On $\pi g(\alpha g)^*$ – continuous maps and $\pi g(\alpha g)^*$ - irresolute maps in Topological Spaces

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Abstract: In this paper, we have introduced the concept of continuous, irresolute and homeomorphism maps of $\pi g(\alpha g)^*$ closed set. Some of the fundamental properties of this set are studied. And their application also given namely, $\pi g(\alpha g)^*$ – $T_{1/2}$ – space.

Keywords: $\pi g(\alpha g)^*$- closed set, $\pi g(\alpha g)^*$- continuous map, $\pi g(\alpha g)^*$- irresolute map, $\pi g(\alpha g)^*$ – $T_{1/2}$ – space

1. Introduction

Levine[5] introduced the class of g-closed sets, a super class of closed sets in 1970. Dontchev and Noiri [19] have introduced the concept of $\pi g$-closed sets and studied their most fundamental properties in topological spaces. Also, Ekici and Noiri [21] have introduced a generalization of $\pi g$-closed sets and $\pi g$-open sets. Recently, a new class of $\pi g(\alpha g)^*$-closed sets in topological spaces introduced and studied by R.Savithiri, A.Manonmani and M.Anandhi [29]. In this paper, we have made a study on $\pi g(\alpha g)^*$- continuous map, $\pi g(\alpha g)^*$- irresolute map and $\pi g(\alpha g)^*$-homeomorphism. Also, Applications of $\pi g(\alpha g)^*$-closed sets are analyzed.

2. Preliminaries

For a subset $H$ of a space $(X,\tau)$, $cl(H)$ and $int(H)$ denote the closure and the interior of $H$ respectively. The class of all closed subsets of a space $(X,\tau)$ is denoted by $C(X,\tau)$. The smallest closed (resp. $\alpha$-closed) set containing a subset $H$ of $(X,\tau)$ is called the closure (resp. $\alpha$-closure) of $H$ and is denoted by $cl(H)$ (resp. $\alpha cl(H)$).

Definition 2.1: 1) A $\pi$ open set [21] of $X$ is a finite union of all $r$-open sets in $(X,\tau)$.

2) A subset $H$ of a space $X$ is called $\alpha$-generalized closed (briefly $\alpha g$-closed) [13] if $\alpha cl(H) \subseteq U$ whenever $H \subseteq U$ and $U$ is open in $X$.

3) A subset $H$ of a space $X$ is called $\pi$-generalized closed set $\pi g(\alpha g)^*$-closed set [29] (briefly $\pi g(\alpha g)^*$- closed set) if $\pi cl(H) \subseteq U$, whenever $H \subseteq U$ and $U$ is $\pi$ open in $X$.
Remark 2.2:

\( g^{(ag)} \)-closed set is independent with the following closed sets: gp-closed set, rg-closed set, s-closed set, wg-closed set, w\( \pi g \)-closed set, b*-closed set, b-closed set, gs-closed set, gb-closed set and \( \pi gp \)-closed set.

3. On \( \pi (ag)^* \)-continuous map.

Definition 3.1:

