Effect of Magnetic Fields on the Boundary Layer Flow of Heat Transfer with Variable Viscosity in the Presence of Thermal Radiation

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DOI: 10.29322/IJSRP.9.05.2019.p8904
http://dx.doi.org/10.29322/IJSRP.9.05.2019.p8904

ABSTRACT
The effect of Magnetic fields on a steady two dimensional boundary layer flow of heat transfer with three different streams of variable viscosity electrically conducting fluid at T_∞ in the presence of thermal radiation was considered. The governing equations which are partial differential equations were transformed into ordinary differential equations using similarity variables, and the resulting coupled ordinary differential equations were solved numerically using collocation method iterated with the aid of MAPLE 18 software. The effect of space variable viscosity and temperature profiles were studied and presented graphically for different values of physical parameters.

Key Words: *Magnetic Fields, Boundary Layer Flow, heat Transfer, Variable Viscosity, Thermal Radiation, and Collocation Method.*

INTRODUCTION

The boundary layer of heat and mass transfer in magnetic fields has attracted the interest of myriads of researchers due to its numerous applications in engineering, industries, space science and aviation and also in Agriculture, to study the water transport in plants and the likes. If the temperature of the surrounding fluid is high, radiation effects plays an important role in and this situation does exists in space technology. When radiative heat transfer takes place, the fluids involve can be electrically conducting since it is ionized due to high operating temperature. Thus, examining the effects of magnetic field on the flow becomes more relevant. Radiation heat transfer becomes more significant with rising temperature levels and may be totally dominant over conduction and convection at very high temperature. Thus, thermal radiation is pertinent in combustion applications (furnaces, rockets, nozzles, engines etc.) in nuclear reactors. Ajala *et al.,* [1] studies a boundary layer flow and heat transfer with variable viscosity in the presence of magnetic field and then concluded after testing the effect of some physical parameters that the velocity profile increases as the boundary layer thickness increases.

Idowu and Falodun, [6] investigated the influence of magnetic field and thermal radiation on steady free convective flow embedded in a porous medium with soret effects and the transformed coupled nonlinear ordinary differential equations are solved using the Spectral Homotopy Analysis Method. Jagadha *et al.* [7] considered a steady laminar viscous incompressible nanofluid flow of mixed convection and mass transfer about an isothermal vertical flat plate embedded in Darcy porous medium in the presence of magnetic field and viscous dissipation, see also [4, 7, 10]. Lavanya and Ratnam [8] investigated the effects of thermal radiation and chemical reaction a steady two dimensional laminar flow of a viscous incompressible electrically conducting micropolar fluid past a vertical isothermal stretching surface embedded in a non-Darcian porous medium in the presence of viscous dissipation and heat generation. The impact of variable properties on heat and mass transfer over a vertical cone filled with nanofluid saturated porous medium with thermal radiation and chemical reaction was analyzed by Chandra Babu *et al***.,** [11] and the transformed governing equations were solved numerically using an optimized, extensively validated, variational Finite element method.

The concern of Makinde [9] was the inherent irreversibility in hydro magnetic boundary layer flow of variable viscosity fluid over a semi-infinite flat plate under the influence of thermal radiation and Newtonian heating. In the study carried out by Reddy *et al.* [12] on MHD boundary layer flow of a non-newtonian power-law fluid on a moving flat plate**.** Ghara *et al.* [5] in their study observed the effect of radiation on MHD free convection flow past an impulse moving vertical plate with ramped wall temperature. Butt *et al.* [3] carried out investigation on irreversibility effects in magneto hydrodynamic flow over an impulsively started plate. Barik's [2] analysis was based on the combined effects of magneto hydrodynamics and thermal radiations on unsteady flow of an electrically conducting fluid past an impulsively started infinite vertical porous plate with variable temperature.

