An Alternative Method to Find the Solution of Zero One Integer Linear Fractional Programming Problem with the Help of θ-Matrix

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Abstract- To minimize the computational effort needed to solve a zero one Integer Linear Fractional programming problem a new approach has been proposed. Here we use matrix for finding the solution of the integer linear fractional programming problems.

Index Terms- one Integer Linear Fractional Programming Problems, matrix and Promising variables

I. INTRODUCTION

Zero-One Integer Linear Fractional Programming Problem is a special case of Integer Linear Fractional Programming Problem. There are various methods used to solve such problems. To reduce the computational effort a new algorithm has been proposed to solve Zero-One Integer Linear Fractional Programming Problem. In this algorithm the decision variables are arranged based on the maximum contribution to the objective function. The arranged variables are then allowed to enter into the basis. This leads to reduce the computational time by means of reducing the iteration. The descriptions of new algorithm have been discussed in this chapter.

II. STRUCTURE OF ZERO-ONE INTEGER LINEAR FRACTIONAL PROGRAMMING PROBLEM

In a Integer Linear Fractional Programming Problem, if all the variables are restricted to take the values either 0 or 1 only, then the given problem is known as Zero-One Integer Linear Fractional Programming Problem.

The general Zero-One Integer Linear Fractional Programming Problems is given by

$$\text{Extremize } Z = \frac{C^T X + c_0}{D^T X + d_0}$$

Subject to

$$AX \leq P^0$$

$$\text{and } X = 0 \text{ or } 1$$

Where
$$A^{mxn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & . & a_{2n} \\ a_{31} & a_{32} & a_{33} & . & . & . & a_{3n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ a_{m1} & a_{m2} & a_{m3} & . & . & . & a_{mn} \end{bmatrix} \quad X^{nx1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \quad P^0 = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ . \\ . \\ b_m \end{bmatrix}$$

Let the columns corresponding to the matrix A be denoted by P^1, P^2, P^3……P^n where

$$P_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ . \\ . \\ . \\ a_{m1} \end{bmatrix} \quad P_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ . \\ . \\ . \\ a_{m2} \end{bmatrix} \quad P_3 = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ . \\ . \\ . \\ a_{m3} \end{bmatrix} \ldots\ldots\ldots\ldots P_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ . \\ . \\ . \\ a_{mn} \end{bmatrix}$$

$C^T = (c_1, c_2, c_3 \ldots\ldots c_n)$, $D^T = (d_1, d_2, d_3 \ldots\ldots d_n)$ and c_0, d_0 are scalars.

III. Zero-One Integer Linear Fractional Programming Algorithm

This algorithm consists of three phases. In first phase promising variables are arranged, second phase arranged variables are allowed to enter into the basis and finally determination of new solution vector to the integer linear fractional programming problem.

The step by step procedure is as given below.

Step 1 : Let iteration = 0

Step 2 : Perform phase I

Step 3 : Perform phase II

Step 4 : If the set J is empty then , Perform phase III

Step 5 : stop.

Phase I - Ordering of Promising variables

Step 1 . Using the intercepts of the decision variables along the respective axes with respect to the chosen basis a matrix is called θ matrix is to be constructed. A typical intercept for the j^{th} variable, x_j due to the i^{th} the resource, b_i is $\left\{\frac{b_i}{a_{ij}}\right\}$ $a_{ij} > 0$

The expanded form of θ matrix is

$$\begin{array}{c} \\ x_1 \\ x_2 \\ \\ x_j \\ \\ x_n \end{array} \begin{array}{c} \begin{array}{ccccccccc} S_1 & S_2 & . & . & S_i & . & . & S_m \end{array} \\ \begin{bmatrix} \frac{b_1}{a_{11}} & \frac{b_2}{a_{21}} & . & . & \frac{b_i}{a_{i1}} & . & . & \frac{b_m}{a_{m1}} \\ \frac{b_1}{a_{12}} & \frac{b_2}{a_{22}} & . & . & \frac{b_i}{a_{i2}} & . & . & \frac{b_m}{a_{m2}} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \frac{b_1}{a_{1j}} & \frac{b_2}{a_{2j}} & . & . & \frac{b_i}{a_{ij}} & . & . & \frac{b_m}{a_{mj}} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \frac{b_1}{a_{1n}} & \frac{b_2}{a_{2n}} & . & . & \frac{b_i}{a_{in}} & . & . & \frac{b_m}{a_{mn}} \end{bmatrix} \end{array}$$

Each row of the θ matrix consists of m number of intercepts of the decision variable along their respective axes and each column consists of intercepts formed by the number of promising decision variables in each of the m constraints.

