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Combination of Two Exponential Ratio Type Estimators for Estimating Population Mean Using Auxiliary Variable with Double Sampling in Presence of Non-Response

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Abstract- We have to proposed combination of two exponential ratio type estimators for estimating population mean using auxiliary variable with double sampling in presence of non-response and study its properties like as mean square error (MSE) and bias. An illustration of the proposed estimator has been made with the relevant class of estimators for optimum value of α.

Index Terms- Double Sampling, Bias and MSE, Non-response, Sampling Technique.

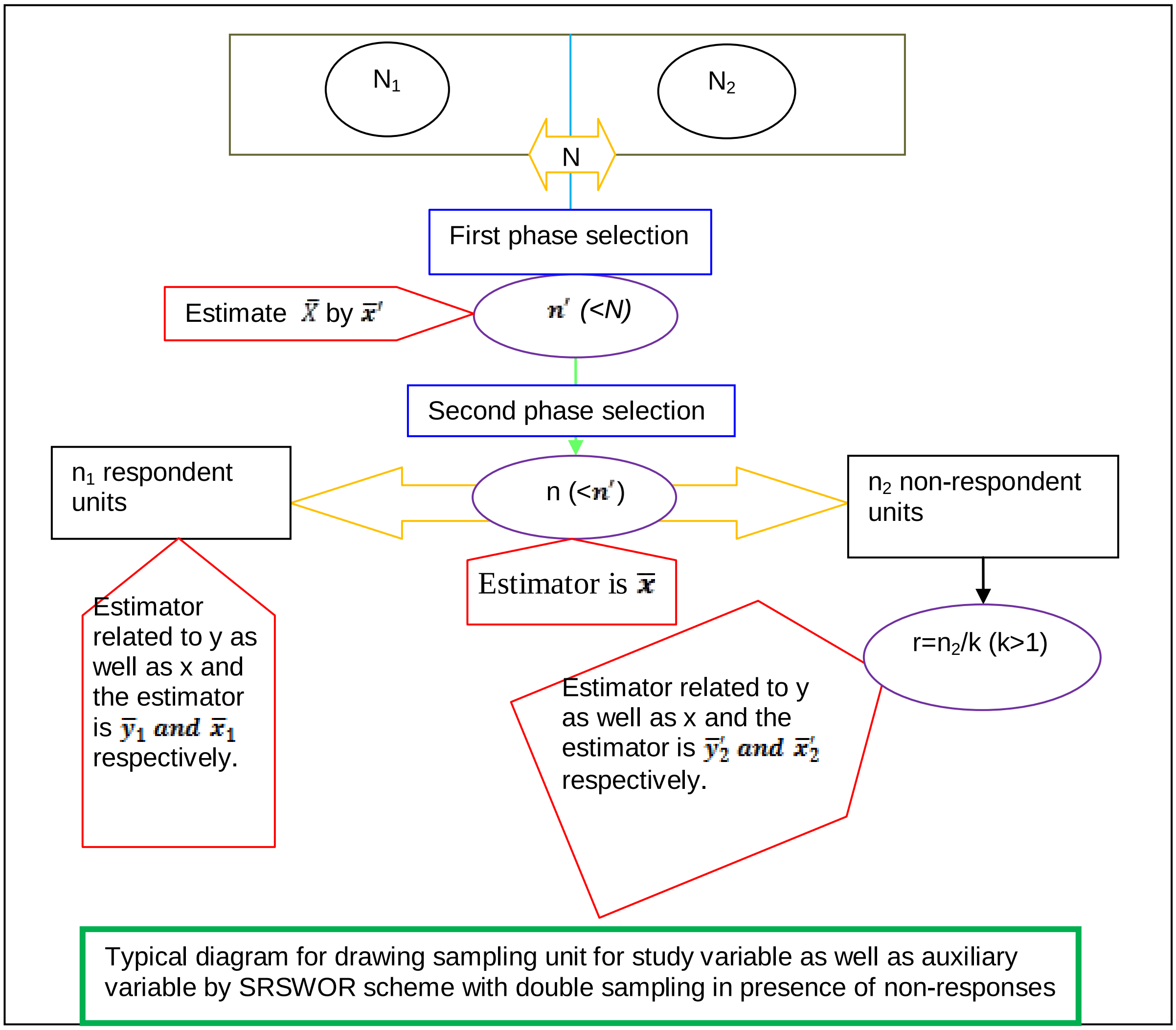
I. Introduction

We know that auxiliary information in study of sample survey gives us an efficient estimate of population parameters like as population mean or total, under some crucial conditions. This information may be used for drawing a random sample using SRSWOR / SRSWR, to stratification, systematic or probability proportional to size sampling strategy or for estimating the population parameter or at both purposes. Auxiliary information gives us a variety of techniques by means of ratio, product, regression and other methods. Dalenius (1934) states that "As demonstrated by the developments in the last half-century, supplementary information may be exploited for all aspects of the sample design, the definition of sampling units, the selection design and the estimation method." In practical situation almost all surveys suffer from non-response. Hansen and Hurwitz (1946), assumed that a sub-sample of initial non-respondents is recontacted with a more expensive method, suggesting the first attempt by mail questionnaire and the second attempt by a personal interview. Sodipo and Obisesan (2007) have considered the problem of estimating the population mean in the presence of non-response, in sample survey with full response of an auxiliary character x. Other authors such as Cochran (1977), Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997), Khare and Rehman (2014), Okafor and Lee (2000) and Tabasum and Khan (2004, 2006) and Singh and Kumar (2008a,b) have studied the problem of non-response under double (two-stage) sampling.

