

Prediction and Optimization of Compressive Strength of Sawdust Ash-Cement Concrete Using Scheffe's Simplex Design

¹D.O. Onwuka, ²L.Anyaogu, ³C.Chijioke and ⁴P.C. Okoye

^{1,2}Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria

³Department of Civil Engineering, Federal Polytechnic, Nekede, Owerri, Nigeria

⁴Department of Mechanical Engineering, Federal Polytechnic, Nekede, Owerri, Nigeria

Abstract- Frequent increase in the price of cement and other building materials across Nigeria has reawakened serious need to relate research to production, especially in the use of locally available materials as alternatives for construction of functional but low-cost dwellings in both rural and urban areas in the country. This article aimed at prediction and optimizing compressive strength of concrete when one of its conventional materials, cement is partially or wholly replaced by Sawdust ash. Sawdust ash (SDA) is a non-toxic construction waste material found in abundance in Nigeria. The effective utilization of this material as a component in concrete depends on the mix proportioning of the various component materials. A mathematical model to predict and optimize compressive of Sawdust ash- cement concrete was developed using Scheffe's five component second degree simplex lattices.

The model was used to optimize the compressive strength of concrete made from water, cement, sawdust ash, sand and granites. The results of the response function compared favourably with the corresponding experimental results and the predictions from the response function were tested for adequacy using the statistical student's t-test and found to be adequate at 95% confidence level. The optimum compressive strength of concrete at twenty-eight (28) days was found to be 20.44N/mm². This strength corresponds to a mix ratio of 0.5: 0.95: 0.05: 2.25: 4 (i.e. water: cement: sawdust ash: sand: granites). With the optimization function developed in this work, any desired compressive strength of sawdust ash-cement concrete can be predicted from known mix proportions and vice versa.

Index Terms- sawdust, optimization, scheffes, concrete, prediction, compressive strength

I. INTRODUCTION

One of the basic needs of man is housing. In any developing country like Nigeria, there is a perpetual problem of accommodation and inadequate housing. A recent investigation showed that more than seven million Nigerians have no accommodation (Punch, 2012). It is important to note that majority of housing units in Nigeria are constructed using concrete.

The construction industry relies heavily on conventional materials such as cement, granite and sand for the production of concrete. The high and increasing cost of these materials has

greatly hindered the development of shelter and other infrastructural facilities in Nigeria and other developing countries. There arises the need for engineering consideration of the use of cheaper and locally available materials to meet desired need enhance self efficiency, and lead to an overall reduction in construction cost for sustainable development.

Attempts have equally been made by various researchers to reduce the cost of its constituents and hence total construction cost by investigating and ascertaining the usefulness of materials which could be classified as agricultural or industrial waste. Some of these wastes include sawdust, pulverized fuel ash palm kernel shells, slag, fly ash etc which are produced from milling stations, thermal power station, waste treatment plants etc.

Sawdust is an industrial waste in the timber industry. It is obtained as loose particles or wood chippings from sawing of timber into standard useable sizes. It poses a nuisance to both the health and environment when not properly managed. It has pozzolanic properties and has been shown to react chemically with the calcium hydroxide released from the hydration of Portland cement, to form cement compounds (Elinwa and Mahmood, 2002). Active pozzolans gain their binding properties when they react with calcium hydroxide in lime or cement in presence of water. The advantages of using SDA for concrete production are numerous. It acts as a retarder prolonging the setting times, reduces the heat of hydration, encourages a healthier environment by reducing green gas emission and abundantly available as a waste. SDA has been used as partial replacement in mortar and concrete works (Elinwa and Mahmood, 2002; Elinwa and Ejeh, 2004). It has also been used as a powder material in the production of self compacting concrete (SCC) (Elinwa and Mamuda, 2008) and in combination with metakaolin as a ternary blend with 3 % added to act as an admixture.

