Modeling of the Compressive Strength of River Sand-Termite Soil Concrete Using Osadebe's Second- Degree Polynomial Function

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Abstract- In this article, a model of the compressive strength of river sand- termite soil concrete was formulated using Osadebe's second degree polynomial equation. The formulated model can be used for determining the compressive strength obtainable from a given mix proportions of its constituents.

Besides, it can be used for determining the mix proportions that can yield a given or desired compressive strength of a five-component concrete containing a percentage of termite soil. The validity of the formulated model was tested Using student's t-test. At 5 percent significant level, the model was found to be valid. This implies that there is no significant difference between the results. The results obtained from the model agrees with the corresponding experimental results.

Index Terms- termite soil, modeling, concrete, compressive strength, river sand, Osadebe polynomial function

I. Introduction

Concrete is a composite material composed primarily of aggregates (fine and coarse), cement and water. Cement and aggregates remain the major constituents while aggregates alone constitute a major portion of the volume of concrete. According to Alexander and Sydney (2005), between 70 to 80 per cent out of the total volume of concrete is occupied by aggregate. With this large proportion of the concrete occupied by aggregate, it is expected that aggregate will have a profound influence on the concrete properties, production cost and its general performance. Aggregates are generally coarse gravel or crushed rocks such as limestone or granite along with fine aggregate such as sand. These are responsible partly for the durability, quality and strength of the resulting concrete. Portland cement and other cementitious materials such as fly ash and slag cement etc. serve as a binder for the aggregates, while various chemical admixtures may be added to achieve varied properties.

Aggregates for concrete are either naturally or artificially produced. Natural aggregates are seen on the sea-shore, in river beds or in deserts while the artificially produced aggregates are found in crusher plants. Sand as fine aggregate is naturally extracted from sand quarries.

In developing countries like Nigeria, the high costs of procuring concrete materials for construction works, have over the years constrained the users to compromise quality, this has resulted in poor performance of infrastructure in service; a major factor that has contributed to the increase in maintenance costs and the series of collapsed structures with attendant loss of lives and properties (Falade, et.al, 2010). For some time now, the Nigerian government has been clamoring for the use of local materials and recycled waste materials in the construction industry to limit costs of construction. There has therefore been a greater call for the sourcing and development of alternative, non-conventional local construction materials.

Amongst the basic constituent of concrete, aggregate and cement are the most expensive; thus, the major determinant factors in the cost of producing concrete are aggregate and cement. Following the fact that concrete is one of the most used man-made material in the world, its demands has been on the increase despite its high cost. Nigeria in the pursuit of vision 2020, has re-awakened serious need to relate research to production especially in the use of local materials as alternatives for the construction of functional, but low-cost dwellings both in the urban and rural areas of Nigeria. Presently, there is need for exploration of means to partially or totally replacing sand in a concrete mixture with a local material while still maintaining the required high compressive strength.

The high demand for concrete in construction industry has resulted to a rapid decrease in natural soil deposit such as the river sand and granite, leading to escalation of environmental problem. The excessive usage of natural soil deposit causes ecological imbalance, thus the need to find and explore an alternative material that could be used as a replacement to the conventional aggregate becoming necessary.

In this work, a mathematical model for the optimization of compressive strength of concrete is developed with different percentages of termite soil as partial replacement of fine aggregate. Concrete produced from the different mix ratios, with fine aggregate partially replacing termite soil were subjected to compression test. Then, the model for determining the compressive strength of river sand-termite soil concrete, was developed using Osadebe's second degree polynomial function and the compression test result.

II. Theoritical Backgroud

According to Osadebe (2003), concrete is a four-component composite produced by mixing water, cement, fine aggregate (sand) and coarse aggregate. These ingredients are mixed in reasonable proportions to achieve desired strength of the concrete. In this paper, the fifth component termite soil shall be added as one of the component materials of concrete.

Let us consider an arbitrary amount 'S' of a given concrete mixture and let the portion of the ith component of the five constituent materials of the concrete be S_i, (where i= 1, 2, 3, 4, 5). This was carried out with the principle of absolute mass.

Thus,

$$S_1 + S_2 + S_3 + S_4 + S_5 = S \qquad (1)$$

where $S_1, S_2, S_3, S_4 \; and \; S_5$ are the quantities of water, cement, river Sand, termite soil and coarse aggregate.

Dividing Eqn(1) through by S, gives:

$$\frac{S_1}{S} + \frac{S_2}{S} + \frac{S_3}{S} + \frac{S_4}{S} + \frac{S_5}{S} = 1 \qquad (2)$$

where $\frac{S_i}{S}$ is the fractional proportion of the ith constituent component of the concrete mixture.

