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Nano Generalised Closed Sets in Nano Topological Space

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Abstract- The basic objective of this paper is to introduce and investigate the properties of Nano generalized closed sets in Nano Topological Spaces which is the extension of Nano closed sets introduced by Lellis Thivagar[2]

Index Terms- Nano topology , Nano closed sets , Nano Interior , Nano closure , Nano generalized closed sets.

I. Introduction

Levine[2] introduced the class of g- closed sets , a super class of closed sets in 1970.This concept was introduced as a generalisation of closed sets in Topological spaces through which new results in general topology were introduced.

LellisThivagar [1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nanotopological space are called Nano open sets He has also defined Nano closed sets, Nano-interior and Nano closure of a set. He also introduced the weak forms of Nano open sets namely Nano – α open sets , Nano semi open sets and Nano preopen sets. In this paper some properties of generalized closed sets in Nano topological spaces are studied.

II. PRELIMINARIES

Definition : 2.1A subset Aof a topological space (X , τ) is called a generalised closed set (briefly g -closed) [2] if Cl(A) $\subseteq$ U Whenever A $\subseteq$ U and U is open in(X , τ)

Definition : 2.2 [1] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X $\subseteq$U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by LR(X).

That is, L R(x) $=\bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where R(X) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by UR(X).That is,UR(X) = $\bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \emptyset\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by BR(x) . That is, BR(x) =UR(X) - L R(X)

Property :2.3[3] If (U, R) is an approximation space and X, Y $\subseteq$ U , then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$
(ii) $L_R(\emptyset) = U_R(\emptyset)$ and $L_R(U) = U_R(U) = U$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L R(X)]^c$ and $L_R(X^c) = [U R(X)]^c$
(ix) $U_R U_R(X) = L_R U R(X) = U R(X)$
(x) L RL R(X) = URL R(X) = L R(X)

Definition : 2.4 [1] Let U be the universe, R be an equivalence relation on U and τ R (X) = { U, ø , L R(X) , UR(X), BR(x) } where X $\subseteq$U . Then by property 2.3 τ R (X) satisfies the following axioms:
(i) U and ø belongs to τ R (X)
(ii) The union of the elements of any subcollection of τ R (X) is in τ R (X)
iii) The intersection of the elements of any finite subcollection of τ R (X) is in τ R (X)
That is, τ R (X) is a topology on U called the Nanotopology on U with respect to X.
We call (U, τR (X)) as the Nanotopological space. The elements of τ R (X) are called as Nano-open sets.

Remark: 2.5[1] If τ R (X) is the Nanotopology on U with respect to X, then the set B = {U,LR(X) , U R(X) }is the basis for τ R (X).

Definition : 2.6 [1] If (U, τ R (X)) is a Nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U , then the *Nano interior* of A is defined as the union of all Nano-open subsets of A and it is denoted by NInt(A). That is, N Int(A) is the largest Nano-open subset of A.

The *Nano closure* of A is defined as the intersection of all Nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest Nano closed set containing A.

III. NANO GENERALIZED CLOSED SET

Throughout this paper (U, τ R (X)) is a Nano topological space with respect to X where X$\subseteq$ U , R is an equivalence Relation on U , U/R denotes the family of equivalence classes of U by R.

Definition :3.1 Let (U, τ R (X)) be a Nano topological space. A subset A of (U, τ R (X)) is called Nanogeneralised closed set (briefly Ng- closed) if NCl(A) $\subseteq$ V where A$\subseteq$ V and V is Nano open.

Example : 3.2 Let U = { x,y,z } with U/R ={ {x} , {y,z} } and X= {x,z}.Then the Nanotopology

τ R (X) = { U, ø , {x} ,{y,z}}. Nano closed sets are {ø , U ,{y,z},{x}}. Let V= {y,z} and

A= {y}. Then NCl(A) = {y,z} $\subseteq$ V . That is A is said to be Ng- closed in (U, τ R (X))

Theorem : 3.3 A subset A of (U, τ R (X)) is Ng –closed if NCl(A) – A contains no nonempty Ng- closed set.

Proof: Suppose if A is Ng- closed. Then NCl(A) $\subseteq$ V where A$\subseteq$ V and V is Nano open. Let Y be a Nanoclosed subset of NCl(A) – A . Then A$\subseteq Y^c$ and Y^c is Nano open. Since A is Ng – closed ,NCl(A)$\subseteq Y^c$ or Y $\subseteq [NCl(A)]^c$. That is Y $\subseteq$NCl(A) andY $\subseteq [NCl(A)]^c$ implies that Y$\subseteq$ ø . So Y is empty.

