Nano Generalised Closed Sets in Nano Topological Space

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Abstract- The basic objective of this paper is to introduce and investigate the properties of Nano generalised closed sets in Nano Topological Spaces which is the extension of Nano closed sets introduced by Lellis Thivagar[2]

Index Terms- Nano topology, Nano closed sets, Nano Interior, Nano closure, Nano generalised closed sets.

I. INTRODUCTION

Levine[2] introduced the class of g-closed sets, a super class of closed sets in 1970. This concept was introduced as a generalization of closed sets in Topological spaces through which new results in general topology were introduced.

Lellis Thivagar [1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nanotopological space are called Nano open sets. He has also defined Nano closed sets, Nano-interior and Nano closure of a set. He also introduced the weak forms of Nano open sets namely Nano semi open sets and Nano preopen sets. In this paper some properties of generalised closed sets in Nano topological spaces are studied.

II. PRELIMINARIES

Definition : 2.1A subset A of a topological space \((X, \tau)\) is called a generalised closed set (briefly g-closed set) [2] if \(C(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).

Definition : 2.2 [1] Let \(U\) be a non-empty finite set of objects called the universe and \(R\) be an equivalence relation on \(U\) named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair \((U, R)\) is said to be the approximation space. Let \(X \subseteq U\).

(i) The lower approximation of \(X\) with respect to \(R\) is the set of all objects, which can be for certain classified as \(X\) with respect to \(R\) and its is denoted by \(LR(X)\).

(ii) The upper approximation of \(X\) with respect to \(R\) is the set of all objects, which can be possibly classified as \(X\) with respect to \(R\) and is denoted by \(UR(X)\).

(iii) The boundary region of \(X\) with respect to \(R\) is the set of all objects, which can be classified neither as \(X\) nor as not-\(X\) with respect to \(R\) and it is denoted by \(BR(x)\). That is, \(BR(x) = \{ \forall \alpha \subseteq X \land X \neq \phi \}\)

Property : 2.3 [3] If \((U, R)\) is an approximation space and \(X, Y \subseteq U\), then

(i) \(L_R(X) \subseteq X \subseteq U \cup R(X)\)
(ii) \(L_R(\phi) = U \cup R(\phi)\) and \(L_R(U) = U \cup R(U)\)
(iii) \(U_{R (XUY)} = U_{R(X)} U_{R(Y)}\)
(iv) \(U_{R(X \cap Y)} \subseteq U_{R(X)} \cap U_{R(Y)}\)
(v) \(L_R(xU Y) \supseteq L_R(x) U L_R(Y)\)
(vi) \(L_R(X \cap Y) = L_R(X) \cap L_R(Y)\)
(vii) \(L_R(x) \subseteq L_R(Y)\) and \(U_R(X) \subseteq U_R(Y)\) whenever \(X \subseteq Y\)
(viii) \(U_{R(X)} = U_{R(X)} \subseteq U_{R(X)}\)
(ix) \(U_{R(X)} = U_{R(X)} \subseteq U_{R(X)}\)
(x) \(L_{RL}R(X) = URLR(X) = L\ R(X)\)

Definition : 2.4 [1] Let \(U\) be the universe, \(R\) be an equivalence relation on \(U\) and \(\tau R(X) = \{ U, \varnothing, L R(X), UR(X), BR(X)\} \) where \(X \subseteq U\). Then by property 2.3 \(\tau R(X)\) satisfies the following axioms:

(i) \(U \varnothing \subseteq \tau R(X)\)
(ii) The union of the elements of any subcollection of \(\tau R(X)\) is in \(\tau R(X)\)
(iii) The intersection of the elements of any finite subcollection of \(\tau R(X)\) is in \(\tau R(X)\)

That is, \(\tau R(X)\) is a topology on \(U\) called the Nanotopology on \(U\) with respect to \(X\).

We call \((U, \tau R(X))\) as the Nanotopological space. The elements of \(\tau R(X)\) are called Nano-open sets.

