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An Introduction to Compressive Sensing and its Applications

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Abstract- Compressed sensing or compressive sensing or CS is a new data acquisition protocol that has been an active research area for nearly a decade. It samples the signal of interest at a rate much below the Shannon nyquist rate and has led to better results in many cases as compared to the traditional Shannon – nyquist sampling theory. This paper surveys the theory of Compressive sensing and its applications in various fields of interest.

Index Terms- Compressive sensing, Compressive sampling, applications of CS, data acquisition

I. Introduction

Compressed sensing involves recovering the speech signal from far less samples than the nyquist rate. Also, as this is a sparse signal recovery algorithm, we can recover the signal which is sparse in nature in presence of noise which is non-sparse. Recently, compressive sensing or compressed sensing (will be referred as CS henceforth) has been an active research area in the field of signal processing and communication. It has been applied to Wireless sensor networks, video processing and image processing and up to some extent on speech signal processing also.

Conventionally, the signal of interest is sampled at nyquist rate and a large part of these samples is eliminated during the compression stage. This leads to unnecessary hardware and software load. Compressed sensing, as the name suggest, samples the signal in a compressed format i.e. it uses very less number of distinct samples of the target signal and is then recovered by using various recovery algorithms. As a result, less number of samples are handled, which leads to reduction in power consumption as well as a reduced load on hardware as well as software

The signal of interest is sampled by taking small number of linear random projections of the signal which contain most of the vital information about the signal. It basically relies on two major assumptions about the signal i.e. Sparsity and Incoherence[3]. Sparsity depends upon the signal of interest and incoherence depends upon the sensing modality. Sparsity means that the amount of information present in the signal is much less than the total bandwidth acquired by the signal. Most of the natural signals are sparse in nature. On the other hand, incoherence means that, signals that can be represented sparsely should be spread out in the domain in which they are acquired.

This paper is organized as follows. Section 2 provides an overview of CS, section 3 provides an idea on recovery algorithm implementation, section 4 describes varios applications of CS and section 5 summarizes the findings.

II. Background

A. *Overview of Compressed Sensing*

The theory of CS was developed by Candes [1] and Donoho [2] in 2004. It involves taking random projections of the signal and recovering it from a small number of measurements using optimization techniques. In a traditional sampling theorem, the signal is sampled using Nyquist rate, whereas with the help of compressive sensing the signal is sampled below the Nyquist rate.

This is possible because the signal is transformed into a domain in which it has a sparse representation. Then the signal is reconstructed from the samples using one of the different optimization techniques available. Figure one shows a block diagram which illustrates the difference in the traditional method of signal acquisition and the CS approach.

It is clear from figure 1 that, traditionally, the signal is sensed, sampled at a nyquist criteria, then, the samples are saved and then compressed where a large amount of samples are discarded. In contrast to all these steps, CS senses the signal in an already compressed format. Hence, a lot of hardware as well as software load is reduced.

For understanding the concept of compressed sensing, we will go through following set of definitions and formulae:

i. **Sparse Signal:** A signal is called sparse in nature if it has only a few significant (large in magnitude) components and a greater number of insignificant (close to zero) components.

ii. **Compressible Signal:** A signal is said to be compressible if it is sparse in nature.

iii. $s = \Psi x$ where, s = Signal to be acquired
Ψ = Sparsifying matrix
x = Real valued Column vector

iv. $y = \Phi s = \Phi\Psi x = Ax = A_k . x_k$ where, y = Compressed Samples
Φ = Sensing Matrix

v. The Solution to above equations is:

$$X_k = (A_k^T A_k)^{-1} A_k^T y$$

vi. Above is an underdetermined problem i.e. projection of an n-dimensional vector into an M dimensional space i.e. Number of equations < Number of Unknowns

vii. To Solve this kind of problems, we use the concept of Norms. Norms are nothing but, they assign strictly positive length to vectors in a vector space. Norms are of following types:

a. L_0 Norm: It simply counts the number of non-zero components in a vector
b. L_1 Norm: It is given by the following equation:

$$\|\mathbf{x}\|_1 = \sum_{i=1}^{N} |x_i|$$

c. L_2 Norm: It is given by following equation:

$$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{N} |x_i|^2 \right)^{\frac{1}{2}}$$

viii. **Designing a Sensing Matrix:** Following conditions need to be strictly satisfied while designing a sensing matrix so that, the signal is recovered faithfully:
a. Universal Incoherence condition: It means, that, the value of cross correlation between two column vectors of a sensing matrix must be minimum.
b. Data Independence: The construction of a random matrix does not depend upon any prior knowledge of data.
c. Robustness: Transmission of randomly projected coefficients is robust to packet loss in the network.

