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Finding the Strongest Path between Two Cities by using Mathematical Model

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***Abstract* -** This work introduces Maximum Weight Link algorithm as a framework for strength of the path between two cities, having weights attached to them .The main goal of this work is how to find the strongest path between two cities using Mathematical model with fuzzy rule.

***Index Terms* -** Strength of the Fuzzy Path, Fuzzy Strongest Path, Mathematical Model, Maximum Weight Link Algorithm.

I. INTRODUCTION

Fuzzy Graph proved to be tremendously useful in modeling the essential features of systems with finite components Graphical models are used to represent telephone network, railway network, communication problems, traffic network etc.

A fuzzy graph is a powerful tool for creating mathematical models of a wide variety of situations. The fuzzy graph has been instrumental in analyzing and solving problems in areas as diverse as computer network design, urban planning, finding shortest paths, and molecular biology. It is also used in game programming.

In such a scenario, the entire network can be considered as an interconnected set of vertices, in which each vertex represents different locations in the network, and an edge represents the link among the locations. Therefore, by looking at the graph, it assertions the best possible route for a network. In this situation, the vertices in the fuzzy graph represent cities and the edges represents the roads or some other links.

This paper carriers the fact, that a heavy weight container is brought from one city to another. On the way many bridges are to be acrossed. Some bridges have a capacity to load the limited weights only. Hence we choose strongest path, instead of carrying the load through shortest path.

Therefore, the strongest path problem would enable to find the vertices in the fuzzy graph such that the maximum strength of the weights of its constituent edges in the path in maximum.

II. BASIC CONCEPTS

Definition 2.1

A fuzzy graph is a pair of fuzzy set $G:(\sigma,\mu)$, where $\sigma: V \to [0,1]$, $\mu: V\times V \to [0,1]$ for all u,v in V, such that $\mu(u,v) \le \min[\sigma(u),\sigma(v)]$.

Example 2.1

The following figure represents a fuzzy graph $g:(\sigma,\mu)$, where $\sigma=\{u/0.2, v/0.7, w/0.4, x/0.6\}$ and $\mu=\{(u,v)/0.3,(u,w)/0.7,(w,v)/0.1,(u,x)/0.2,(w,x)/0.6,(x,v)/0.6\}$

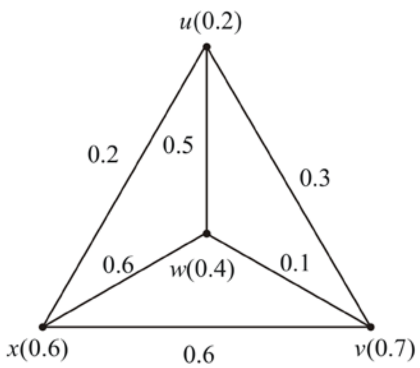

Fig.1. Fuzzy Graph

Definition 2.2

Let $G:(\sigma,\mu)$ is a fuzzy graph and $P:(u,v)$ is a fuzzy path between u and v in G. Minimum membership value of among all the edges in P is called strength of the fuzzy path.

Example 2.2

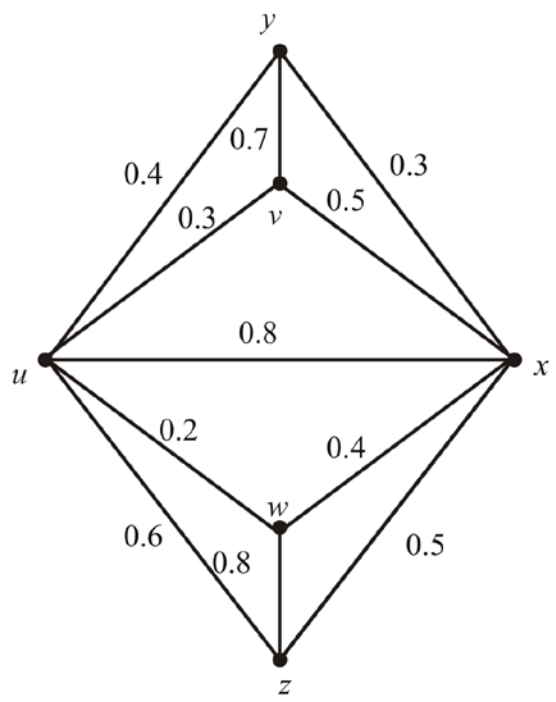

Fig.2. Fuzzy strength of the path

The above graph shows six path between $u-x$.

