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Roman Primary and Auxiliary Domination Number of a Graph

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***Abstract*-** Let $G=(V,E)$ be a graph. The two disjoint proper subsets $D_1 \& D_2$ of $V(G)$ are said to be roman primary dominating set and auxiliary dominating set of G respectively, if every vertex not in $D_1 \& D_2$ is adjacent to at least one vertex in $D_1 \& D_2$. The domination number $\gamma_{pri}(G)$ is the minimum cardinality of a primary dominating set and the domination number $\gamma_{aux}(G)$ is the minimum cardinality of an auxiliary dominating set. Note that $\gamma_{pri}(G) \le \gamma_{aux}(G)$. In this paper, we initiate a study of this new parameter.

***Index Terms*-** domination number, primary domination number, and auxiliary domination number.

I. INTRODUCTION

The graphs considered here are simple, finite, nontrivial, undirected, connected, without loops or multiple edges or isolated vertices. For undefined terms or notations in this paper may be found in Harary [1].

Let $G=(V,\ E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex not in D is adjacent to at least one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set.

While various parameters related to domination in graphs have been studied extensively. Let $G=(V,E)$ be a graph. The two disjoint proper subsets $D_1 \& D_2$ of $V(G)$ are said to be roman primary dominating set and auxiliary dominating set of G respectively, if every vertex not in $D_1 \& D_2$ is adjacent to at least one vertex in $D_1 \& D_2$. The domination number $\gamma_{pri}(G)$ is the minimum cardinality of a roman primary dominating set and the domination number $\gamma_{aux}(G)$ is the minimum cardinality of an auxiliary dominating set. Note that $\gamma_{pri}(G) \le \gamma_{aux}(G)$. In this paper, we initiate a study of this new parameter.

For example, Airspace control increases operational effectiveness by promoting the safe, efficient, and flexible use of airspace while minimizing restraints on airspace users (pilot and co-pilot). Airspace control (dominate) includes coordinating, integrating, and regulating airspace to increase operational effectiveness. Effective airspace control reduces the risk of unintended engagements against friendly, civil, and neutral aircraft, enhances air defense operations, and permits greater flexibility of joint operations. This motivates to our primary and auxiliary dominating sets. And another example, parallel processing is the simultaneous use of more than one processor core to execute a program or multiple computational threads. Ideally, parallel processing makes programs run faster because there are more engines (cores) running it. In practice, it is often difficult to divide a program in such a way that separate cores can execute different portions without interfering with each other. Most computers have just one Processing unit, but some models have several, and multi-core processor chips are becoming the norm. Thus, we initiate a study of this new parameter.

II. RESULTS

We use the notation P_p, W_p, K_p, and C_p to denote, respectively, the path graph, wheel, complete graph, cycle graph with p vertices; the star graph with $p+1$ vertices, and $K_{m,n}$ the complete bipartite graph with $m+n$ vertices (see **[1]**).

We obtain the exact value of $\gamma_{pri}(G)$, $\gamma_{aux}(G)$ and $|V-(D_1+D_2)|$ for some standard graphs, which are simple, to observe, hence we omit the proof. As usual, for a real number s, let $\lfloor s \rfloor$ be the greatest integer less than or equal to s, and $\lceil s \rceil$ be the least integer greater than or equal to s.

Proposition1. For any complete graph K_p with $p \ge 3$ vertices,

$$\gamma_{pri}(K_p)=\gamma_{aux}(K_p)=\gamma(G)=1.$$

Proof. Suppose G is a complete graph with $p \ge 3$ vertices, since every pair of its vertices is adjacent. Clearly, by definition it proves that $\gamma_{pri}(K_p)=\gamma_{aux}(K_p)=\gamma(G)=1$.

Proposition 2. For any cycle C_p, with $p \ge 4$ vertices, $\gamma_{pri}(C_p)=\left\lfloor \frac{p}{4} \right\rfloor$.

and $\gamma_{aux}(C_p)=p-3\left\lfloor \frac{p}{4} \right\rfloor$.

