# A Comparative Study of Interpolation Using the Concept of Mathematical Norm With a Proposed Model

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**ABSTRACT**

Interpolation is a method of constructing new data point within the range of a discrete set of known data points. Different methods have been developed to construct useful interpolation formulae, for evenly as well as unevenly spaced data points. This paper is aimed at developing a central difference interpolation formula which is derived from Gauss's Backward Formula and another formula in which we retreated the subscript in Gauss's Forward Formula by two units and replacing ***u*** by ***u+2***. Also we made the comparisons of the developed interpolation formula with the existing interpolation formulae based on differences. The result from the error analysis carried out using the concept of Mathematical Norm, shows that the New (proposed) formula is much more efficient and have good accuracy for resolving functional values between given data.

**Keywords:** Interpolation, Central Difference, Mathematical Norm, Gauss's Formula.

## 1. INTRODUCTION

Interpolation is an estimation of a value within two known values in a sequence of values. Polynomial interpolation is a method of estimating values between known data points. When graphical data contains a gap, but data is available on either side of the gap or at a few specific points within the gap, interpolation allows for estimation of the values within the gap.

Over the years, many methods have been devised to build expedient interpolation formulae for even and unevenly spaced data points. Such methods include: Newton's divided difference formula (e.g. Atkinson, 1989; Carl and Boor, 1980) and Lagrange's formula (e.g. Burden, and Faires, 2001; Suli and Mayers, 2003;) are the most popular interpolation formulae for polynomial interpolation to any arbitrary degree with finite number of points.

Muthumalai(2008) studied new iterative methods for interpolation of,numerical differentiation and numerical integration formular for evenly and unevenly spaced data using Neville's and Aitken's concept of algorithms.

Muthumala and Uthra(2014) examined a new interpolation formular that generalized both Newton's and langrange's interpolation formula and futher derived new other ones based on differences and divided differences. The modified formulas, when compared with the former interpolation formulas (Newton's,Guass's, Sterling's, Bessel's) were more efficient and of good accuracy.

Garnero and Godone(2013) compared different interpolation tecchniques to analyse the digital terrain models which are used for environment and land-related applications.

Das and Chakrabarty(2016) derived a formular from Langrange's interpolation method and this was used to obtain a numerical data for total population of India.The work was extended by deriving other methods for the same purpose. See Das and Chakrabarty(2016) and Das and Chakrabarty(2016). Abdulla et al. (2004), Proposed a new central difference interpolation formula which was gotten from difference interpolation formula and ours is derived from Gauss's Backward Formula and another formula in which we retreat the subscripts in Gauss's Forward Formula by one unit and replacing ***u*** by ***u+1***. Also, we make the comparisons of the developed interpolation formula with the existing interpolation formulas based on differences. Results show that the new formula is very efficient and possess good accuracy for evaluating functional values between given data.

Singh and Bhadauria. (2009), used Lagrange's Interpolation Formula to develop Finite Difference Formulae for unequal Sub-interval. General finite difference formulae and corresponding error terms were derived considering unequally spaced grid points, and using Lagrange's interpolation formula. Further, the finite difference formulae and the error terms for equally spaced sub-intervals were obtained as their special case of study.

Bater et al. (2009), used interpolation in evaluating errors associated with Lidar-Derived DEM. They discovered that light detecting and ranging (lidar) technology is capable of precisely measuring a variety of vegetation matrices, the estimates of which are usually based on relative heights above a digital evaluation model (DEM). They tested seven interpolation routines, using small footprint lidar data, collected over a range of vegetation classes on Vancourver island.

Reuter et al. (2007), proposed an evaluation of void-filling Interpolation method for Shuttle Radar Topography Mission (SRTM). Based on a sample of 1304 artificial but realistic voice across six terrain types are eight void size classes, they found that the choice of void- filling algorithm is dependent on both size and terrain type of the data.
Liu et al. (2006), proposed a radial point interpolation based on finite difference method (FRDM). In their novel method, redial points interpolation using local irregular nodes is used together with the convolutional finite difference procedure to achieve both adaptability to irregular domain and the stability in the solution that is often encountered in the collection method. A least-square technique was adopted, which lead to a system matrix with good properties such as symmetry and positive definiteness.