In this work, a mathematical model for the prediction and optimization of Compressive Strength of concrete is developed with different percentages of saw dust ash as partial replacement of cement. This involves compression test of concrete from the different mix ratios where cement is partially replaced with sawdust ash. The results were used to develop scheffe's mathematical model. This model would be used to predict the compressive strength of concrete given any mix ratio or predict mix ratios given a particular Compressive Strength of concrete.

II. MATERIALS

The concrete test is a five component composite consisting cement, sawdust ash, fine aggregate, coarse aggregate and water. The cement used in this work is Dangote brand of ordinary Portland cement. It conforms to the requirement of BS 12 (1978). The coarse aggregate consists of crushed granite rock having a maximum size of 20mm. It was obtained from crushed rock industries quarry located in Ishiagu in Ebonyi state, Nigeria. Sand was obtained from Otamiri River was the fine aggregate. The sand was sharp and free from clay, debris and other deleterious materials. The grading of the sand was carried out to BS 812:103 (BS 812: Part 1, 1975). The sand belongs to grading zone C (Neville, A.M., 1996).

Sawdust is a by-product from timber, it is a waste product obtained during sawing of timber into standard sizes. The sawdust was obtained from timber milling market (ogbosisi) Owerri. This material was first dried to remove the natural moisture. The waste was burnt in an enclosure (i.e. open drum) at temperature of about 400-500°C to obtain sawdust ash. The ash was allowed to cool; thereafter the ash was sieved with 150µm sieve aperture to obtain the finest particle of material which approximates to the fineness of that of cement used

III. METHODS

The objective of this work was achieved by employing Scheffe's experimental mixture equation and experimental results obtained after compression test of the concrete cube of sawdust ash-cement concrete. These methods were used in the formulation of the model for optimization of the compressive strength of sawdust ash-cement concrete. The model developed was tested for adequacy using statistical student's t-test.

According to Scheffe (1958), a five-component mixture like Sawdust ash-cement concrete, the proportion, X_i of the i^{th} component of the mixture must satisfy the following constraint:

$$X_i \geq 0 \quad (i = 1, 2, 3, 4, 5) \quad (1)$$

And the sum of all proportions of the constituents of the five-component of Sawdust ash-cement concrete must be equal to unity,

i.e.

$$\sum_{i=1}^q X_i = 1 \quad (2)$$

For the five- component sawdust ash-cement concrete,

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (3)$$

The response sought for the performance criterion of interest (i.e. compressive strength of the Sawdust ash-cement concrete) is presented using a polynomial function of pseudo components. According to Onwuka et al (2011), the equation of response represented by a polynomial function is given by Eqn(4):

$$Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \dots + e \quad (4)$$

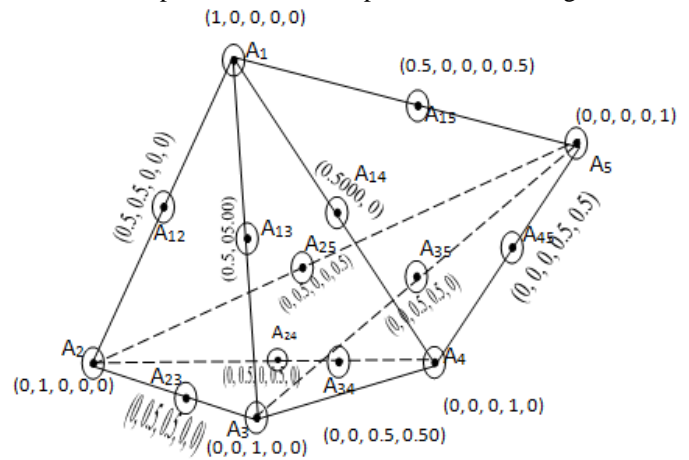
Where;

b_i, b_{ij}, b_{ijk} are constants; X_i, X_j, X_k are pseudo components and e is the random error term which represents the combined effect of all variables not included in the model.

