Solution of Partial Differential Equations with Variables Coefficients Using Double Sumudu Transform

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Abstract- In this paper, we study the properties of sumudu transform and double Sumudu Transform and solve the partial differential equation with variables Coefficient By using Double sumudu transforms. The applicability of this relatively double sumudu transform is demonstrated using some special functions. Double sumudu transform method is a strong method to solve such PDEs

Index Terms- doubles Sumudu Transform, Inverse doubles Sumudu Transform, Laplace transform, partial differential equations,

I. Introduction

Sumudu transform is the integral transform of similar Laplace transform part, introduced in the early 1990s by Gamage K. Watugala [4] to solve differential equation. It is equivalent to the Laplace transform replaced with p = 1/u. The Sumudu transform, from these features, displayed in this paper, is yet not widely known, nor used. The Sumudu transform may be applied to determine problems without resorting to a different frequency domain. In 2003, Belgacem et al. gave it to be the theoretical dual to the Laplace transform; also, therefore .It comes from this fact our interest in this new doubles Sumudu transform. Certainly can cure most problems (may be of an integral, differential, or the nature of the control engineering). The hat would normally be handled by the well-known and used widely Laplace transform [3]. Since this is a new transform it has very distinctive and useful properties. It can help with complex applications in science and engineering. The transform would also be a natural choice for solving the problems scale and units preserving requirements. For that, our goal is applications. We point out that the Laplace transform is defined by,

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)dt, \mathrm{Re}(s) > 0, \tag{1}$$

And the sumudu transform

$$A = \left\{ f(t) \exists M, \tau_1, \tau_2 > 0, \left| f(t) < M e^{\frac{t}{\tau}} j, \text{if } t \in (-)^j \times [0, \infty), \right| \right\} \tag{2}$$

By $$F(u) = S[f(t)] = \int_0^\infty f(ut) e^{-t} dt, u \in (-\tau_1, \tau_2) \tag{3}$$

The Double Sumudu Transform

Sumudu transform gives a clear and relatively unified way for the introduction of a double Sumudu transform, gave that function has a power series transformation concerning its variables. Double Laplace Transform technique of function of two variables identified in the positive quadrant of the xy–plane [see 4] is given by

$$\mathcal{L}_2[f(x,y);(p,q)] = \int_0^\infty \int_0^\infty f(x,y) e^{-(px+qy)} dxdy, \tag{4}$$

As p and q are the transforms of the variables x and y, straight.

Definition

Let $f(t,x); t, x \in \mathbb{R}_+$ be a function that can be expressed shaped an infinite series close, then, the double Sumudu r transform is given by

$$F(u,v) = S_2[f(t,x);(u,v)] = S[S\{f(t,x);t \to u\};x \to v]$$

$$= \frac{1}{uv}\int_0^\infty \int_0^\infty e^{-\left(\frac{t}{u}+\frac{x}{v}\right)} f(t,x)dtdx \tag{5}$$

We offer Sumudu double applications converted to some functions that relate to those obtained in the solution formulas for the dynamics of population age structure. However, it is a frivolous practice to demonstrate that double Sumudu and Laplace transformations are also a dual theory. That is

$$F(u,v) = \mathcal{L}_2\left[f(x,y);\left(\frac{1}{u},\frac{1}{v}\right)\right] \tag{6}$$

As $\mathcal{L}_2$ is run double Laplace Transform technique

Theorem

let $f(x,y), x,y \in R_+$ be a real valued function ,then

$$S_2[f(x+y);(u,v)] = \frac{1}{u-v}\{uF(u) - vF(v)\} \tag{7}$$

The case of (x - y) is even more interesting from the viewpoint of biology, where often encountered these jobs in Mathematical Biology, with f representing the population density, x the age, and y the time, or vice-versa. The proof for the case $x \geq y$ is simple and enough, but with a tedious manipulation. Therefore, geometrically, if the line dividing the first quadrant into two equal parts represents the η-axis (the lower part being represented by Q1, and the upper part Q2), although that dividing both the second and fourth quadrants represents the ζ-axis (arrow pointing upwards) and ξ- axis (arrow from origin into the fourth quadrant), sequentially, then the test is as follows: Let f be an even function, then