Fritsch et al. (1980), derived a necessary and sufficient conditions for a cubic to be monotone on an interval. These conditions are used to develop an algorithm which constructs a visually pleasing monotone piecewise cubic interpolant to monotone data. Several examples were given which compares the algorithm with other interpolation methods.

Akima (1970), developed a new mathematical method for interpolation from a given set of data points in a plane and for fitting a smooth curve to the point. The method was developed in such a way that the resultant curve will pass through the given point and will appear smooth and natural. In this method, the slope of the curve was determined at each given point locally, and each polynomial representing a portion of the curve between a pair of given point, was determined by the coordinates of the slope at that point. Comparison indicates that the curve obtained by the new method was closer to a manually drawn curve than those drawn by other mathematical methods.

In this paper, we try to develop a central difference interpolation formula which is derived from Gauss's Backward Formula and another formula in which we retreated the subscript in Gauss's Forward Formula by two units and replacing ***u*** by ***u+2***. Also we will carry out a comparisons of the developed interpolation formula with the existing interpolation formulae, (Gauss's, Stirling's and Bessel's etc) based on differences and use the concept of mathematical norm to select which method is best suitable for evaluating functional values between data.

## 2. NEW (PROPOSED) AND EXISTING INTERPOLATION FORMULAE

Given below are the Gauss's Central-Difference Formulae (see James B. Scarborough, 1966)

**Gauss's Forward Formula:**

$$y = y_0 + u\Delta y_0 + u(u-1)\frac{\Delta^2 y_{-1}}{2!} + u(u^2-1)\frac{\Delta^3 y_{-1}}{3!} + u(u^2-1)(u-2)\frac{\Delta^4 y_{-2}}{4!} + u(u^2-1)(u^2-2^2)\frac{\Delta^5 y_{-2}}{5!} + \ldots \quad (1)$$

**Gauss's Backward Formula**

$$y = y_0 + u\Delta y_{-1} + u(u+1)\frac{\Delta^2 y_{-1}}{2!} + u(u^2-1)\frac{\Delta^3 y_{-2}}{3!} + u(u^2-1)(u-2)\frac{\Delta^4 y_{-2}}{4!} + u(u^2-1)(u^2-2^2)\frac{\Delta^5 y_{-3}}{5!} + \ldots \quad (2)$$

**Sterling Interpolation Formula**

We have the Sterling Interpolation Formula by taking the mean of the Gauss's Forward and Gauss's Backward Formula i.e. adding equations (1) and (2) and dividing by 2 (check james B.Scarborough,!966):

$$y = y_0 + u\frac{(\Delta y_{-1} + \Delta y_0)}{2} + \frac{u^2}{2!}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\frac{(\Delta^3 y_{-2} + \Delta^2 y_{-1})}{2} + \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2} + \frac{u(u^2-1)(u^2-2^2)}{5!}\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \ldots \quad (3)$$

**Bessel's Interpolation Formula**

To derive the  Bessel's Interpolation Formula, we take the Gauss's Formula. To derive this formula, we take the third and advance the subscripts in Gauss's Backward Formula (i.e. Equation (1)) by one unit and replace *u* by *u-1* to obtain:

$$y = y_1 + (u-1)\Delta y_0 + u(u-1)\frac{\Delta^2 y_{-1}}{2!} + u(u-1)(u-2)\frac{\Delta^3 y_{-2}}{3!} + u(u^2-1)(u-2)\frac{\Delta^4 y_{-2}}{4!} + u(u^2-1)(u-2)(u-3)\frac{\Delta^5 y_{-2}}{5!} + \ldots \quad (4)$$

The mean of the Gauss's Forward Formula and Third Gauss's Formula gives the Bessel's Formula as

(see, James B. Scarborough, 1966):

$$y = \frac{y_0 + y_1}{2} + \left(u - \frac{1}{2}\right)\Delta y_0 + \frac{u(u-1)}{2!}\frac{(\Delta^2 y_{-1} + \Delta^2 y_0)}{2} + \frac{u\left(u-\frac{1}{2}\right)(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u^2-1)(u-2)}{4!}\frac{(\Delta^4 y_{-2} + \Delta^4 y_{-1})}{2}$$

$$+ \frac{u\left(u-\frac{1}{2}\right)(u^2-1)(u-1)}{5!}\Delta^5 y_{-2} + \ldots \quad (5)$$