The number of coefficients, k, of the polynomial, is determined using Eqn(5).

3.1 SIMPLEX DESIGN

Simplex is a factor space or a polygon. The simplest simplex is a straight line. A straight line is a one dimensional factor space. Other factor spaces could be two dimensional (a plane), three dimensional (a tetrahedron i.e. solids) or any other imaginary factor space, whose dimension is above three. Scheffe's factor was considered in this work for formulation of the response function. Scheffe's method of optimization is applicable to mixtures in which the desired response depends on the proportion of components present in the mixture, rather than on the quality of the mixture (Scheffe, H., 1958). Sawdust – cement concrete is a five-[component mixture consisting of water, cement, sawdust ash, sand and crushed granite rock. This was analyzed using four-dimensional simplex lattice. Four-dimensional simplex lattice factor space is shown in figure 1:



$$k = \frac{(q + m - 1)!}{(q - 1)! * m!} \quad (5)$$

Where q is the number of components of the mixture, and, m is the degree of the polynomial.

For the five pseudo component mixture with two degrees of reaction, the number of coefficients is fifteen. The equation of the response, Y, for the five-pseudo component mixture is given as:

$$Y = b_0 + \sum b_i X_i + \sum b_{ij} X_i X_j + \dots + e \quad (6)$$

Where $0 \leq i \leq j \leq 5$

Expanding Eqn(6) by substituting the values of 'i' and 'j' yields Eqn(7)

$$Y = b_0 + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1 X_5 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} X_2 X_5 + b_{34} X_3 X_4 + b_{35} X_3 X_5 + b_{45} X_4 X_5 + b_{11} X_1^2 + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 + b_{55} X_5^2 \quad (7)$$

Multiplying Eqn(3) by b_0 yields Eqn (8)

$$b_0 X_1 + b_0 X_2 + b_0 X_3 + b_0 X_4 + b_0 X_5 = b_0 \quad (8)$$

Multiplying Eqn(3) successively by X_1, X_2, X_3, X_4 and X_5 and rearranging the products, gives Eqn(9).

$$\left. \begin{aligned} X_1^2 &= X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4 - X_1 X_5 \\ X_2^2 &= X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4 - X_2 X_5 \\ X_3^2 &= X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4 - X_3 X_5 \\ X_4^2 &= X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4 - X_4 X_5 \\ X_5^2 &= X_5 - X_1 X_5 - X_2 X_5 - X_3 X_5 - X_4 X_5 \end{aligned} \right\} \quad (9)$$

Substituting Eqns (8) and (9) into Eqn (7) and simplifying the result, gives Eqn (10)

$$Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (10)$$

Where β_i and X_i are the coefficient of response equation and pseudo components of the mix respectively.

The coefficients β_i and β_{ij} are defined as follows:

$$\text{And } \left. \begin{aligned} \beta_i &= b_0 + b_i + b_{ii} \\ \beta_{ij} &= b_{ij} - b_{ii} - b_{jj} \end{aligned} \right\} \quad (11)$$

Eqn(5) can be represented in the form:

$$Y = \sum_{i=1}^5 \beta_i X_i + \sum_{i \leq j \leq 5} \beta_{ij} X_i X_j \quad (12)$$

If the response function is represented by Y, the response function for the pure component, Y_i , and that of binary mixture components, Y_{ij} , are Y_i and Y_{ij} respectively.