While for $f(0)$ odd, we have

$$S_2[f(x-y);(u,v)] = \frac{1}{uv}\int\int_{Q_1} f(x-y)e^{-\left(\frac{x}{u}+\frac{y}{v}\right)}dxdy - \frac{1}{uv}\int\int_{Q_2} f(x-y)e^{-\left(\frac{x}{u}+\frac{y}{v}\right)}dxdy \tag{8}$$

Let

$$x = \frac{1}{2}(\zeta+\eta); y = \frac{1}{2}(\zeta-\eta)$$

Then

$$\int\int_{Q_1} f(x-y)e^{-\left(\frac{x}{u}+\frac{y}{v}\right)}dxdy = \frac{1}{2}\int_0^\infty f(\zeta)d\zeta \int_\zeta^\infty e^{-\frac{1}{2}\left(\frac{1}{u}+\frac{1}{v}\right)\zeta - \frac{1}{2}\left(\frac{1}{u}-\frac{1}{v}\right)\eta} d\eta$$

$$= \frac{uv}{u-v}\int_0^\infty e^{-\frac{\zeta}{v}} f(\zeta)d\zeta$$

$$= \frac{uv^2}{u - v} F(u)$$

Similarly,

$$\int\int_{Q_4} f(x - y) e^{-\left(\frac{x}{u} + \frac{y}{v}\right)} dxdy = \frac{v^2 u}{u - v} F(v)$$

Hence , for f even

$$S_2[f(x - y)]; (u, v) = \frac{uF(u) + vF(v)}{u - v} \tag{9}$$

And for odd function

$$S_2[f(x - y)]; (u, v) = \frac{uF(u) - vF(v)}{u + v} \tag{10}$$

From equations (3) and (9), it is obvious that if f is an even function, then

$$(u + v) S_2[f(x - y)] = (u - v) s_2[f(x + y)] \tag{11}$$

Where a and b are positive constants (double Sumudu transform one like its equivalent is to extend the range to maintain).
Lemma Let $f(x), x \in \mathbb{R}_+$ and $H(0)$ represents the Heaviside function then

$$\text{(i)} S_2\left[f(x)H(x - y); (u, v) = F(u) + \left(\frac{\mathcal{W}}{v} - 1\right) F(\mathcal{W})\right]$$

$$\mathcal{W} = \frac{uv}{u + v}$$

$$\text{(ii)} S_2\left[f(x)H(x - y); (u, v) = \left(1 - \frac{\mathcal{W}}{v}\right) F(\mathcal{W})\right]$$

The proof is easy, for the case, by writing the left-hand side of the equation in (ii) as

$$\frac{1}{uv} \int_0^\infty f(x) e^{\frac{x}{u}} \int_x^\infty e^{\frac{y}{v}} dxdy$$

Integration and performance, taking into account the fact that Fubini theory, the result followed.
Corollary 4.5

$$(i)S_2[H(x-y);(u,v)] = \frac{v}{u+v}$$

$$(ii)S_2[H(y-x);(u,v)] = \frac{u}{u+v}$$

Consequently,

$$S_2[f(x)H(y-x)] + S_2[f(x)H(x-y)] = F(u) \quad (12)$$

Apply DST to partial derivatives as follows: Let f(0, a) = F_0(a), then

$$S_2\left[\frac{df}{dt}f(t,a);(u,v)\right] = \frac{1}{uv}\int_0^\infty\int_0^\infty e^{-\left(\frac{t}{u}+\frac{a}{v}\right)}\frac{\partial}{\partial t}f(t,a)\,dt\,da = \frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}\left\{\frac{1}{u}\int_0^\infty e^{-\frac{t}{u}}f(t,a)dt\right\}da$$