Proof. Let $G=(V,E)$ be a graph. The two disjoint proper subsets D_1 & D_2 of $V(G)$ are said to be primary dominating set and auxiliary dominating set of G respectively, Suppose Every vertex of D_1 set is contributing twice the number of vertices in $V-(D_1 \cup D_2)$ …..

Proposition 3. For any wheel W_p, with $p \geq 5$ vertices, $\gamma_{pri}(P_p)=1$, and $\gamma_{aux}(G)=P-4.$

Prof. Suppose the graph is a wheel then we know that a wheel W_p having one vertex say v of maximum degree Δ, and remaining all other vertices are having degree three. Now, a vertex v and another two non adjacent vertices of W_p are also belongs to $V-(D_1 \cup D_2)$. Since primary domination number having exactly one vertex and $V-(D_1 \cup D_2)$ contains three vertices. Therefore auxillary domination number is $P-4.$

Proposition 4. If G is a star, then $\gamma_{pri}(G)=1$ and $\gamma_{aux}(G)=\Delta(G)-1$.

Proof. Suppose G is a star and v being a vertex of maximum degree, then $\Delta(G)=$ number of pendent vertices. By the definition every vertex not in D_1 & D_2 is adjacent to at least one vertex in D_1 & D_2, such that every pendant vertex of G must be belongs to D_1 or D_2. Hence $\gamma_{pri}(G)=1$ and $\gamma_{aux}(G)=\Delta(G)-1$.

Proposition 5. If G is a complete bipartite graph with $m \leq n$ vertices, where, $m \geq 2$, then $\gamma_{pri}(K_{m,n})=1$, $\gamma_{aux}(K_{m,n})=m-1$.

Prof. Obviously follows from the definition; Hence we omit its proof.

Proposition 6. If G is a complete bipartite graph with $m \leq n$ vertices, then $\gamma_{pri}(G)+\gamma_{aux}(G)=\gamma(G)=m$.

Proof. Obviously, $\gamma(G)=m.$ It is easy to notice that $\gamma_{pri}(G)+\gamma_{aux}(G)=n$ =m. Hence the theorem is proved.

Proposition 7. Let G be a complete bipartite graph with $m \leq n$ vertices, where, $m \geq 2$, then $\gamma_{pri}(K_{m,n})<\gamma(K_{m,n})$.

Proof. Obviously follows from the Proposition 4.

Proposition 8. If x be a pendent vertex of a graph G, then $x \in D_1$ *or* D_2.

Proof. Clearly, every vertex not in D_1 & D_2 is adjacent to at least one vertex in D_1 & D_2. Hence every pendant vertex of G must be belongs to D_1 or D_2.

Next, we obtain a relationship between primary domination numbers with the domination number, since their proof are trivial, we omit the same.

Theorem 9. If G is a star or complete graph, then $\gamma_{pri}(G)=\gamma(G)$.

Theorem 10. If G be a complete bipartite graph with $m \leq n$ vertices, where, $m \geq 2$, then $\gamma_{pri}(G)<\gamma(G)$.

Proposition 11. If G is a complete bipartite graph with $m \leq n$ vertices, then $\gamma_{pri}(G)+\gamma_{aux}(G)=\gamma(G)=m$.

REFERENCES

[1] F. Harary. Graph Theory, Addison-Wesley, Reading Mass.(1969).

[2] Haynes, T. W., Hedetniemi, S.T. and Slater, P.J. (1998) - Domination in Graphs: Advanced Topics, Marcel Dekker, Inc., New York.

[3] Haynes, T. W., Hedetniemi, S.T. and Slater, P.J.(1998) - Fundamentals of domination in graphs, Marcel Dekker, Inc., New York.

[4] Allan, R.B. and Laskar, R.C. (1978) – On domination, independent domination numbers of a graph, Discrete Math., 23, pp.73 – 76.

[5] Cockayne, E.J. and Hedetniemi, S.T. (1977) - Towards a theory of domination in graphs, Networks, 7, pp.247 – 261.

[6] Cockayne, E. J., Dreyer, P.A., Hedetniemi, S.M. and Hedetniemi, S.T. (2004) - Roman domination in graphs, Discrete Math., 278, 11 – 22.

[7] Ian Stewart (1999)-Defend the Roman Empire!, Scientific American, 281(6),136–139.

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