Equation (5) is the derived Bessel's Interpolation Formula

**Previously Proposed Formula by Abdulla et al.**

This formula was derived by retreating the subscripts in Gauss's Forward Formula by one unit and replacing *u* by *u*+1, to obtain a third Gauss's Formula and then, the mean of the third formula and the Gauss's Backward Formula, to obtain the New Formula.

$$y = \frac{y_{-1} + y_0}{2} + \left(u + \frac{1}{2}\right)\Delta y_{-1} + \frac{u(u+1)}{2!}\frac{(\Delta^2 y_{-2} + \Delta^2 y_{-1})}{2} + \frac{u\left(u+\frac{1}{2}\right)(u+1)}{3!}\Delta^3 y_{-2} + \frac{u\left(u^2-1\right)(u+2)}{4!}\frac{\left(\Delta^4 y_{-3} + \Delta^4 y_{-2}\right)}{2} +$$

$$+u\frac{\left(u+\frac{1}{2}\right)\left(u^2-1\right)(u+2)\Delta^5 y_{-3}}{5!} + \ldots \quad (6)$$

**New (Proposed) Interpolation Formula**

To derive the proposed formula, we retreat the subscript in Gauss's Forward Interpolation Formula by two units and replacing *u* by *u+2*.

Recall from equation (1) above

$$y = y_0 + u\Delta y_0 + u(u-1)\frac{\Delta^2 y_{-1}}{2!} + u(u^2-1)\frac{\Delta^3 y_{-1}}{3!} + u(u^2-1)(u-2)\frac{\Delta^4 y_{-2}}{4!} + u(u^2-1)(u^2-2^2)\frac{\Delta^5 y_{-2}}{5!} + \ldots \quad (7)$$

So we obtain,

$$y = y_{-2} + (u+2)\Delta y_{-2} + (u+1)(u+2)\frac{\Delta^2 y_{-3}}{2!} + (u+1)(u+2)(u+3)\frac{\Delta^3 y_{-3}}{3!} + u(u+1)(u+2)(u+3)\frac{\Delta^4 y_{-4}}{4!} + u(u+1)(u+2)(u+3)(u+4)\frac{\Delta^5 y_{-5}}{5!} + \ldots \quad (8)$$

Also recall from equation (2) above,

$$y = y_0 + u\Delta y_{-1} + u(u+1)\frac{\Delta^2 y_{-1}}{2!} + u(u^2-1)\frac{\Delta^3 y_{-2}}{3!} + u(u^2-1)(u-2)\frac{\Delta^4 y_{-2}}{4!} + u(u^2-1)(u^2-2^2)\frac{\Delta^5 y_{-3}}{5!} + \ldots \quad (9)$$

Taking the mean of equations (8) and (9) we get the New (proposed) Interpolation Formula

$$y = \frac{y_{-2}+y_0}{2} + \left[1 - \frac{u}{2}\left(1 + \frac{\Delta y_{-1}}{\Delta y_{-2}}\right)\right]\Delta y_{-2} + \frac{(u+1)}{2!}\left[\frac{(u+2)(\Delta^2 y_{-3}) + u\Delta^2 y_{-1})}{2}\right] + \frac{(u+1)}{3!}\left[\frac{(u+2)(u+3)\Delta^2 y_{-3} + u(u-1)\Delta^3 y_{-2}}{2}\right] +$$

$$+ \frac{u(u+1)(u+2)}{4!}\left[\frac{(u+3)\Delta^4 y_{-4} + (u-1)\Delta^4 y_{-2}}{2}\right] + \frac{u(u+1)(u+2)}{5!}\left[\frac{(u+3)(u+4)\Delta^5 y_{-5} + (u-1)(u-2)\Delta^5 y_{-5}}{2}\right] + \ldots \quad (10)$$

## 3. Comparisons of the Formulas by Examples

In order to compare our proposed formula of interpolation with the existing formulas we consider different examples. They are discussing in below.