3.1.1 DETERMINATION OF THE COEFFICIENTS OF THE POLYNOMIAL FUNCTION

If the response function is represented by Y , the response function for the pure component, i , and that of binary mixture components, ij , are Y_i and Y_{ij} respectively, then,

$$Y_i = \sum_{i=1}^5 \beta_i X_i \tag{13}$$

And

$$Y_{ij} = \sum_{i=1}^5 \beta_i X_i + \sum_{i \leq j \leq 5} \beta_{ij} X_i X_j \tag{14}$$

The substitution of the values of the pseudo components X_1, X_2, X_3, X_4 and X_5 at the i^{th} on the lattice into Eqn (13), yields Eqn(15)

$$Y_i = \beta_i \tag{15}$$

And the substitution of the pseudo components X_1, X_2, X_3, X_4 and X_5 at the point ij , into Eqn (14), yields Eqn (16)

$$Y_{ij} = \frac{1}{2} \beta_i + \frac{1}{2} \beta_j + \frac{1}{4} \beta_{ij} \tag{16}$$

Rearrangement of Eqns (15) and (16) gives:

$$\beta_i = Y_i \tag{17}$$

$$\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \tag{18}$$

Let $n_i = Y_i$ and $n_{ij} = Y_{ij}$, hence, Eqns (17) and (18) will be:

$$\beta_i = n_i \tag{19}$$

$$\beta_{ij} = 4n_{ij} - 2n_i - 2n_j \tag{20}$$

Substituting Eqns(19) and (20) into Eqn(10) and simplifying further, gives

$$Y = n_1 X_1 (2X_1 - 1) + n_2 X_2 (2X_2 - 1) + n_3 X_3 (2X_3 - 1) + n_4 X_4 (2X_4 - 1) + n_5 X_5 (2X_5 - 1) + 4n_{12} X_1 X_2 + 4n_{13} X_1 X_3 + 4n_{14} X_1 X_4 + 4n_{15} X_1 X_5 + 4n_{23} X_2 X_3 + 4n_{24} X_2 X_4 + 4n_{25} X_2 X_5 + 4n_{34} X_3 X_4 + 4n_{35} X_3 X_5 + 4n_{45} X_4 X_5 \tag{21}$$

Eqn(21) is the response function for optimization of Sawdust ash-cement concrete consisting of five components. The terms n_i and n_{ij} are the responses (i.e. compressive strengths) at the points i and ij . The values of these responses are determined by carrying out compression tests on cubes obtained using sawdust ash as one of the components of concrete.

3.1.2 CONCRETE MIX RATIOS

Five mixed ratios (real and pseudo) that defined the vertices of the four-dimensional simplex lattice used in this study are shown in Table 1

Table 1: Five Mix Ratios (Actual and Pseudo) Obtained From Scheffe's (5, 2) Factor Space

points	Real Mix ratios					Pseudo Mix ratios				
	Water S ₁	Cement S ₂	SDA S ₃	Sand S ₄	Granite S ₅	Water X ₁	Cement X ₂	SDA X ₃	Sand X ₄	Granite X ₅
N ₁	0.500	0.95	0.05	2.25	4.00	1.0	0.0	0.0	0.0	0.0
N ₂	0.55	0.90	0.10	1.75	3.50	0.0	1.0	0.0	0.0	0.0
N ₃	0.60	0.85	0.15	2.25	4.25	0.0	0.0	1.0	0.0	0.0

N ₄	0.45	0.80	0.20	1.50	3.00	0.0	0.0	0.0	1.0	0.0
N ₅	0.65	0.75	0.25	2.50	5.00	0.0	0.0	0.0	0.0	1.0

Legend: SDA= Sawdust Ash

According to Osadebe, et.al (2008) the actual mix ratios relate with pseudo mix ratios in defined by the following equation:

$$\{S\} = [A]\{X\} \tag{22}$$

where S, A and X represent the real mix ratio, coefficient of relation matrix, and pseudo mix ratio respectively. According to Osadebe et.al(2008), matrix A can be taken to be the transpose of the first five real mix ratios shown in Table 1, and this resulted to matrix A:

$$[A] = \begin{bmatrix} 0.50 & 0.55 & 0.60 & 0.45 & 0.65 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 2.25 & 1.75 & 2.25 & 1.50 & 2.50 \\ 4.00 & 3.50 & 4.25 & 3.00 & 5.00 \end{bmatrix} \tag{23}$$

The five real and pseudo mix ratios in Table 1 corresponds to points of observations, N₁, N₂, N₃, N₄, N₅ located at the five vertices of the four-dimensional simplex lattice. For a (5, 2) simplex design, ten other observations are needed to add up to the first five to get a total of fifteen observations needed for the

development of the response function. The remaining ten points are located at the mid points of the lines joining the five vertices. On successive substitution of these ten pseudo mix ratios into Eqn(22), the real mix ratios corresponding to the pseudo ones were obtained. Their values are shown in Table 2.