In the present paper, we have to proposed combination of two exponential ratio type estimators for estimating population mean using auxiliary variable with double sampling in presence of non-response. We have obtained the expressions for bias and mean square errors of the proposed estimators for the fixed value of n' and n, also for the optimum values of the constants. An illustration of the proposed estimators has been made with the relevant class of estimators.

II. SAMPLING SCHEME

The double sampling in presence of non-response sampling scheme is that; let a population have N units and the population is divided in two parts say N_1 of responding units and N_2 of non-responding units. Here we first estimate population mean of auxiliary variable(x) is in the first stage sample of size n' (<N) from population of size N by using SRSWOR scheme and estimate population mean $\bar{X}$ by $\bar{x}'$, which is the mean of the values of x on the first phase sample. Now we draw a second stage sample of size n is selected from n' by SRSWOR scheme and on the second stage sample of size n in which n_1 units respond and n_2 units non-respondent for study variable (y) and as well as auxiliary variable (x). From the n_2 non-respondent units a sample of size $r = n_2/k$; $k > 1$ units is draw by SRSWOR scheme, where k is the inverse sampling rate at the second stage sample of size n. All the r units respond at this time now.



III. NOTATION AND TERMINOLOGY

In this paper we used sufficient number of notations and terms, they are defined as follows ($\bar{y}_1, \bar{y}_2'$) be the sample mean based on n_1 and r units of study variable and ($\bar{x}_1, \bar{x}_2', \bar{x}', \bar{x}$) be the sample mean of auxiliary variable based on n_1, r, n', n units respectively. $\bar{Y}$ and $\bar{X}$ be the population mean of study variable and auxiliary variable based on population size $N=N_1+N_2$. Also $\bar{Y}_2$, $\bar{X}_2$ be the population mean of study variable and auxiliary variable based on population size N_2 (non-response part).

$$S_y^2 = \sum_{i=1}^{N}(y_i-\bar{Y})^2 \Big/ (N-1),\ S_x^2 = \sum_{i=1}^{N}(x_i-\bar{X})^2 \Big/ (N-1),\ S_{y2}^2 = \sum_{i=1}^{N_2}(y_i-\bar{Y}_2)^2 \Big/ (N_2-1),$$

$$S_{x2}^2 = \sum_{i=1}^{N_2}(x_i-\bar{X}_2)^2 \Big/ (N_2-1),\ C_y^2 = S_y^2/\bar{Y}^2,\quad C_{y2}^2 = S_{y2}^2/\bar{Y}^2,\quad C_x^2 = S_x^2/\bar{X}^2,$$

$$C_{x2}^2 = S_{x2}^2/\bar{X}^2$$

$$S_{yx} = \frac{\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})}{N-1},\ S_{yx2} = \frac{\sum_{i=1}^{N_2}(y_i-\bar{Y})(x_i-\bar{X})}{N_2-1},\ \rho_{yx} = \frac{S_{yx}}{S_y S_x},\ \rho_{yx2} = \frac{S_{yx2}}{S_{y2} S_{x2}},\ \beta_{yx} = \frac{S_{yx}}{S_x^2},$$