MATHEMATICAL FORMULATION

In this study, the effect of magnetic fields on a steady two-dimensional boundary layer flow of heat transfer with different stream of variable viscosity electrically conducting fluid at temperature T_∞ in the presence of thermal radiation were considered. It is assumed that the lower surface of the plate is heated by convection from a hot fluid at temperature T_f which provides a heat transfer coefficient h_f. A uniform transverse magnetic field B_0 is imposed along the y-axis. The induced magnetic field due to the polarization of charges are assumed to be neglected.

Under the usual boundary layer approximations, Makinde [9] stressed that the flow is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2}{\rho}(u-U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho c_p}(u-U_\infty)^2 \tag{3}$$

Where equation (1) is the continuity equation; (2) is the momentum equation; and (3) is the energy equation; with μ = $\mu_0 e^{\frac{1}{1+y}}$

U_∞ = free stream velocity; c_p = specific heat at constant pressure; α = thermal diffusivity;
σ = fluid electrical conductivity; ρ = fluid density, μ =dynamic viscosity. The fluid variable dynamical viscosity μ is assumed to be a non- linear function.

$$\mu=\mu_0 e^{\frac{1}{1+y}} \tag{4}$$

Where μ_0 is the cold fluid viscosity and y is the space variable.

Following the Roseland approximation for radiation, the radiative heat flux is simplified as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stephan–Boltzmann constant and
k^* is the mass absorption coefficient.

The temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as linear function of temperature T using a truncated Taylor series about the free stream temperature T_∞ i.e.

$$T^4\approx 4T_\infty^3 T-3T_\infty^4\ . \tag{6}$$

The boundary condition at the plate surface and far into the free stream may be written as:

$$u(x,0)=0,\ \ v(x,0)=0,\ -k\frac{\partial T}{\partial y}(x,0)=h_f[T_f-T(x,0) \tag{7}$$

$$u(x,\infty)=U_\infty,\ \ T(x,\infty)=T_\infty$$

Where, k is the thermal conductivity coefficient. The stream function ψ satisfies the continuity equation (1) automatically with:

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x} \tag{8}$$

A similarity solution of equations was obtained by defining an independent variable η and a dependent variable f in terms of the stream function ψ as:

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}}, \quad \psi = \sqrt{\upsilon x U_\infty}\, f(\eta), \upsilon = \frac{\mu}{\rho}, \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}. \quad (9)$$

After introducing equation (9) into Equations (1)-(8), we obtained

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{2} y\sqrt{\frac{U_\infty^3}{\upsilon x^3}}.f''(\eta) + \frac{1}{2} y\sqrt{\frac{U_\infty^3}{\upsilon x^3}}.f''(\eta) = 0. \quad (10)$$

$$e^{\frac{1}{1+y}} f''' + \frac{1}{2} ff'' - \frac{1}{(1+y)^2} e^{\frac{1}{1+y}} \delta f'' - Ha(f' - 1) = 0. \quad (11)$$

$$\theta'' + \frac{1}{2}\beta \Pr f\theta' + \beta Br e^{\frac{1}{1+y}} (f'')^2 + \beta Ha.Br(f' - 1)^2 = 0. \quad (12)$$

$$f(0) = 0, f(0) = 0, f'(\infty) = 1. \quad (13)$$

$$\theta'(0) = Bi[\theta(0) - 1], \quad \theta(\infty) = 0. \quad (14)$$

Where the prime denotes differentiation with respect to η and the flow is in y-direction. υ is the kinetic viscosity, Ha is the local magnetic field parameter, δ is the boundary layer thickness, Ra is the thermal radiation parameter, $\frac{1}{\kappa}$ is the thermal conductivity, Pr and Br is the Prandtl and Brinkman number respectively. Hence, these parameters and dimensionless numbers are defined as follows:

$$\beta = \frac{3Ra}{3Ra+4}, \quad \upsilon = \frac{\mu_0}{\rho}, \quad \text{Ha} = \frac{\sigma B_0^2 x}{\rho U_\infty}, \quad \delta \approx \sqrt{\frac{\upsilon x}{U_\infty}}, \quad \Pr = \frac{\upsilon}{\alpha}, \quad \frac{\Pr}{\mu_0 c_p} = \frac{1}{k}, \quad Ra = \frac{kk^*}{4\sigma^* T_\infty^3},$$

$$Br = \frac{\mu_0 U_\infty^2}{k(T_f - T_\infty)}$$

Numerical procedure

An efficient Collocation Weighted Residual method was devised in solving couple of ordinary differential equations (10)-(12) with boundary conditions (13)-(14) for different values of controlling parameters. Taking $\infty \approx 5$ and imposing boundary condition on a polynomial.

