Pre-service teachers' procedural and conceptual knowledge of fractions

Abstract- The study examined pre-service elementary teachers and newly accepted secondary non-math and science major students at a university. The study examined weaknesses of students in conceptual and procedural understanding of fractions. It also examined whether there were differences in math fractional knowledge for both groups. The study found students' weaknesses were in solving word fraction problems, in dividing, multiplying, adding and subtracting fractions. Moreover, the study found that on procedural knowledge of fractions, newly accepted students significantly had higher knowledge than pre-service elementary teachers. The study did not find significant differences on conceptual understanding of fractions or knowledge of over-all fractions.

Index Terms- pre-service teachers, fractions, conceptual understanding, procedural understanding

I. Introduction

Researchers have found that pre-service teachers lacked the sufficient mathematics content knowledge to teach fractions (Cramer, et al., 2002) and had difficulty teaching this subject (Chinnappan, 2000), or lacked the knowledge of correct procedures to get answers and were not able to explain reasons for their answers (Becker & Lin, 2005). Moreover, fractions are considered to be one of the most complicated concepts in students' lives (Boulet, 1998). Having difficulty with fractions is not just for American students. For Chinese students, Anderson and Kim (2003) discovered that students in that Asian country have indicated that fractions are the most difficult concepts. However, studies have shown that Asian students or teachers have a better comprehension of fractions (Cai & Wang 2006). For example, Ma (1990) has found that Chinese teachers have a significantly a deeper understanding of fractions. Ma also states that lack of conceptual understanding could be contributing to American teachers' inability in understanding the concept of dividing of fractions. Furthermore, Ma (1990) states that less than half of American teachers were able to use proper algorithm in dividing fractions, and none of teachers could explain the reasons for that operation while 90% of Chinese teachers could provide explanations for their answers.

The National Mathematics Advisory Panel (2008) has mentioned that students' proficiency with fractions should be an essential goal for students in k-8, because it is vital for performing well in high school math courses such as algebra. Moreover, a study has discovered that students in high school who are competent in working with fractions will have higher achievement in mathematics in the United States and United Kingdom (Siegler, et al., 2012). Additionally, the Panel states that at least 40% of middle school students had difficulty with fractions and 50% of high school and middle school students had some problems with fractions.

The purpose of this study is to investigate pre-service teachers' knowledge of procedural and conceptual understanding of fractions and concepts related to them such as decimal and percentage. The study will examine whether there is a difference in knowledge of conceptual and procedural fractions between two groups of students, pre-service teachers and the newly accepted students to the college of education. The study will also discover whether there is a difference in both groups' over-all knowledge of fractions and conceptual understanding of them. Additionally, the study will investigate the types of errors students are making while they are working with fractions. This study is important because mathematics content knowledge has great potential on students' performance in their future academic achievement (Kulm, 2008). Furthermore, this study is significant because teachers' level of math knowledge could significantly affect students' knowledge of mathematics (Shirvani, 2008). In addition, knowledge of math will give students more opportunity in selecting careers with higher pay.

II. Why fractions are challenging

Literature review of the study has shown that many students who attend universities have a naïve understanding of fractions (Tichá & Hošpesová, 2009). Moreover, pre-service teachers have indicated they have difficulty when solving problems involving fractions (Tichá & Hošpesová, 2009). Research has documented several reasons that have contributed to students having difficulty in understanding fractions. First, fractions have different interpretations, or constructs, which could be quotient, part to whole, ratio, measures, rate, and operations (Brousseau et al., 2004; Nickson (2000). Second, students' prior knowledge of fractions could interfere with their understanding of fractions. For example, when students encounter the fraction ¾, they do not see it as one number; they see it as two numbers, 3 and 4. Therefore, when they encounter ¾+4/5, they will add the numerator with numerator and the denominator with denominator because they have learned that when they encounter two numbers, they should add them (Grégoire, et al., 2010). Furthermore, they will have difficulty to believe when multiplying two fractions, the result could be less than original numbers, and they do not know that they should not follow the properties of adding whole numbers (English & Halford, 1995). Third, teachers have failed to link fractions or rational numbers to their students' out of school real experiences (Greer, 1994). Fourth, due to shortage of math instructors, colleges in the United States tend to hire instructors with no real life experience in teaching elementary or high school, so they either lack pedagogical math content teaching or they lack knowledge of the math being taught in k-12 schools