The inner integral gives (see [6]),

$$\frac{F(u,a)-f(0,a)}{u} \quad (13)$$

$$S_2\left[\frac{\partial f(t,a)}{\partial t};(u,v)\right] = \frac{1}{u}\left\{\frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}F(u,a)da - \frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}f_0(a)da\right\}$$

$$= \frac{1}{u}\left\{\frac{1}{v}F(u,v) - F_0(v)\right\} \quad (14)$$

Also,

$$S_2\left[\frac{\partial f(t,a)}{\partial a};(u,v)\right] = \frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}\left\{\frac{1}{u}\int_0^\infty e^{-\frac{t}{u}}\frac{\partial}{\partial a}f(t,a)dt\right\}da$$

$$= \frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}\frac{\partial}{\partial a}F(u,a)da$$

$$= F_u(u,v) \quad (15)$$

Alternatively,

$$S_2\left[\frac{\partial f(t,a)}{\partial t};(u,v)\right] = \frac{1}{u}\int_0^\infty e^{-\frac{a}{v}}\left(\frac{1}{v}\int_0^\infty e^{-\frac{a}{v}}\frac{\partial f}{\partial a}da\right)dt$$

$$= \frac{1}{u}\int_0^\infty e^{-\frac{t}{u}}\frac{1}{v}[F(t,v)-f(t,0)]dt$$

$$=\frac{1}{v}\left(F(u,v)-F_0(u)\right) \tag{16}$$

Where F (u, 0) = $F_0(u)$ and F(0, v) = $F_0(v)$. It is obvious from equations (4.14) and (5.15) that,

$$F_v(u,v)=\frac{F(u,v)-F_0}{v}$$

If u and v are equal, there is the case of DST, here in the is unique that Sumudu frequency converter, Compared to the Laplace transform [3]. Thus, the iterative Sumudu transform any particular function of two variables f (x, y), for example, is defined by

$$[f(x,y);(v,u)]=S_2[f(x,y);(u,u)]=\frac{1}{u^2}\int_0^\infty\int_0^\infty e^{-\left(\frac{x+y}{u}\right)}f(x,y)dxdy. \tag{17}$$

Therefore, the SD of the general convolution function

$$f^{(2)}(x)=\int_0^x f(x-y,y)dy$$

is related to the iterated Sumudu transform as follows:

$$S[f^2(x);u]=\frac{1}{u}\int_0^\infty e^{-\frac{x}{u}}f^{(2)}(x)dx=\frac{1}{u}\int_0^\infty e^{-\frac{x}{u}}\int_0^\infty f(x-y,y)dxdy \tag{18}$$

By letting x = s + t and y = t in equation (18), weObtain

$$S\left[f^{(2)}(x)\right]=\frac{1}{u}\int\int_\Omega f(s,t)e^{-\left(\frac{s+t}{u}\right)}dsdt=uS_2[f(s,t);(u,u)]=uJ[f(s,t);(u,v)]$$

with the convolution integral taken in the classical sense, and $x,y,s\in\mathbb{R}^+$.

Theorem 4.1: Sumudu transform inflate the power series of factors,

$$f(t)=\sum_{n=0}^{\infty}a_n t^n,$$

By sending it to the power series function,

$$G(u)=\sum_{n=0}^{\infty}n!\,a_n u^n$$

Proof: Let $f(t)$ be in $G(u)=\sum_{n=0}^{\infty}a_n t^n$ in some interval Proof. $I\subset\mathbb{R}$, then by Taylor's function theorem,

$$f(t)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}t^n$$

Therefore, by (4.3), and that of the gamma function Γ (see Table 4.1), we have

$$S[f(t)] = \int_0^\infty \sum_{k=0}^{\infty} \frac{f^{(n)}(0)}{n!}(ut)^n e^{-t} dt = \sum_{k=0}^{\infty} \frac{f^{(n)}(0)}{n!} u^n \int_0^\infty t^n e^{-t}\, dt$$

$$= \sum_{k=0}^{\infty} \frac{f^{(n)}(0)}{n!} u^n dt\Gamma(n+1) = \sum_{k=0}^{\infty} f^{(n)}(0) u^n$$

consequently

$$S[(1+t)^m] = S\sum_{n=0}^{m} C_n^m t^n = S\sum_{n=0}^{m} \frac{m!}{n!\,(m-n)!} u^n = \sum_{n=0}^{m} \frac{m!}{(m-n)!} u^n = \sum_{n=0}^{m} P_n^m u^n$$