Step 2 . The minimum intercept and its position in each row of θ matrix is find out. If there are more one minimum intercept then one of them is selected arbitrarily. Multiply the minimum intercept of the variable corresponding to a row with the corresponding contribution coefficient in the objective function both in the numerator and denominator and the objective function value $\left(\frac{c_j x_j + c_0}{d_j x_j + d_0}\right)$ is calculated.

Step 3 . Repeat step2 till the minimum for each row as well as its contribution to the objective function are calculated.

Step 4 . Let λ = 0. J is a set consisting of the subscript of the promising variables.

Step 5 . Select the variable whose $\left(\frac{c_j x_j + c_0}{d_j x_j + d_0}\right)$ value is the largest. If the same largest $\left(\frac{c_j x_j + c_0}{d_j x_j + d_0}\right)$ occurs, for more than one variable then the variable that has maximum contribution including the fractional value is taken as the promising variable.If that is also same then select any one arbitrarily.

Step 6 . Let it be x_R.Then x_R is selected as the promising variable.

Step 7. Increment λ by 1.The subscript of the variable x_R is stored as the l^{th} element in set J.

Step 8. The row corresponding to the variable x_R as well as the other rows whose minimum occurs in the column at which the minimum for x_R occurs are deleted.

Step 9. Step 5 to 8 are repeated till either all the rows or all the columns are deleted.

Step 10. The set of variables collected in Steps 5 to 8 are the ordered promising variables.

Let J= { Subscripts of the promising variables arranged in the descending order $\left(\frac{c_j x_j + c_0}{d_j x_j + d_0}\right)$ value }.

Let λ be the total number of elements in the set J.

Phase II – Arranged variables are allowed to enter into the basis

The arranged promising variables are allowed to enter into the basis one by one based on the entering criteria.The step by step procedure is given below.

Step 1. Let $k = 1$, pos = 0 and X^B is the solution vector and flag(=0) is the flag vector.

$$\text{flag} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}_{n\times 1} ; \qquad P^{0-old} = P^0$$

Step 2. If Iteration = 1 then $P^{0-old} = P^0$

Step 3. Iteration is incremented by 1.

Step 4. The k^{th} element in the set J is selected and let it be j. Then the entering variable is x_j.

Step 5. Assign 1 to the j^{th} element of the vector X^B

Step 6. P^0 vector is modified using the relation

$$P^{0-new} = (P^{0-old})^i - (P^j)^i$$
i = 1,2,3….m

Step 7. Replace P^{0-old} by P^{0-new}

Step 8. If pos = 0 then t = pos = k, else t = pos.

If pos = λ then Perform Phase I

Step 9. t = t + 1

Step 10. Let the t^{th} element in the set J be r

$$\theta_r = \min \left\{ \text{int} \left\{ \frac{(P_{0-old})_i}{(P_j)_i} \right\} ; \quad (P^j)^i > 0 \right\}$$

$$\varepsilon_1 = \left(\frac{c_r \theta_r + c_0}{d_r \theta_r + d_0}\right)$$

Pos = t

Step 11. If $\theta_r \geq 1$ then goto Step 12.

Else if t < λ then goto Step 9.

Step 12. Let the subscripts of the variable corresponding to pos be r and goto Step 5.

Phase III – Determination of new (improved) solution vector to the Zero-One Integer Linear Fractional Programming Problems

Except for the most promising variable in the solution set obtained in phase II the values of remaining variables are set to zero. Taking this as starting solution, phase I and II are performed until improved solution is obtained. If there is no improvement the next promising variable value along with the most promising variable also is retained and the remaining basic variables made to zero. Phase III is repeated until the basic variables list exhausted.

Algorithm

Stage I. The basic variables are arranged according to the descending order of their contribution to the objective function

Step 1. $\lambda = 0$, k =0, n^1 is is the number of basic variables having non zero values in the solution

Step 2. X is the solution vector obtained in phase II.