Table2: Additional Ten Mix Ratios (Real and Pseudo) for formulation of the Optimization Function

points	Real Mix ratios					Pseudo Mix ratios				
	Water S ₁	Cement S ₂	SDA S ₃	Sand S ₄	Granite S ₅	Water X ₁	Cement X ₂	SDA X ₃	Sand X ₄	Granite X ₅
N ₁₂	0.525	0.925	0.075	2	3.75	0.5	0.5	0.0	0.0	0.0
N ₁₃	0.55	0.90	0.10	2.25	4.125	0.5	0.0	0.5	0.0	0.0
N ₁₄	0.475	0.875	0.125	1.875	3.5	0.5	0.0	0.0	0.5	0.0
N ₁₅	0.575	0.85	0.150	2.375	4.5	0.5	0.0	0.0	0.0	0.5
N ₂₃	0.575	0.875	0.125	2	3.875	0.0	0.5	0.5	0.0	0.0
N ₂₄	0.50	0.85	0.150	1.625	3.25	0.0	0.5	0.0	0.5	0.0
N ₂₅	0.60	0.825	0.175	2.125	4.25	0.0	0.5	0.0	0.0	0.5
N ₃₄	0.525	0.825	0.175	1.875	3.625	0.0	0.0	0.5	0.5	0.0
N ₃₅	0.625	0.80	0.20	2.375	4.625	0.0	0.0	0.5	0.0	0.5
N ₄₅	0.55	0.775	0.225	2	4	0.0	0.0	0.0	0.5	0.5

Legend: SDA = Sawdust ash

In order to validate the optimization function, extra fifteen points (C₁, C₂, C₃, C₄, C₅, C₆, C₇, C₈, C₉, C₁₀, C₁₁, C₁₂, C₁₃, C₁₄, and C₁₅) of observations were used. These observations provided control mix ratios needed to test the validity of the response function. The mix ratios (actual and Pseudo) for the entire work are shown in Table 3.

Table 3: Mix Ratios for Thirty Observations (Actual and Pseudo) Obtained From Scheffe’s Factor Space

points	Real Mix ratios					Pseudo Mix ratios				
	Water S ₁	Cement S ₂	SDA S ₃	Sand S ₄	Granite S ₅	Water X ₁	Cement X ₂	SDA X ₃	Sand X ₄	Granite X ₅
N ₁	0.500	0.95	0.05	2.25	4.00	1.0	0.0	0.0	0.0	0.0
N ₂	0.55	0.90	0.10	1.75	3.50	0.0	1.0	0.0	0.0	0.0
N ₃	0.60	0.85	0.15	2.25	4.25	0.0	0.0	1.0	0.0	0.0
N ₄	0.45	0.80	0.20	1.50	3.00	0.0	0.0	0.0	1.0	0.0
N ₅	0.65	0.75	0.25	2.50	5.00	0.0	0.0	0.0	0.0	1.0
N ₁₂	0.525	0.925	0.075	2	3.75	0.5	0.5	0.0	0.0	0.0
N ₁₃	0.55	0.90	0.10	2.25	4.125	0.5	0.0	0.5	0.0	0.0