Also a requirement that S[f (t)] Coincide, in an interval containing u = 0, is

Provided by the following conditions when satisfied, namely, that

$$(i) f^{(n)}(0) \longrightarrow 0 \text{ as } n \longrightarrow \infty$$
$$(ii) \lim_{n\to\infty} \left| \frac{f^{(n+1)}(0)}{f^n(0)} u \right| < 1$$

This means that the convergence radius r of S[f (t)] depends on the sequence f (n)(0),
Since

$$r = \lim_{n\to\infty} \left| \frac{f^n(0)}{f^{(n+1)}(0)} u \right|$$

It is clear that Sumudu converter can be used in signal processing or disclosures

Theorem: Let G (u) be the sumudu transform of f (t) such that
(i)G ((1/s))/s is a monomorphic function, with characteristics having Re(s) < γ, and
(ii)There survives a circular area with radius R and positive steady, M and K, with

$$\left| \frac{G(1/s)}{s} \right| < MR^{-k}$$

Then the function f (t) is given on using the Cauchy Residue TheoremAs follows

$$S^{-1}[G(s)] = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} G\left(\frac{1}{s}\right) \frac{ds}{s} = \sum \text{residues} \left[e^{st} \frac{G\left(\frac{1}{s}\right)}{s} \right]$$

Definition 1: (Hassan Eltayeb,2010)
Let f (t, x) and g (t, x) having double sumudu transform. Then double Sumudu transform of the double convolution of the f (t,x) and g(t,x),

$$(f ** g)(t,x) = \int_0^x \int_0^t f(\zeta,\eta) g(t-\zeta, x-\eta)\, d\zeta d\eta$$

Is given by

$$S_2[(f ** g)(t,x); u,v] = uvF(u,v)G(u,v)$$

Also, in the following we discuss double Sumudu transform of partial derivative with respect to x of double convolution,

$$S_2\left[\frac{\partial}{\partial x}(f ** g)(t,x);u,v\right] = uvS_2\left[\frac{\partial}{\partial x}(t,x);u,v\right] = S_2[g(t,x);u,v]$$

Or

$$uvS_2[f(t,x);u,v]S_2\left[\frac{\partial}{\partial x}g(t,x);u,v\right]$$

Similar to t. Thus we can conclude that the relation between double Sumudu Of convolution and double Laplace transform of double convolution can be given by

$$S_2[(f ** g)(t,x);u,v] = \frac{1}{uv}L_tL_x[(f ** g)(t,x)]$$

To use Sumudu Transform to solve PDEs we need the partial derivatives of This Transform. Thus, application of double sumudu transform to second Partial derivatives with respect to x is given by

$$S_2\left[\frac{\partial^2}{\partial x^2}f(t,x);u,v\right] = \frac{1}{v^2}F(u,v) - \frac{1}{v^2}F(u,0) - \frac{1}{v}\frac{\partial}{\partial x}\ F(u,0)$$

Similarly, second partial derivative with respect to t is given by

$$S_2\left[\frac{\partial^2}{\partial t^2}f(t,x);u,v\right] = \frac{1}{u^2}F(u,v) - \frac{1}{u^2}F(0,v) - \frac{1}{u}\frac{\partial}{\partial t}\ F(0,v)$$

Applications

The following two samples explain the application of the double Sumudu transform. It well recognized that to get the solution of partial differential Equations by integral transform techniques we need the following two actions: Beginning, we transform the partial differential equations to algebraic equations By employing double new integral transform technique. Further, on applying Inverse Double new integral transform, we see the solution of PDEs. Respect The general linear telegraph equation (Tarig.M.Elzaki et al..2012) in the form

$$U_{tt} + aU_t + bU = c^2U_{xx} \text{ where } a, b, c \text{ are constant} \tag{19}$$