A map \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) is called

1) continuous[5] if the inverse image of every closed set of \((Y, \tau_2)\) is a closed set of \((X, \tau_1)\).
2) \( g \)-continuous[5] if the inverse image of every closed set of \((Y, \tau_2)\) is \( g \)-closed set of \((X, \tau_1)\).
3) \( g \)-continuous[4] if the inverse image of every closed set of \((Y, \tau_2)\) is \( g \)-closed set of \((X, \tau_1)\).
4) \( ag \)-continuous[13] if the inverse image of every closed set of \((Y, \tau_2)\) is \( ag \)-closed set of \((X, \tau_1)\).
5) \( ga \)-continuous[13] if the inverse image of every closed set of \((Y, \tau_2)\) is \( ga \)-closed set of \((X, \tau_1)\).
6) \( \pi g \)-continuous[21] if the inverse image of every closed set of \((Y, \tau_2)\) is \( \pi g \)-closed set of \((X, \tau_1)\).
7) \( \pi ga \)-continuous [25] if the inverse image of every closed set of \((Y, \tau_2)\) is \( \pi ga \)-closed of \((X, \tau_1)\).
8) s-continuous [6] if the inverse image of every closed set of \((Y, \tau_2)\) is s- closed set of \((X, \tau_1)\).
9) gp-continuous[23] if the inverse image of every closed set of \((Y, \tau_2)\) is gp- closed set of \((X, \tau_1)\).
10) rg-continuous[28] if the inverse image of every closed set of \((Y, \tau_2)\) is rg- closed set of \((X, \tau_1)\).
11) wg-continuous[15] if the inverse image of every closed set of \((Y, \tau_2)\) is wg- closed set of \((X, \tau_1)\).
12) w\( \pi g \)-continuous[15] if the inverse image of every closed set of \((Y, \tau_2)\) is w\( \pi g \)-closed of \((X, \tau_1)\).
13) b*-continuous[26] if the inverse image of every closed set of \((Y, \tau_2)\) is b*-closed set of \((X, \tau_1)\).
14) b-continuous[16] if the inverse image of every closed set of \((Y, \tau_2)\) is b- closed set of \((X, \tau_1)\).
15) gs-continuous[29] if the inverse image of every closed set of \((Y, \tau_2)\) is gs- closed set of \((X, \tau_1)\).
16) gb-continuous[27] if the inverse image of every closed set of \((Y, \tau_2)\) is gb-closed set of \((X, \tau_1)\).
17) \( \pi gp \)-continuous[24] if the inverse image of every closed set of \((Y, \tau_2)\) is \( \pi gp \)-closed of \((X, \tau_1)\).
18) \( \alpha \)-continuous [11] if the inverse image of every closed set of \((Y, \tau_2)\) is \( \alpha \)-closed set of \((X, \tau_1)\).

Definition 3.2:

A map \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) is called a \( \pi (ag)^* \)-continuous if the inverse image of every closed set of \((Y, \tau_2)\) is \( \pi (ag)^* \)-closed set of \((X, \tau_1)\).

Theorem 3.3:

Every continuous map, \( g \)-continuous map, \( \alpha \)-continuous map and \( ag \)-continuous map is \( \pi (ag)^* \)-continuous.

Proof:

(i) Take \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) be continuous map. Let \( W \) be closed set of \((Y, \tau_2)\) then inverse image of \( W \) is closed of \((X, \tau_1)\). Inverse image of \( W \) is \( \pi (ag)^* \)-closed of \((X, \tau_1)\), since closed \( \rightarrow \pi (ag)^* \)-closed. Hence \( \theta \) is \( \pi (ag)^* \)-continuous of \((X, \tau_1)\).

(ii) Take \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) be \( g \)-continuous map. Let \( W \) be closed set of \((Y, \tau_2)\) then inverse image of \( W \) is \( g \)-closed of \((X, \tau_1)\). Inverse image of \( W \) is \( \pi (ag)^* \)-closed of \((X, \tau_1)\), since \( g \)-closed \( \rightarrow \pi (ag)^* \)-closed. Hence \( \theta \) is \( \pi (ag)^* \)-continuous of \((X, \tau_1)\).

(iii) Take \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) be \( \alpha \)-continuous map. Let \( W \) be closed set of \((Y, \tau_2)\) then inverse image of \( W \) is \( \alpha \)-closed of \((X, \tau_1)\). Inverse image of \( W \) is \( \pi (ag)^* \)-closed of \((X, \tau_1)\), since \( \alpha \)-closed \( \rightarrow \pi (ag)^* \)-closed. Hence \( \theta \) is \( \pi (ag)^* \)-continuous of \((X, \tau_1)\).

(iv) Take \( \theta: (X, \tau_1) \rightarrow (Y, \tau_2) \) be \( ag \)-continuous map. Let \( W \) be closed set of \((Y, \tau_2)\) then inverse image of \( W \) is \( ag \)-closed of \((X, \tau_1)\). Inverse image of \( W \) is \( \pi (ag)^* \)-closed of \((X, \tau_1)\), inverse image of \( W \) is \( \pi (ag)^* \)-closed. Hence \( \theta \) is \( \pi (ag)^* \)-continuous of \((X, \tau_1)\).
since \(\alpha g\)-closed \(\rightarrow\) \(\pi g(\alpha g)^*\) -closed . Hence \(\theta\) is \(\pi g(\alpha g)^*\) -continuous of \((X, \tau_1)\).

The converse of the above theorem need not be true from the following example.