$\beta_{yx2} = \frac{S_{yx2}}{S_{x2}^2}, K_{yx} = \frac{\rho_{yx} C_y}{C_x}, K_{yx2} = \frac{\rho_{yx2} C_{y2}}{C_{x2}}, b^* = \frac{\hat{S}_{yx}}{\hat{S}_x^2}, b^{**} = \frac{\hat{S}_{yx}}{s_x^2}, s_x^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}, C_{yx} = \frac{S_{yx}}{\bar{X}\bar{Y}},$

$C_{yx2} = \frac{S_{yx2}}{\bar{X}\bar{Y}}$, $\hat{S}_{yx}$ and $\hat{S}_x^2$ are estimates of S_{yx} *and* S_x^2 respectively based on n_1+r units. ρ_{yx} and ρ_{yx2} are respectively the correlation coefficient of response and non-response group between study variable y and auxiliary variable x.

$w_1 = n_1/n$, $w_2 = n_2/n$, $f = n/N$, $W_2=N_2/N$, $\lambda = (1-f)/n$, $\lambda' = (1-f')/n'$, $\lambda^* = \frac{W_2(k-1)}{n}$, $f' = n'/N$. We also use the constant a.

IV. SOME WELL KNEW ESTIMATORS AND ITS MSE

A usual unbiased estimator for the population mean $\bar{Y}$ of the study variable y, proposed by Hansen and Hurwitz (1946), is defined by

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_2'$$

The variance of $\bar{y}^*$ is given by

$$Var(\bar{y}^*) = \bar{Y}^2(\lambda C_y^2 + \lambda^* C_{y2}^2)$$

It is well known that in estimating the population mean, sample survey experts sometimes use auxiliary information to improve the precision of the estimates. Let x denote an auxiliary variable with population mean $\bar{X}$. The Hansen and Hurwitz (1946) estimator is

$$\bar{x}^* = w_1 \bar{x}_1 + w_2 \bar{x}_2'$$

The variance of $\bar{x}^*$ is given by

$$Var(\bar{x}^*) = \bar{X}^2(\lambda C_x^2 + \lambda^* C_{x2}^2)$$

Khare and Srivastava (1993), Tabasum and Khan's (2004) defined ratio estimator in presence of nonresponse $T_1 = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}$ and its MSE is

$$MSE(T_1) = \bar{Y}^2[(\lambda - \lambda')\{C_y^2 + (1-2K_{yx})C_x^2\} + \lambda^*\{C_{y2}^2 + (1-2K_{yx2})C_{x2}^2\} + \lambda' C_y^2)]$$

Khare and Srivastava (1993), Tabasum and Khan's (2006) defined a ratio type estimator in presence of non-response as $T_2 = \bar{y}^* \frac{\bar{x}'}{\bar{x}}$ and its MSE is

$$MSE(T_2) = \bar{Y}^2[(\lambda - \lambda')\{C_y^2 + (1-2K_{yx})C_x^2\} + \lambda^* C_{y2}^2 + \lambda' C_y^2)]$$

Singh and Kumar's (2008a) defined a ratio type estimator in presence of non-response as

$$T_3 = \bar{y}^* \left(\frac{\bar{x}'}{\bar{x}^*}\right)\left(\frac{\bar{x}'}{\bar{x}}\right)$$

and its MSE is

$$MSE(T_3) = \bar{Y}^2[(\lambda - \lambda')\{C_y^2 + 4(1-K_{yx})C_x^2\} + \lambda^*\{C_{y2}^2 + (1-2K_{yx2})C_{x2}^2\} + \lambda' C_y^2)]$$

Singh and Kumar's (2008a) defined a product type estimator in presence of non-response as

$$T_4 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}'}\right)\left(\frac{\bar{x}}{\bar{x}'}\right)$$

and its MSE is

$$MSE(T_4) = \bar{Y}^2[(\lambda - \lambda')\{C_y^2 + 4(1-K_{yx})C_x^2\} + \lambda^*\{C_{y2}^2 + (1+2K_{yx2})C_{x2}^2\} + \lambda' C_y^2)]$$