With initial condition:

$$U(x,0) = f_1(x), \qquad U_t(x,0) = g_1(x) \tag{20}$$

And boundary conditions:

$$U(t,0) = f_2(t), \qquad U_x(0,t) = g_2(t) \tag{21}$$

Solution:

$$k_2[U_{tt}(x,t);(u,v)] = \frac{A(u,v)}{v^4} - \frac{A(u,0)}{v^3} - \frac{1}{v}\frac{\partial A(u,0)}{dt}$$

$$k_2[U_t(x,t);(u,v)] = \frac{A(u,v)}{v^2} - \frac{A(u,0)}{v}k_2[U_t(x,t);(u,v)] = A\ (u,v)$$

$$k_2[U_{xx}(x,t);(u,v)] = \frac{A(u,v)}{u^4} - \frac{A(0,v)}{u^3} - \frac{1}{u}\frac{\partial A(u,0)}{dx}$$

$$A(u,0) = F_1(u) = \frac{1}{u}\int_0^\infty e^{\frac{x}{u^2}} U(x,0)dx = \frac{1}{u}\int_0^\infty e^{\frac{x}{u^2}} f_1(x)dx \tag{22}$$

$$A(0,v) = F_2(v) = \frac{1}{v}\int_0^\infty e^{\frac{x}{v^2}} U(0,t)dt = \frac{1}{v}\int_0^\infty e^{\frac{t}{v^2}} f_2(t)dt \tag{23}$$

$$\frac{\partial A(u,0)}{dt} = G_1(u) = \frac{1}{u}\int_0^\infty e^{\frac{x}{u^2}} \frac{dU(x,0)}{dt}dx = \frac{1}{u}\int_0^\infty e^{\frac{x}{u^2}} g_1(x)dx \tag{24}$$

$$\frac{\partial A(0,v)}{dx} = G_1(v) = \frac{1}{v}\int_0^\infty e^{\frac{t}{v^2}} \frac{\partial U(0,t)}{dx}dt = \frac{1}{v}\int_0^\infty e^{\frac{t}{v^2}} g_1(t)dt \tag{25}$$

Use the double new integral transform of Eq (19) and single new integral the transform of conditions, and then we have::

$$\left(\frac{A(u,v)}{v^4} - \frac{A(u,0)}{v^3} - \frac{1}{v}\frac{\partial A(u,0)}{\partial t}\right) + a\left(\frac{A(u,v)}{v^2} - \frac{A(u,0)}{v}\right) + bA(u,v) = c^2\left(\frac{A(u,v)}{u^4} - \frac{A}{\ }\right.$$

Substituting initial and boundary conditions (2) – (3) in equation above we Have

$$\frac{A(u,v)}{v^4} - \frac{1}{v^3}F_1(u) - \frac{1}{v}G_1(u) + \frac{a}{v^2}A(u,v) - \frac{a}{v}F_1(u) + bA(u,v) - \frac{c^2}{u^4}A(u,v) + \frac{c^2}{u^3}F_2(,v)$$

After some simple algebraic operations we get :

$$A(u,v) = \frac{F_1(u)(vu^3 + au^3v^3) - c^2v^4F_2(v) - c^2u^2v^4G_2(v) + u^3v^3G_1(u)}{u^3 + au^3v^2 + bu^3v^4 - c^2u^4} \tag{25}$$

Take the inverse of double new integral transform to get the solution of general Linear telegraph Eq (19) in the form

$$U(x,t) = K_2^{-1}[A(u,v);(x,t)] = K_2^{-1}[N(u,v);(x,t)] = D(x,t) \tag{26}$$

Example 1.1: Consider the linear telegraph equation in the form:

$$U_{xx} = U_{xx} + U_t + U \tag{27}$$

With initial conditions:

$$U\,(x,0) = f_1(x) = e^x\,, U_t(x,0) = g_1(x) = -e^x \tag{28}$$

And boundary conditions:

$$U\,(0,t) = f_2(t) = e^{-t}\,, U_x(0,t) = g_1(t) = e^{-t} \tag{29}$$

Solution: We take a new double transform on integral part of the equation (10) and single new integral transform of conditions, and then we have:

$$\frac{A(u,v)}{u^4}-\frac{A(0,v)}{u^3}-\frac{1}{u}\frac{\partial A(0,v)}{\partial x}=\frac{A(u,v)}{v^4}-\frac{A(u,0)}{v^3}-\frac{1}{v}\frac{\partial A(u,0)}{dt}+\frac{A(u,v)}{v^2}-\frac{A(u,0)}{v}+A(u,v)$$

$$A(u,0)=\frac{u}{1-u^2}\,;A(0,v)=\frac{v}{1-v^2}\,;\frac{\partial A(u,0)}{\partial t}=\frac{u}{-1-u^2}\,;\frac{\partial A(0,v)}{\partial x}=\frac{v}{1-v^2}$$