Let $\frac{S_i}{S} = Z_i$, (3)

Substituting Eqn (3) into Eqn (2) yields:

$$Z_1 + Z_2 + Z_3 + Z_4 + Z_5 = 1 \qquad (4)$$

where Z_1, Z_2, Z_3, Z_4 and Z_5 are fractional proportions of water, cement, river sand, termite soil and coarse aggregate respectively. In general, for any given concrete mixture, exists a vector Z (Z_1, Z_2, Z_3, Z_4). In this paper where five component materials are considered, the vector is transformed to Z (Z_1, Z_2, Z_3, Z_4, Z_5) whose elements satisfy Eqn (4). Also, for each value of Z_i, the following inequality holds:

$$Z_i > 0 \qquad (5)$$

It is important to note that the proportion of relative constituent ingredient of concrete govern the strength of the concrete at its hardened state. Thus, the compressive strength, Y, of concrete can be expressed mathematically using Eqn (6) as:

$$Y = f(Z_1, Z_2, Z_3, Z_4, Z_5) \qquad (6)$$

where $f(Z_1, Z_2, Z_3, Z_4, Z_5)$ is a multi-variate response function whose variables Z_i are subject to the constraints defined in Eqns (4) and (5).

2.1 Osadebe's regression equation

This theory assumed that the response function is continuous and differentiable with respect to its variables, Z_i, hence, it can be expanded using Taylor's series in the neighbourhood of a chosen point $Z^{(0)} = Z_1^{(0)} + Z_2^{(0)} + Z_3^{(0)} + Z_4^{(0)} + Z_5^{(0)}$ as follows:

$$f(Z) = \sum f^m\left(Z^{(0)}\right) + \frac{\left(Z_i - Z^{(0)}\right)}{m!}$$

for $0 \le m \le \infty$

where m is the degree of polynomial of the response function and $f(Z)$ is the response function. Expanding Eqn (7) to the second order yields:

$$f(Z) = f\left(Z^{(0)}\right) + \sum_{i=1}^{5} \frac{\partial f\left(Z^{(0)}\right)}{\partial Z_i}\left(Z_i - Z_i^{(0)}\right) + \frac{1}{2}! \sum_{i=1}^{4}\sum_{j=1}^{5} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i \partial Z_j}\left(Z_i - Z_i^{(0)}\right)\left(Z_j - Z_j^{(0)}\right) + \frac{1}{2}! \sum_{i=1}^{5} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i^2}(Z_i - Z_i^{(0)})^2 + \cdots \qquad (8)$$

The point, $Z^{(0)}$ will be chosen as the origin for convenience sake without loss of generality of the formulation. The predictor, Z_i is not the actual portion of the mixture component, rather, it is the ratio of the actual portions to the quantity of concrete. For convenience sake, let Z_i be called the term of "fractional portion". The actual portions of the mixture components are S_i.

Consequently, the origin, $Z^{(0)} = 0$, implies that:

$$Z_1^{(0)} = 0,\; Z_2^{(0)} = 0,\; Z_3^{(0)} = 0,\; Z_4^{(0)} = 0,\; Z_5^{(0)} = 0 \qquad (9)$$

Let: $b_0 = f(0)$, $b_i = \frac{\partial f(0)}{\partial Z_i}$, $b_{ij} = \frac{\partial^2 f(0)}{\partial Z_i \partial Z_j}$ and $b_{ii} = \frac{\partial^2 f(0)}{\partial Z_i^2}$

Then, Eqn (8) can be rewritten as follows:

$$f(0) = b_0 + \sum_{i=1}^{5} b_i Z_i + \sum_{i=1}^{4}\sum_{j=1}^{5} b_{ij} Z_i Z_j + \sum_{i=1}^{5} b_{ii} Z_i^2 + \cdots \qquad (10)$$

Multiplying Eqn (4) by b_0, gives the following expression:

$$b_0 = b_0Z_1 + b_0Z_2 + b_0 Z_3 + b_0Z_4 + b_0Z_5 \qquad (11)$$

Similarly, multiplying Eqn (4) by Z_i will give the following expression:

$$Z_1 = Z_1^2 + Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_5 \qquad (12a)$$
$$Z_2 = Z_1 Z_2 + Z_2^2 + Z_2Z_3 + Z_2Z_4 + Z_2Z_5 \qquad (12b)$$
$$Z_3 = Z_1 Z_3 + Z_2Z_3 + Z_3^3 + Z_3Z_4 + Z_3Z_5 \qquad (12c)$$
$$Z_4 = Z_1 Z_4 + Z_2Z_4 + Z_4Z_3 + Z_4^2 + Z_4Z_5 \qquad (12d)$$
$$Z_5 = Z_1 Z_5 + Z_2Z_5 + Z_3Z_5 + Z_4Z_5 + Z_5^2 \qquad (12e)$$

