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Exponential – Half Logistic Additive Failure Rate Model

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Abstract- A combination of exponential and half logistic failure rate model for reliability studies is paid much attention. An attempt is made to present the distributional properties, estimation of parameters , testing of hypothesis and the power of likelihood ratio criterion about the proposed model .

I. Introduction

It is well-known that in the theory of distributions, normal distribution and exponential distribution are the basic models exemplified in a number of theoretical results. Specifically exponential distribution is an invariable example for a number of theoretical concepts in reliability studies. It is characterized as CFR model also. In case of necessity for an IFR model, the choice falls on Weibull model with shape parameter more than 1 (>1), in particular taken as 2. Similar in shape, with common characteristics of Weibull, we have half logistic distribution as another model. Though not as popular as Weibull for reliability studies, half logistic distribution has its own prominence as a life testing model. A half logistic distribution is an IFR model and it is also a weighted exponential distribution. In this paper, we propose to combine an exponential model (CFR) and a half logistic model (IFR) through their hazard functions to get a two component series system reliability.

Studies related to half logistic distribution can be found in Balakrishnan (1985) introducing half logistic distribution by folding the wellknown logistic distribution at its median. His main contribution in the paper is tabulation of means, variances and covariances of order statistics in samples of size n=1 (1) 15 drawn from half logistic distribution. The modes of all order statistics, percentiles of the extreme order statistics are also given. Balakrishnan and Puthenpura (1986) obtained the coefficients to get the best linear unbiased estimates of location and scale parameters in half logistic distribution. Approximate M.L. estimation of location and scale parameters in half logistic model is considered by Balakrishnan and Wong (1991). Balakrishnan and Chan (1992) considered a scaled half logistic distribution and developed theory of linear estimation for its scale parameter in small and large samples. Kantam and Dharma rao (1993) suggested a modification to estimate the scale parameter of half logistic distribution in ML method of estimation to get simpler and more efficient estimator. Rosaiah et al (1997) studied reliability estimation in half logistic distribution from complete samples. Acceptance sampling plans based on life test data following half logistic model for a given consumer's risk are worked out by Kantam and Rosaiah (1998). Estimation of stress-strength reliability where stress, strength variates are assumed to follow half logistic distribution is discussed by Kantam et al (2000). Rosaiah et al (2003) derived optimum group limits for ML estimation of scale parameter of half logistic distribution from a grouped data. Ananda et al (2006) viewed half logistic distribution as a folded distribution and discussed its distributional properties and ML estimation of its parameters.

Because such a combination and the related works are not published in the available literature, we made an attempt to consider such a model for our study. In reliability studies, combinations of components forming series, parallel, k out of 'n' systems are quite popular. The survival probabilities of such systems are evaluated either by the system as a whole or through the survival probabilities of the components that define the system. It is well known that in a series system of a finite number of components with independent life time random variables, the system reliability is equal to the product of the component reliabilities. If f(x), F(x), h(x) respectively indicate the failure density, failure probability, failure rate of a component with life time random variable 'X', then we know that

The reliability $R(x)= 1-F(x)$

$$R(x)= e^{-\int_0^x h(x)dx}$$

If a series system has two components with independent but non-identical life patterns explained by two distinct random variables say $X_1, X_2,$ with respective failure densities, failure probabilities, failure rates as $f_1(x)$, $f_2(x)$; $F_1(x)$, $F_2(x)$; $h_1(x)$, $h_2(x)$ then the system reliability is given by

$$R(x) = e^{-\int_0^x [h_1(x)+h_2(x)]dx} \tag{1.1}$$

From the above expression we can get the failure density and failure rate of the series system whose reliability is given by (1.1).

Specifically we consider a series system of two components with their respective life times modeled by exponential and half logistic distribution.

The hazard functions of the exponential distribution with parameter 'θ' and a half logistic distribution with parameter σ.

i.e. $$h_1(x) = \theta \ \ \forall x, \ \ x > 0$$

$$h_2(x) = \frac{1}{1+e^{-\sigma x}} \ .$$

The corresponding reliabilities are

$$e^{-\theta x} \text{ and } 2e^{-\sigma x} \Big/ (1+e^{-\sigma x})$$

The series system reliability is

$$R(x) = \frac{2^{\frac{1}{\sigma}} e^{-\theta x}}{(1+e^{\sigma x})^{\frac{1}{\sigma}}}, \quad x>0, \quad \theta,\sigma>0 \tag{1.2}$$

we consider the failure density corresponding to (1.2) as our exponential half logistic additive failure rate model (EHLAFRM).

The distributional properties, graphical natures for different choices of θ, σ are discussed in section 2. Estimation of parameters from ungrouped and grouped data is presented in section 3 and section 4 respectively. Likelihood ratio criterion and power of likelihood ratio criterion are given in section 5 and section 6 respectively. Summary and conclusions are in section 7.