**Example 3.4:**

Take \(X = Y = \{a, b, c\}\) and \(\tau_1 = \{X, \emptyset, \{a\}\}\), \(\tau_2 = \{Y, \emptyset, \{a\}\}\). Define \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) as \(\theta(a) = a\), \(\theta(b) = b\), \(\theta(c) = c\). Here inverse image of all \(\tau^c\) are \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\) but not closed, \(g\)-closed, \(\alpha\)-closed and \(\alpha g\)-closed of \((X, \tau_1)\). This implies converse not true.

**Theorem 3.5:**

Every \(g\) -continuous map and \(g\alpha\)-continuous map is \(\pi g(\alpha g)^*\) -continuous.

**Proof:**

(i) Take \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) be \(g\) -continuous map. Let \(W\) be closed set of \((Y, \tau_2)\) then inverse image of \(W\) is \(g\) -closed of \((X, \tau_1)\). Inverse image of \(W\) is \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\), since \(g\) -closed \(\rightarrow\) \(\pi g(\alpha g)^*\) -closed . Hence \(\theta\) is \(\pi g(\alpha g)^*\) -continuous of \((X, \tau_1)\).

(ii) Take \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) be \(g\alpha\)-continuous map. Let \(W\) be closed set of \((Y, \tau_2)\) then inverse image of \(W\) is \(g\alpha\)-closed of \((X, \tau_1)\). Inverse image of \(W\) is \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\), since \(g\alpha\)-closed \(\rightarrow\) \(\pi g(\alpha g)^*\) -closed . Hence \(\theta\) is \(\pi g(\alpha g)^*\) -continuous of \((X, \tau_1)\).

The converse of the above theorem need not be true from the following example.

**Example 3.6:**

Take \(X = Y = \{a, b, c\}\) and \(\tau_1 = \{X, \emptyset, \{c\}\}, \tau_2 = \{Y, \emptyset, \{b\}\}\). Define \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) as \(\theta(a) = a\), \(\theta(b) = b\), \(\theta(c) = c\). Here inverse image of all \(\tau^c\) are \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\) but not \(g\alpha\)-closed and \(g\) -closed of \((X, \tau_1)\). This implies converse not true.

**Theorem 3.7:**

Every \(\pi g\)-continuous map is \(\pi g(\alpha g)^*\) -continuous.

**Proof:**

Take \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) be \(\pi g\)-continuous map. Let \(W\) be closed set of \((Y, \tau_2)\) then inverse image of \(W\) is \(\pi g\)-closed of \((X, \tau_1)\). Inverse image of \(W\) is \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\), since \(\pi g\)-closed \(\rightarrow\) \(\pi g(\alpha g)^*\) -closed . Hence \(\theta\) is \(\pi g(\alpha g)^*\) -continuous of \((X, \tau_1)\).

The converse of the above theorem need not be true from the following example.

**Example 3.8:**

Take \(X = Y = \{a, b, c, d\}\) and \(\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}, \tau_2 = \{Y, \emptyset, \{a\}, \{b\}\}\). Define \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) as \(\theta(a) = a\), \(\theta(b) = b\), \(\theta(c) = c\), \(\theta(d) = d\). Here inverse image of all \(\tau^c\) are \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\) but not \(\pi g\) -closed of \((X, \tau_1)\). This implies converse not true.

**Theorem 3.9:**

Every \(\pi g\alpha\)-continuous map is \(\pi g(\alpha g)^*\) -continuous but not converse.

**Proof:**

Take \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) be \(\pi g\alpha\)-continuous map. Let \(W\) be closed set of \((Y, \tau_2)\) then inverse image of \(W\) is \(\pi g\alpha\)-closed of \((X, \tau_1)\). Inverse image of \(W\) is \(\pi g(\alpha g)^*\) -closed of \((X, \tau_1)\), since \(\pi g\alpha\)-closed \(\rightarrow\) \(\pi g(\alpha g)^*\) -closed . Hence \(\theta\) is \(\pi g(\alpha g)^*\) -continuous of \((X, \tau_1)\).

**Remark 3.10:**

The composition of two \(\pi g(\alpha g)^*\)-continuous map is need not be a \(\pi g(\alpha g)^*\)-continuous map.