**Problem 1:**

Consider the function $y = 3x^2 + 2x + 1$ whose value of $y$ for some equidistantly spaced values of $x$

| $x$ | **1** | **3** | **5** | **7** | **9** | **11** |
|---|---|---|---|---|---|---|
| $y = 3x^2 + 2x + 1$ | 6 | 34 | 86 | 162 | 262 | 386 |

**Table 1:** Difference Table for Problem 1

| x | y | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| **1** | 6 | | |
| | | 28 | |
| **3** | 34 | | 24 |
| | | 52 | |
| **5** | 86 | | 24 |
| | | 76 | |
| **7** | 162 | | 24 |

| | | | |
|---|---|---|---|
| | | 100 | |
| **9** | 262 | | 24 |
| | | 124 | |
| **11** | 386 | | 24 |
| | | 148 | |
| **13** | 534 | | |

Here we take $x = 6, x_0 = 7$ , $h = 2$ , and $u = \frac{x-x_0}{h} = \frac{6-7}{2} = -0.5$

Actual value is: $y(6) = 3(6)^2 + 2(6) + 1 = 121$

Gauss's Forward Interpolation Formula gives

$$y(6) = 162 + (-0.5)(100) + \frac{(-0.5)(-0.5-1)(24)}{2!} = 121$$

Gauss's Backward Interpolation Formula gives:

$$y(6) = 162 + (-0.5)(76) + \frac{(-0.5)(-0.5+1)(24)}{2!} = 121$$

Stirling's Interpolation Formula gives:

$$y(6) = 162 + (-0.5)\frac{(100+76)}{2} + \frac{(-0.5)^2}{2!}(24) = 121$$

Bessel's Interpolation Formula gives:

$$y(6) = \frac{162+262}{2} + \left(-0.5 - \frac{1}{2}\right)(100) + \frac{(-0.5)(-0.5-1)}{2}\left(\frac{24+24}{2}\right) = 121$$

Previously Proposed Formula by Faruq Abdulla et. al. gives:

$$y(6) = \frac{86+162}{2} + \left(-0.5 + \frac{1}{2}\right)(76) + \frac{(-0.5)(-0.5+1)}{2!}\left(\frac{24+24}{2}\right) = 121$$

Proposed Interpolation Formula gives:

$$y(6) = \frac{34+162}{2} + \left[1 - \frac{0.5}{2}\left(1 + \frac{76}{52}\right)\right](52) + \frac{(-0.5+1)}{2!}\left[\frac{(-0.5+2)(24)-(0.5)24}{2}\right] = 121$$

**Problem 2:**

The following table gives the value of the function $y = x^3 - 2x^2 + 7x - 3$ for certain equidistant values of $x$

| $x$ | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| $y = x^3 - 2x^2 + 7x - 3$ | -3 | 3 | 11 | 27 | 57 | 107 |

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**Table 2:** Difference Table for Problem 3

| x | $y=x^3-2x^2+7x-3$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| **0** | -3 | | | |
| | | 6 | | |
| **1** | 3 | | 8 | |
| | | 8 | | 6 |
| **2** | 11 | | 8 | |
| | | 16 | | 6 |
| **3** | 27 | | 14 | |
| | | 30 | | 6 |
| **4** | 57 | | 20 | |
| | | 50 | | |
| **5** | 107 | | | |

Here we take $x=2.3$, $x_0=3$ and since $h=1$, we have, $u=\frac{x-x_0}{h}=\frac{2.3-3}{1}=-0.7$

Actual value is: $y=x^3-2x^2+7x-3=(2.3)^3-2(2.3)^2+7(2.3)-3=14.687$

Again, Gauss's Forward Formula gives:

$$y(2.3)=27+(-0.7)(30)+(-0.7)(-0.7-1)\left(\frac{14}{2!}\right)+\frac{(-0.7)\left((-0.7)^2-1\right)(6)}{3!}=14.687$$

Gauss's Backward Interpolation Formula gives

$$y(2.3)=27+(-0.7)(16)+(-0.7)(-0.7+1)\left(\frac{14}{2!}\right)+\frac{(-0.7)\left((-0.7)^2-1\right)(6)}{3!}=14.687$$

