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Roman Domination Number of a Graph

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Abstract- A dominating set D of a graph $G=(V,\ E)$ is *roman dominating* set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in $V-D$ as well as adjacent to some vertex in T. The roman domination number of $\gamma_{rds}(G)$ of G is the minimum cardinality of a roman dominating set of G. In this paper many bounds on $\gamma_{rds}(G)$ are obtained and its exact values for some standard graphs are found. Also relationship with other parameter is investigated. We introduce Roman dominating set in which the interest is in dominating contains two types of subset.

Index Terms- Domination, Roman domination.

I. Introduction

The graphs considered here are simple, finite, nontrivial, undirected, connected, without loops or multiple edges or isolated vertices. For undefined terms or notations in this paper may be found in Harary [1].

Let $G=(V,\ E)$ be a graph. A set $D\subseteq V(G)$ is a dominating set of G if every vertex not in D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set.

Roman Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army, which would stay and protect the home. Thus, there are two types of armies, stationary and traveling. Each vertex with no army must have a neighboring vertex with a traveling army. Stationary armies then dominate their own vertices, and its stationary army dominates a vertex with two armies, and the traveling army dominates its open neighborhood, which motivates to our roman domination number of a graph.

A dominating set D of a graph $G=(V,\ E)$ is *roman dominating* set if S,T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in $V-D$ as well as adjacent to some vertex in T. The roman domination number of $\gamma_{rds}(G)$ of G is the minimum cardinality of a roman dominating set of G.

II. Results

We use the notation P_p, W_p, K_p, and C_p to denote, respectively, the *path*, *wheel*, *complete graph*, and *cycle* with p vertices; $K_{1,p}$ *star* with p + 1 vertices, and $K_{m,n}$ *complete bipartite graph* with $(m+n)$ vertices (see [4]).

Theorem 1.

(1). $\gamma_{rds}(C_p)=\left\lfloor \frac{p}{2}\right\rfloor+1\ ,\quad p\geq 3$

(2). $\gamma_{rds}(K_p)=p-1\ ,\quad p\geq 3$

(3). $\gamma_{rds}(W_p)=3\ ,\quad p\geq 4$

(4). $\gamma_{rds}(P_p)=\left\lceil \frac{p}{2}\right\rceil,\ p\geq 3$,

(5). $\gamma_{rds}(K_{m,n})=m,$ where $m<n$,

(6). $\gamma_{rds}(K_{1,\ p})=p=q=\Delta(K_{1,\ p})$

Proof. Since, follows directly from the definition.

Theorem 2. For any graph G, $\gamma_{rds}(G)\leq p-1.$ equality holds when G a complete graph or a star.

Proof. Let D be a roman dominating set of G, such that S and T are two subsets of D, clearly, by the definition of roman dominating set the number of vertices in S are adjacent to a number of vertices in $(V-D)$ are equal. Then the number of vertices are in T are adjacent to a number of vertices in $(V-D)$ are greater than or equal to zero. Hence $\gamma_{rds}(G)\leq p-1.$

Further, the complete graph or star achieves this bound.

Theorem 3. For any graph G, $p\geq 3$ vertices, a set S of independent $|m|$ cut vertices and each cut vertex is adjacent to a pendent vertex. Then $\gamma_{rds}(T)=\sum_{i=1}^{m}\deg m_i$

Proof. We know that, a dominating set D of a graph $G=(V,\ E)$ is *roman dominating* set if S,T are two subsets of D. Obviously, every cut vertex of S is adjacent to a pendent vertex u in $V-D$ as well as adjacent in t.

Thus,

$|N[s]|-|u|$

$\Rightarrow|N(S)|$

$$\Rightarrow \gamma_{rds}(T) = \sum_{i=1}^{p-2} \deg reeU_i \in S.$$

Theorem 4. For any graph G, $\gamma_{rds}(G) \geq \gamma(G)$.

Proof. Clearly, by the definition, roman dominating set is also a dominating set.

Theorem 5. For any graph G, $\frac{p}{\Delta+1} \leq \gamma_{rds}(G)$.

Proof. It is known that $\frac{p}{\Delta+1} \leq \gamma(G)$ and since $\frac{p}{\Delta+1} \leq \gamma_{rds}(G)$, result holds

Theorem 6. For any graph G, $\gamma_{rds}(G) \leq p-1$. And this bound is sharp.

Proof. Let D be a minimum dominating set of G. Then every vertex of $V-D$ except one vertex $v \in (v-D)$, then $D \cup (V-D)-1$ is a roman dominating set. Thus

$$D \cup (V-D)-1$$
$$\gamma + P - \gamma - 1$$
$$P-1.$$

The Complete graph K_p achieves this bound.

Theorem 7. If T is a tree with m cut vertices and if each cut vertex is adjacent to at least one end vertex except one is adjacent to exactly one end vertex, then $\gamma_{rds}(G) = m$.

Proof. Let C be the set of all cut vertices of tree T with $|C| = m$. Then $V-C$ is a dominating set of all end vertices with $|V-C| > m$. We know that S and T be the subset Stationary and travelling respectively of a roman dominating set. Here a cut vertex u which is adjacent to one end vertex is belongs to stationary set and $|C-u|$ cut vertices are belongs to travelling set, therefore C is a roman dominating set. Hence, the roman domination number of G $\gamma_{rds}(G) = m$.

Corollary 8. For any tree T, with $m \geq 2$ cut vertices, then $\gamma_{rds}(T) \leq m.$

Proof. Suppose, the tree T contain m number of cut vertices. Follows form the theorem 6. If some cut vertices is belongs $V-D$, by the definition $\gamma_{rds}(T) < m.$

The following propositions, corollary are straight forward and the proofs are omitted.

Propositions 9. If a graph G has cut vertex v and is adjacent to an end vertex, then

$$2 \leq \gamma_{rds}(G) \geq P-1.$$

Corollary 10. If a graph G has cut vertex, $P \geq 4$ vertices and every cut vertex has adjacent to at most two end vertices, then

$\gamma_{rds}(G) = P - e$, where e be the end vertex.

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