**Example 3.11**

Take \(X = Y = Z = \{a, b, c\}\) and \(\tau_1 = \{X, \emptyset, \{a\}, \{b\}\}, \tau_2 = \{X, \emptyset, \{a\}, \{b\}\}\) and \(\tau_3 = \{Y, \emptyset, \{a\}, \{b\}\}\). Define \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) and \(h : (Y, \tau_2) \rightarrow (Z, \tau_3)\) be an identity maps. Let \(\theta\) and \(h\) be a \(\pi g(\alpha g)^*\)-continuous maps . But \((h \circ \theta)^{-1}(\{a\}) = \theta^{-1}((h^{-1}(\{a\}))=\{a\}\) is not \(\pi g(\alpha g)^*\)-closed of \((X, \tau_1)\). Hence \(\theta\) is not \(\pi g(\alpha g)^*\)-continuous.

**Theorem 3.12:**

A map \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) is \(\pi g(\alpha g)^*\)-continuous and \(h : (Y, \tau_2) \rightarrow (Z, \tau_3)\) is continuous , then \(h \circ \theta : (X, \tau_1) \rightarrow (Z, \tau_3)\) is \(\pi g(\alpha g)^*\)-continuous.

**Proof:**

Take \(W\) be any closed set in \((Z, \tau_3)\) and.so \(h^{-1}(W)\) of \(W\) is closed of \((Y, \tau_2)\). Since \(h\) is continuous. \((h \circ \theta)^{-1}(W) = h^{-1}(W)\) is \(\pi g(\alpha g)^*\)-closed of \((X, \tau_1)\). Since \(h \circ \theta\) is \(\pi g(\alpha g)^*\)-continuous . Hence \(h \circ \theta\) is \(\pi g(\alpha g)^*\)-continuous.

**Theorem 3.13:**

A map \(\theta : (X, \tau_1) \rightarrow (Y, \tau_2)\) is \(\pi g(\alpha g)^*\)-continuous if and only if inverse image of every open set in
(Y, τ₂) is πg(ag)*-open set in (X, τ₁).

**Proof:**

Take θ : (X, τ₁) → (Y, τ₂) be πg(ag)*-continuous map and W be open set in (Y, τ₂) then W is closed in (Y, τ₂). Inverse image of W is πg(ag)* -closed of (X, τ₁) since θ is πg(ag)*-continuous . But θ⁻¹(W⁻) = (θ⁻¹(W))². Hence inverse image of W is πg(ag)*-open of (X, τ₁).

Conversely, Assume, For every open set W of (Y, τ₂), inverse image of W is πg(ag)*-open of (X, τ₁). If W of (Y, τ₂) be a closed set, then W⁻ is closed set of (Y, τ₂) be a open set. By assumption , inverse image of W⁻ is πg(ag)*-open set of (X, τ₁). But θ⁻¹(W⁻) = (θ⁻¹(W))². Hence inverse image of W is πg(ag)*-closed set of (X, τ₁). This implies θ is πg(ag)*-continuous

**Diagram-I**

```
  a-continuous
     |
   g-continuous
     |
  gα-continuous
     |
  πgα-continuous
```

4. On πg(ag)*-open map.

**Definition 4.1:**

A map θ : (X, τ₁) → (Y, τ₂) is called a πg(ag)*-open map if the image of every open set of (X, τ₁) is πg(ag)*-open set of (Y, τ₂).

**Theorem 4.2:**

Every open map, g-open map, gα-open map, πg-open map, and gα-open map is πg(ag)*-open map.

**Proof:**

Follows from the fact that “Every open set, g-open set, gα-open set, πg-open set, and gα-open set is πg(ag)*-open set”.

The converse of the above theorem need not be true from the following example.

**Example 4.3:**

Take X = Y = {a,b,c} and τ₁ = {X,Φ, {a},{b,c}}, τ₂ = {Y,Φ, {a}}. Define θ : (X, τ₁) → (Y, τ₂) as θ(a) = a, θ(b) = b, θ(c) = c. Here W = {b,c} be a be open set of (X, τ₁) . But image of W is πg(ag)*-open set of (Y, τ₂) but not open set, g-open set, α-open set, gα-open set of (Y, τ₂) . This implies converse of above theorem not true.

5. On πg(ag)*-irresolve map.

**Definition 5.1:**

A map θ : (X, τ₁) → (Y, τ₂) is called a πg(ag)*-irresolve if the inverse image of every πg(ag)*-closed set of (Y, τ₂) is πg(ag)*-closed set of (X, τ₁).

**Theorem 5.2:**

Every πg(ag)*-irresolve map is πg(ag)*-continuous map.