Singh and Ruiz Espejo (2007) defined an estimator in presence of nonresponse as $T_5 = \bar{y}^* \left\{ b \frac{\bar{x}'}{\bar{x}^*} + (1-b) \frac{\bar{x}^*}{\bar{x}'} \right\}$

where b is any suitably chosen constant and its MSE is

$$D = \{ \lambda' S_x^2 + \lambda^* S_{x2}^2 \}, \qquad D^* = \{ (\lambda - \lambda') K_{yx} S_x^2 + \lambda^* K_{yx2} S_{x2}^2 \}$$

$$MSE(T_5) = \left[\lambda' C_y^2 + (\lambda - \lambda') \left\{ S_y^2 + \frac{D^*}{D} \left(\frac{D^*}{D} - 2\beta_{yx} \right) S_x^2 \right\} + \lambda^* \left\{ S_{y2}^2 + \frac{D^*}{D} \left(\frac{D^*}{D} - 2\beta_{yx2} \right) S_{x2}^2 \right\} \right]$$

Khare and Srivastava (1995), defined a regression type estimator in presence of non-response
$T_6 = \bar{y}^* + b^{**} (\bar{x}' - \bar{x})$
and its MSE is

$$MSE(T_6) = Var(\bar{y}^*) - \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right) \rho_{yx}^2 C_y^2$$

Khare and Rehman (2014) defined generalized ratio in regression type estimators of non-response as
$T_7 = \bar{y}^* \left(\frac{\bar{x}'}{\bar{x}^*} \right)^{\alpha} + b^* (\bar{x}' - \bar{x}^*)$
and its MSE is

$$MSE(T_7) = Var(\bar{y}^*) + \left(\frac{1}{n} - \frac{1}{n'} \right) \{ \bar{Y}^2 \alpha^2 C_x^2 + \bar{X}^2 B^2 C_x^2 - 2\bar{Y}^2 \alpha C_{yx} - 2\bar{X}\bar{Y} B C_{yx} + 2\bar{X}\bar{Y} B \alpha C_x^2 \} +$$
$$\lambda^* \{ \bar{Y}^2 \alpha^2 C_{x2}^2 + \bar{X}^2 B^2 C_{x2}^2 - 2\bar{Y}^2 \alpha C_{yx2} - 2\bar{X}\bar{Y} B C_{yx2} + 2\bar{X}\bar{Y} B \alpha C_{x2}^2 \}$$
where
$$\alpha^{opt} = \left[\left(\frac{1}{n} - \frac{1}{n'} \right) (\bar{Y} C_{yx} - \bar{X} B C_x^2) + \lambda^* (\bar{Y} C_{yx2} - \bar{X} B C_{x2}^2) \right] \Big/ \left[\left(\frac{1}{n} - \frac{1}{n'} \right) \bar{X} C_x^2 + \lambda^* \bar{Y} C_{x2}^2 \right] \text{ and } B = \frac{\rho_{yx} S_y}{S_x}$$

V. PROPOSED ESTIMATOR WITH BIAS AND MSE

In the given sampling scheme we have to proposed combination of two exponential ratio type estimators for estimating population mean using auxiliary variables with double sampling in the presence of non-response, which is given as follows:

$$\mathrm{T_P} = \bar{y}^* \left\{ \alpha \exp\left(\frac{\bar{x}'}{\bar{x}^*} - 1 \right) + (1-\alpha) \exp\left(\frac{\bar{x}'}{\bar{\bar{x}}} - 1 \right) \right\} \qquad (1)$$

where α is constant.

To obtain the bias and variance of the estimator T_P, we write

$\bar{y}^* = \bar{Y}(1+\varepsilon_0), \bar{x}^* = \bar{X}(1+\varepsilon_1), \bar{x}' = \bar{X}(1+\varepsilon_1'), \bar{x} = \bar{X}(1+\varepsilon_2)$
such that,

$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_1') = E(\varepsilon_2) = 0$ and

$$E(\varepsilon_0^2) = \lambda C_y^2 + \lambda^* C_{y2}^2, E(\varepsilon_1^2) = \lambda C_x^2 + \lambda^* C_{x2}^2, E\left(\varepsilon_1'^2\right) = \lambda' C_x^2, E(\varepsilon_2^2) = \lambda C_x^2,$$

$$E(\varepsilon_0\varepsilon_1) = \lambda \rho_{yx} C_y C_x + \lambda^* \rho_{yx2} C_{y2} C_{x2}, E(\varepsilon_0\varepsilon_2) = \lambda \rho_{yx} C_y C_x, E(\varepsilon_1\varepsilon_1') = \lambda' C_x^2, E(\varepsilon_1\varepsilon_2) = \lambda C_x^2,$$
$$E(\varepsilon_2\varepsilon_1') = \lambda' C_x^2, E(\varepsilon_0\varepsilon_1') = \lambda' \rho_{yx} C_y C_x$$