Rearranging Eqns (12a) to (12e), the expression for Z_i^2 becomes;

$$Z_1^2 = Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5 \qquad (13a)$$
$$Z_2^2 = Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 - Z_2Z_5 \qquad (13b)$$
$$Z_3^2 = Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 - Z_3Z_5 \qquad (13c)$$
$$Z_4^2 = Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 - Z_4Z_5 \qquad (13d)$$
$$Z_5^2 = Z_5 - Z_1Z_5 - Z_2Z_5 - Z_3Z_5 - Z_4Z_5 \qquad (13e)$$

Substituting Eqn (13a) to (13e) into Eqn (10) and setting $f(0) = Y$ will give in the expanded form below:

$$\begin{aligned} Y = {} & b_0Z_1 + b_0Z_2 + b_0Z_3 + b_0Z_4 + b_0Z_5 + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4 + b_5Z_5 + b_{12}Z_1Z_2 + \\ & b_{13}Z_1Z_1Z_3 + b_{14}Z_1Z_4 + b_{15}Z_1Z_5 + b_{23}Z_2Z_3 + b_{24}Z_2Z_4 + b_{25}Z_2Z_5 + b_{34}Z_3Z_4 + \\ & b_{35}Z_3Z_5 + b_{45}Z_4Z_5 + b_{11}(Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 - Z_1Z_5) + \\ & b_{22}(Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 - Z_2Z_5) + b_{33}(Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 - Z_3Z_5) + \\ & b_{44}(Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 - Z_4Z_5) - b_{55}(Z_5 - Z_1Z_5 - Z_2Z_5 - Z_3Z_5 - Z_4Z_5) \end{aligned} \qquad (14a)$$

Factorizing Eqn (14a) gives

$$\begin{aligned} Y = {} & Z_1(b_0 + b_1 + b_{11}) + Z_2(b_0 + b_2 + b_{22}) + Z_3(b_0 + b_3 + b_{33}) + Z_4(b_0 + b_4 + b_{44}) + Z_5(b_0 + b_5 + b_{55}) \\ & + Z_1Z_2(b_{12} - b_{11} - b_{22}) + Z_1Z_3(b_{13} - b_{11} - b_{33}) + Z_1Z_4(b_{14} - b_{11} - b_{44}) \\ & + Z_1Z_5(b_{15} - b_{11} - b_{55}) + Z_2Z_3(b_{23} - b_{22} - b_{33}) + Z_2Z_4(b_{24} - b_{22} - b_{44}) \\ & + Z_2Z_5(b_{25} - b_{22} - b_{55}) + Z_3Z_4(b_{34} - b_{33} - b_{44}) + Z_3Z_5(b_{35} - b_{33} - b_{55}) \\ & + Z_4Z_5(b_{45} - b_{44} - b_{55}) \end{aligned} \qquad (14b)$$

The summation of the constants is equal to a constant thus; let

$$\propto_i = b_0 + b_i + b_{ii} \ and \ \ \propto_{ij} = b_{ij} + b_{ii} + b_{jj} \qquad (15)$$

Eqn (14b) becomes:

$$\begin{aligned} Y = {} & \propto_1 Z_1 + \propto_2 Z_2 + \propto_3 Z_3 + \propto_4 Z_4 + \propto_5 Z_5 + \propto_{12} Z_1Z_2 + \propto_{13} Z_1Z_3 + \propto_{14} Z_1Z_4 + \propto_{15} Z_1Z_5 + \propto_{23} Z_2Z_3 + \\ & \propto_{24} Z_2Z_4 + \propto_{25} Z_2Z_5 + \propto_{34} Z_3Z_4 + \propto_{35} Z_3Z_5 + \propto_{45} Z_4Z_5 \end{aligned} \qquad (16a)$$

Rewriting Eqn (16a) in a compact form, gives:

$$Y = \sum_{i=1}^{5} \alpha_i Z_i + \sum_{1 \le i \le j}^{5} \alpha_{ij} Z_i Z_j \qquad (16b)$$

And, Y is the response function at any point of observation Z_i and Z_j are the predictors, and $\propto_i$ and $\propto_{ij}$ are the coefficients of the regression equation.