Stirling's Interpolation Formula gives:

$$y(2.3)=27+(-0.7)\frac{16+30}{2}+\frac{(-0.7)^2}{2!}(14)+\frac{(-0.7)\left((-0.7)^2-1\right)}{3!}\left(\frac{6+6}{2}\right)=14.687$$

Bessel's Interpolation Formula gives

$$y(2.3)=\frac{57+27}{2}+\left(-0.7-\frac{1}{2}\right)30+\frac{-0.7(-0.7-1)}{2!}\frac{(14+20)}{2}+\frac{-0.7\left(-0.7-\frac{1}{2}\right)(-0.7-1)(6)}{3!}=14.687$$

Previously Proposed Formula by Abdulla et. al. gives:

$$y(2.3)=\frac{11+27}{2}+\left(-0.7-\frac{1}{2}\right)(16)+\frac{-0.7(-0.7-1)}{2!}\frac{(8+14)}{2}+\frac{(-0.7)\left(-0.7-\frac{1}{2}\right)(-0.7+1)(6)}{3!}=14.687$$

Proposed Interpolation Formula gives: $y(2.3)=\frac{3+27}{2}+\left[1-\frac{-0.7}{2}\left(1+\frac{16}{8}\right)\right]8+\frac{(-0.7+1)}{2!}\left[\frac{(-0.7+2)(2)+(-0.7)(14)}{2}\right]+$

$$+\frac{(-0.7+1)}{3!}\left[\frac{(-0.7+2)(-0.7+3)(6)+(-0.7)(-0.7-1)(6)}{2}\right]=14.687$$

**Problem 3:**

Consider the function $y=\sin x$ for some equidistantly spaced values of $x$

| $x$ | $45^0$ | $50^0$ | $55^0$ | $60^0$ |
|---|---|---|---|---|
| $y=\sin x$ | 0.7071 | 0.7660 | 0.8192 | 0.8660 |

**Table 3:** Difference Table for Problem 4

| x | $y=\sin x$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| $45^0$ | 0.7071 | | | |
| | | 0.0589 | | |
| $50^0$ | 0.7660 | | -0.0057 | |
| | | 0.0539 | | -0.0007 |
| $55^0$ | 0.8192 | | -0.0064 | |
| | | 0.0468 | | |
| $60^0$ | 0.8660 | | | |

Here we take $x=52^0$, $x_0=55$ and since $h=5$, we have, $u=\frac{x-x_0}{h}=\frac{52-55}{5}=-0.6$

Actual value is: $\sin 52^0=0.788010$

Gauss's Forward Interpolation Formula gives:

$$y(52^0) = 0.8192 + (-0.6)(0.0468) + (-0.6)\left((-0.6)^2 - 1)\right)\left(\frac{-0.0064}{2!}\right) + \frac{(-0.6)\left((-0.6)^2 - 1\right)(0)}{3!} = 0.7898912$$

Gauss's Backward Formula gives:

$$y(52^0) = 0.8192 + (-0.6)(000539) + (-0.6)\left(-0.6+1\right)\left(\frac{-0.0064}{2!}\right) + \frac{(-0.6)\left((-0.6)^2 - 1\right)(-0.0007)}{3!} = 0.787583$$

Stirling's Interpolation Formula gives:

$$y(52^0) = 0.8192 + (-0.6)\frac{0.0539 + 0.0468}{2} + \frac{(-0.6)^2}{2!}(-0.0064) + \frac{(-0.6)\left((-0.6)^2 - 1\right)}{3!}\left(\frac{-0.0007+0}{2}\right) = 0.7877036$$

Bessel's Interpolation Formula gives:

$$y(52^2) = \frac{0.8660 + 0.8192}{2} + \left(-0.6 - \frac{1}{2}\right)(0.0468) + \frac{-0.6(-0.6-1)}{2!}\frac{(-0.0064+0)}{2} + \frac{-0.6\left(-0.6 - \frac{1}{2}\right)(-0.6-1)(-0.0007)}{3!} = 0.7897072$$

Previously Proposed Formula by Faruq Abdulla et al. gives:

$$y(52^0) = \frac{0.7660 + 0.8192}{2} + \left(-0.6 + \frac{1}{2}\right)(0.0539) + \frac{-0.6(-0.6+1)}{2!}\frac{(-0.0057 - 0.0064)}{2} + \frac{(-0.6)\left((-0.6)^2 - \frac{1}{2}\right)(-0.6+1)(-0.0007)}{3!} = 0.78866172$$