Assuming the trial function residual is defined as $f = a + b\eta + c\eta^2 + d\eta^3 + ...$

$$\theta = a + b\eta + c\eta^2 + d\eta^3 + ...$$

and substituting the trial function into the resulting differential equation, we have the Residual:

$$R := 336\eta^5 C_8 + 210\eta^4 C_7 + 120\eta^3 C_6 + 60\eta^2 C_5 + 24\eta C_4 + 6C_3 + 0.5(C_0 + C_1\eta + C_2\eta^2 + C_3\eta^3 + C_4\eta^4 + C_5\eta^5 + C_6\eta^6 + C_7\eta^7 + C_8\eta^8)(56\eta^6 C_8 + 42\eta^5 C_7 + 30\eta^4 C_6 + 20\eta^3 C_5 + 12\eta^2 C_4 + 6\eta C_3 + 2C_2) + 14\eta^6 C_8 + 10.50\eta^5 C_7 + 7.50\eta^4 C_6 + 5.00\eta^3 C_5 + 3\eta^2 C_4 + 1.50\eta C_3 + 0.50C_2 - 0.8C_8\eta^7 - 0.7C_7\eta^6 - 0.6C_6\eta^5 - 0.5C_5\eta^4 - 0.4C_4\eta^3 - 0.3C_3\eta^2 - 0.2C_2\eta - 0.1C_1 + 0.1.$$

The residual above was collocated at various points within the domain and after collocating, the residual within the points η =0.5, 2.0, 2.5, 5.0 …, were chosen arbitrarily. This gives system of non - linear equation in C_0 to C_8, which were solved simultaneously in order to obtain the value of the constants as follows: C_0 =0, C_1=0, C_2 =0.2654771707, C_3= -0.03256433016, C_4 =0.003696748910, C_5 = -0.001143783758, C_6 =0.0002466764944, C_7 = -0.0002471358822, C_8 = 0.000000946187656710.

Following the above stated procedures, the subsequent residuals were generated and the corresponding collocations were executed with the aid of Maple 18 software pseudo code.

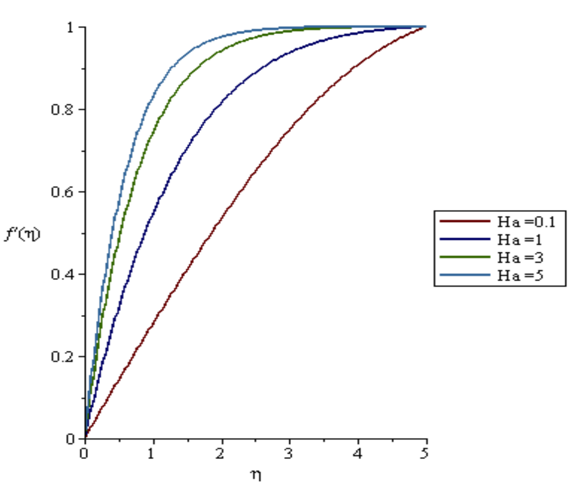
Result and discussion

The non-linear ordinary differential equations (10)-(12) subject to the boundary conditions (13)-(14) were solved numerically by an inbuilt pseudocode computer software MAPPLE 18. Following the process of numerical computations, the plate surface temperature, the local skin- friction coefficient and the Nusselt number in proportionality to $\theta(0)$, $f''(0)$ and $-\theta'(0)$, were analyzed and their respective numerical values were represented in the table 1 below:

Table 1: Computation showing $f''(0)$, θ(0), and -θ′(0) for various values of key parameters.

