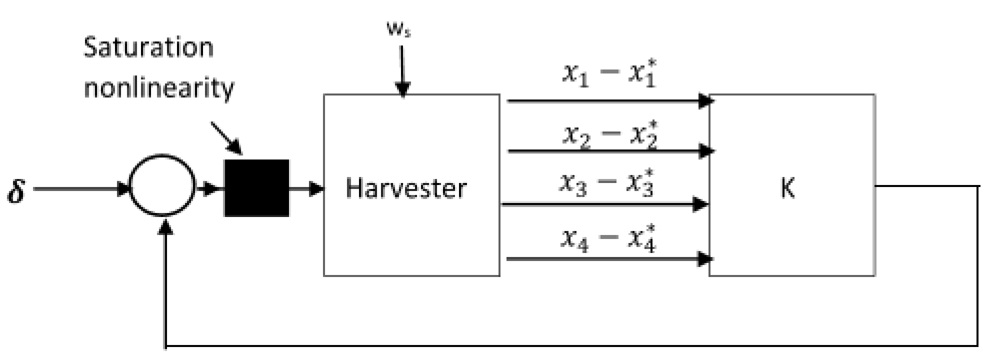


**Figure 2:** ***Implementation of State feedback***

The saturation nonlinearity was incorporated to ensure that the control $\delta*(t) = \delta - K(xi - x*i)\ \epsilon\ [0\ 1]$ for all $t \geq 0$. i =1,2,3,4. Where $x*i$ represents the equilibrium states. The equilibria are gotten by equating $\dot{xi} = 0$ from (35) i = 1,2,3,4. When this is done, the following is obtained:

$$x1 = \frac{\pi}{2}$$
$$x2 = 0$$
$$x3 = \frac{x4}{RL(1-\delta)}$$
$$x4 = \frac{kE(ws-x2)RL(1-\delta)}{Ra+(1-\delta)^2RL}$$

ISSN 2250-3153

The logical implication of the control law $\delta*(t) = \delta - K(xi - x*i)$ is that when the states of the harvester, $xi$, becomes equal to the equilibrium, $x*i$, then $\delta*(t) = \delta$ which assures global asymptotic stability.

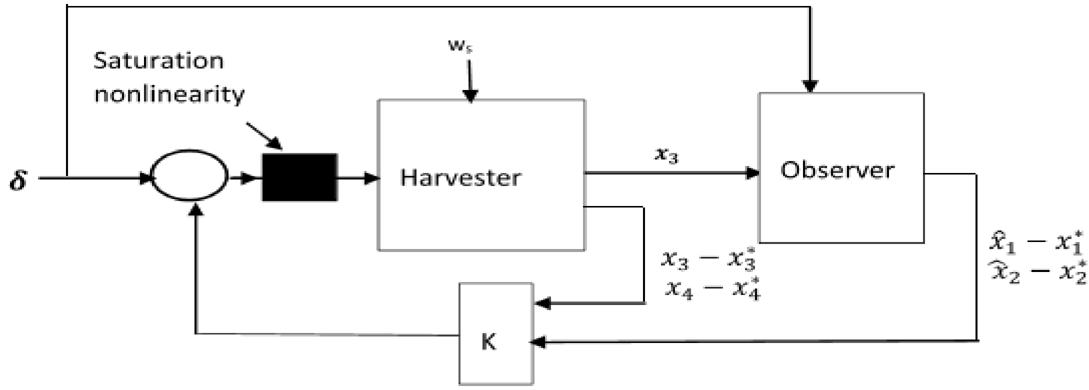


**Figure 3:** ***Implementation of output feedback***

## IV. RESULTS AND DISCUSSION

Figures 4 - 7 examines the plots from the nonlinear model of the harvester described by equation (1). These plots were obtained with a duty cycle of 0.7 and source rotation speed of 75 rad/sec. They show instability. The oscillation in the angular position and angular velocity causes oscillations in the current and voltage. Figures 6 and 7 in particular show that very little power ($P = I_L.V_c$) is produced in the absence of a controller.

Figures 12 and 13 represent the performance of the linear controller together with the constant controller. An angle of 90 degree was maintained with an angular velocity of 0 rad/sec. This satisfies the conditions of optimality. As observed in figures 9 and 10, the controller eliminates all the oscillations in the inductor current and capacitor voltage. It also shows that a voltage of 4.2V was produced with a current of 0.48mA. Hence, maximum power is harnessed. It is important to note that the numerical values of the harnessed current and voltage depends on the numerical values of harvester model parameters.