II. Distributional Properties

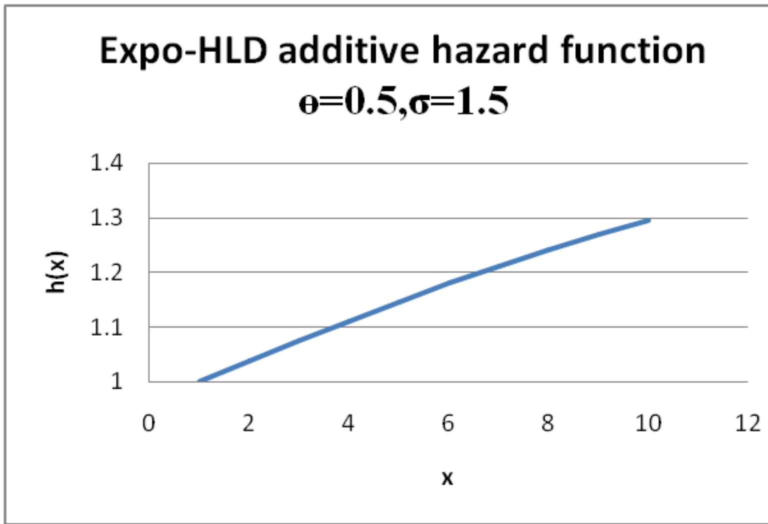
The probability density function, the CDF, failure rate of EHLAFRM are respectively given by

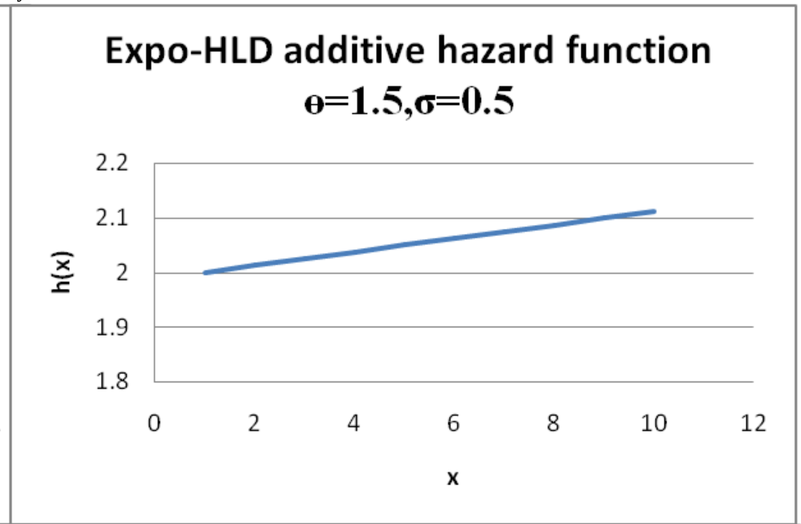
$$f(x) = \frac{2^{\frac{1}{\sigma}} e^{-\theta x}}{(1+e^{\sigma x})^{\frac{1}{\sigma}+1}} \left[\theta(1+e^{\sigma x}) + e^{\sigma x}\right], \; x>0, \; \theta,\sigma>0 \tag{2.1}$$

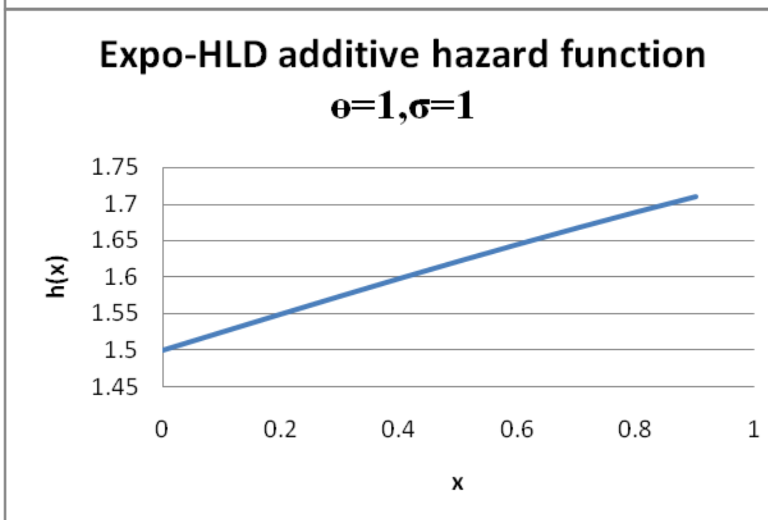
$$F(x) = 1 - \frac{2^{\frac{1}{\sigma}} e^{-\theta x}}{(1+e^{\sigma x})^{\frac{1}{\sigma}}}, \; x>0, \; \theta,\sigma>0 \tag{2.2}$$