N ₁₄	0.475	0.875	0.125	1.875	3.5	0.5	0.0	0.0	0.5	0.0
N ₁₅	0.575	0.85	0.150	2.375	4.5	0.5	0.0	0.0	0.0	0.5
N ₂₃	0.575	0.875	0.125	2	3.875	0.0	0.5	0.5	0.0	0.0
N ₂₄	0.50	0.85	0.150	1.625	3.25	0.0	0.5	0.0	0.5	0.0
N ₂₅	0.60	0.825	0.175	2.125	4.25	0.0	0.5	0.0	0.0	0.5
N ₃₄	0.525	0.825	0.175	1.875	3.625	0.0	0.0	0.5	0.5	0.0
N ₃₅	0.625	0.80	0.20	2.375	4.625	0.0	0.0	0.5	0.0	0.5
N ₄₅	0.55	0.775	0.225	2	4	0.0	0.0	0.0	0.5	0.5
CONTROL										
C ₁	0.550	0.900	0.100	2.083	3.92	0.333	0.333	0.333	0	0
C ₂	0.520	0.867	0.133	2.000	3.75	0.333	0.00	0.333	0.333	0
C ₃	0.533	0.833	0.167	2.083	4.00	0.333	0.00	0	0.333	0.333
C ₄	0.525	0.8375	0.125	2.9375	3.688	0.25	0.25	0.25	0.25	0.00
C ₅	0.55	0.8375	0.1625	2.125	4.0625	0.25	0.00	0.25	0.25	0.25
C ₆	0.575	0.8625	0.1375	2.1875	4.1875	0.25	0.25	0.25	0.00	0.25
C ₇	0.05375	0.9125	0.0875	2.125	3.9375	0.50	0.25	0.25	0.00	0.00
C ₈	0.60	0.825	0.175	2.375	4.5625	0.25	0.00	0.25	0.00	0.50
C ₉	0.52	0.890	0.11	2	3.75	0.40	0.20	0.20	0.20	0.00
C ₁₀	0.55	0.85	0.15	2.05	3.95	0.20	0.20	0.20	0.20	0.20
C ₁₁	0.545	0.855	0.145	2.10	4.0	0.30	0.10	0.20	0.20	0.20
C ₁₂	0.545	0.835	0.165	1.975	3.85	0.10	0.20	0.20	0.30	0.20
C ₁₃	0.57	0.8675	0.1325	2.2375	4.2375	0.35	0.15	0.25	0.00	0.25
C ₁₄	0.545	0.855	0.145	2.05	3.9375	0.25	0.20	0.15	0.20	0.20
C ₁₅	0.5375	0.8575	0.1425	2.15	4.075	0.45	0.05	0.00	0.20	0.30

3.2 COMPRESSIVE STRENGTH TEST

Compressive strength tests were carried out in order to determine the responses needed to formulate and validate the optimization function. The sawdust ash-cement concrete specimen, were concrete cubes measuring 150 x 150x 150 mm in size. A total of ninety cubes were produced from the thirty mix ratios given in Table 3, three cubes from each mix. The first set of 45 cubes made from first set of fifteen mix ratios, were used in

formulating the final optimization model, while the second set of 45 cubes from the second set of fifteen mix ratios, were used as control test for validating the optimization model. The concrete cubes were cured in water for 28 days, and tested in compression thereafter. The compression load at failure were recorded and used in Eqn(24) to determine the compressive strength of the river sand-termite soil concrete and presented in Table 4:

$$\text{Compressive strength} = \frac{\text{compressive load of cube at failure (N)}}{\text{cross sectional area of mould (mm}^2\text{)}} \quad (24)$$

IV. RESULTS AND ANALYSIS

The compressive strength of the cubes obtained from the experiment and model are given in Table 4

Table 4: Compressive strength (N/mm²) of the 28 day old concrete cubes

Points	Compressive Strength (N/mm ²)			Mean Experiment compressive strength (N/mm ²)	Predicted Compressive Strength of Concrete cubes (N/mm ²)
	Replicate 1	Replicate 2	Replicate 3		
N ₁	20.69	20.00	20.44	20.34	20.34
N ₂	19.56	19.33	19.11	19.30	19.30
N ₃	18.44	18.22	17.78	18.13	18.13
N ₄	13.33	12.00	15.56	14.45	14.45