Step 3. If λ^{th} element in X, ie $X^{\lambda} > 0$, then

Multiply $\left(\frac{c_\lambda x_\lambda + c_0}{d_\lambda x_\lambda + d_0}\right)$, let it be stored as k^{th} row 0^{th} column element of array W and λ is stored as k^{th} row 1^{th} column element of array W. k is incremented by one.

Step 4. λ is incremented by one

Step 5. If $\lambda < n^1$ then goto step 3.

Step 6. The array W is sorted in the descending order based on the 0^{th} column values of W

Stage II. Finding the solution by assigning all the variable values except one in the basis to zero level.

Step 7. k = 0

Step 8. $\lambda = 0$

Step 9. i = 0

Step 10. If i > k then

$J = W^{i1}$

$X^{j} = 0$

Step 11. i is incremented by one

Step 12. If $i < n^1$ then goto step 10

Step 13. Now P^c is the current resource vector or (RHS) and corresponding objective function value Z^1 is calculated.

Stage III. Find the new solution

Step 14. Use phase I and phase II and find the new solution X which is stored as Y(λ) and the corresponding objective function value Z^2 is stored as V (λ).

Step 15. λ is incremented by one

Step 16. If $\lambda < n^1$ then goto step 9

Step 17. Find the largest of V (λ) and its position pos, where ($0 \le \lambda < n^1$), Let it be stored in Z^3.

Step 18. If $Z^3 > Z$, then Replace X by Y (pos)
goto step 1.

else if $k < n^1$ then
increment k by 1.
goto step 8.

IV. Numerical Example

Solve the following Integer linear Programming Problem.

Maximize $Z = \dfrac{4x_1 + 17x_2 + 24x_3 + 23x_4 + 19x_5 + 3}{2x_1 + 3x_2 + 4x_3 + 6x_4 + 3x_5 + 50}$

Subject to the constraints

$4x_1 + 2x_2 + 6x_3 + 7x_4 + 2x_5 \le 25$

$3x_1 + 5x_2 + 4x_3 + 15x_4 + 3x_5 \le 20$

$9x_1 + 3x_2 + x_3 + 4x_4 + 7x_5 \le 22$

Where $x_1, x_2, x_3, x_4, x_5 \ge 0$ and all are integers

Solution

$A = \begin{pmatrix} 4 & 2 & 6 & 7 & 2 \\ 3 & 5 & 4 & 15 & 3 \\ 9 & 3 & 1 & 4 & 7 \end{pmatrix}$ $P_0 = \begin{pmatrix} 25 \\ 20 \\ 22 \end{pmatrix}$, $P_1 = \begin{pmatrix} 4 \\ 3 \\ 9 \end{pmatrix}$, $P_2 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$, $P_3 = \begin{pmatrix} 6 \\ 4 \\ 1 \end{pmatrix}$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$, $P_4 = \begin{pmatrix} 7 \\ 15 \\ 4 \end{pmatrix}$, $P_5 = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}$

$C^T = (4,17,24,23,19)$, $D^T = (2,3,4,6,3)$,$C_0 = 3$, $D_0 = 50$.

<u>Phase - I</u>

To find θ Matrix

d_j c_j x_j $\dfrac{c_j x_j + c_0}{d_j x_j + d_0} = \dfrac{c_j x_j + 3}{d_j x_j + 50}$

d_j	c_j	x_j					
2	4	x_1	6.25	6.67	2.44	=	0.23
3	17	x_2	12.50	4.00	7.33	=	1.15
4	24	x_3	4.17	5.00	22.00	=	1.55
6	23	x_4	3.57	1.33	5.50	=	0.58
θ = 3	19	x_5	12.50	6.67	3.14	=	1.05

Arrangement of promising variables

J = {3, 2, 5}

Phase – II

$X_0 = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}$ $P_0 = \begin{pmatrix}25\\20\\22\end{pmatrix}$

X_3 enter into the basis with value 1.