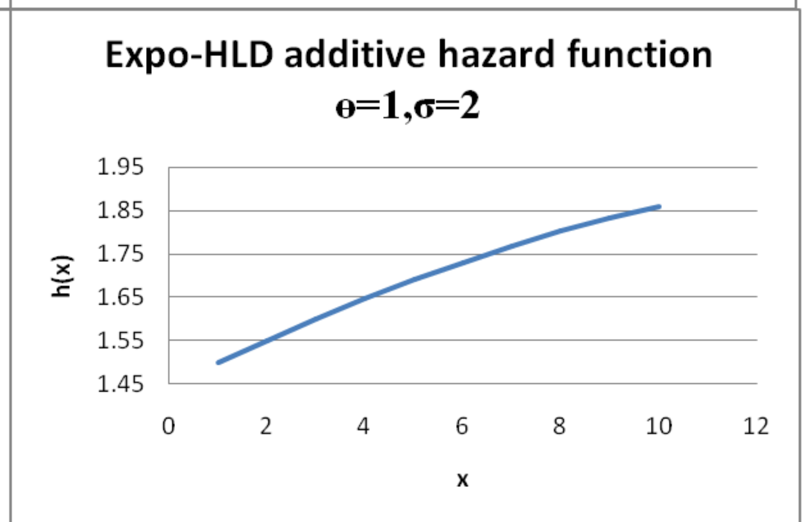
$$h(x) = \theta + \frac{e^{\sigma x}}{1+e^{\sigma x}}, \; x>0 \; ; \; \theta,\sigma>0 \tag{2.3}$$

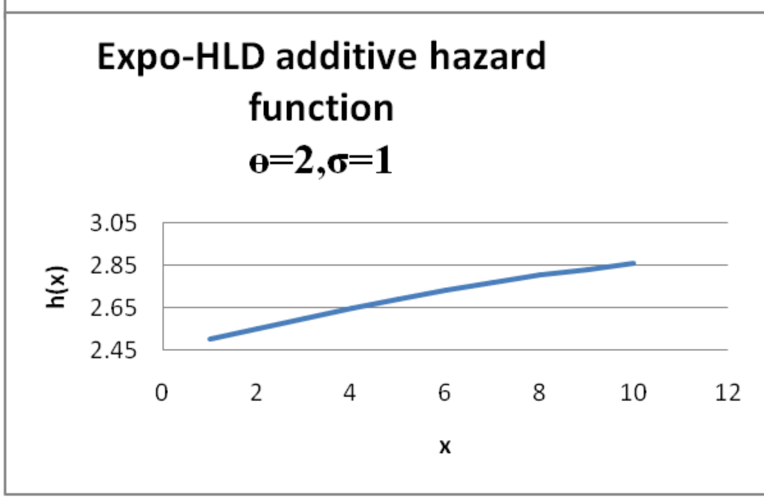
The shape of the frequency curve is as shown in the following graphs for various values of θ=0.5,1,1.5,2 and σ =0.5,1,1.5,2. The probability function appears to be a decreasing function for all the 6 cases given below.

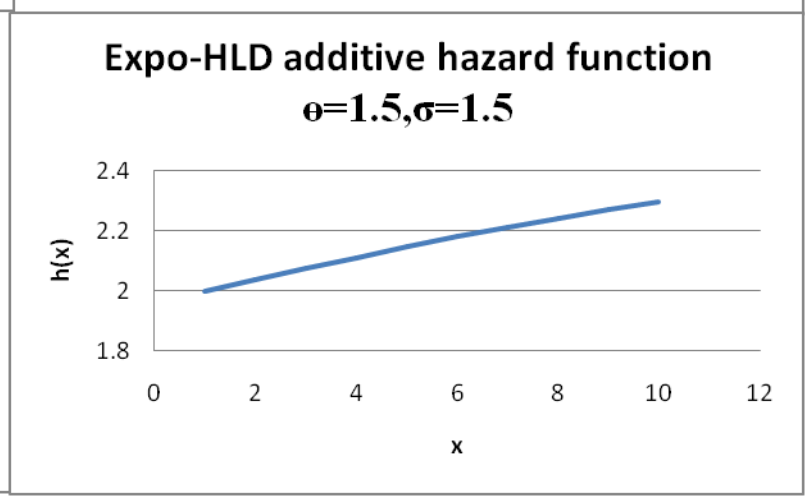












III. Maximum Likelihood Estimation from Ungrouped Data

Let $x_1, x_2 \ldots\ldots x_n$ be a random sample of size n from a population with density function $f(x, \theta)$, then the likelihood function of the sample values $x_1, x_2 \ldots\ldots x_n$ is given by

$$L = \prod_{i=1}^{n} f(x_i, \theta)$$

where $f(x_i, \theta)$ is the probability density function.