Remark: 2.5[1] If \(\tau R(X)\) is the Nanotopology on \(U\) with respect to \(X\), then the set \(B = \{ U, LR(X), UR(X) \}\) is the basis for \(\tau R(X)\).

Definition : 2.6 [1] If \((U, \tau R(X))\) is a Nano topological space with respect to \(X\) where \(X \subseteq U\) and if \(A \subseteq U\), then the Nano interior of \(A\) is defined as the union of all Nano-open subsets of \(A\) and it is denoted by \(NInt(A)\). That is, \(N Int(A)\) is the largest Nano-open subset of \(A\).
The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by $NCl(A)$. That is, $NCl(A)$ is the smallest Nano closed set containing A.

### III. NANO GENERALIZED CLOSED SET

Throughout this paper $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, $R$ is an equivalence relation on $U$, $U/R$ denotes the family of equivalence classes of U by R.

**Definition : 3.1** Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called Nanogeneralised closed set (briefly Ng- closed) if $NCl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open.

**Example : 3.2** Let $U = \{x,y,z\}$ with $U/R = \{\{x\}, \{y,z\}\}$ and X= $\{x,z\}$. Then the Nanotopology $
\tau_R(X) = \{\emptyset, \{x\}, \{x,y,z\}\}$. Nano closed sets are $\emptyset, U, \{y,z\}, \{x\}$. Let $V = \{y,z\}$ and $A = \{y\}$. Then $NCl(A) = \{y,z\} \subseteq V$. That is A is said to be Ng- closed in $(U, \tau_R(X))$.

**Theorem : 3.3** A subset A of $(U, \tau_R(X))$ is Ng–closed if $NCl(A) \cap A$ contains no nonempty Ng- closed set.

**Proof:** Suppose if A is Ng- closed. Then $NCl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open. Let Y be a Nanoclosed subset of $NCl(A) - A$. Then $A \subseteq NC\cap V$ and $B \subseteq NC\cap V$ is Nano open. Since A is Ng- closed, $NCl(A) \cap A \subseteq V$ or $Y \subseteq [NCl(A)]^e$. That is $Y \subseteq NCl(A)$ and $Y \subseteq [NCl(A)]^e$ implies that $Y \subseteq \emptyset$. So Y is empty.

**Theorem : 3.4** If A and B are Ng- closed, then A U B is Ng- closed.

**Proof:** Let A and B are Ng- closed sets. Then $NCl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open and $NCl(B) \subseteq V$ where $B \subseteq V$ and V is Nano open. Since A and B are subsets of $V$, (A U B) is a subset of $V$ and V is Nano open. Then $NCl(A U B) = NCl(A) U NCl(B) \subseteq V$ which implies that (AUB) is Ng- closed.

**Remark : 3.5** The Intersection of two Ng- closed sets is again an Ng- closed set which is shown in the following example.

**Example : 3.6** Let $U = \{a,b,c\}$, $X = \{a,b\}$, $U/R = \{(a,b), \{b,a\}, \{c\}\}$. $
\tau_R(X) = \{U, \emptyset, \{a,b\}\}$. Let $A = \{b,c\}$, $B = \{a,c\}$ and $A \cap B = \{c\}$. Here $NCl(A \cap B) \subseteq V$ when $(A \cap B) \subseteq V$ and V is Nano open.

**Theorem : 3.7** If A is Ng- closed and $A \subseteq B \subseteq NCl(A)$, then B is Ng- closed.

**Proof:** Let $B \subseteq V$ where V is Nano open in $\tau_R(X)$. Then $A \subseteq B$ implies $A \subseteq V$. Since A is Ng-closed, $NCl(A) \subseteq V$. Also B $\subseteq NCl(A)$ implies $NCl(B) \subseteq NCl(A)$. Thus $NCl(B) \subseteq V$ and so B is Ng- closed.

**Theorem : 3.8** Every Nano closed set is Nano generalized closed set.

**Proof:** Let $A \subseteq V$ and V is Nano open in $\tau_R(X)$. Since A is Nano closed, $NCl(A) \subseteq A$. That is $NCl(A) \subseteq \tau_R(X)$ and B is Nano generalized closed set.

### REFERENCES


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