S. No.	$u-x$ paths	Membership value	Strength of the path
1.	$u-w-x$	0.2, 0.4	0.2
2.	$u-z-x$	0.6, 0.5	0.5
3.	$u-w-z-x$	0.2, 0.8, 0.5	0.2
4.	$u-v-x$	0.3, 0.5	0.3
5.	$u-y-x$	0.4, 0.3	0.3
6.	$u-v-y-x$	0.3, 0.7, 0.3	0.3

Definition 2.3

Let $G:(\sigma,\mu)$ be a fuzzy graph and u,v be any two vertices in G. A fuzzy path from u to v with maximum strength of the path is called Fuzzy Strongest Path.

Example 2.3

The above graph a fuzzy path from $u-x$ with the maximum value of strength of the path is 0.5.

Therefore, this is the Fuzzy Strongest Path.

Definition 2.4

- Models are abstractions of reality.
- Models are representation of a particular thing, idea or condition.
- Mathematical model a self-contained set of formulas (equations) based on an approximate quantitative description of real phenomena and created in the hope that the behaviour it predicts will be consistent with the real behaviour on which it is based.

Example 2.4

If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82. Find the ages of father and son.

Solution

Introduce required variables for the information given in the question.

Let x be the Father's age.

Let y be the Son's age.

Then the data of the problem gives twice the age of son is added to age of father, the sum is 56.

(i.e.) $2y+x=56$ … (1)

Twice the age of the father is added to the age of son, the sum is 82.

(i.e.) $2x+y=82$ … (2)

Solve the equation (1) and (2)

We get $x=36,\ y=10$.

Therefore, The age of father's is 36 years

The age of son's is 10 years.

Equation (1) and (2) is given a mathematical model of the biological situation, so that the biological problem of ages is reduced to the mathematical problem of the solution of a system of two algebraic equations. The solution of the equations is finally interpreted biologically to give the ages of the father and son.

Definition 2.5

A fuzzy rule generally assumes the form

$$\text{R : IF } x \text{ is } A, \text{ THEN } y \text{ is } B$$

where A and B are linguistic values defined by fuzzy sets on universe of discourse X and Y, respectively. the rule is also called a "fuzzy implication" or fuzzy conditional statement.

The part x is A is called the "antecedent" or "Premise", while y is B is called the "consequence" or "conclusion".

In general, the antecedent and consequence are represented by the form of linguistic variables.

$$\text{R : IF } x \text{ is } A, \text{ THEN } y \text{ is } B$$

which is sometimes abbreviated as

$$R : A \to B.$$

Example 2.5

IF temperature is high, THEN humidity is fairly high.

III. FIND THE STRENGTH PATH BETWEEN TWO CITIES WITH FUZZY RULE USING MATHEMATICAL MODEL

Maximum Weight Link Algorithm

Step 1. If any starting point (source) is chosen, then multiple different paths will be exist to destination.

Step 2. If starting from the source vertex, then to find all paths from source to destination.

Step 3. If given a source vertex in the weighted fuzzy graph, then to find the path weights to all other vertices in the fuzzy graph.

Step 4. If every path to be found, then finding the minimum membership value of among all the edges in path.

Step 5. This value will be strength of the path.

Step 6. If finding a strength path from source to destination, then to find the maximum value of strength of the path. This is the strongest path.

Example

The problem is to maximize the new line using Maximum link algorithm with Fuzzy Graph and to find the strength of the path for carrying the load from one city to another city.

There are many bridges to reach from one city to another. But, if we know concretely which way is strength path, then we can carry heavy load in it.

Consider each city as vertex such as
X_1 - Mumbai, X_2 - Delhi, X_3 - Bangalore, X_4 - Hyderabad, X_5 - Chennai, X_6 - Kolkata, X_7 - Jaipur, X_8 - Lucknow.