Bi	Y	Br	Ra	Pr	Ha	$f''(0)$	-θ′(0)	θ(0)
0.1	0	0.1	0.7	0.72	0.1	0.39556967	0.059697304	0.403026965
1.0	0	0.1	0.7	0.72	0.1	0.41360353	0.150591663	0.849408337
10	0	0.1	0.7	0.72	0.1	0.44669845	0.166022899	0.983397710
0.1	0.5	0.1	0.7	0.72	0.1	0.52697089	0.056049908	0.439500915
0.1	1.0	0.1	0.7	0.72	0.1	0.55237890	0.051115131	0.488848687
0.1	0	1.0	0.7	0.72	0.1	0.64903194	0.004640636	0.953593635
0.1	0	10	0.7	0.72	0.1	0.66691220	0.539383019	6.393830197
0.1	0	0.1	5.0	0.72	0.1	0.47201485	0.060894110	0.391058897
0.1	0	0.1	10.0	0.72	0.1	0.47200510	0.060955826	0.390441735
0.1	0	0.1	0.7	3.0	0.1	0.46925141	0.076048970	0.283711183
0.1	0	0.1	0.7	7.10	0.1	0.47879905	0.077897764	0.221022358
0.1	0	0.1	0.7	0.72	0.5	0.68805234	0.053319518	0.466804824
0.1	0	0.1	0.7	0.72	1.0	1.02755678	0.046165161	0.538348381
0.1	0	0.1	0.7	0.72	2.0	1.34986477	0.031856450	0.681435496

Below is the graphical representation of physical parameters involved on the flow and thermal field.



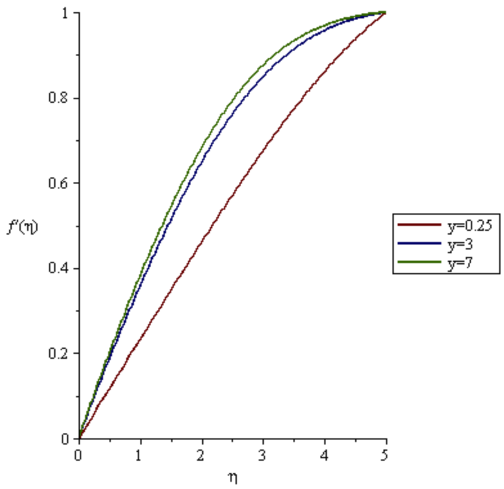


Figure 1 Effect of magnetic field parameter, Ha, with Ha = 0.1, δ = 0.25.

Figure 2: Effect of space variable, y, on the velocity profile on the velocity profile with y = 0, δ = 0.25

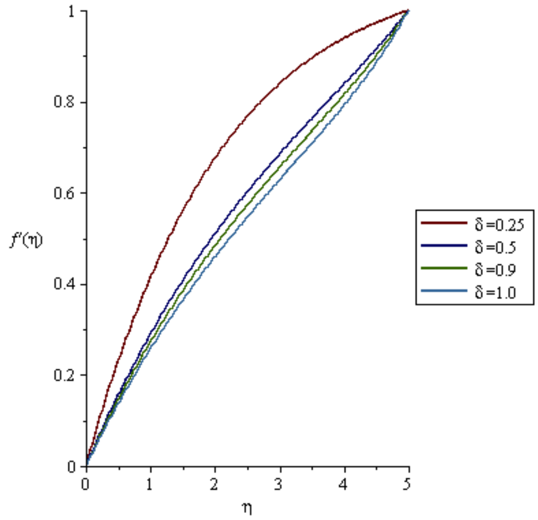


Figure 3: Effect of boundary layer thickness, δ, on the velocity profile with y = 0, Ha = 0.1