L gives the relative likelihood that the random variable assume a particular set of values $x_1, x_2 \ldots\ldots x_n$ for a given sample $x_1, x_2 \ldots\ldots x_n$, L becomes a function of θ, the parameter.

If there exists a function $\hat{\theta} = \hat{\theta}(x_1, x_2 \ldots . x_n)$ of the sample values which maximizes L for variations in θ, then $\hat{\theta}$ can be taken as estimator of θ and usually called maximum likelihood estimator (MLE).

Hence $\hat{\theta}$ is the solution of

$$\frac{\partial L}{\partial \theta} = 0 \text{ and } \frac{\partial^2 L}{\partial \theta^2} < 0$$

Let x_1, x_2 ……..x_n is a random sample of size 'n' drawn from the EHLAFRM with pdf $f(x;\theta,\sigma)$ then the likelihood function is given by

$$L = \prod_{i=1}^{n} f(x_i;\theta,\sigma)$$

$$\Rightarrow L = \prod_{i=1}^{n} \frac{2^{\frac{1}{\sigma}} e^{-\theta x_i}}{(1+e^{-\sigma x_i})} \left[\theta(1+e^{\sigma x_i}) + e^{\sigma x_i}\right] \tag{3.1}$$

$$\Rightarrow \log L = \sum_{i=1}^{n} \log\left\{ \frac{2^{\frac{1}{\sigma}} e^{\theta x_i}}{(1+e^{\sigma x_i})^{\frac{1}{\sigma}+1}} \left[\theta(1+e^{\sigma x_i}) + \right] e^{\sigma x_i} \right\} \tag{3.2}$$

The MLEs of θ,σ can be obtained by solving the following likelihood equations

$$\frac{\partial \log L}{\partial \theta} = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left\{ \frac{(1+e^{\sigma x_i})(1-\theta x_i) - x_i e^{\sigma x_i}}{\theta(1+e^{\sigma x_i}) + e^{\sigma x_i}} \right\} = 0 \tag{3.3}$$

and

$$\frac{\partial \log L}{\partial \sigma} = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left\{ \frac{\log(1+e^{\sigma x_i}) - \log 2}{\sigma^2} - \frac{x_i e^{\sigma x_i}(\theta-\sigma)}{\sigma\left[\theta(1+e^{\sigma x_i}) + e^{\sigma x_i}\right]} - \frac{x_i e^{2\sigma x_i}(1+\sigma)}{\sigma(1+e^{\sigma x_i})[\theta(1+e^{\sigma x_i}] + e^{\sigma x_i}} \right\} = 0 \tag{3.4}$$

The equations (3.3) and (3.4) have to be solved through iteration only with some well known numerical methods and get the MLEs of θ and σ say θ^* and σ^* respectively.

However, by using a simple successive method, the ML equations (3.3) and (3.4) can be further simplified and get the following estimators (not ML estimators) for θ,σ say $\hat{\theta}$, $\hat{\sigma}$ are obtained

$$\hat{\theta} = \sum_{i=1}^{n} \left\{ \frac{1}{x_i} - \frac{e^{\sigma x_i}}{1+e^{\sigma x_i}} \right\} \tag{3.5}$$

$$\hat{v} = \frac{2\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i^2} - 1 \tag{3.6}$$

Accordingly the exact variances of the MLEs are not mathematically tractable. However, the asymptotic variance, covariance of the estimates of the parameters are obtained using the following elements of the information matrix :

$$I_{11} = -E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)$$

$$I_{11} = -E\left[\sum_{i=1}^{n}\left\{\frac{(1+e^{\sigma x_i})^2}{(\theta(1+e^{\sigma x_i})+e^{\sigma x_i})^2}\right\}\right] \tag{3.7}$$

$$I_{12} = I_{21} = -E\left(\frac{\partial^2 \log L}{\partial \theta \partial \sigma}\right)$$

$$I_{12} = I_{21} = -E\left[\sum_{i=1}^{n}\left\{\frac{x_i e^{\sigma x_i}}{(\theta(1+e^{\sigma x_i})+e^{\sigma x_i})^2}\right\}\right] \tag{3.8}$$

$$I_{22} = -E\left(\frac{\partial^2 \log L}{\partial \sigma^2}\right)$$

$$= -E\left[\sum_{i=1}^{n}\left\{\frac{(\lambda x_i + 2vx_i)^2 - (\lambda(1+vx_i)+v^2x_i)2x_i}{(\lambda(1+vx_i)+v^2x_i)^2}\right\}\right] \tag{3.9}$$

The estimated information matrix elements are

$$\hat{I}_{11} = -\frac{\partial^2 \log L}{\partial \theta^2}\Big|_{\theta=\theta^*}$$

$$\hat{I}_{12} = \hat{I}_{21} = -\frac{\partial^2 \log L}{\partial \theta \partial \sigma}\Big|_{|\theta=\theta^*,\ \sigma=\sigma^*}$$

$$\hat{I}_{22} = -\frac{\partial^2 \log L}{\partial \sigma^2}\Big|_{\sigma=\sigma^*}$$