Another factor contributing to students having difficulty in math is teachers' lack of knowledge in teaching math. Teachers should know that there are several strategies in teaching fractions. However, most teachers tend to use only part-whole interpretation of fractions. While it is good approach at the beginning of teaching fractions, it may have undesirable consequences when students promote to upper-level math courses. Therefore, this could result in misunderstanding of fractions because students have not deeply understood applications of fractions in different situations (Lamon, 2005). Furthermore, many teachers lack the basic knowledge of fractions; for example, research has shown that teachers have difficulty writing fractions in descending or ascending orders. Additionally, Shirvani (2008) administered a 6th grade test to pre-service teachers enrolled in math and science methodology class and examined math concepts where t they had the lowest scores. The investigator then examined 6th grade students' performance on the same test and found both teachers and students showed with similar weaknesses in mathematical concepts.

III. Common Error with Fractions

One common mistake children make in addition and subtraction of fractions is disregarding the differences in values of the denominators. Therefore, they will add or subtract fractions without finding a common denominator (Brown & Quinn 2006). In division and multiplication, the common mistake was that learners found common denominators before multiplying and dividing. Moreover, research has shown when dividing fractions, students tried to inverse the dividend rather than the divisor (Newton, 2008). These errors in multiplication and division have been verified by other studies (Ashlock, 1998).

Research has shown when learners are given two fraction numbers and asked to write a word problem that requires dividing these numbers, almost 90% had difficulty responding with correct answers (Ball, 1990). An example of this type of problem is giving fractions 3 1/3 and 1/6, and writing a word problem that requires dividing these fractions in order to get answers. Moreover, DeWolf and Vosniadou (2011) found students have difficulty when ordering a pair of fractions. Their study discovered when two fractions are given in a situation where the numerator and denominator of the first fraction are larger than the second fraction, students had little difficulty finding correct answers. For example, they had little problem finding which one is larger for fractions 4/6 and 3/5. They chose 4/6 because 4 is larger than 3 and 6 is bigger than 5. However students had difficulty when such patterns did not exist; for example, in fractions, 5/7 and 6/5, most students had difficulty finding correct responses. Additionally, Gallistel and Gelman (1992) have stated that children' prior knowledge of whole numbers could hamper student learning of fractions and negatively impact their understanding of how fractions could be shown on a number line, because they see a fraction as two whole numbers.

Conceptual understanding vs. procedural understanding

There are two types of understandings in learning fractions: procedural understanding and conceptual understanding. Conceptual understanding is defined as employing a strategy that link different related concepts to fractions (Kilpatrick et al., 2001). Conceptual understanding also helps students apply learners' knowledge of fraction in non-routine situations. This understanding is about explaining meaningful explanations that support their answers (Siegler et al., 2010). Therefore, conceptual understanding is based on deep and meaningful understanding of mathematical concepts. Procedural understanding is about algorithm and steps that students employ to find answers, and some suggest that this strategy encourages learners to memorize mathematics rather than truly understand it (Prediger, 2008). Since procedural understanding of fractions is based on rote memory of algorithm, students may forget steps needed to find answers, so students may have difficulty in finding correct answers. Moreover, students forget rules required in procedural understanding whereas when they understand fractions conceptually, learners rarely will have difficulty finding correct answers. Ball (1990) found perspective teachers who could divide fractions by inverting divisors (procedural understanding) had difficulty using pictorial representations (conceptual understanding) to explain reasons that support their answers. Additionally, some researchers believe procedural understanding could hinder students' comprehension of fraction concepts (Hallett et al., 2010)

IV. Teaching strategy

To teach fractions effectively, teachers need to use concrete and hands-on activities so students can learn them. When teachers use visual representations, students can better understand number sense, decimal, and fractions (Cramer & Henry, 2002). Unfortunately, traditional teaching style focuses on procedural method and uses symbols to represent these numbers; however, this strategy could result in students' misunderstanding of working with fractions and hamper students' developing conceptual understanding that supports procedural strategy of working with fractions (Huinker, 2002). Some studies recommend teachers connect fractions to other mathematical concepts. For example, a study found while decimal numbers are easier for students to learn, teachers should connect this concept to fractions (Heibert & Wearne, 1986). Ashlock (1998) recommends teachers use students' error patterns in math to remediate them.