$\therefore x_B = \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}$

$Z = \frac{27}{54} = 0.5$

$P_{0\text{-}new} = P_{0\text{-}old} - P_j = \begin{pmatrix}25\\20\\22\end{pmatrix} - \begin{pmatrix}6\\4\\1\end{pmatrix} = \begin{pmatrix}19\\16\\21\end{pmatrix}$

$P_{0\text{-}old} = P_{0\text{-}new} = \begin{pmatrix}19\\16\\21\end{pmatrix}$

2nd element in the set J is r (ie) r = X_2

θ_2 = minimum $\left\{int\left(\frac{14}{2}, \frac{16}{5}, \frac{21}{3}\right)\right\}$ = minimum (9,3,7) = 3

$\epsilon_1 = \frac{c_2\ \theta_2 + c_0}{d_2\ \theta_2 + d_0} = 0.92$

θ_2 is greater than 1

So, X_2 enter into the basis with value 1.

$\therefore x_B = \begin{pmatrix}0\\1\\1\\0\\0\end{pmatrix}$

$Z = \frac{17+27}{3+54} = \frac{44}{57} = 0.77$

$P_{0\text{-}new} = P_{0\text{-}old} - P_j = \begin{pmatrix}19\\16\\21\end{pmatrix} - \begin{pmatrix}2\\5\\3\end{pmatrix} = \begin{pmatrix}17\\11\\18\end{pmatrix}$

$P_{0\text{-}old} = P_{0\text{-}new} = \begin{pmatrix}17\\11\\18\end{pmatrix}$

3rd element in the set J is r (ie) r = X_5

θ_5 is greater than 1

So, X_5 enter into the basis with value 1.

$\therefore x_B = \begin{pmatrix}0\\1\\1\\0\\1\end{pmatrix}$, $Z = \frac{19+44}{3+57} = \frac{63}{60} = 1.05$, $P_{0\text{-}new} = P_{0\text{-}old} - P_j = \begin{pmatrix}15\\8\\11\end{pmatrix}$, $P_{0\text{-}old} = P_{0\text{-}new} = \begin{pmatrix}15\\8\\11\end{pmatrix}$

Phase - I

To find θ Matrix

d_j	c_j	x_j						$\frac{c_j x_j + 3}{d_j x_j + 50}$
2	4	x_1	3.75	2.67	1.22	$\frac{7.88}{52.44}$	=	0.15
3	17	x_2	7.50	1.60	3.67	–	=	–
4	24	x_3	2.50	2.00	11.00	–	=	–
6	23	x_4	2.14	0.53	2.75	–	=	–
θ = 3	19	x_5	7.50	2.67	1.57	–	=	–

Arrangement of promising variables

J = {1}

Phase – II

$X_B = \begin{pmatrix}0\\1\\1\\0\\1\end{pmatrix}$ $P_{0\text{-old}} = P_0 = \begin{pmatrix}15\\8\\11\end{pmatrix}$

X_1 enter into the basis with value 1.

$\therefore x_B = \begin{pmatrix}1\\1\\1\\0\\1\end{pmatrix}$

$Z = \frac{4+63}{2+60} = \frac{67}{62} = 1.08$

$P_{0\text{-new}} = P_{0\text{-old}} - P_j = \begin{pmatrix}11\\5\\2\end{pmatrix}$

Phase - I

To find θ Matrix

$d_j \quad c_j \quad x_j \qquad \frac{c_j x_j + 3}{d_j x_j + 50}$

Phase - I

To find θ Matrix

$$\begin{matrix} d_j & c_j & x_j \\ 2 & 4 & x_1 \\ 3 & 17 & x_2 \\ 4 & 24 & x_3 \\ 6 & 23 & x_4 \\ \theta = 3 & 19 & x_5 \end{matrix} \begin{bmatrix} 5.75 & 5.67 & 1.67 \\ 11.50 & 3.40 & 5.00 \\ 3.83 & 4.25 & 15.00 \\ 3.29 & 1.13 & 3.75 \\ 11.50 & 5.67 & 2.14 \end{bmatrix} \begin{matrix} \frac{c_j x_j + 3}{d_j x_j + 50} \\ \\ = 0.18 \\ = 1.01 \\ = 1.45 \\ = 0.51 \end{matrix}$$

Arrangement of promising variables

J = { 3,2,1}

Phase – II

$X_B = \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$ $P_{0\text{-old}} = P_0 = \begin{pmatrix}23\\17\\15\end{pmatrix}$