The estimated asymptotic dispersion matrix of the MLEs is given by the inverse of

$$\begin{bmatrix} \hat{I}_{11} & \hat{I}_{12} \\ \hat{I}_{21} & \hat{I}_{22} \end{bmatrix}$$

IV. Maximum Likelihood Estimation from Grouped Data

The probability density function (p.d.f) and the distribution function of EHLAFRM are respectively given by

$$f(x) = \frac{2^{\frac{1}{\sigma}} e^{-\theta x}}{(1+e^{\sigma x})^{\frac{1}{\sigma}+1}}\left[\theta(1+e^{\sigma x})+e^{\sigma x}\right],\ x>0,\ \theta,\sigma>0 \tag{4.1}$$

$$F(x) = 1 - \frac{2^{\frac{1}{\sigma}} e^{-\theta x}}{(1+e^{\sigma x})^{\frac{1}{\sigma}}},\ \ x>0,\ \theta,\sigma>0 \tag{4.2}$$

where θ, σ are the parameters.

Suppose that a raw sample x_1, x_2 ….. x_n drawn from (4.1) is distributed into k(≥ 2) unequispaced groups $(t_{i-1}, t_i), i = 1,2, \ldots, k,$ such that the i-th group (t_{i-1}, t_i) includes 'n_i' observations. The points t_i's $(i = 0,1,2 \ldots, k)$ are called group (class) limits. We take t_0 and t_k as '0' and '∞' respectively. Then we have the data as

Class interval :	$(0-t_1)$	(t_1-t_2)	…	$(t_{i-1}-t_i)$	..	$(t_{k-1}-\infty)$
Frequency :	n_1	n_2	…	n_i	..	n_k

The likelihood function of the grouped sample is given by

$$L = \prod_{i=1}^{k} [p_i]^{n_i} \tag{4.3}$$

where p_i is the probability of an observation falling in the i-th group, i.e.,

$$p_i = F(t_i) - F(t_{i-1}), \qquad i = 1,2, \ \ldots, k.$$

F(.) is the standard EHLAFRM function given by

$$F(t_i) = 1 - \frac{2^{\frac{1}{\sigma}} e^{-\theta t_i}}{(1+e^{\sigma t_i})^{\frac{1}{\sigma}}} \qquad t > 0, \theta, \sigma > 0$$

The log-likelihood equations, to get ML estimates of the parameters θ and σ, on simplification, are respectively,

$$\sum_{i=1}^{k} n_i \left[\frac{-A_{i-1}(t_{i-1}) + B_i t_i}{(A_{i-1} - B_i)} \right] = 0 \tag{4.4}$$

and

$$\sum_{i=1}^{k} n_i \left[\frac{-A_{i-1}\left\{ \frac{\log 2}{\sigma^2} + \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma(1+e^{\sigma t_{i-1}})} - \frac{\log(1+e^{\sigma t_{i-1}})}{\sigma^2} \right\} + B_i \left\{ \frac{\log 2}{\sigma^2} + \frac{t_i e^{\sigma t_i}}{\sigma(1+e^{\sigma t_i})} + \frac{\log(1+e^{\sigma t_i})}{\sigma^2} \right\}}{(A_{i-1} - B_i)} \right] = 0 \tag{4.5}$$

where $A_{i-1} = e^{-\theta t_{i-1}}(1+e^{\sigma t_i})^{\frac{1}{\sigma}}$ and $B_i = e^{-\theta t_{i-1}}(1+e^{\sigma t_i})^{\frac{1}{\sigma}}$.

We shall often consider the special but not unusual case when the group limits $0, t_i, t_2, \ldots, t_{k-1}$ [except the last group (t_{k-1}, ∞)] are equidistant, i.e., $t_i = it_1$ for i=0,1,2, . . , k-1. This situation will be referred to as 'equispaced grouping'. Then the log-likelihood equations (4.4) and (4.5) respectively reduce to

$$\sum_{i=1}^{k} n_i \left[\frac{-A^*_{i-1}(i-1)t_1 + B^*_i it_1}{(A^*_{i-1} - B^*_i)} \right] = 0 \tag{4.6}$$

$$\sum_{i=1}^{k} n_i \left[\frac{-A_{i-1}\left\{ \frac{\log 2}{2} + \frac{(i-1)t_i e^{\sigma(i-1)t_i}}{\sigma(1+e^{\sigma(i-1)t_i})} - \frac{\log(1+e^{\sigma(i-1)t_i})}{\sigma^2} \right\} + B_i \left\{ \frac{\log 2}{\sigma^2} + \frac{it_i e^{\sigma it_i}}{\sigma(1+e^{\sigma it_i})} + \frac{\log(1+e^{\sigma it_i})}{\sigma^2} \right\}}{(A_{i-1} - B_i)} \right] = 0$$

and
(4.7)

where $A^*_{i-1} = e^{-(i-1)t_1}(1+e^{\sigma it_i})^{\frac{1}{\sigma}}$ and $B^*_i = e^{-\theta it_1}(1+e^{\sigma(i-1)t_i})^{\frac{1}{\sigma}}$.