N ₅	11.55	9.33	11.11	11.33	11.33
N ₁₂	18.22	20.00	20.66	19.63	19.63
N ₁₃	13.78	13.78	13.33	13.63	13.63
N ₁₄	16.44	10.67	16.89	16.67	16.67
N ₁₅	8.66	9.33	9.33	9.11	9.11
N ₂₃	21.33	15.56	16.00	17.66	17.66
N ₂₄	15.56	16.00	13.30	15.78	15.78
N ₂₅	14.67	16.00	8.89	15.34	15.34
N ₃₄	13.78	15.11	11.58	13.48	13.48
N ₃₅	14.67	14.22	16.44	15.10	15.10
N ₄₅	20.00	18.22	17.78	18.67	18.67
C ₁	17.47	18.08	16.89	17.48	18.42
C ₂	13.55	15.56	15.11	14.74	13.57
C ₃	15.90	15.33	15.51	15.58	14.62
C ₄	16.89	16.11	16.35	16.45	16.44
C ₅	14.34	13.89	14.28	14.17	13.64
C ₆	14.78	14.56	14.67	14.67	15.24
C ₇	17.22	17.11	17.09	17.14	18.87
C ₈	11.604	11.33	11.51	11.48	10.70
C ₉	16.82	16.36	16.00	16.39	17.23
C ₁₀	14.39	14.29	14.22	14.30	15.59
C ₁₁	13.47	13.31	13.72	13.50	14.64
C ₁₂	14.80	14.88	14.72	14.80	15.78
C ₁₃	12.88	13.03	13.24	13.05	14.01
C ₁₄	14.68	14.68	14.38	14.58	15.79
C ₁₅	13.11	13.04	13.18	13.11	14.29

4.1 OPTIMIZATION FUNCTION FOR PREDICTING THE COMPRESSIVE STRENGTH OF THE CONCRETE

The final optimization function is obtained by substituting compressive strength of concrete cubes from the first fifteen points of observations (N₁, N₂, N₃, N₄, N₅, N₆, N₇, N₈, N₉, N₁₀, N₁₁, N₁₂, N₁₃, N₁₄, and N₁₅) into Eqn (21) to obtain:.

$$\begin{aligned}
 Y = & 20.34X_1 + 19.30X_2 + 18.13X_3 + 4.45X_4 + 11.33X_5 + 19.24X_1X_2 - 22.42X_1X_3 - 2.90X_1X_4 \\
 & - 26.9X_1X_5 - 4.22X_2X_3 - 4.38X_2X_4 + 0.1X_2X_5 - 11.24X_3X_4 + 1.48X_3X_5 \\
 & + 23.12X_4X_5 \qquad \qquad \qquad (25)
 \end{aligned}$$

The Eqn (25) is the final function for the optimization of compressive strength of sawdust ash-cement concrete.

Test of Adequacy of the Model

The test for adequacy of second degree polynomial was done using statistical student's t-test at 95% accuracy level. The compressive strength at the control points (i.e. C₁, C₂, C₃, C₄, C₅, C₆, C₇, C₈, C₉, C₁₀, C₁₁, C₁₂, C₁₃, C₁₄, C₁₅) were used for the test. The following two hypotheses were tested using statistical student's t-test.

- a) **Null Hypothesis:** There is no significant difference between the laboratory concrete cube strengths and predicted compressive strength results at 95% accuracy level.
- b) **Alternative Hypothesis:** There is a significant difference between the laboratory concrete cube strengths and predicted strength compressive strength results at 95% accuracy level.