Proposed Formula gives:

$$y(2.3) = \frac{0.7071 + 0.8192}{2} + \left[1 - \frac{0.6}{2}\left(1 + \frac{0.0539}{0.0589}\right)\right]0.0589 + \frac{(-0.6+1)}{2!}\left[\frac{(-0.6+2)(0) + (-0.6)(-0.0064)}{2}\right] +$$

$$+\frac{-0.6+1}{3!}\left[\frac{(-0.6+2)(-0.6+3)(0) + (-0.6)(-0.6-1)(-0.0007)}{2}\right] = 0.7885492$$

**Problem 4:**

The following table, gives the value of $e^x$ for certain equidistant values of $x$. We find the value of x when $x = 1.7489$

| $x$ | 1.72 | 1.73 | 1.74 | 1.75 | 1.76 | 1.77 | 1.78 |
|---|---|---|---|---|---|---|---|
| $y = e^x$ | 5.5845285 | 5.6406539 | 5.6973434 | 5.7546027 | 5.8124374 | 5.8708534 | 5.9298564 |

**Table 4:** Difference Table for Problem 2

| x | $y=e^x$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| **1.72** | 5.5845285 | | | | |
| | | 0.056125444 | | | |
| **1.73** | 5.6406539 | | 0.00056407 | | |
| | | 0.056689514 | | 0.000005669 | |
| **1.74** | 5.6973434 | | 0.000569739 | | 0.00000005697 |
| | | 0.057259253 | | 0.00000572597 | |
| **1.75** | 5.7546027 | | 0.000575465 | | 0.00000005755 |
| | | 0.057834718 | | 0.00000578352 | |
| **1.76** | 5.8124374 | | 0.000581249 | | 0.00000005813 |
| | | 0.058415967 | | 0.00000584165 | |
| **1.77** | 5.8708534 | | 0.00058709 | | |
| | | 0.059003057 | | | |
| **1.78** | 5.9298564 | | | | |

Here we take $x=1.7489$, $x_0=1.75$ and since $h=0.01$, we have, $u=\dfrac{x-x_0}{h}=\dfrac{1.7489-1.75}{0.01}=-0.11$

Actual value is: $y=e^x=e^{1.7489}=5.748276093$

Now, Gauss's Forward Interpolation Formula gives

$$y(1.7489)=5.7546027+(-0.11)(0.057834718)+(-0.11)(-0.11-1)\left(\frac{(0.000575465)}{2!}\right)+$$

$$+(-0.11)\left((+0.11)^2-1\right)\left(\frac{0.00000578352}{3!}\right)+(-0.11)\left((+0.11)^2-1\right)(-0.11-2)\left(\frac{0.00000005755}{4!}\right)=5.748277091$$

Gauss's Backward Interpolation Formula gives

$$y(1.7489)=5.7546027+(-0.11)(0.057259253)+(-0.11)(-0.11+1)\left(\frac{(0.000575465)}{2!}\right)+$$

$$+(-0.11)\left((+0.11)^2-1\right)\left(\frac{0.00000572597}{3!}\right)+(-0.11)\left((+0.11)^2-1\right)(-0.11+2)\left(\frac{0.00000005813}{4!}\right)=5.748271047$$

Stirling's Interpolation Formula gives:

$$y(1.7489)=5.7546027+(-0.11)\frac{(0.057259253+0.07834718)}{2}+\frac{(-0.11)^2}{2!}(0.000575465)+$$

$$+\frac{(-0.11)\left((-0.11)^2-1\right)}{3!}\left(\frac{0.0000057259+0.0000057835}{2}\right)+\frac{(-0.11)^2\left((-0.11)^2-1\right)}{4!}(0.0000000575)=5.748276106$$