The ML estimates of the parameters θ and σ say $\hat{\theta}_G$ and $\hat{\sigma}_G$ from unequispaced and $\hat{\theta}_g$ and $\hat{\sigma}_g$ from equispaced grouped samples are simultaneous iterative solutions of the pairs of equations (4.4), (4.5) and (4.6), (4.7) respectively. In the case of an unequispaced grouped sample, the elements of information matrix are given by

$$I_{11} = -E[\partial^2 \log L / \partial\theta^2]$$

$$= -E\left\{ \sum_{i=1}^{k} n_i \frac{\left[\left((t_{i-1})^2 A_{i-1} - t_i^2 B_i \right)(A_{i-1} - B_i) - (B_i t_i - t_{i-1} A_{i-1})^2 \right]}{(A_{i-1} - B_i)^2} \right\} \qquad (4.8)$$

$$I_{22} = -E[\partial^2 \log L / \partial\sigma^2]$$

$$\frac{\partial^2 \log L}{\partial\sigma^2} =$$

$$-E\left\{ \sum_{i=1}^{k} n_i \left[-A_{i-1}B_i \left\{ \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma(1+e^{\sigma t_{i-1}})} - \frac{\log(1+e^{\sigma t_{i-1}})}{\sigma^2} - \frac{t_i e^{\sigma t_i}}{\sigma(1+e^{\sigma t_i})} + \frac{\log(1+e^{\sigma t_i})}{\sigma^2} \right\} \right.\right.$$

$$\left\{ \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma(1+e^{\sigma t_{i-1}})} + \frac{\log(1+e^{\sigma t_{i-1}})}{\sigma^2} + \frac{t_i e^{\sigma t_i}}{\sigma(1+e^{\sigma t_i})} + \frac{\log(1+e^{\sigma t_i})}{\sigma^2} \right\}$$

$$-(A_{i-1}^2 - A_{i-1}B_i)\left[\frac{2\log 2}{\sigma^3} - t_{i-1}\left\{ \frac{(1+e^{\sigma t_{i-1}})\left[e^{\sigma t_{i-1}}(\sigma^{t_{i-1}-1})\right] - \sigma^{t_{i-1}} e^{2\sigma t_{i-1}}}{\sigma^2(1+e^{\sigma t_{i-1}})^2} \right\} + \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma^2(1+e^{\sigma t_{i-1}})} - \frac{2\log(1+e^{\sigma t_{i-1}})}{\sigma^3} \right]$$

$$\left.\left. +(A_{i-1}B_i - B_i^2)\left[\frac{2\log 2}{\sigma^3} - t_i\left\{ \frac{(1+e^{\sigma t_{i-1}})[e^{\sigma t_i}(\sigma t_{i-1})] - \sigma t_{i-1} e^{2\sigma t_i}}{\sigma^2(1+e^{\sigma t_i})^2} \right\} + \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma^2(1+e^{\sigma t_i})} - \frac{2\log(1+e^{\sigma t_{i-1}})}{\sigma^3} \right]\right] \Big/ (A_{i-1} - B_i)^2 \right\} \qquad (4.9)$$

and

$$I_{12} = I_{21} = -E[\partial^2 \log L / \partial\theta\partial\sigma]$$

$$\frac{\partial^2 \log L}{\partial\theta\partial\sigma} = -E\left\{ \sum_{i=1}^{k} n_i \left[(-A_{i-1}B_i ti \right] + A_{i-1}t_{i-1}B_i) \right.$$

$$\left. \left\{ \frac{t_{i-1}e^{\sigma t_{i-1}}}{\sigma(1+e^{\sigma t_{i-1}})} - \frac{\log(1+e^{\sigma t_{i-1}})}{\sigma^2} - \frac{t_{i-1}e^{\sigma t_i}}{\sigma(1+e^{\sigma t_i})} - \frac{\log(1+e^{\sigma t_i})}{\sigma^2} \right\} \right] \Big/ (A_{i-1} - B_i)^2 \right\} \qquad (4.10)$$

where $A_{i-1}^* = e^{-\theta(i-1)t_1}(1+e^{\sigma i t_1})^{\frac{1}{\sigma}}$ and $B_i^* = e^{-\theta i t_1}(1+e^{-\sigma(i-1)t_1})^{\frac{1}{\sigma}}$.