So the estimator T_P can be expressed in terms of ε's as follows

$$T_P = \bar{Y}(1+\varepsilon_0)[\alpha\, exp\{(\varepsilon_1' - \varepsilon_1)(1+\varepsilon_1)^{-1}\} + (1-\alpha)\, exp\{(\varepsilon_1' - \varepsilon_2)(1+\varepsilon_2)^{-1}\}] \qquad (2)$$

If we assume that $|\varepsilon_0| < 1,\ |\varepsilon_1'| < 1, |\varepsilon_2| < 1$ then the right hand side of (2) is expandable. Now, expanding the right hand side of (2) to the second degree of approximation, we have

$$T_P = \bar{Y}(1+\varepsilon_0)[\alpha\, exp\{(\varepsilon_1' - \varepsilon_1)(1-\varepsilon_1+\varepsilon_1^2)\} + (1-\alpha)\, exp\{(\varepsilon_1' - \varepsilon_2)(1-\varepsilon_2+\varepsilon_2^2)\}]$$

$$T_P - \bar{Y} = \bar{Y}(1+\varepsilon_0)[\alpha\, exp(\varepsilon_1' - \varepsilon_1 - \varepsilon_1\varepsilon_1' + \varepsilon_1^2) + (1-\alpha) exp(\varepsilon_1' - \varepsilon_2 - \varepsilon_1'\varepsilon_2 + \varepsilon_2^2)] - \bar{Y}$$

$$T_P - \bar{Y} = \bar{Y}(1+\varepsilon_0)\begin{bmatrix} \alpha\left(1+\varepsilon_1' - \varepsilon_1 - \varepsilon_1\varepsilon_1' + \varepsilon_1^2 + \frac{\varepsilon_1^2 + \varepsilon_1'^2 - 2\varepsilon_1\varepsilon_1'}{2}\right) + \\ (1-\alpha)\left(1+\varepsilon_1' - \varepsilon_2 - \varepsilon_2\varepsilon_1' + \varepsilon_2^2 + \frac{\varepsilon_2^2 + \varepsilon_1'^2 - 2\varepsilon_2\varepsilon_1'}{2}\right)\end{bmatrix} - \bar{Y}$$

$$T_P - \bar{Y} = \bar{Y}(1+\varepsilon_0)\begin{bmatrix} \alpha\left(1+\varepsilon_1' - \varepsilon_1 - 2\varepsilon_1\varepsilon_1' + \frac{3\varepsilon_1^2}{2} + \frac{\varepsilon_1'^2}{2}\right) + \\ (1-\alpha)\left(1+\varepsilon_1' - \varepsilon_2 - 2\varepsilon_2\varepsilon_1' + \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_1'^2}{2}\right)\end{bmatrix} - \bar{Y}$$

$$T_P - \bar{Y} = \bar{Y}\begin{bmatrix} \alpha\left(1+\varepsilon_0+\varepsilon_1' + \varepsilon_0\varepsilon_1' - \varepsilon_1 - \varepsilon_0\varepsilon_1 - 2\varepsilon_1\varepsilon_1' + \frac{3\varepsilon_1^2}{2} + \frac{\varepsilon_1'^2}{2}\right) + \\ (1-\alpha)\left(1+\varepsilon_0+\varepsilon_1' + \varepsilon_0\varepsilon_1' - \varepsilon_2 - \varepsilon_0\varepsilon_2 - 2\varepsilon_2\varepsilon_1' + \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_1'^2}{2}\right)\end{bmatrix} - \bar{Y}$$