**Proof:**

Take a map θ : (X, τ₁) → (Y, τ₂) be πg(ag)*-irresolve map. Let W be closed set of (Y, τ₂) then W be a πg(ag)*-closed set of (Y, τ₂), since closed → πg(ag)*-closed. But inverse image of W is πg(ag)*-closed of (X, τ₁). Hence θ is πg(ag)*-continuous.

The converse of the above theorem need not be true from the following example.

**Example 5.3:**

Take X = Y = {a,b,c} and τ₁ = {X,Φ, {a},{b,c}}, τ₂ = {Y,Φ, {a}}. Define θ : (X, τ₁) → (Y, τ₂) as θ(a) = a, θ(b) = b, θ(c) = c, θ(d) = d. Here inverse image of all τ₂⁻ are of πg(ag)*-closed of (X, τ₁) so θ is πg(ag)*-continuous. But inverse image of all πg(ag)*-closed of (X, τ₁) are not πg(ag)*-closed of (Y, τ₂) are not πg(ag)*-irresolve map.

**Theorem 5.4:**

A map θ : (X, τ₁) → (Y, τ₂) is πg(ag)*-irresolve and h : (Y, τ₂) → (Z, τ₃) is πg(ag)*-continuous , then h o θ : (X, τ₁) → (Z, τ₃) is πg(ag)*-continuous.

**Proof:**

Take W be any closed set of (Z, τ₃) and so h⁻¹ of W is πg(ag)*-closed of (Y, τ₂). Since h is πg(ag)*-continuous. (h o θ)⁻¹(W) = θ⁻¹ (h⁻¹(W)) is πg(ag)*-closed of (X, τ₁). Since θ is πg(ag)*-irresolve. Hence h o θ is πg(ag)*-continuous.

**Theorem 5.5:**
A map \( \theta : (X, \tau_1) \rightarrow (Y, \tau_2) \) is \( \pi g(\alpha g)^* \)-irresolute and h : (Y, \tau_2) \rightarrow (Z, \tau_3) is \( \pi g(\alpha g)^* \)-irresolute , then h o \( \theta : (X, \tau_1) \rightarrow (Z, \tau_3) \) is \( \pi g(\alpha g)^* \)-irresolute.

**Proof:**

Take W be any \( \pi g(\alpha g)^* \)-closed set of (Z, \tau_3) and so \( h^{-1}(W) \) is \( \pi g(\alpha g)^* \)-closed of (Y, \tau_2), Since h is \( \pi g(\alpha g)^* \)-irresolute. (h o \( \theta \))\(^{-1}\)(W) is \( \pi g(\alpha g)^* \)-closed of (X, \tau_1). Since \( \theta \) is \( \pi g(\alpha g)^* \)-irresolute. Hence h o \( \theta \) is \( \pi g(\alpha g)^* \)-irresolute.

**Theorem 5.6:**

A map \( \theta : (X, \tau_1) \rightarrow (Y, \tau_2) \) is \( \pi g(\alpha g)^* \)-irresolute if and only if inverse image of every \( \pi g(\alpha g)^* \)-open set in (Y, \tau_2) is \( \pi g(\alpha g)^* \)-open set in (X, \tau_1).

**Proof:**

Similar to Theorem 3.13.

### 6. On \( \pi g(\alpha g)^* \)-homeomorphisms.

**Definition 6.1:**

A bijective map \( \theta : (X, \tau_1) \rightarrow (Y, \tau_2) \) is called a \( \pi g(\alpha g)^* \)-homeomorphism if a map is both \( \pi g(\alpha g)^* \)-continuous and \( \pi g(\alpha g)^* \)-open.

**Remark 6.2:**

For a bijective map \( \theta : (X, \tau_1) \rightarrow (Y, \tau_2) \), the following statement are equivalent:

(i) \( \theta^{-1} \) is \( \pi g(\alpha g)^* \)-continuous , (ii) \( \theta \) is \( \pi g(\alpha g)^* \)-open ,

(iii) \( \theta \) is \( \pi g(\alpha g)^* \)-closed .

**Theorem 6.3:**

Every homeomorphism is a \( \pi g(\alpha g)^* \)-homeomorphism.

**Proof:**

Follows from the fact that “Every continuous map is \( \pi g(\alpha g)^* \)-continuous map and Every open map is \( \pi g(\alpha g)^* \)-open map”.

The converse of the above theorem need not be true from the following example.

**Example 6.4:**

Take X = Y= \{a,b,c\} and \( \tau_1 = \{X,\Phi,\{a\},\{b\},\{a,b\}\} \), \( \tau_2 = \{Y,\Phi,\{a\},\{a,b\}\} \).