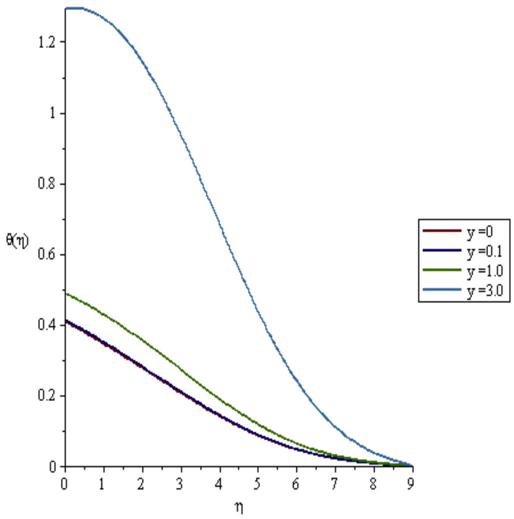


Figure 4: Temperature profiles for *Pr* =0.72, *Ha* =0.1,*Bi* =0.1, *Ra* =0.1,*Br* =0.1

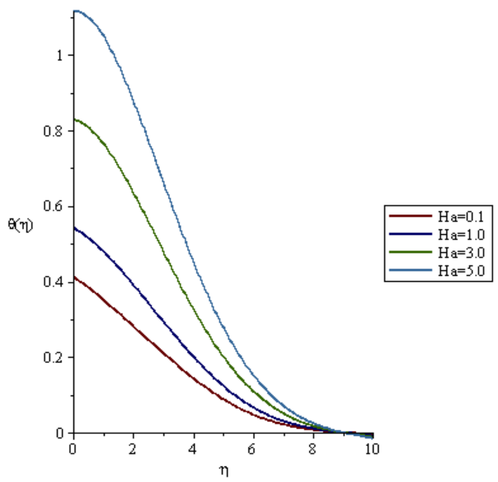


Figure 6: Temperature profiles for *Pr* = 0.72, *Br* =0.1 ,*Ra* = 0.7, y=0,*Bi* =0.1.

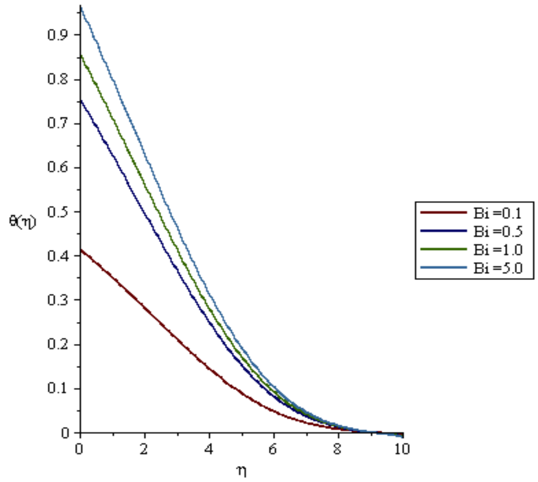


Figure 5: Temperature profile for *Pr* =0.72, *Ha* =0.1, *y*=0, *Ra* =0.7,*Br* =0.1.