$Z = \frac{22}{53} = 0.42$

X_3 enter into the basis with value 1.

$$\begin{matrix} 2 & 4 & x_1 \\ 3 & 17 & x_2 \\ 4 & 24 & x_3 \\ 6 & 23 & x_4 \\ \theta = 3 & 19 & x_5 \end{matrix} \begin{bmatrix} 2.75 & 1.67 & 0.22 \\ 5.50 & 1.00 & 0.67 \\ 1.00 & 1.25 & 2.00 \\ 1.57 & 0.33 & 0.50 \\ 5.50 & 1.67 & 0.29 \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \end{matrix}$$

Arrangement of promising variables

J = { }

The current solution is $x_1 = 1, x_2 = 1, x_3 = 1, x_5 = 1$

Maximum $Z = \frac{4+17+24+19+3}{2+3+4+3+50} = \frac{67.00}{62.00} = 1.08$

Phase-III

x_5 is retained 1 and remaining variables are set to zero (ie) is $x_1 = 0, x_2 = 0 \; and \; x_3 = 0$

Now $P_0 = \begin{pmatrix}23\\17\\15\end{pmatrix}$

$\therefore x_B = \begin{pmatrix}0\\0\\1\\0\\1\end{pmatrix}$

$Z = \frac{24+22}{4+53} = 0.81$, $P_{0\text{-new}} = P_{0\text{-old}} - P_j = \begin{pmatrix}17\\13\\14\end{pmatrix}$

2nd element in the set J is r (ie) r = X_2

θ_2 = minimum $\left\{int\left(\frac{17}{2}, \frac{13}{5}, \frac{14}{3}\right)\right\}$ = minimum (8,2, 4) = 2

$\in_1 = \frac{c_2 \; \theta_2 + c_0}{d_2 \; \theta_2 + d_0} = 0.66$

θ_2 is greater than 1

So, X_2 enter into the basis with value 1.

$$\therefore x_B = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$Z = \frac{17+46}{3+57} = 1.05$$

$$P_{0\text{-}new} = P_{0\text{-}old} - P_j = \begin{pmatrix} 15 \\ 8 \\ 11 \end{pmatrix}$$

3^{rd} element in the set J is r (ie) r = X_1

$$\theta_1 = \text{minimum}\left\{int\left(\frac{15}{4}, \frac{8}{3}, \frac{11}{9}\right)\right\} = \text{minimum } (3,2,1) = 1$$

$$\epsilon_1 = \frac{c_2\ \theta_2 + c_0}{d_2\ \theta_2 + d_0} = 0.13$$

θ_1 is greater than or equal to 1

So, X_1 enter into the basis with value 1.

$$\therefore x_B = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$Z = \frac{4+63.00}{2+60.00} = 1.08, P_{0\text{-}new} = P_{0\text{-}old} - P_j = \begin{pmatrix} 11 \\ 5 \\ 2 \end{pmatrix}, P_{0\text{-}old} = P_{0\text{-}new} = \begin{pmatrix} 11 \\ 5 \\ 2 \end{pmatrix}$$

<u>Phase - I</u>

To find θ Matrix

d_j	c_j	x_j			$\frac{c_j x_j + 3}{d_j x_j + 50}$	
2	4	x_1	2.75	1.67	0.22	
3	17	x_2	5.50	1.00	0.67	—
4	24	x_3	1.00	1.25	2.00	—
6	23	x_4	1.57	0.33	0.50	—
θ = 3	19	x_5	5.50	1.67	0.29	—

Arrangement of promising variables

J = { }

Here the Phase III will not improve the solution.

So , the Optimal Solution is

$$x_1 = 1, x_2 = 1, x_3 = and\ x_5 = 1$$

1) Research Elaborations
2) Results or Finding
3) Conclusions

$$\text{Maximum } Z = \frac{4+17+24+19+3}{2+3+4+3+50} = \frac{67.00}{62.00} = 1.08$$

V. Conclusion

In this paper a new approach to solve Zero One Integer Linear Fractional Programming problem has been discussed. The above algorithm rendered best optimal solution. In Future this method can be applied in different types of Linear Fractional programming problem to get a better optimal solution.

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