2.2 The Coefficients of the Regression Equation

Let the n^{th} response (compressive strength at n^{th} observation point) be $Y^{(n)}$ and the vector of the corresponding set of variables be as follows:

$$Z^{(n)} = (Z_1^{(n)}, Z_2^{(n)}, Z_3^{(n)}, Z_4^{(n)}, Z_5^{(n)})$$

Different points of observation will have different predictor at constant coefficient. At n^{th} observation point, the response function, $Y^{(n)}$, will correspond with the predictors $Z_i^{(n)}$.
Thus,

$$Y^{(n)} = \sum_{i=1}^{5} \alpha_i Z_i^{(n)} + \sum_{1\leq i\leq j}^{5} \alpha_{ij} Z_i Z_j^{(n)} \qquad (16)$$

Where $1\leq i\leq j\leq 5$ and $n = 1, 2, 3, \ldots, 15$
Eqn (16) can be written in a matrix form as,

$$[Y^{(n)}] = [Z^{(n)}]\,[\propto] \qquad (17)$$

Expanding Eqn (17) yields:

$$\begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ \cdot \\ \cdot \\ Y^{(15)} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \ldots & Z_4^{(1)}Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \ldots & Z_4^{(2)}Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \ldots & Z_4^{(3)}Z_5^{(3)} \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \ldots & Z_4^{(15)}Z_5^{(15)} \end{pmatrix} \cdot \begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ \propto_{45} \end{pmatrix} \qquad (18)$$

The actual mixture proportions $S_i^{(n)}$ and the corresponding fractional portions, $Z_i^{(n)}$ are shown in Table 1. And the values of the constant coefficient $\propto$ in Eqn (17) are determined with the values of $Y^{(n)}$ and $Z^{(n)}$. Rearranging Eqn (17) gives

$$[\propto] = [Z^{(n)}]^{-1}\,[Y^{(n)}] \qquad (19)$$

Expressing equation (19) in expanded form yields:

$$\begin{pmatrix} \propto_1 \\ \propto_2 \\ \propto_3 \\ \cdot \\ .\propto_{45} \end{pmatrix} = \begin{pmatrix} Z_1^{(1)} & Z_2^{(1)} & Z_3^{(1)} & \ldots & Z_4^{(1)}Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & \ldots & Z_4^{(2)}Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & \ldots & Z_4^{(3)}Z_5^{(3)} \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & \ldots & Z_4^{(15)}Z_5^{(15)} \end{pmatrix}^{-1} \cdot \begin{pmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ Y^{(15)} \end{pmatrix} \qquad (20)$$

The values of $\propto_1$ to $\propto_{45}$ are obtained from Eqn (20) and substituted into Enq. (16a) to obtain the regression equation. The values of $Z^{(n)}$] matrix are shown in Table 2 and the values of the inverse of [$Z^{(n)}$] matrix are presented in Table 3; while the values of [$Y^{(n)}$] matrix are obtained from the experimental investigation.

III. Materials

The materials used for the laboratory test included:

(i) Water that is good for drinking obtained from a borehole at the premises of Federal University of Technology Owerri , Imo State, Nigeria. The water was clean, fresh and free from dirt, unwanted chemicals or rubbish that may affect the desired quality of concrete.

(ii) Ibeto cement, a brand of ordinary Portland cement that conforms to BS 12(1978)

(iii) The fine aggregate, river sand used for this research work were obtained from a flowing river (Otamiri). As at the time of purchase the sharp river sand was wet but free from debris and deleterious matter and clay.

(iv) The coarse aggregate used for this research work was granite chippings quarried from a quarry in Ishiagu, along Enugu-Port Harcourt express way, a town in Ebonyi state, Nigeria. The granite was sun dried for seven days so they can be free from water. They were sieved through a 20mm British test sieve and the materials passing through the sieve were used to produce concrete with various proportions of termite soil.

(v) The termite soil was obtained from termite mound above ground termite nest, which was located at the strategic places in the premises of Federal University of Technology Owerri. The soil was sun- dried for two weeks; they were broken down into finer, and sieved to sand-sized particles before it was used in the preparation of concrete.

The mix ratios used for the simplex design points were obtained using pentahedron factor space for five –component mixture.

