A New Approximate Bayesian Decision Rule About the Mean of a Gaussian Distribution Versus the Classical Decision Rule

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Decision- Rules about the mean of a Gaussian distribution are presented and compared. Considering the square error loss function, our approximate Bayesian decision rule for the mean of a normal population is derived. Using normal data and SAS software, our approximate Bayesian test results were compared to their classical counterparts obtained with the well-known classical decision rule. It is shown that the proposed approximate Bayesian decision rule relies only on observations that are under study. The classical decision rule, which uses the standard normal and the student-t statistics, does not always yield the best results; the proposed approach performs often better.

Index Terms- Hypothesis testing, loss functions, statistical analysis.

I. Introduction

Life testing in reliability has gainedd interest from theorists as well as reliability engineers.
distribution - which has been and is still widely used in industry and in academia - is considered. The normal distribution is defined as follows:

$$f\,(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)2};$$

$$-\infty \prec x \prec \infty\,,\ -\infty \prec \mu \prec \infty, \sigma \succ 0. \quad (1)$$

A test of hypothesis consists in testing a given theory or belief related to n parameter based on some sample information. Whenever the underlying model is found to be normal or approximately normal, the classical approach considers the following decision rule for a level of significance of alpha and a sample of size n (Mario F. Triola, 2007):

For a normal population with known variance, the classical method uses the following test statistic: $z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$

wth the following rejection and non rejection regions for a Two-Tailed Test
Hypotheses:

$H_0: \mu = c$

$H_A: \mu \neq c$

Non Rejection Region: (Z1-α/2, Zα/2)

Rejection Region: The complement orf (Z1-α/2, Zα/2) in the set of the real numbers R.

For a normal opulation with unknown variance, the classical method uses the following test statistic wth the following rejection and non rejection regions for a Two-Tailed Test:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

Two-Tailed Test
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New Approximate Bayesian Decision Rule
About the mean of a Gaussian Distribution
Two-Tailed Test
Hypotheses:

II. Methodology

Even thouh no specific analytical procedure exists that allows identification of the appropriate loss function to be used in Bayesian analysis, the most frqently used is the square error loss function. One of the reasons for selecting this loss function is due to its analytical tractability in Bayesian analysis. The square error loss function places a small weight on estimates near the true value and proportionately more weight on extreme deviation from the true value of the parameter. The square error loss is defined

(2) $$L_{SE}(\hat{\theta},\theta)=\left(\hat{\theta}-\theta\right)^2$$

The use of the square error loss function along with a suitable approximation of the Pareto prior leads to the following approximate Bayesian confidence bounds for the normal population variance (Camara, 2003)

New Approximate Bayesian Decision Rules
About the Mean of a Gaussian distribution

To obtain the approximate Bayesian decision rule for mean of a normal population, the close relationship that exists between confidence intervals and hypothesis testing is used. Considering the following approximate Bayesian confidence intervals the following approximate Bayesian decision rule is derived:

The use of the square error loss function along with a suitable approximation of the Pareto prior led to the following Approximate Bayesian confidence bounds: for the population mean
(Camara, 2003):

$$U_{\mu(SE)}=\left(\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}+\bar{y}^2-\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-2-2\ln(\alpha/2)}\right)^{0.5}-a$$

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Hence we can easily infer the following test statistic:

$$T=\frac{(x^2+s^2-u^2)}{(n-1))s^2}$$

With the following lower and upper Approximate Bayesian critical values:

$$Lc==\frac{1}{(n-2-2Ln(\alpha/2)}$$

$$Uc=\frac{1}{(n-2-2Ln(1-\alpha/2)}$$

III. Numerical Results

To compare the proposed approximate Bayesian decision rule with the classical approach, samples obtained from normally distributed populations (e.g., 1, 2, 3, .4, 7) as well as approximately normal populations (e.g., 5, 6) are considered. SAS software was used to obtain the normal population parameters corresponding to each sample data set.

The observed value, which is the value of the test statistic under the assumption that the null hypothesis is true, will be denoted by toto for the classimal method abd by To for the approximate Bayesian method.. If the observed value, falls into the rejection region, the null hypothesis will be rejected at a level of significance selected beforehand. If the observed value falls into the non-rejection region, the null hypothesis will not be rejected at the selected level of significance

Data Set #1:
24, 28, 22, 25, 24, 22, 29, 26, 25, 28, 19, 29 (Mann, 1998, p. 504).

Normal population distribution obtained with SAS:

$N(\mu=25.083,\sigma=3.1176)$.

The population and sample variances are: $\sigma^2=9.71943$, and $s^2=9.719696$. For the following test of hypothesis, classical and approximate Bayesian non-rejection regions are presented in Table 1. Table 1 was used to conduct the following five tests of hypothesis about the normal population variance corresponding to the first data set.