Hence, the estimated asymptotic dispersion matrix of the MLEs $\hat{\lambda}_G, \hat{v}_G$ of λ, v from an unequispaced grouped sample is

$$D[\hat{\theta}_G, \hat{\sigma}_G] = [I_{11}\ I_{22} - I_{12}^2]^{-1} \begin{bmatrix} I_{22} & -I_{12} \\ -I_{12} & I_{11} \end{bmatrix} \qquad (4.11)$$

In the case of an equispaced grouped sample, the elements of information matrix are obtained from (4.8) through (4.10) with $t_i = it_1,\ i = 1, 2, \ldots, k-1$, i.e.,

$$\hat{I}_{11} = -E[\partial^2 \log L / \partial\theta^2]$$

$$= -E\left\{\sum_{i=1}^{k} n_i \left[\frac{[(i-1)t_1)^2 A^*_{i-1} - (it_1)^2 B^*_i](A^*_{i-1} - B^*_i) - [B^*_i it_1 - (i-1)t_1 A^*_{i-1}]^2}{(A^*_{i-1} - B^*_i)^2}\right]\right\} \quad (4.12)$$

$$\hat{I}_{22} = -E[\partial^2 \log L / \partial\sigma^2]$$

$$\frac{\partial^2 \log L}{\partial\sigma^2} = \;= -E\left\{\sum_{i=1}^{k} n_i \left[A_{i-1}B_i \left\{\frac{(i-1)t_1 e^{\sigma(i-1)t_1}}{\sigma(1+e^{\sigma(i-1)t_1})} - \frac{\log(1+e^{\sigma(i-1)t_1})}{\sigma^2} - \frac{it_1 e^{\sigma it_1}}{\sigma(1+e^{\sigma it_1})} + \frac{\log(1+e^{\sigma it_1})}{\sigma^2}\right\}\right.\right.$$

$$\left\{\frac{-(i-1)t_1 e^{\sigma(i-1)t_1}}{\sigma(1+e^{\sigma(i-1)t_1})} + \frac{\log(i+e^{\sigma(i-1)t_1})}{\sigma^2} + \frac{it_1 e^{\sigma it_1}}{\sigma(1+e^{\sigma it_1})} - \frac{\log(i+e^{\sigma it_1})}{\sigma^2}\right\}$$

$$+ (A^2_{i-1} - A_{i-1}B_i)\left[\frac{2\log 2}{\sigma^3} - (i-1)t_1\left\{\frac{(1+e^{\sigma(i-1)t_1})[e^{\sigma(i-1)t_1}(\sigma(i-1)t_{1-1})] - \sigma(i-1)t_1 e^{2\sigma(i-1)t_i}}{\sigma^2(1+e^{\sigma(i-1)t_1})^2}\right\}\right.$$

$$\left. + \frac{(i-1)t_1 e^{\sigma(i-1)t_1}}{\sigma(1+e^{\sigma(i-1)t_1})} - \frac{2\log(i+e^{\sigma(i-1)t_1})}{\sigma^3}\right] - (A_{i-1} - A_{i-1}B_i)^2\left[\frac{2\log 2}{\sigma^3} - it_1\left\{\frac{(1+e^{\sigma it_1})[e^{\sigma it_1}(\sigma it_{1-1})] - \sigma it_1 e^{2\sigma it_i}}{\sigma^2(1+e^{\sigma it_1})^2}\right\}\right.$$

$$\left.\left.\left. + \frac{it_1 e^{\sigma it_1}}{\sigma^2(1+e^{\sigma it_1})} - \frac{2\log(1+e^{\sigma it_1})}{\sigma^3}\right]\right] \right/ (A_{i-1} - B_i) \quad (4.13)$$

$$I^*_{12} = I^*_{21} = -E[\partial^2 \log L / \partial\theta\partial\sigma]$$

$$\frac{\partial^2 \log L}{\partial\theta\partial\sigma} =$$

$$= -E\left\{\sum_{i=1}^{k} n_i \{((i-1)t_1)^2(-A_{i-1}(i-1)t_1 B_i + A_{i-1}B_i it_1\right.$$

$$\left.\left\{\frac{(i-1)t_1 e^{\sigma(i-1)t_1}}{\sigma(1+e^{\sigma(i-1)t_1})} - \frac{\log(1+e^{\sigma(i-1)t_1})}{\sigma} - \frac{it_1 e^{\sigma it_1}}{\sigma(1+e^{\sigma it_1})} + \frac{\log(1+e^{\sigma it_1})}{\sigma^2}\right\} \right/ (A_{i-1} - B_i)^2 . \quad (4.14)$$

where $A^*_{i-1} = e^{-\theta(i-1)t_1}(1+e^{\sigma it_1})^{\frac{1}{\sigma}}$ and $B^*_i = e^{-\theta it_1}(1+e^{-\sigma(i-1)t_1})^{\frac{1}{\sigma}}$.