IV. Compressive Strength Test

Batching of the ingredients was done by mass. Cement was thoroughly mixed together in the dry state with a mixture of river sand/termite soil and granite and then, water added. The mixing continued until a uniform and consistent concrete mix is obtained. The entire concrete was cast in concrete mould of sizes 150 x 150x 150 mm. In all, sixty concrete cubes, two from each mix incorporating various proportions of termite soil, were cast and cured in a curing water tank for 28 days, and then crushed in a universal testing machine. Thirty of the concrete cubes served as control test. Compressive strength of the cubes was calculated using Eqn (21):

$$\text{Compressive strength} = \frac{\textit{compressive load of cube at failure } (N)}{\textit{cross sectional area of mould } (mm^2)} \quad (21)$$

The results of the compressive strength test of the concrete cubes are presented in Table 4

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Table 1: Selcted mix proportions, S, and their corresponding fractional portions,Z, based on Osadebe's second-degree polynomial

OP	Mix Proportions, S_i					$\sum_{i=1}^{n=5} S_i$	FractionaL Portions, Z_i				
	S_1	S_2	S_3	S_4	S_5	S	Z_1	Z_2	Z_3	Z_4	Z_5
1	0.50	1.0	1.42	0.08	2.50	5.500	0.090909	0.181818	0.258182	0.014545	0.454545
2	0.55	1.0	1.8	0.2	3.00	6.550	0.083969	0.152672	0.274809	0.030534	0.458015
3	0.6	1.0	1.70	0.30	4.00	7.600	0.078947	0.131579	0.223684	0.039474	0.526316
4	0.45	1.0	1.60	0.40	4.25	7.700	0.058442	0.129870	0.207792	0.051948	0.551948
5	0.65	1.0	1.80	0.60	3.50	7.550	0.086093	0.132450	0.238411	0.079470	0.463576
12	0.525	1.0	1.61	0.14	2.75	6.025	0.087137	0.165975	0.267220	0.023237	0.456432
13	0.55	1.0	1.56	0.19	3.25	6.550	0.083969	0.152672	0.238168	0.029008	0.496183
14	0.475	1.0	1.51	0.24	3.375	6.600	0.071970	0.151515	0.228788	0.036364	0.511364
15	0.575	1.0	1.61	0.34	3.00	6.525	0.088123	0.153257	0.246743	0.052107	0.459770
23	0.575	1.0	1.75	0.25	3.500	7.075	0.081272	0.141343	0.247350	0.035336	0.494700
24	0.5	1.0	1.7	0.3	3.625	7.125	0.070175	0.140351	0.238596	0.042105	0.508772
25	0.6	1.0	1.8	0.4	3.250	7.050	0.085106	0.141844	0.255319	0.056738	0.460993
34	0.525	1	1.65	0.35	4.125	7.650	0.068627	0.130719	0.215686	0.045752	0.539216
35	0.625	1.0	1.75	0.45	3.750	7.575	0.082508	0.132013	0.231023	0.059406	0.495050
45	0.55	1.0	1.7	0.5	3.875	7.625	0.072131	0.131148	0.222951	0.065574	0.508197
CONTROL											
AC_1	0.546	0.993	1.828	0.192	3.143	6.702	0.0815	0.148	0.273	0.029	0.469
AC_2	0.513	0.993	1.562	0.258	3.556	6.882	0.075	0.144	0.227	0.037	0.517
AC_3	0.530	0.993	1.595	0.357	3.391	6.866	0.077	0.145	0.232	0.052	0.494
AC_4	0.525	1.0	1.630	0.245	3.438	6.838	0.077	0.146	0.238	0.036	0.503
AC_5	0.550	1.0	1.630	0.345	3.563	7.088	0.078	0.141	0.230	0.049	0.503
AC_6	0.575	1.0	1.680	0.295	3.250	6.800	0.085	0.147	0.247	0.043	0.478
AC_7	0.538	1.0	1.585	0.165	3.000	6.288	0.086	0.159	0.252	0.026	0.477
AC_8	0.600	1.0	1.680	0.395	3.375	7.050	0.085	0.142	0.238	0.056	0.479
AC_9	0.520	1.0	1.588	0.212	3.250	6.570	0.079	0.152	0.242	0.032	0.495
AC_{10}	0.550	1.0	1.664	0.316	3.450	6.980	0.079	0.143	0.238	0.045	0.494
AC_{11}	0.545	1.0	1.626	0.304	3.400	6.875	0.079	0.145	0.237	0.044	0.494
AC_{12}	0.545	1.0	1.682	0.348	3.625	7.200	0.076	0.139	0.233	0.048	0.503
AC_{13}	0.570	1.0	1.642	0.283	3.200	6.695	0.085	0.149	0.245	0.042	0.478
AC_{14}	0.545	1.0	1.650	0.305	3.375	6.875	0.079	0.145	0.240	0.044	0.491
AC_{15}	0.538	1.0	1.589	0.306	3.175	6.608	0.081	0.151	0.240	0.046	0.480