### 4.1 Nonlinear Model Simulations

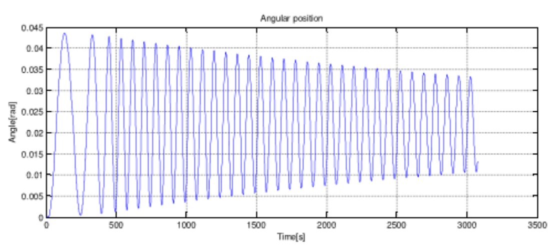


**Figure 4:** ***Variation of the angular position (ϑ = x1) with time***

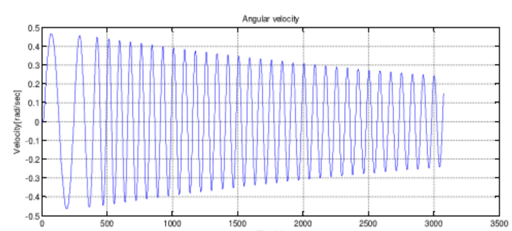


**Figure 5:** ***Variation of the angular velocity (wr = x2) with time***

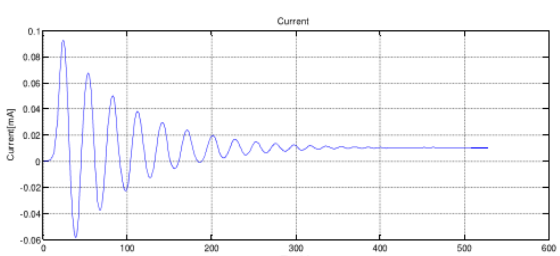


**Figure 6:** ***Variation of current (IL = x3) with time***

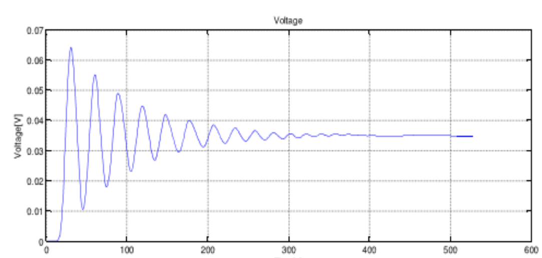


**Figure 7:** ***Variation of voltage (Vc = x4) with time***

### 4.2 Constant Controller Simulations

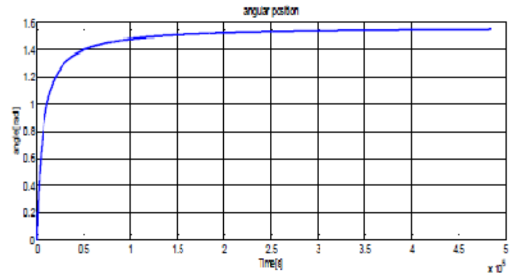


**Figure 8:** ***Angle of offset mass maintained at 90 degree***

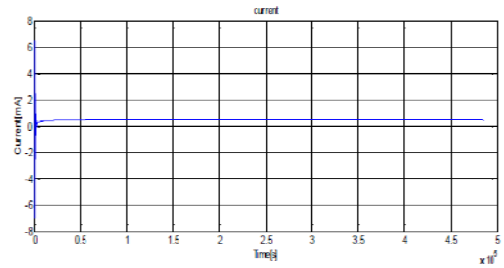


**Figure 9*: Variation of inductor current with time***

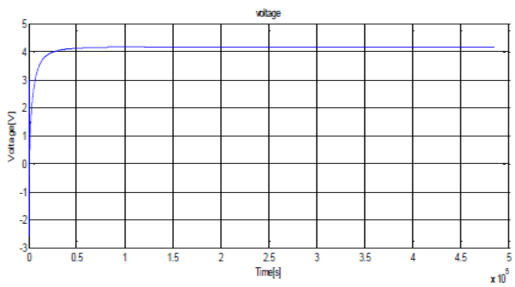


**Figure 10*: Variation of output voltage with time***

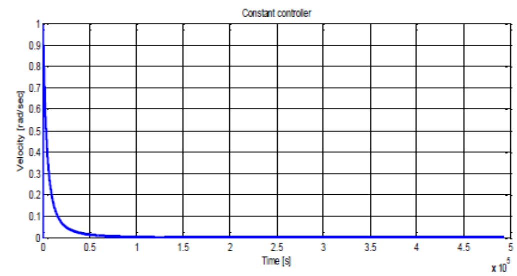


**Figure 11*: Velocity of offset mass maintained at 0***

### 4.3 Linear Controller Simulations

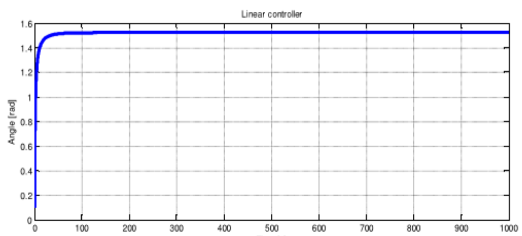


**Figure 12:** ***Angle of offset mass maintained at pi/2***

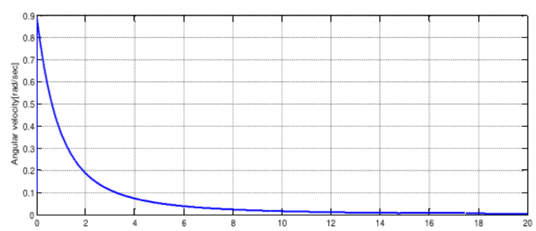


**Figure 13:** ***Angular velocity of offset mass maintained at 0 rad/sec.***

## V. CONCLUSION

In conclusion, a linear feedback controller was designed. This controller worked well with the nonlinear model as shown in the matlab plots. However, a nonlinear controller is better. Nonlinear controllers provide better tracking and regulation of the state, reduction of transients and finally reduction of the sensitivity to plant parameters. For local applications where transients and sensitivity to plant parameters is not a strong requirement, a linear controller can be utilized.

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ISSN 2250-3153

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