V. Teachers' knowledge of fractions

Research studies have discovered that a high rate of elementary teachers do not have the sufficient knowledge of mathematics teaching in helping students to learn the concept of fractions (Hill, 2010). Teachers should be aware of the seriousness of their problems with the lack of their understanding of fractions. A study from a sample of 1000 American high school teachers found that they rated their mathematics knowledge of fractions as the most challenging concepts among 14 mathematics topics in preparing to teach for their algebra course (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). In addition, a study of pre-service teachers examined pre-service teacher's knowledge of procedural and conceptual understanding of fractions and discovered that there is a lack of understanding of fractions. They also found that there was no significant difference in fraction knowledge between first year pre-service teachers and last year pre-service teachers (Tirosh, 2000).

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Moreover, a study has mentioned that the errors that teachers make in fractions are similar to the students' errors (Newton, 2008). Simon (1993) also found that 70% of the pre-service elementary teachers in the study were not able to create an appropriate word problem.

VI. Methods

The participants in this study included three investigator's classes. In one class, there were 31 students who were in math and science methods for elementary pre-service teachers. The researcher calls them the pre-service group. For these students, almost all of them were females and were in their last semester of their course work. Next semester all of them will be doing their student teaching. The participants for the second group included two classes with about 40% male and 60% female. The second group, which is called new students, included 43 students. It must be noted that students in the new group were preparing to be future secondary school teachers, and their majors were in non-math or sciences. Students in the new group had attended the university for two years, and they are either newly accepted or planned to choose education as their major. A test of math fractions included 15 questions, three of which evaluated students' conceptual knowledge of math fractions. These questions involved word problems; they had to find the type of operation they needed to use to solve them. The remaining 12 questions examined students' procedural math fractions. Procedural questions included questions that did include word problems and students were instructed the type of procedure they have need to use. The assessment of knowledge of fractions included adding, multiplying, dividing, and subtracting. It also included changing fractions to percent, decimal to fractions, fractions to decimal, definition of fractions, and finding a fraction on a number line. Most questions were multiple choice types. Students were given 25 minutes to complete the test and were asked to show their work on a space provided below each question. The researcher scored the test based on correct answers, and no partial credit was given to any question. To grade the test, the researcher found the percentage of students who answered each question correctly. Then, for each student, the investigator found the percentage of correct answers for all questions, the percentages of correct answers on 12 procedural questions, and the percentage of correct answers for all three conceptual questions.

VII. Results

Table 1 shows the percentage of corrected answers for 11 construct in fractions

Math questions	Pre-service teachers	New students
1. Adding or subtracting fractions	44	41
2. Multiplying fractions	20	32
3. Dividing fraction	18	32
4. Converting a fraction to a percent	70	83
5. What does 3 2/3 mean?	89	81
6. Definition of a fraction	48	70
7. Converting a fraction to a decimal	41	81
8. Converting decimal to a fraction	56	74
9. Finding a fraction on a number line	56	50
10. Word problem fractions	35	45
11. Writing a word problem when two fractions are given	3.7	2

For pre-service teachers and new students

Table 1 shows that when given two fractions (component #11) and asked to write a word problem that requires dividing these fractions, only 3.7% of pre-service teachers were able to correctly answer them. The Table also shows the component #11 was the weakest for groups, pre-service teachers and new students. The results also show that students' second weakest area for both groups is in multiplying and dividing fractions. Moreover, the results show that only19% of pre-service teachers responded to these questions correctly (20+18)/2. For new students, the study discovered that 32% of them answered these questions correctly. The third weakest area was solving fraction word problems, 32% for the pre-service teachers and 45% for the new students. Furthermore, the study showed that less than half of students were able to answer addition and subtraction fractions correctly. Participants' area of strength in fractions was the component #5, which asks students what 3 2/3 means. Over 80% of students responded that it means 3 plus 2/3. Moreover, at least 70% of students in both groups were able to answer components #4, which was about converting fractions to percentages.