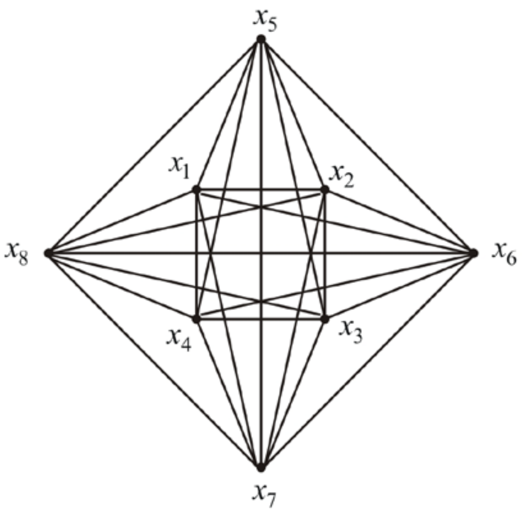

Fig.3.

The above graph shows many bridges between Mumbai to Kolkata. In this graph, it should be found that how many longest bridges are there.

Now we can bring heavy weight container from Mumbai to Kolkata. Because some bridges cannot bear the overload. Hence we can choose strength path instead shortest path.

-	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
X_1	-	0.02	0.17	0.35	0.28	0.1	0.07	0.24
X_2	0.25	-	0.28	0.16	0.4	0.17	0.02	0.01
X_3	0.17	0.28	-	0.07	0.18	0.26	0.47	0.11
X_4	0.35	0.16	0.07	-	0.04	0.25	0.08	0.4
X_5	0.28	0.4	0.18	0.04	-	0.24	0.04	0.19
X_6	0.1	0.17	0.28	0.25	0.14	-	0.8	0.24
X_7	0.07	0.02	0.47	0.08	0.04	0.08	-	0.15
X_8	0.24	0.01	0.11	0.4	0.19	0.22	0.15	-

Table 3.1

Solution

S. No.	Path	$X_1 - X_6$ paths	Weight of the edge of the path	Strength of the path
1.	P_1	$X_1 - X_4 - X_3 - X_6$	0.35, 0.08, 0.26	0.07
2.	P_2	$X_1 - X_2 - X_6$	0.02, 0.17	0.02
3.	P_3	$X_1 - X_8 - X_4 - X_3 - X_6$	0.24, 0.4, 0.07, 0.26	0.07
4.	P_4	$X_1 - X_5 - X_2 - X_6$	0.28, 0.4, 0.12	0.02
5.	P_5	$X_1 - X_4 - X_7 - X_6$	0.35, 0.08, 0.08	0.08
6.	P_6	$X_1 - X_5 - X_6$	0.28, 0.24	0.24
7.	P_7	$X_1 - X_8 - X_7 - X_6$	0.24, 0.15, 0.08	0.08
8.	P_8	$X_1 - X_5 - X_7 - X_6$	0.28, 0.04, 0.08	0.04
9.	P_9	$X_1 - X_4 - X_7 - X_3 - X_6$	0.35, 0.08, 0.47, 0.26	0.08
10.	P_{10}	$X_1 - X_8 - X_6$	0.24, 0.22	0.22
11.	P_{11}	$X_1 - X_3 - X_2 - X_6$	0.17, 0.28, 0.02	0.02
12.	P_{12}	$X_1 - X_3 - X_6$	0.17, 0.26	0.17
13.	P_{13}	$X_1 - X_3 - X_7 - X_6$	0.17, 0.47, 0.08	0.08
14.	P_{14}	$X_1 - X_7 - X_3 - X_6$	0.07, 0.47, 0.26	0.07

By calculating the strength of the path from X_1 to X_6 with the maximum value is 0.24.

Therefore $X_1 - X_5 - X_6$ in this bridge through heavy load things can be brought.

IV. CONCLUSION

It is calculated after analysis strength of the path can be used in Mathematical model with fuzzy rule to carry heavy load to travel on the bridge, which is proved successfully.

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