ix. **Incoherence condition:** The sensing matrix should be as different from the sparsifying basis. Time and frequency basis are maximally incoherent. Following equation signifies the incoherence condition:

$\mu < 1/(2K-1)$

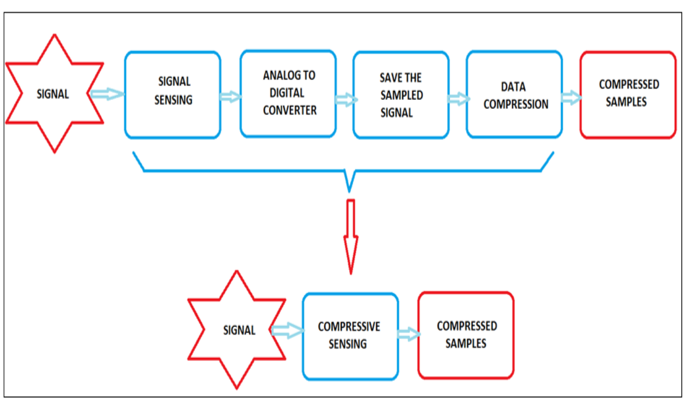


Fig. 1: Traditional sensing Vs. compressed sensing

III. Recovery Algorithms

There are basic two types of approaches as follows:

1. Greedy Type – Orthogonal Matching Pursuit

OMP is a greedy-type algorithm because it selects the one index regarded as the optimal decision at each iteration. Thus, its performance is dominated by its ability to find the sparse set exactly. If the sparse set is not correctly reconstructed, OMP's solution could be wrong. It is mostly used when the number of common components is more.

2. Gradient Type – Primal Dual Interior Point

This algorithm is nothing but making some changes into the L_1 norm and is mainly used when the innovation components are more.

IV. Applications of CS

In this section, we see applications of CS to various areas of signal processing that have been done up till now. We will also see few results based on these applications.

1. Wireless Sensor Networks

Wireless sensor networks are usually placed in field e.g. seismic sensors, fire, temperature and humidity detectors in forest etc. These sensors are usually battery operated and cannot be easily replaced. Hence, an efficient data acquisition system is needed in order to optimize the data transferred from these sensors as well as minimize the computational complexity of these sensors in order to increase their battery life. Compressed sensing can very well fit into such situations as it samples the signal of interest at a very low rate than the nyquist criteria and as a result it has an effective computational capacity.

Wireless sensors collect their individual data and send this data from the sensor node to the collaboration location, from which they are transmitted through wireless channel to the fusion center. Conventionally, the intra-sensor correlation takes place at the sensor node and the inter sensor correlation takes place at the collaboration location. When we apply CS to WSN [14], the intra sensor correlation takes place at sensor node, the output is directly transmitted over wireless channel and the intersensor correlation takes place at the fusion center which is rich in resources. This is possible because, the data that is transmitted using CS is intelligent and can be sent using very less number of bits as compared to traditional method.

The advantages of applying CS to WSN are listed as follows.
i. CS can be used to save transmittal and computational power significantly at the sensor node.
ii. This CS based signal acquisition and compression scheme is very simple, so it is suitable for inexpensive sensors.
iii. The number of compressed samples required for transmission from each sensor to the FC is significantly small, which makes it perfect for sensors whose operational power is drawn from onboard battery.
iv. The joint CS recovery at the FC exploits signal correlation and enables Distributed Compressive Sensing.

2. Wireless video transmission

Recently there has been an alarming increase in demand for wireless video streaming in applications such as Home entertainment, Home security, Mobile video etc. Hence, the need to provide the required quality of service (QoS) to support video applications is very crucial. This alarming increase in utilization and number of users with different QoS requirements increase the computational complexity and time.

Following are the main concerns for transmitting wireless video signal:

i. Wireless nodes need to send data out in a timely and energy efficient way.
ii. We need to jointly consider perceived video quality, quality variation, power consumption and Transmission delay requirements.