Table1: Two tailed tests of hypothesis- Classical and Approximate Bayesian Non-**Rejection**

Non-Rejection Regions

Regions

C. L. %		
	Classical Method	*Approximate Ba-yesian Approach*
80	*-1.3634_ 1.3634*	*0.0685_ 0.0979*
90	*-1.7960_ 1.7960*	*0.0623_ 0.0990*
95	-2.2010_ 2.010	*0.0575_ 0.0995*
99	*-3.1060 _3.1060*	*0.0486_ 0.0999*

Null Hyp((Ho		
	Observed value **Classsical Method**	Observed value **Aprox. Bayesian Method**
$u = 25.083$	0.000367	0.09103
$u = 25.1$	-0.074111	0.08305275.
$u = 25$	0.09259	0.1299102
$u = 28$	-2.917	-1.357

Example 2

13, 11, 9, 12, 8, 10, 5, 10, 9, 12, 13.

Normal population distribution obtained with SAS : $N(\mu = 10.182, \sigma = 2.4008)$

Population and sample variances: $\sigma^2 = 5.76384$, $s^2 = 5.763636$

Datata 2

Table2: Two tailed tests of hypothesis- Classical and Approximate Bayesian Non-**Rejection Regions**

C. L. %	Non-Rejection Regions	
	Classical Method	Approximate Ba-yesian Approach
80	-1.3722_1.3722	
90	-1.8125_18125	0.7350__0.1086
95	-2.2281_2.2281	0.06111_0.1105
99	-3.1693___3.1693	0,0512_0.ii01

	Observed value Classsical Method	Observed value Aprox. Bayesian Method
$u = 10.182 =$	-0,0027778	0.o9925
$u = 10.2$	-0.141667	0.009288
u =10.3	-0.2805556	0.05729
u =12.4	-3.0833	-0.7703

Example 3

16, 14, 11, 19, 14, 17, 13, 16, 17, 18, 19, 12.

Normal population distribution obtained with SAS: $N(\mu = 15.5, \sigma = 2.6799)$

Population and sample variances: $\sigma^2 = 7.18186$, $s^2 = 7.181818$

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99	-3.1060 _3.1060	0.0486_ 0.0999

Null Hyp((Ho)		
	Observed value **Classsical Method**	Observed value **Aprox. Bayesian Method**
$u = 15.5$	0	0.09093
$u = 15.6$	-012987	0.05153
U=15.7	-0.2597	0.0119
$u = 17.8$	-2.987	-0.8788

Example 4

27, 31, 25, 33, 21, 35, 30, 26, 25,31.33.30, 28.

Normal population distribution obtained with SAS: $N(\mu = 28.846, \sigma = 3.9549)$

Population and sample variances: $\sigma^2 = 15.64123$, $s^2 = 15.641025$

C. L. %	Non-Rejection Regions	
	Classical Method	Approximate Ba-yesian Approach
80	-1.3562_1.3562	0.0569_0.0757
90	-1.78229_1.78229	0.0527__0.07
95	-2.1788_2.1788	0.0497_0.0767
99	-3.0545_3.0545	0.0424_0.0424

Null Hyp((Ho)		
	Observed value **Classsical Method**	Observed value **Aprox. Bayesian Method**
$u = 28.85$	0.08383373	0.0833
$-u$=28.90	-0.04545	0.06799
u=30	-1.045	-0.277
u=32	--2.86	-0.937\\

Example 5

52, 33, 42, 44, 41, 50, 44, 51, 45, 38,37,40,44, 50, 43.

Normal population distribution obt ained with SAS: $N(\mu = 43.6, \sigma = 5.4746)$

Population and sample variances: $\sigma^2 = 29.97124$, $s^2 = 29.971428$

C. L. %	Non-Rejection Regions	
	Classical Method	Approximate Ba-yesian Approach
80	-1.3450_1.3450	0.0568_0.0757
90	-1.7613_1.7613	0.0527 _0.0766
95	-2.1448_2.1448	0.04903_0.076625
99	-2.9768__2.9768	0.04237_0.07686

Null Hyp((Ho)		
	Observed value **Classsical Method**	Observed value **Aprox. Bayesian Method**
$u = 43.60$	0	0.6666
u=43.55	0.0355	0.0764
u43.58	0.01418	0.0705
$u = 47$	2.41	-0.6185

Example 6
52, 43, 47, 56, 62, 53, 61, 50, 56, 52, 53, 60, 50, 48, 60, 55.

Normal population distribution obtained with SAS: $N(\mu = 53.625, \sigma = 5.4145)$

Population and sample variances: $\sigma^2 = 29.31681$, $s^2 = 29.316666$

Table 6

C. L. %	Non-Rejection Regions	
	Classical Method	Approximate Bayesian Approach
80	-1.3406_1.3406	0.0537_0.0703
90	-1.75331_1.7531	0.0500_0.07100.0468_0.0710
95	-2.1314_2.1314	0.0468_0.0712

80	--1.341_1.345
90	-1.7530__1.7530

	99	-3.9467_2.9467	0.0407__0.0714

Null Hyp((Ho)		
	Observed value **Classsical Method**	Observed value **4Aprox. Baysian Method**
***u* =53.625**		0.06667
***u*53.6**		0.07276
***u*53.62**	0.0671	0.0678
***u*=56**	-1.7556	-0.0525

Example 7

The following observations have been obtained from the collection of SAS data sets.