$$T_P - \bar{Y} = \bar{Y}\begin{bmatrix} \alpha\begin{pmatrix}\varepsilon_0+\varepsilon_1' + \varepsilon_0\varepsilon_1' - \varepsilon_1 - \varepsilon_0\varepsilon_1 - 2\varepsilon_1\varepsilon_1' + \frac{3\varepsilon_1^2}{2} + \frac{\varepsilon_1'^2}{2} - \\ \varepsilon_0 - \varepsilon_1' - \varepsilon_0\varepsilon_1' + \varepsilon_2 + \varepsilon_0\varepsilon_2 + 2\varepsilon_2\varepsilon_1' - \frac{3\varepsilon_2^2}{2} - \frac{\varepsilon_1'^2}{2}\end{pmatrix} + \\ \left(\varepsilon_0+\varepsilon_1' + \varepsilon_0\varepsilon_1' - \varepsilon_2 - \varepsilon_0\varepsilon_2 - 2\varepsilon_2\varepsilon_1' + \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_1'^2}{2}\right)\end{bmatrix}$$

$$T_P - \bar{Y} = \bar{Y}\begin{bmatrix} \alpha\left(\varepsilon_2 - \varepsilon_1 - \varepsilon_0\varepsilon_1 - 2\varepsilon_1\varepsilon_1' + \varepsilon_0\varepsilon_2 + 2\varepsilon_2\varepsilon_1' - \frac{3\varepsilon_2^2}{2} + \frac{3\varepsilon_1^2}{2}\right) + \\ \left(\varepsilon_0+\varepsilon_1' + \varepsilon_0\varepsilon_1' - \varepsilon_2 - \varepsilon_0\varepsilon_2 - 2\varepsilon_2\varepsilon_1' + \frac{3\varepsilon_2^2}{2} + \frac{\varepsilon_1'^2}{2}\right)\end{bmatrix} \qquad (3)$$

Taking expectation on both side of (3), we get the bias of T_P to the first degree of approximation is given by

$$B(T_P) = \bar{Y}\begin{bmatrix} \alpha\left(-\lambda\rho_{yx}C_yC_x - \lambda^*\rho_{yx2}C_{y2}C_{x2} - 2\lambda' C_x^2 + \lambda\rho_{yx}C_yC_x + 2\lambda' C_x^2 - \frac{3\lambda C_x^2}{2} + \frac{3(\lambda C_x^2 + \lambda' C_{x2}^2)}{2}\right) + \\ \left(\lambda'\rho_{yx}C_yS_x - \lambda\rho_{yx}C_yS_x - 2\lambda' C_x^2 + \frac{3\lambda C_x^2}{2} + \frac{\lambda' C_x^2}{2}\right)\end{bmatrix}$$

$$B(T_P) = \bar{Y}\left[\alpha\left(-\lambda^*\rho_{yx2}C_{y2}C_{x2} + \frac{3\lambda' C_{x2}^2}{2}\right) + \left(\frac{3C_x^2}{2}(\lambda - \lambda') - \rho_{yx}C_yS_x(\lambda - \lambda')\right)\right] \qquad (4)$$

So our estimator T_P is approximately unbiased if the value of the constant is

$$\alpha = \left[\frac{\left(\frac{3C_x^2}{2}(\lambda - \lambda') - \rho_{yx}C_yS_x(\lambda - \lambda')\right)}{\left(\lambda^*\rho_{yx2}C_{y2}C_{x2} - \frac{3\lambda' C_{x2}^2}{2}\right)}\right]$$

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Rewrite (3) we have

$$T_P - \bar{Y} = \bar{Y}\left[\alpha(\varepsilon_2 - \varepsilon_1) + \left(\varepsilon_0 + \varepsilon_1' - \varepsilon_2\right)\right] \qquad (5)$$

Squaring both sides of (5) and neglecting terms of $\varepsilon's$ involving power greater than two, we have

$$(T_P - \bar{Y})^2 = \bar{Y}^2\left[\alpha(\varepsilon_2 - \varepsilon_1) + \left(\varepsilon_0 + \varepsilon_1' - \varepsilon_2\right)\right]^2$$

$$(T_P - \bar{Y})^2 = \bar{Y}^2\left[\alpha^2(\varepsilon_2 - \varepsilon_1)^2 + \left(\varepsilon_0 + \varepsilon_1' - \varepsilon_2\right)^2 + 2\alpha(\varepsilon_2 - \varepsilon_1)\left(\varepsilon_0 + \varepsilon_1' - \varepsilon_2\right)\right]$$

$$(T_P - \bar{Y})^2 = \bar{Y}^2\begin{bmatrix}\alpha^2({\varepsilon_2}^2 + {\varepsilon_1}^2 - 2\varepsilon_1\varepsilon_2) + \left({\varepsilon_0}^2 + {\varepsilon_1'}^2 + {\varepsilon_2}^2 + 2\varepsilon_0\varepsilon_1' - 2\varepsilon_0\varepsilon_2 - 2\varepsilon_1'\varepsilon_2\right) + \\ 2\alpha(\varepsilon_0\varepsilon_2 - \varepsilon_0\varepsilon_1 + \varepsilon_1'\varepsilon_2 - \varepsilon_1\varepsilon_1' - {\varepsilon_2}^2 + \varepsilon_1\varepsilon_2)\end{bmatrix} \qquad (6)$$