Legend: OP is Observation Point

Table 2: $Z^{(n)}$ Matrix

OP	Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
1	0.0909	0.1818	0.2582	0.0145	0.4545	0.0165	0.0235	0.0013	0.0413	0.0469	0.0026	0.0826	0.0038	0.1174	0.0066
2	0.0840	0.1527	0.2748	0.0305	0.4580	0.0128	0.0231	0.0026	0.0385	0.0420	0.0047	0.0699	0.0084	0.1259	0.0140
3	0.0789	0.1316	0.2237	0.0395	0.5263	0.0104	0.0177	0.0031	0.0416	0.0294	0.0052	0.0693	0.0088	0.1177	0.0208
4	0.0584	0.1299	0.2078	0.0519	0.5519	0.0076	0.0121	0.0030	0.0323	0.0270	0.0067	0.0717	0.0108	0.1147	0.0287
5	0.0861	0.1325	0.2384	0.0795	0.4636	0.0114	0.0205	0.0068	0.0399	0.0316	0.0105	0.0614	0.0189	0.1105	0.0368
12	0.0871	0.1660	0.2672	0.0232	0.4564	0.0145	0.0233	0.0020	0.0398	0.0444	0.0039	0.0758	0.0062	0.1220	0.0106
13	0.0840	0.1527	0.2382	0.0290	0.4962	0.0128	0.0200	0.0024	0.0417	0.0364	0.0044	0.0758	0.0069	0.1182	0.0144
14	0.0720	0.1515	0.2288	0.0364	0.5114	0.0109	0.0165	0.0026	0.0368	0.0347	0.0055	0.0775	0.0083	0.1170	0.0186
15	0.0881	0.1533	0.2467	0.0521	0.4598	0.0135	0.0217	0.0046	0.0405	0.0378	0.0080	0.0705	0.0129	0.1134	0.0240
23	0.0813	0.1413	0.2473	0.0353	0.4947	0.0115	0.0201	0.0029	0.0402	0.0350	0.0050	0.0699	0.0087	0.1224	0.0175
24	0.0702	0.1404	0.2386	0.0421	0.5088	0.0098	0.0167	0.0030	0.0357	0.0335	0.0059	0.0714	0.0100	0.1214	0.0214
25	0.0851	0.1418	0.2553	0.0567	0.4610	0.0121	0.0217	0.0048	0.0392	0.0362	0.0080	0.0654	0.0145	0.1177	0.0262
34	0.0686	0.1307	0.2157	0.0458	0.5392	0.0090	0.0148	0.0031	0.0370	0.0282	0.0060	0.0705	0.0099	0.1163	0.0247
35	0.0825	0.1320	0.2310	0.0594	0.4950	0.0109	0.0191	0.0049	0.0408	0.0305	0.0078	0.0654	0.0137	0.1144	0.0294
45	0.0721	0.1311	0.2230	0.0656	0.5082	0.0095	0.0161	0.0047	0.0367	0.0292	0.0086	0.0666	0.0146	0.1133	0.0333

Table 3: Inverse of $Z^{(n)}$ Matrix

S/N	Z_1	Z_2	Z_3	Z_4	Z_5	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_2Z_3	Z_2Z_4	Z_2Z_5	Z_3Z_4	Z_3Z_5	Z_4Z_5
1	80.9	1287.0	5558.1	3805.1	238.1	-650.1	1376.1	-1141.1	284.5	-5378.7	4454.9	-1112.7	-9198.1	2300.7	-1903.8
2	1143.7	772.1	1798.2	399.4	22.1	-1845.6	-2926.6	1686.7	593.1	2046.6	-1178.2	-415.0	-1822.3	-642.8	369.7
3	350.9	1306.3	144.3	2.2	29.7	-1364.5	461.9	-56.6	209.4	-873.0	106.9	-396.1	-35.3	131.0	-16.0
4	0.6	114.7	877.0	169.2	782.8	-16.2	-45.6	20.1	43.0	637.8	-280.4	-602.3	-770.5	-1657.2	728.0
5	5.7	7.8	77.1	3.7	16.9	13.5	-43.2	-9.5	20.2	-49.2	-10.8	23.0	33.9	-72.3	-15.9