Define \( \theta : (X, \tau_1) \rightarrow (Y, \tau_2) \) as bijective map. Now \( \theta \) is \( \pi g(\alpha g)^* \)-homeomorphism but not a homeomorphism. Since \( \theta (\{b\}) = \{b\} \) is not in open set of (Y, \tau_2).

**Remark 6.5:**

The composition of two \( \pi g(\alpha g)^* \)-homeomorphism map is need not be a \( \pi g(\alpha g)^* \)-homeomorphism map.

### 7. Applications of \( \pi g(\alpha g)^* \)-closed set.

**Definition 7.1:**

1) A topological space (X,\( \tau \)) is called \( T_{1/2} \) - space [5] if every g-closed in (X,\( \tau \)) is closed in (X,\( \tau \)).

2) A topological space (X,\( \tau \)) is called \( T_b \) - space [14] if every g\( s \)-closed in (X,\( \tau \)) is closed in (X,\( \tau \)).

3) A topological space (X,\( \tau \)) is called \( T_b \)-space[17] if every \( \pi g(\alpha g) \)-closed in (X,\( \tau \)) is closed in (X,\( \tau \)).

**Definition 7.2:**

A topological space (X, \( \tau \)) is called \( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space if every \( \pi g(\alpha g)^* \)-closed set of (X, \( \tau \)) is closed of (X, \( \tau \)).

**Theorem 7.3:**

Every \( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space is a \( T_{1/2} \) - space.

**Proof:**

Assume that (X, \( \tau \)) is a \( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space. Let H be a g-closed set. But every g-closed set is a \( \pi g(\alpha g)^* \)-closed set. By assumption, (X, \( \tau \)) is a \( T_{1/2} \) - space.

**Theorem 7.4:**

Every \( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space is a \( T_b \) - space.

**Proof:**

Assume that (X, \( \tau \)) is a \( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space. Let H be a a\( g \)-closed set. But every a\( g \)-closed set is an \( \pi g(\alpha g)^* \)-closed set. By assumption, (X, \( \tau \)) is a \( T_b \) - space.

**Remark 7.5:**

\( \pi g(\alpha g)^* \)-\( T_{1/2} \) - space and \( T_b \) - space are independent spaces.

**Theorem 7.6:**
A space \((X, \tau)\) is a \(\pi g(\alpha g)^* -T_{1/2}\) space if and only if every singleton of \(X\) is either \(\pi\)-closed set or \(\alpha g\) –open.

**Proof:**

Assume that \((X, \tau)\) is a \(\pi g(\alpha g)^* -T_{1/2}\) space. Let \(y\) be an element in \(X\) and \(\{y\}\) is not in \(\pi\)-closed, then \(X\)-\(\{y\}\) is not in \(\pi\)-open and then \(X\)-\(\{y\}\) is \(\pi g(\alpha g)^*\) –closed. By assumption , \(X\)-\(\{y\}\) is \(\alpha g\)-closed. Hence \(\{y\}\) is \(\alpha g\)-open. The converse is similar.

**Theorem 7.7:**

A map \(\theta : (X, \tau_1) \to (Y, \tau_2)\) and \(h : (Y, \tau_2) \to (Z, \tau_3)\) be two maps and if \(\theta\) is \(\alpha g\)-irresolute and \(h\) is a \(\pi g(\alpha g)^*\) –continuous and \(Y\) is a \(\pi g(\alpha g)^* -T_{1/2}\) – space, then \(h \circ \theta : (X, \tau_1) \to (Z, \tau_3)\) is \(\alpha g\)-continuous.

**Proof:**

Take \(W\) be any closed set in \((Z, \tau_1)\). Here \((Y, \tau_2)\) is a \(\pi g(\alpha g)^* -T_{1/2}\) – space and \(h^{-1}\) of \(W\) is \(\pi g(\alpha g)^*\)-closed of \((Y, \tau_2)\). Since \(h\) is \(\pi g(\alpha g)^*\)-continuous. But \((h \circ \theta)^{-1}(W) = \theta^{-1}(h^{-1}(W))\) is \(\alpha g\)-closed \((X, \tau_1)\), Since \(\theta\) is \(\alpha g\)- irresolute. Hence \(h \circ \theta\) is \(\alpha g\)-continuous.

**References**


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