The following fig 2 gives us an idea about how CS can be applied to video signal:

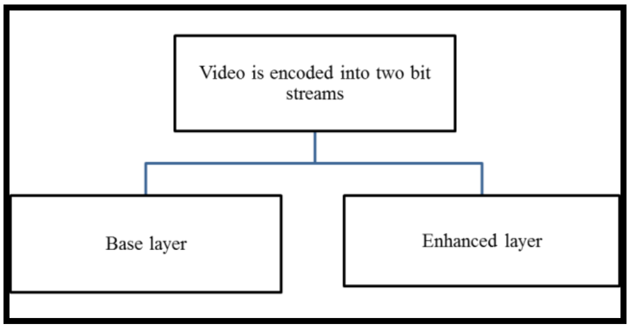


Fig 2 : CS applied to layered architecture of video signal

Base layer – The Discrete cosine transform is used to encode the given video signal in the base layer. As the name suggests, it extracts the necessary information required to describe the given video signal. It consists mainly of the basic information which is required to describe a video signal. It is the necessary information that should be present to represent the video signal; irrespective of anything.

Enhanced layer – Compressed Sensing technology is used to encode the enhanced layer bit stream. It consists mainly of additive features used to enhance the video quality. These measurements can be transmitted depending upon the availability of channel and the required latency and QoS [13].

3. *Speech Signal*

The following figure 3 explains the basic block diagram of compressed sensing applied to speech signal.

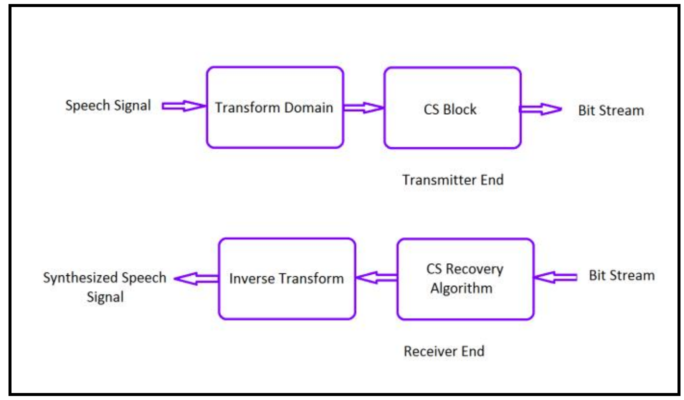
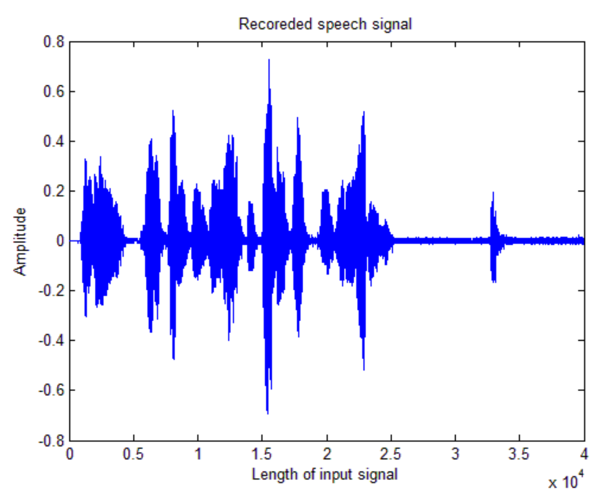


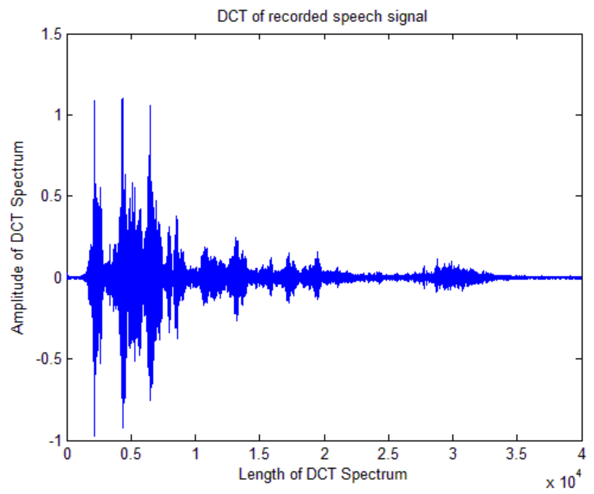
Fig. 3: Block diagram of CS applied to speech signal