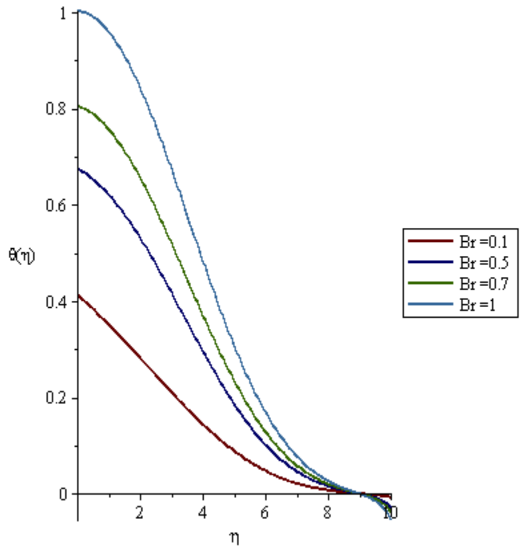


Figure 7. Temperature for *Pr* =0.72, *Bi* =0.1, *Ra* = 0.7, *y*=0 *Ha* =0.1

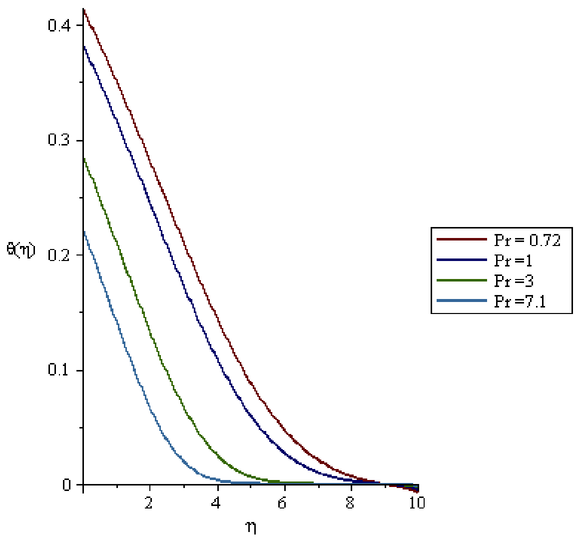


Figure 8 .Temperature profiles for *Bi* =0.1, *Ha* =0.1,*Br* 0.1, *Ra* =0.7,y=0

Discussion of Results

In figure 1, the fluid velocity is lowest at the plate surface and increases to the free stream value, satisfying the far field boundary condition. Application of the magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid motion, thus causing the velocity of the fluid to overshoot towards the plate surface. Figure 2 shows the variation of the velocity profile as a function of η at different values of space variable, *y* and as y increases, the velocity profile increases. Figure 3 shows that as boundary layer increases, the velocity profile decreases. Figure 4 shows that the thermal boundary layer decreases due to a decrease in the fluid viscosity thereby causing decrease in the temperature of the fluid. It was observed from figure 5 that the fluid temperature increases with increase in *Ha* accordingly leading to an increase in thermal boundary layer. The transverse magnetic field gives rise to a resistive force known as the Lorentz force of an electrically conducting fluid. This force makes the fluid experience a resistance by increasing the friction between its layers and thus increases its temperature. Figure 6, shows that the thermal boundary layer increases as The *Bi* increases, and this leads to an increase in fluid temperature. From Figure 7, it was shown that as *Br* increases, thermal boundary layer also increases thereby causing increment in the temperature of the fluid. Figure 8 shows that the temperature profiles for different values of the Prandtl number. The fluid temperature decreases with increase in Prandtl number, consequently the thermal boundary layer decreases. Hence, prandtl number can be used to increase the rate of cooling.

Conclusion

In this article, a mathematical and theoretical pattern was presented for the effect of magnetic fields on the boundary layer flow of heat and mass transfer with variable viscosity in the presence of thermal radiation. The partial differential equations were nondimensionalized using suitable similarity terms and the resulting non-linear equations were solved using collocation method in MAPLE 18. The velocity and temperature profiles were studied graphically for different physical parameters of space variable, y, Biot number, *Bi*, Brankmann number, *Br*, Radiation parameter, *Ra*, Prandtl number, *Pr*, and Hartmann number, *Ha*. It was shown that the skin friction, $f''(0)$, increases as Biot number, *Bi*, Brankmann number, *Br*, Space variable, *y*, and Hartmann number, *Ha* increase, the skin friction, $f''(0)$ decreases with increase in Radiation parameter, *Ra*, and Prandtl number, *Pr*. Also, Nuselt number increases as Biot number, Bi, Brankmann number, *Br*, Radiation parameter, *Ra*, and Prandtl number, *Pr*, increases, Nuselt number decreases as Space variable, *y*, and Hartmann number decreases. The plate surface temperature $\theta(0)$ increases as Biot number, *Bi*, Brankmann number, *Br*, Space variable, *y*, Hartmann number, *Ha* increases, plate surface temperature $\theta(0)$ decreases as Radiation parameter, *Ra*, and Prandtl number, *Pr*, increases.

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