The asymptotic dispersion matrix of the corresponding MLEs - $\hat{\theta}_g, \hat{\sigma}_g$ of θ, σ is

$$\hat{D}[\hat{\theta}_g, \hat{\sigma}_g] = [\hat{I}_{11}\ \hat{I}_{22} - \hat{I}^2_{12}]^{-1}\begin{bmatrix}\hat{I}_{22} & -\hat{I}_{12}\\ -\hat{I}_{12} & \hat{I}_{11}\end{bmatrix} \quad (4.15)$$

V. Likelihood Ratio Criterion and Critical Values

Let us designate our distribution EHLAFRM as a null population say P_0. We call exponential distribution as alternate population say P_1. We propose a null hypothesis H_0 : " A given sample belongs to the population P_0" against an alternative hypothesis H_0 : " the sample belongs to population P_1".

Let L_1, L_0 respectively stand for the likelihood function of the sample with population P_1 and P_0. Both L_1 and L_0 contain the respective parameters of the population. The given sample is used to get the parameters of P_1, P_0, so that for the given sample the value of $\frac{L_1}{L_0}$ is now estimated. If H_0 is true, $\frac{L_1}{L_0}$ must be small, therefore for accepting H_0 with a given degree of confidence $\frac{L_1}{L_0}$ is compared with a critical value with the help of the percentiles in the sampling distribution of $\frac{L_1}{L_0}$. We have seen in section 4 how to get the estimates of parameters. But the sampling distribution of $\frac{L_1}{L_0}$ is not analytical, we therefore resorted to the empirical sampling distribution through simulation.

We have generated random samples of size 5(1)10, 15, 20, 25, 30 from the population P_0 with various parameter combinations and got the value of L_1, L_0 along with the estimates of respective parameters for each sample.

The percentiles of $\frac{L_1}{L_0}$ at various probabilities are computed and are given in Table 5.1.

Table : 5.1
Percentiles of L_1/L_0 for various values of λ and v

n	θ=1, σ=1					
	0.99	**0.975**	**0.95**	**0.05**	**0.025**	**0.01**
5	4619176.5	**10664.53**	**300.05**	1.493	1.4156	1.3273
6	485274.62	**3008.14**	**467.07**	1.6316	1.5641	1.4533
7	85986.66	**4144.9**	**742.15**	1.7808	1.6841	1.5473
8	368121.53	**6009.11**	**972.65**	1.9544	1.8182	1.6405
9	53868.42	**5818.12**	**1203.75**	2.1989	1.9845	1.8557
10	226564.95	**13751.23**	**1513.11**	2.3252	2.1098	1.9397
15	371196	**23611.12**	**4502.76**	3.8728	3.3998	3.0233
20	260060.65	**44157.11**	**10065.15**	6.3438	5.6431	4.9735
25	1605025.37	**271543.9**	**33833.48**	10.9535	9.7752	8.5964
30	4192773.25	**322768.93**	**38841.87**	18.0275	15.6319	12.8386

VI. POWER OF LIKELIHOOD RATIO CRITERION

In testing of hypothesis we know that the power of a test statistic is the complementary probability of accepting a false hypothesis at a given level of significance. Let us conventionally fix 2.5% and 5% level of significance, so that the percentiles in Table 5.1 under the column 0.975 and 0.95 shall become the critical values. We generate a random sample of sizes 5(1)10,15,20,25, 30 from the population P_1 namely exponential. At this sample we find the estimates of the parameters of P_1 and P_0 using the respective probability models. Accordingly we got the estimates of L_1, L_0 for the sample from P_1.

Over repeated simulation runs we got the proportion of values of $\frac{L_1}{L_0}$ that fall below the respective critical values of Table 5.1. These proportions would give the value of β, the probability of type II error. If the test statistic has a discriminating power β must be small so that the power l- β must be large. Various power values are given in Table 6.1. We see that as 'n' increases β is decreasing and hence l- β increases. We conclude that as long as n is less than 10, a given sample can distinguish between the populations P_1 and P_0 only with a probability of 10%. This probability is increasing with sample size 'n'. We therefore conclude that exponential can be a reasonable alternative to our model in small samples

Table: 6.1
Power of Likelihood Ratio Criterion

n	0.975	0.95
5	0.9790	0.9730
6	0.9820	0.9770
7	0.9840	0.9790
8	0.9880	0.9800
9	0.9910	0.9850
10	0.9930	0.9860
15	0.9950	0.9900
20	0.9960	0.9910
25	0.9980	0.9950
30	0.9900	0.9960

VII. Summary and Conclusions

Exponential and half logistic failure rate models are combined for the reliability studies and is named as Exponential - half logistic additive failure rate model . The distributional properties, estimation of parameters, testing of hypothesis and the power of likelihood ratio criterion about the proposed model are discussed. This paper can further be used for applying the likelihood ratio criterion with the proposed distribution as null population and the distribution under consideration as an alternative

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