Theorem : 3.4 If A and B are Ng- closed , then A U B is Ng- closed.

Proof: Let A and B are Ng- closed sets . Then NCl(A) $\subseteq$ V where A $\subseteq$ V and V is Nano open and NCl(B) $\subseteq$ V where B $\subseteq$ V and V is Nano open . Since A and B are subsets of V , (A U B) is a subset of V and V is Nano open . Then NCl(A U B) = NCl(A) U NCl(B) $\subseteq$V which implies that (AUB) is Ng- closed.

Remark : 3.5 The Intersection of two Ng- closed sets is again an Ng-closed set which is shown in the following example.

Example: 3.6 Let U = {a,b,c} , X = {a,b} , U/R = {{a,b} ,{b.a}, {c}},τ R (X)= {U,ø,{a,b}}.Let A= {b,c} , B= {a,c} and A$\cap$B = {c}.Here NCl(A$\cap$B) $\subseteq$ V when (A$\cap$B) $\subseteq$V and V is Nano open.

Theorem : 3.7 If A is Ng – closed andA$\subseteq$B$\subseteq$NCl(A) , then B is Ng-closed.

Proof: Let B$\subseteq$ V where V is Nano open in τ R (X) .Then A$\subseteq$ B implies A$\subseteq$ V. Since A in Ng-closed ,NCl(A) $\subseteq$ V. Also B $\subseteq$NCl(A) implies NCl(B) $\subseteq$NCl(A). Thus NCl(B) $\subseteq$ V and so B is Ng- closed.

Theorem : 3.8 Every Nano closed set is Nano generalized closed set.

Proof: Let A$\subseteq$V and V is Nano open in τ R (X). Since A is Nano closed ,NCl(A) $\subseteq$A. That is NCl(A) $\subseteq$A$\subseteq$V. Hence A is Nano generalized closed set.The converse of the above theorem need not be true as seen from the following example.

Example : 3.9 Let U = {a,b,c,d} with X= {a,c} with U/R = {{a},{b},{c,d}}.

τ R (X) = {U, ø, {a},{a,c,d},{c,d}}. Nano closed sets are U,ø, {b,c,d},{b},{a,b}. Let A= {a,b,c} and A$\subseteq${a,b,c,d},NCl(A) $\subseteq${a,b,c,d} which implies that A is Ng-closed , but A is not Nano closed.

Theorem : 3.10 An Ng- closed set A is Nano closed if and only if NCl(A) – A is Nano closed.

Proof:(Necessity) Let A is Nano closed . Then NCl(A) = A and so NCl(A) – A = ø which is Nano closed.

(Sufficiency) Suppose NCl(A) – A is Nano closed. Then NCl(A) – A = ø since A is Nano closed. That is NCl(A) = A or A is Nano closed.

Theorem : 3.11 Suppose that B$\subseteq$A$\subseteq$U,B is an Ng-closed set relative to A and that A is an Ng-closed subset of U. Then B is Ng-closed relative to U.

Proof: Let B$\subseteq$ V and suppose that V is Nanoopen in U.Then B$\subseteq$ A$\cap$V. Therefore NClA(B)$\subseteq$A$\cap$V. It follows then that A$\cap$NCl(B) $\subseteq$A$\cap$V and A $\subseteq$ V $\cup$NCl(B). Since A is Ng- closed in U , we have NCl(A)$\subseteq$ V $\cup$NCl(B). Therefore NCl(B) $\subseteq$NCl(A)$\subseteq$ V $\cup$NCl(B).and so NCl(B)$\subseteq$ V. Then B is Ng-closed relative to V.

Corralary: 3.12 Let A be a Ng-closed set and suppose that F is a Nanoclosedset . Then A$\cap$F is an Ng-closed set which is given in the following example.

Example : 3.13 Let U = {a,b,c,d} with X= {a,b} with U/R = {{a},{c},{b,d}}.

τ R (X) = {U, ø, {a},{a,b,d},{b,d}}. Nano closed sets are U,ø, {b,c,d},{c},{a,c}. Let A= {a,b,c} and F= {b,c,d}. Then A$\cap$F= {b,c} is an Ng closed set.

Theorem : 3.14 For each a$\in$ U , either {a} is Nano closed (or) $\{a\}^c$ is Nano generalized closed in τ R (X)

Proof: Suppose {a} is not Nano closed in U. Then$\{a\}^c$is not nano open and the only nano open set containing $\{a\}^c$ is V $\subseteq$ U. That is $a^c \subseteq$ U. Therefore NCl($\{a\}^c \subseteq$ U which implies $\{a\}^c$ is Nano generalized closed set in τ R (X).

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