12	-615.3	-58.5	-13691.8	-6692.7	-93.4	-557.2	6297.3	-4667.4	404.6	2370.4	-1754.9	152.4	19344.5	-1682.1	1244.2
13	-768.7	-5186.4	-7493.3	-3988.5	-436.0	4024.0	-4926.7	3602.1	-1187.2	12537.1	-9156.0	3022.8	10934.3	-3615.2	2637.8
14	-67.9	-633.3	-10850.7	-5579.3	-157.5	418.0	-1762.5	1266.6	212.1	5271.9	-3784.1	-634.8	15562.0	2614.5	-1874.9
15	-129.7	-1095.0	-4326.0	-4047.0	-382.0	759.6	-1537.8	1490.6	-456.5	4376.9	-4237.7	1300.0	8368.7	-2571.0	2486.9
23	-2763.8	-4093.9	-921.7	-342.3	-110.8	6784.3	3112.2	-2404.0	-1513.2	-3449.0	2661.2	1677.9	1190.8	751.9	-579.6
24	-1093.5	-289.6	-158.6	-43.9	-502.4	1050.1	733.5	-712.9	1431.4	-304.5	295.6	-594.5	201.4	-405.6	393.4
25	-1311.9	-624.6	-2621.4	-325.2	-83.5	1754.7	3811.3	-1647.9	-955.2	-2226.0	961.3	558.2	2036.8	1184.4	-511.0
34	-379.6	-2195.0	-309.9	-133.2	-1117.6	1839.6	704.0	-462.5	-1335.6	-1658.5	1088.4	3148.4	406.3	1177.0	-771.7
35	-266.9	-1515.3	-432.3	-0.2	-1.8	1281.5	-697.2	-15.5	-44.8	1627.7	36.2	104.6	-19.2	-55.6	-1.2
45	-9.9	-62.8	-1474.2	-122.7	-1030.0	50.3	248.0	-71.7	-207.1	-611.9	176.7	511.2	850.7	2464.5	-711.2

Table 4: Compressive strength in N/mm^2 of 28^{th} day old concrete cubes

OP	Replicate 1 Compressive strength(N/mm^2)	Replicate 1 Compressive strength(N/mm^2)	Replicate 1 Compressive strength(N/mm^2)	Average Laboratory Compressive strength(N/mm^2)	Osadebe's Model compressive strength result (N/mm^2)
1	25.77	26.22	20.44	24.14	24.14
2	24.44	21.33	27.11	24.29	24.29
3	24.89	26.67	27.78	26.45	26.45
4	24.22	23.33	23.11	23.55	23.55
5	25.11	22.22	23.56	23.63	23.63
12	24.89	25.33	23.56	24.59	24.59
13	22.67	24.89	23.56	23.71	23.71
14	22.22	23.56	25.33	23.70	23.70
15	24.00	24.00	22.67	23.56	23.56
23	29.11	30.22	31.11	30.15	30.15
24	21.78	19.11	19.11	20.00	20.00
25	20.89	18.22	18.67	18.96	18.96
34	16.89	17.33	17.33	17.19	17.19
35	16.89	17.11	16.89	16.96	16.96
45	11.56	11.11	11.11	11.26	11.26
AC_1	25.40	25.20	25.30	25.30	26.87
AC_2	19.78	19.56	19.64	19.66	19.80
AC_3	18.98	19.00	19.08	19.02	16.92
AC_4	21.83	22.12	22.20	22.05	22.06
AC_5	21.78	20.00	20.89	20.89	15.86
AC_6	22.91	22.53	22.78	22.74	21.70
AC_7	23.78	25.11	24.22	24.37	25.57
AC_8	20.44	20.02	20.44	20.30	19.32
AC_9	22.54	22.33	22.69	22.52	22.52
AC_{10}	18.24	18.25	18.59	18.36	18.11
AC_{11}	19.98	20.07	20.07	20.04	18.09
AC_{12}	17.33	18.67	17.33	17.78	16.42
AC_{13}	22.35	22.35	22.20	22.30	21.42
AC_{14}	18.63	18.67	18.62	18.64	18.38
AC_{15}	21.00	21.04	21.20	21.08	19.22

Legend: OP is the Observation points

5.1 The Regression Equation

The solution of Eqn (20),using the responses in Table 4 and the Z matrix, gives the unknown coefficients of the regression equation as follows:

α_1 = 159629.7, α_2 = 15708.88, α_3 = -3495.77, α_4 = 2978.45, α_5 = 353.809

α_{12} = -307398, α_{13} = -206593, α_{14} = -83697.6, α_{15} = -214636, α_{23} = -34564.4

α_{24} = -61995.4, α_{25} = 19052.46, α_{34} = -42807.7, α_{35} = 37643.54, α_{45} = -6773.03

Hence, the regression equation is given by Eqn (16a) becomes:

$$Y = 159629.7Z_1 + 15708.88Z_2 - 3495.77Z_3 + 2978.45Z_4 + 353.809 Z_5 - 307398Z_1Z_2 - 206593Z_1Z_3 - 83697.6Z_1 Z_4 \; 214636Z_1Z_5 - 34564.4Z_2Z_3 - 61995.4Z_2Z_4 + 19052.46Z_2Z_5 - 42807.7Z_3Z_4 + 37643.54Z_3Z_5 - 6773.03Z_4Z_5 \quad (22)$$

Eqn (22) is the final second degree polynomial function for the optimization of compressive strength of river sand-termite soil concrete based on Osadebe's second-degree equation.