Taking expectation on both sides on (6), we get the MSE of the estimator T_P to the first degree of approximation, we get

$$MSE(T_P) = \bar{Y}^2\begin{bmatrix}\alpha^2\left(\lambda C_x^2 + \lambda C_x^2 + \lambda^* C_{x2}^2 - 2\lambda C_x^2\right) + \\ \left(\lambda C_y^2 + \lambda^* C_{y2}^2 + \lambda' C_x^2 + \lambda C_x^2 + 2\lambda'\rho_{yx}C_yC_x - 2\lambda\rho_{yx}C_yC_x - 2\lambda' C_x^2\right) + \\ 2\alpha\left(\lambda\rho_{yx}C_yC_x - \lambda\rho_{yx}C_yC_x - \lambda^*\rho_{yx2}C_{y2}C_{x2} + \lambda' C_x^2 - \lambda' C_x^2 - \lambda C_x^2 + \lambda C_x^2\right)\end{bmatrix}$$

$$MSE(T_P) = \bar{Y}^2\begin{bmatrix}\alpha^2\left(\lambda^* C_{x2}^2\right) + \left(\lambda C_y^2 + \lambda^* C_{y2}^2 + (\lambda - \lambda')C_x^2 - 2(\lambda - \lambda')\rho_{yx}C_yC_x\right) - \\ 2\alpha\left(\lambda^*\rho_{yx2}C_{y2}C_{x2}\right)\end{bmatrix} \qquad (7)$$

The MSE (7) is minimized for

$$\alpha = \frac{\left(\lambda^*\rho_{yx2}C_{y2}C_{x2}\right)}{\lambda^* C_{x2}^2}$$

Hence the optimal value of $\boldsymbol{\alpha}$ is

$$\alpha^{opt} = \frac{\left(\rho_{yx2}C_{y2}\right)}{C_{x2}}$$

The optimal variance is

$$MSE(T_P)^{opt} = \bar{Y}^2\left[\left(\lambda C_y^2 + \lambda^* C_{y2}^2 + (\lambda - \lambda')C_x^2 - 2(\lambda - \lambda')\rho_{yx}C_yC_x\right) - \lambda^*\rho_{yx2}^2C_{y2}^2\right]$$

VI. EMPERICAL STUDY

To illustrate for the proposed results we considered the data earlier consider by Khare and Sinha(2009), Khare and Rehman (2014). The description of the population is given below:

Here we study 96 village wise population of rural area under Police-station – Singur, District -Hooghly, West Bengal from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labors in the village is taken as study character (y) while the area (in hectares) of the village is taken as auxiliary variable x. The values of the different parameters of the population are given below:

Table-1: Calculation for different term.

Term	Value	Term	Value	Term	Value	Term	Value
$\bar{Y}$	137.9271	C_{y2}^2	4.34246	S_{yx2}	28362.05463	f'	0.729167
$\bar{Y}^2$	19023.88	C_{x2}^2	0.885105	C_{yx2}	1.419396	λ	0.014583
$\bar{X}$	144.872	ρ_{yx}	0.773	K_{yx2}	1.603649	λ'	0.003869
$\bar{X}^2$	20987.9	ρ_{yx}^2	0.597529	N	96	$\lambda - \lambda'$	0.010714
S_y^2	33306.69	S_{yx}	16585.1	n	40	B	1.19998
C_y^2	1.750783	C_{yx}	0.830012	n'	70	B^2	1.43994
S_x^2	13821.21	K_{yx}	1.260396	$\frac{1}{n} - \frac{1}{n'}$	0.010714	β_{yx}	1.19997
C_x^2	0.658532	ρ_{yx2}	0.724	f	0.416667	β_{yx2}	1.52677
	k=2	k=3	k=4	k=5		S_y	182.5012
λ^*	0.00625	0.0125	0.01875	0.025		S_{y2}	287.4202