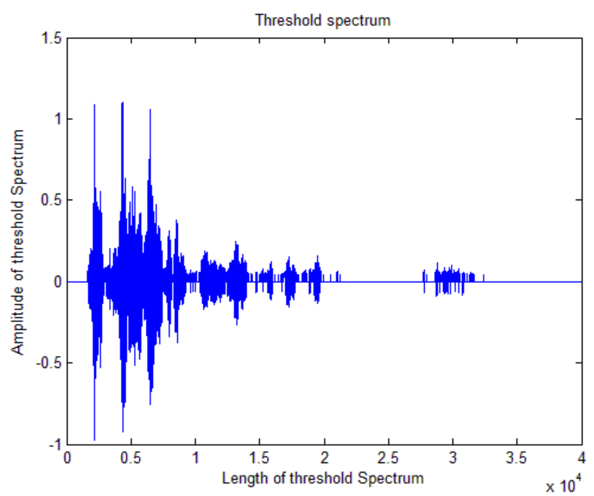
The recorded speech signal is first transformed into a domain in which it is sparse. Here, we have taken the transform domain as Discrete Cosine Transform (DCT). Once we find the DCT of the recorded speech signal, the next step is to design a window function. The function of the window function will be to multiply all the components in that window by zero. This will make the transformed signal sparser. The resulting signal is ready to be sensed compressively. For sensing the signal compressively, we multiply it with a random matrix of size K by N. here, K signifies the level of sparsity and N is the total number of samples in the transformed and windowed function.

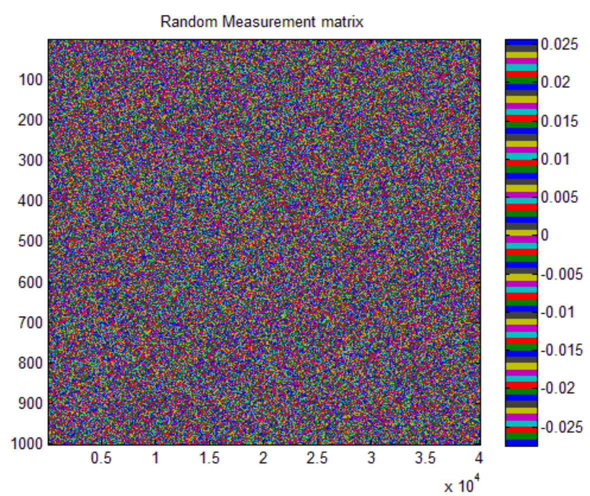
By using random matrix, we make random linear projections on the sparse signal in order to take very few components of the sensed signal. Thus, the signal to be recovered becomes robust for any errors. Hence we now have the compressively sensed signal y.

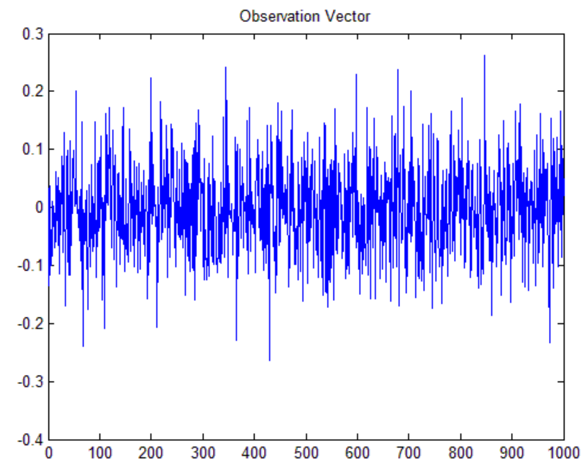
Following figures explain the step by step results for obtaining the compressively sensed signal y:











The above results are by using values of k (sparse number) as 1000 out of total number of samples present i.e. 40000. Hence the degree of compression is huge. We can vary the sparse number and observe the results. The next step is to apply OMP and Basis pursuit algorithm and inverse DCT in order to recover original speech signal.

Compressed sensing can be applied to speech signal by using short time Fourier transform as the transform domain instead of discrete cosine transform [4].

V. CONCLUSION

CS can prove to be a revolutionary technique for signal acquisition and recovery. The key advantages are:

- Fast acquisition of data with fewer samples
- Decreased computational complexity
- Lower transmission power
- Small traffic volume
- Small time delay
- Sampling matrix need not be adaptive to signal
- The desired resolution for recovering the compressively sensed signal can be achieved by manipulating the sparse number K.

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