Test of Adequacy of the Model

The test for adequacy of second degree polynomial was done using statistical student's t-test at 95% accuracy level. The compressive strength at the control points (i.e. AC_1, AC_2 , AC_3 ,AC_4, AC_5, AC_6 ,AC_7, AC_8 ,AC_9 ,AC_{10}, AC_{11}, AC_{12}, AC_{13}, AC_{14}, AC_{15}) were used for the test . The following two hypotheses were tested using statistical student's t-test.

a) **Null Hypothesis:** There is no significant difference between the laboratory concrete cube strengths and predicted compressive strength results at 95% accuracy level.

b) **Alternative Hypothesis:** There is a significant difference between the laboratory concrete cube strengths and l predicted strength compressive strength results at 95% accuracy level.

The test is carried out and presented in Table 5 using the following equations:

Let:

Y_E = Responses(compressive strength)from the experi

Y_M = Responses(compressive strength) from the Second

N = Number of observations

Di = Difference of Y_E and Y_M

$D_A = \frac{\sum Di}{N}$ =Mean of difference of Y_E and Y_M

$S^2 = \frac{\sum (DA - Di)^2}{N-1}$ = Variance of difference of Di and DA

$t = \frac{DA * N^{0.5}}{S}$ = Calculated value of t

Table 5: Statistical Student's t-test for Osadebe's Regression Model

		TWO-TAILED t-TEST			
OP	Y_E	Y_M	$Di = Y_E - Y_M$	$DA - Di$	$(D_A - D_i)^2$
AC_1	25.30	26.87	-1.57	2.4227	5.8695
AC_2	19.66	19.80	-0.14	0.9927	0.9855
AC_3	19.02	16.92	2.10	-1.2473	1.5558
AC_4	22.05	22.06	-0.01	0.8627	0.7443
AC_5	20.89	15.86	5.03	-4.1773	17.450
AC_6	22.74	21.70	1.04	-0.1873	0.0351
AC_7	24.37	25.57	-1.20	2.0527	4.2136
AC_8	20.30	19.32	0.98	-0.1273	0.0162
AC_9	22.52	22.52	0.00	0.8527	0.7271
AC_{10}	18.36	18.11	0.25	0.6027	0.3632
AC_{11}	20.04	18.09	1.95	-1.0973	1.2041
AC_{12}	17.78	16.42	1.36	-0.5073	0.2574
AC_{13}	22.30	21.42	0.88	-0.0273	0.0007
AC_{14}	18.64	18.38	0.26	0.5927	0.3513

AC_{15}	21.08	19.22	1.86	-1.0073	1.0147
		□□□□□□□Di =	12.79	□□(DA - Di)^2 =	34.7885

Legend: OP is the observation point

Here,

□□Di = 12.79

N = 15

$DA = \frac{\sum Di}{N} = 0.8527$

□□(DA - Di)^2 = 34.7885

$S^2 = \frac{\sum (DA - Di)^2}{N-1} = 2.4849$

$S = \sqrt{S^2} = 1.5746$

Actual value of total variation in t-test is:

$t = \frac{DA * N^{0.5}}{S} = 2.09$

Allowable value of total variation in t-test:

Degree of freedom = N-1 = 15-1 = 14

5 % significance for Two-Tailed Test = 2.5 %

1 - 2.5% = 97.5% = 0.975

Allowable Total Variation in t-test = $t_{(0.975, N-1)} = t_{(0.975, 14)} = 2.14$ (Obtained from statistical table).

From table 5, the calculated t = 2.09 Thus, $t_{(table)} > t_{(calculated)}$

This implied that difference between the two set of cubes compressive strength is less than allowable difference. Hence null hypothesis is accepted and alternative hypothesis rejected. Hence, Osadebe regression model is adequate.

V. CONCLUSION

Using Osadebe's second degree polynomial regression equation, mix design model for a five component river sand-termite soil concrete cube was developed. This model could predict the compressive strength of concrete cube when the mix ratios are known and vice versa. The predictions from this model were tested at 95% accuracy level using statistical student's t-test and found to be adequate.

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