The test is carried out and presented in Table 5 using the following equations:

Let:

Y_E = Responses (compressive strength) from the experiment

Y_M = Responses (compressive strength) from the Second degree polynomial equation

N = Number of observations

D_i = Difference of Y_E and Y_M
 $\sum D_i$

$D_A = \frac{\sum D_i}{N}$ = Mean of difference of Y_E and Y_M

$$S^2 = \frac{\sum(DA - D_i)^2}{N-1} = \text{Variance of difference of } D_i \text{ and } D_A$$

$$t = \frac{S D_i}{S} = \text{Calculated value of } t$$

Table 5: Statistical Student's t-test for the model

TWO-TAILED t-TEST					
OP	Y_E	Y_M	$D_i = Y_E - Y_M$	$D_A - D_i$	$(D_A - D_i)^2$
C ₁	17.48	18.42	-0.94	0.447	0.1998
C ₂	14.74	13.57	1.17	-1.663	2.7656
C ₃	15.58	14.62	0.96	-1.453	2.1112
C ₄	16.45	16.44	0.01	-0.503	0.2530
C ₅	14.17	13.64	0.53	-1.023	1.0465
C ₆	14.67	15.24	-0.57	0.077	0.0059
C ₇	17.14	18.87	-1.73	1.237	1.5302
C ₈	11.48	10.70	0.78	-1.273	1.6205
C ₉	16.39	17.23	-0.84	0.347	0.1204
C ₁₀	14.30	15.59	-1.29	0.797	0.6352
C ₁₁	13.50	14.64	-1.14	0.647	0.4186
C ₁₂	14.80	15.78	-0.98	0.487	0.2372
C ₁₃	13.05	14.01	-0.96	0.467	0.2181
C ₁₄	14.58	15.79	-1.21	0.717	0.5141
C ₁₅	13.11	14.29	-1.18	0.687	0.4720
		S D _i	= -7.39	S (D _A - D _i) ²	= 12.1483

Legend: OP is the observation point

Here,

$$S D_i = -7.39$$

$$N = 15$$

$$D_A = \frac{\sum D_i}{N} = -0.493$$

$$S^2 = \frac{\sum(DA - D_i)^2}{N-1} = 0.8677$$

$$S = \sqrt{S^2} = 0.9315$$

Actual value of total variation in t-test

$$t = \frac{S D_i}{S} = -2.05$$

Allowable total variation in t-test

$$\text{Degree of freedom} = N-1 = 15-1 = 14$$

$$5\% \text{ significance for Two-Tailed Test} = 2.5\%$$

$$1 - 2.5\% = 97.5\% = 0.975$$

Allowable Total Variation in t-test = $t_{(0.975, N-1)} = t_{(0.975, 14)} = 2.14$ (Obtained from statistical table).

From table 5, the calculated $t = 2.05$. Thus, $t_{(table)} > t_{(calculated)}$

This implied that difference between the two set of cubes compressive strength is less than allowable difference. Hence null hypothesis is accepted and alternative hypothesis rejected. Hence, the model is adequate.

V. CONCLUSION

Using scheffe's five component second degree polynomial regression equation, mix design model for a five component saw dust ash-cement concrete cube was developed. This model could predict the compressive strength of concrete cube when the mix ratios are known and vice versa. The predictions from this model were tested at 95% accuracy level using statistical student's t-test and found to be adequate. The maximum strength predicted by this model is 20.44N/mm². This strength is from mix ratio 0.5: 0.95: 0.05: 2.25: 4.0 (corresponding to the water: cement: sawdust ash: sand: granites).

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AUTHORS

First Author – D.O. Onwuka, Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria
Second Author – L.Anyaogu, Department of Civil Engineering, Federal University of Technology, Owerri, Nigeria
Third Author – C.Chijioke, Department of Civil Engineering, Federal Polytechnic, Nekede, Owerri, Nigeria
Fourth Author – P.C. Okoye, Department of Mechanical Engineering, Federal Polytechnic, Nekede, Owerri, Nigeria