Substituting initial and boundary conditions (11)-(12) above we have:

$$\frac{A(u,v)}{u^4}-\frac{1}{u^3}\left(\frac{v}{1+v^2}\right)-\frac{1}{u}\left(\frac{v}{1+v^2}\right)=\frac{1}{v^4}A(u,v)-\frac{1}{v^3}\left(\frac{u}{1+u^2}\right)+\frac{1}{v}\left(\frac{u}{1-u^2}\right)+\frac{1}{v^2}A(u,v)-\frac{1}{v}\left(\right.$$

After some simple algebraic operations we get:

$$A(u,v)=\frac{uv}{(1-u^2)1(1+v^2)} \quad (30)$$

Take inverse of double new integral transform to obtain the solution of linear telegraph Eq.(17) in the form :

$$A(x,t)=K_2^{-1}[A(u,v);(x,t)]=e^{x-t} \quad (31)$$

Example 1.2: Consider wave equation in the form

$$U_{tt}-U_{xx}=3(e^{x+2t}-e^{2x+t})(x,t)\in R_+^2 \quad (32)$$

With initial conditions:

$$U(x,0)=f_1(x)=e^{2x}+e^{x},U_t(x,0)=g_1(x)=e^{2x}+2e^{x} \quad (33)$$

And boundary conditions:

$$U(0,t)=f_2(t)=e^{t}+e^{2t},U_x(0,t)=g_2(t)=2e^{t}+e^{2t} \quad (34)$$

Solution:

Use the double new integral transform of Eq.(32) and single new integral transform of conditions, and then we have:

$$\left(\frac{A(u,v)}{v^4}-\frac{A(u,0)}{v^3}-\frac{1}{v}\frac{\partial A(u,0)}{dt}\right)-\left(\frac{A(u,v)}{u^4}-\frac{A(0,v)}{u^3}-\frac{1}{u}\frac{\partial A(0,v)}{\partial x}\right)=3\left(\frac{uv}{(1-u^2)(1-2v^2)}-\frac{uv}{(1-2u^2)(1-v^2)}\right) \quad (35)$$

$$A(u,0)=\frac{u}{1-2u^2}+\frac{u}{1-u^2};A(ov)=\frac{v}{1-v^2}+\frac{v}{1-2v^2}$$
$$\frac{\partial A(u,0)}{dt}=\frac{u}{1-2u^2}+\frac{2u}{1-u^2};\frac{\partial A(0,v)}{dx}=\frac{2v}{1-v^2}+\frac{v}{1-2v^2}$$

Substituting initial and boundary conditions (33)-(34) in Eq. (35) and after some simple algebraic operations we have::

$$A(u,v) = \frac{uv}{(1-2u^2)(1-v^2)} + \frac{uv}{(1-u^2)(1-2v^2)} \quad (36)$$

Take the inverse of double new integral transform to reach the solution of wave Eq.(32) in the form

$$U(x,t) = K_2^{-1}[A(u,v);(x,t)] = e^{2x+t} + e^{x+2t} \quad (36)$$

II. Conclusion

This paper discusses the properties of sumudu transform and double Sumudu Transform and solves the partial differential equation with variables Coefficient By using Double sumudu transforms. The applicability of this relatively double sumudu transform is demonstrated using some special functions. Double sumudu transform method is the strong method to solve such PDEs. We may conclude that double sumudu transform is so powerful and efficient in getting the logical answer for a broad range of beginning Value boundary problems. The relationship of the double sumudu transforms with double Laplace transform performs is much deeper, and we can detect Other relatives of the Double sumudu transform

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