50, 65, 100, 45, 111, 32, 45, 28, 60, 66, 114, 134, 150, 120, 77, 108, 112, 113, 80, 77, 69, 91, 116, 122, 37, 51, 53, 131, 49, 69, 66, 46, 131, 103, 84, 78.

Normal population distribution obtained with SAS: $N(\mu = 82.861, \sigma = 33.226)$

Population and sample variances: $\sigma^2 = 1103.96716$, $s^2 = 1103.951587$.

	Non-Rejection Regions	
	Classical Method	*Approximate Bayesian Approach*
80	*-1.3062_1.3062*	*0.0259_0.0292*
90	*-1.6896_1.6896*	*0.0250__0.o2932*
95	*-2.0301_2.0301*	*0.0242_0.0293*
99	*-2.7238_2.7238*	*0.0224__0.0294*

Null Hyp((Ho)		
	Observed value **Classsical Method**	Observed value **Aprox. Bayesian Method**
***u* =82.861**	0	0.0286
***u*=82**	0.1557	0.03224
***u* =82.6**	0.00398	0.02968
***u* = 90**	-1.2988	-0.0072

In the present study, a new approximate Bayesian decision rule for the mean of a normal population has been derived with the use of the square error loss function. Based on the above numerical results we can conclude the following:

All randomly selected tests of hypothesis show that the classical and the proposed approximate Bayesian decision rules perform well as they do not reject null hypotheses corresponding to claims that are equal to the values of the parameters under study.

1. The classical decision rule for the mean of a normal population does not always yield the best results. In fact, contrary to our proposed Appoximate Bayesian decision rule, the classical approach fails, at times to reject claims that are far from being good estimates of the population mean.
2. The classical decision rule does not always yield a smaller Type II error than the approximate Bayesian decision rule. In fact the numerical results show that our new Approximate Bayesian approach performs better when it comes to rejecting a wrong null hypothesis.

3. Contrary to the classical rejection and non-rejection regions that are defined with the use the normal table and the t-table, their approximate Bayesian counterparts rely only on the observations that are under study.

4. The approximate Bayesian decision rule can be easily applied to any normal or approximately normal data, irrespective of the size of the sample size and the confidence level that are used for the study.

5. The tests corresponding to our approximate Bayesian Decision rule have great power, since the probability that they fail to reject a wrong null hypothesis is extremely small.

Bayesian analysis contributes to enhancing ng well-known statistical theories such as the Estimation and Decision Theories.

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Hence we can easily infer the following test statistic: $T = \frac{(x^2 + s^2 - u^2)}{(n-1))s^2}$

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$-a$

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Example 6
52, 43, 47, 56, 62, 53, 61, 50, 56, 52, 53, 60, 50, 48, 60, 55.

Normal population distribution obtained with SAS: $N(\mu = 53.625, \sigma = 5.4145)$

Population and sample variances: $\sigma^2 = 29.31681$, $s^2 = 29.316666$

Table 6

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u53.6		0.07276
u53.62	0.0671	0.0678
u=56	-1.7556	-0.0525

Example 7

The following observations have been obtained from the collection of SAS data sets.

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		ıe **esian**
***u* =82.6**	0.00398	0.02968
***u* = 90**	-1.2988	-0.0072

In the present study, a new approximate Bayesian decision rule for the mean of a normal population has been derived with the use of the square error loss function. Based on the above numerical results we can conclude the following:

All randomly selected tests of hypothesis show that the classical and the proposed approximate Bayesian decision rules perform well as they do not reject null hypotheses corresponding to claims that are equal to the values of the parameters under study.

6. The classical decision rule for the mean of a normal population does not always yield the best results. In fact, contrary to our proposed Appoximate Bayesian decision rule, the classical approach fails, at times to reject claims that are far from being good estimates of the population mean.
7. The classical decision rule does not always yield a smaller Type II error than the approximate Bayesian decision rule. In fact the numerical results show that our new Approximate Bayesian approach performs better when it comes to rejecting a wrong null hypothesis.

8. Contrary to the classical rejection and non-rejection regions that are defined with the use the normal table and the t-table, their approximate Bayesian counterparts rely only on the observations that are under study.

9. The approximate Bayesian decision rule can be easily applied to any normal or approximately normal data, irrespective of the size of the sample size and the confidence level that are used for the study.

10. The tests corresponding to our approximate Bayesian Decision rule have great power, since the probability that they fail to reject a wrong null hypothesis is extremely small.

Bayesian analysis contributes to enhancing ng well-known statistical theories such as the Estimation and Decision Theories.

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