Table 2 shows the mean scores for all questions answered correctly for both groups

Students	N	Mean	Std deviation
Pre-service	31	44.48	21.27
New students	43	51.72	16.99

Table 3. Independent t-test for significance of performance of students on all questions

	Levene's Test for Equality of variances F Sig.	t	df	Sig
Equal Variances assumed	3.18 .079	-1.57	72	.12
Equal Variances not assumed		-1.49	46.39	.14

The study investigated whether students in both groups performed differently on all fraction questions. Table 3 shows that there were no significant differences between the two groups on knowledge of fractions. The study also examined whether these two groups performed differently on procedural questions. Table 5. Shows the *p* value of .00 means there are significant differences on procedural knowledge of fractions between two groups. Table 4 shows that the mean for the pre-service students is 47.88 while the mean for the new students is 65.09, which is significantly higher than the pre-service's mean.

Table 4 shows the mean for both groups on questions related to procedural understanding

Students	N	Mean	Std deviation
Pre-service	31	47.88	21.19
New students	43	65.09	20.01

Table 5 Shows the Independent t-test for significance of the mean of procedural understanding

	Levene's Test for Equality of variances F	Sig.	t	df	Sig
Equal Variances assumed	5.91	.614	-3.44	72	.00
Equal Variances not assumed			-3.39	46.39	.00

This research also investigated whether the two groups performed differently on conceptual questions of fractions and found there were no significant differences between the two groups because the *p* value in table 7 is .767. Table 6 also indicates that the mean for both groups are 18.44 and 20.02, which are close to each other.

Table 6 shows the mean for both groups on questions related to conceptual understanding

Students	N	Mean	Std deviation
Pre-service	31	18.44	23.27
New students	43	20.02	20.01

Table 7 shows the Independent t-test for significance of the mean of conceptual understanding

	Levene's Test for Equality of variances F	Sig.	t	df	Sig
Equal Variances assumed	.675	.414	-.296	72	.767
Equal Variances not assumed			-.288	50.34	.774

Other findings

The investigator found several errors when evaluating students' work. When dividing fractions, students either inverse the first fraction (divisor) or both fractions (divisor and dividend). Another problem with division of fractions by participants was that they divided the first numerator by the second numerator and divided the first denominator with the second denominator. Also, another error in division was that they cross multiply, multiplying the denominator of the divisor to the numerator of the dividend, which becomes the numerator of the resultant fraction numerator. Then, they multiply denominator of the dividend to numerator of the divisor, which became the denominator of the resultant fraction. In multiplying fractions, they attempted to find common denominators, and many of them simply forgot the operation of multiplication or division, so they left them unanswered. Moreover, in multiplication, students flipped the second fraction and confused this with division.

In addition and subtractions, they added numerators and denominators, regardless of having different denominators. Some students in addition or subtraction found common denominator; however, when they found two fractions with the same denominators, they added the denominator too.

VIII. Summary

The study examined participants' conceptual and procedural knowledge of fractions. This research included two groups of participants: those who were in a math methodology for elementary math students and those who were in a general education course for newly accepted students (new students). The study found over 50% of students having difficulty with adding or subtracting fractions. The study also reported that students had even more difficulty with dividing or multiplying fractions than adding or subtracting fractions. Moreover, the results showed the majority of students lacked the ability to solve fraction word problems. Furthermore, the study found pre-service elementary teachers had significantly lower procedural knowledge of fractions than newly accepted students who will become secondary school teachers, excluding math and science majors. Additionally, this research found that on conceptual knowledge, there were no significant differences between pre-service teachers and new students. Furthermore, the results documented several common errors on working with fractions that participants have made.

The findings from this study suggest most pre-service teachers lack the necessary knowledge of fractions needed for upper-grade elementary students. Additionally, several studies mentioned in this study have found student mathematics achievement could be negatively impacted when teachers' knowledge of fractions is very limited. Therefore, college mathematics departments should work with college teacher training programs in creating a curriculum that guarantees students have sufficient knowledge of basic math concepts, such as fractions.

There are several limitations for this study. First, the sample size for the pre-service teachers needed to be larger. Second, participants came from one instructor in one university. Third, over 90% of students were from Hispanic ethnicity so the findings from this study may not be applicable to other learners from different ethnicities. Fourth, there needs to be more questions in each component of fractional concepts; most fractional constructs for this study included only three questions.

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