D	169.5779727	285.6811	401.7841	517.8871711		C_{y2}.	2.08386
D^*	208.9057936	231.167	253.42746	275.6882877		C_x	0.8115
$\frac{D^*}{D}$	1.231915857	0.80918	0.630756	0.532332722		S_x	117.5636
a	0.146709	0.20556	0.237288	0.257133		C_{x2}	0.9408
a^2	0.021524	0.042255	0.056306	0.066117		S_{x2}	136.2956

Table-2: Relative efficiency of the estimators (in %) and MSE with respect to $\bar{y}^*$ for fixed values of n, n' and different values of k (N =96, n'= 70 and n =40).

MSE					Relative efficiency(%)				
Estimator	k=2	k=3	k=4	k=5	Estimator	k=2	k=3	k=4	k=5
$\bar{y}^*$	1002.0	1518.4	2034.7	2551.0	$\bar{y}^*$	100	100	100	100
T_1	565.6	849.6	1133.7	1417.7	T_1	177	179	179	180
T_2	797.9	1314.2	1830.5	2346.9	T_2	126	116	111	109
T_3	629.9	914.0	1198.0	1482.0	T_3	159	166	170	172
T_4	1305.0	2264.1	3223.2	4182.3	T_4	77	67	63	61
T_5	399.6	777.2	1208.3	1651.6	T_5	251	195	168	154
T_6	788.8	1305.1	1821.4	2337.8	T_6	127	116	112	109
T_7	568.3	824.9	1069.4	1308.7	T_7	176	184	190	195
T_p	527.3	772.9	1018.6	1264.3	T_p	190	196	200	202

The figure-1 and table-2 shows that

a) If k is increase then the relative efficiency (%) of the estimatorsT_2, T_4, T_5, T_6 decrease.
b) If k is increase then the relative efficiency (%) of the estimatorsT_1, T_3, T_7, T_p increase.
c) We see that proposed estimator is more efficient than the other estimators.
d) Also we see that at k=2 the estimator T_5 is more efficient than the proposed estimator T_p, but after k>2 i.e., increasing of k the proposed estimator goes to more efficient than the T_5.

Therefore the proposed estimator should be preferred for the estimation of population mean using auxiliary variable with double sampling in presence of non-response.

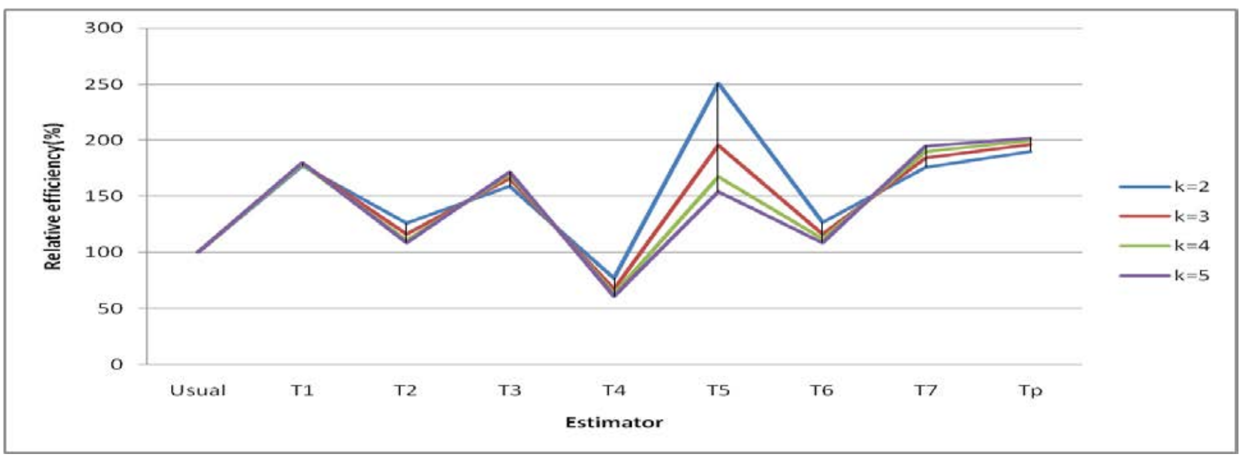

Figure-1: Relative efficiency (%) with respect to different estimator.

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