Bessel's Interpolation Formula gives:

$$y(1.7489)=\frac{5.7546027+5.8124374}{2}+\left(-011-\frac{1}{2}\right)0.057834718+\frac{-0.11(-0.11-1)}{2!}\frac{(0.000575465+0.000581249)}{2}+\frac{-0.11\left(-0.11-\frac{1}{2}\right)(0.11-1)}{3!}(0.0000057838)+$$

$$+\frac{(-0.11)\left((-0.11)^2-1)\right)(-0.11-2)}{4!}\frac{(0.0000000575+0.0000000581)}{2}=5.748276093$$

Previously Proposed Formula by Faruq Abdulla et al. gives:

$$y(1.7589)=\frac{5.7546027+5.8124374}{2}+\left(-0.11+\frac{1}{2}\right)(0.057834718)+\frac{-0.11(-0.11+1)}{2!}\frac{(0.000575465+0.000581249)}{2}+$$

$$\frac{(-0.11)\left((-0.11)^2-\frac{1}{2}\right)(-0.11+1)(0.00000578352)}{3!}+\frac{(-0.11)\left((-0.11)^2-1\right)(-0.11+2)}{4!}\left(\frac{0+0.813\times10^{-8}}{2}\right)=5.806047233$$

Proposed Interpolation Formula gives:

$$y(1.7589)=\frac{5.6406539+5.7546027}{2}+\left[1-\frac{-0.11}{2}\left(1+\frac{0.05725953}{0.056689514}\right)\right]0.056689514+\frac{(-0.11+1)}{2!}\left[\frac{(-0.11+2)(0.00056407)+(-0.11)(0.000575465)}{2}\right]+$$

$$+\frac{(-0.11+1)}{3!}\left[\frac{(-0.11+2)(-0.11-3)(0.000005669)+(-0.11)(-0.11-1)(0.000569739)}{2}\right]+$$

$$\frac{(-0.11)(-0.11+1)(-0.11+2)}{4!}\left[\frac{(-0.11+3)(0)+(-0.11-1)(0.00000005755)}{2}\right]=5.748276011$$

**Problem 5:**

The following table gives the value of the function $y=\sqrt{x}$ for some equidistantly spaced values of $x$

| $x$ | **1** | **3** | **5** | **7** | **9** |
|---|---|---|---|---|---|
| $y=\sqrt{x}$ | 0.1 | 1.73205 | 2.23607 | 2.64575 | 3 |

**Table 5:** Difference Table for Problem 5

| x | $y=\sqrt{x}$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|

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| | | | | | |
|---|---|---|---|---|---|
| **1** | 1.0 | | | | |
| | | 0.73205 | | | |
| **3** | 1.73205 | | -0.22803 | | |
| | | 0.50402 | | 1.3694 | |
| **5** | 2.23607 | | -0.09434 | | -0.03935 |
| | | 0.40968 | | 1.33039 | |
| **7** | 2.64575 | | 1.23605 | | |
| | | 1.64573 | | | |
| **9** | 3 | | | | |

Here we take $x = 3.8$, $x_0 = 5$ and since $h = 2$, we have, $u = \frac{x - x_0}{h} = \frac{3.8 - 5}{2} = -0.6$

Actual value is: $y = \sqrt{x} = \sqrt{3.8} = 1.949358869$

Gauss's Forward Interpolation Formula gives:

$$y(3.8) = 2.23607 + (-0.6)(0.40968) + (-0.6)(-0.6-1)\left(\frac{-0.09434}{2!}\right) + \frac{(-0.6)\left((-0.6)^2 - 1\right)(1.33039)}{3!} +$$

$$+\frac{(-0.6)\left((-0.6)^2 - 1\right)(-0.6-2)(-0.03935)}{4!} = 2.044588174$$

Gauss's Backward Interpolation Formula gives:

$$y(3.8) = 2.23607 + (-0.6)(0.50402) + (-0.6)(-0.6+1)\left(\frac{-0.09434}{2!}\right) + \frac{(-0.6)\left((-0.6)^2 - 1\right)(1.36974)}{3!} +$$

$$+\frac{(-0.6)\left((-0.6)^2 - 1\right)(-0.6+2)(-0.03935)}{4!} = 2.03173096$$

Stirling's Formula Interpolation gives:

$$y(3.8) = 2.23607 + (-0.6)\frac{0.50402 + 0.40968}{2} + \frac{(-0.6)^2}{2!}(-0.22803) + \frac{(-0.6)\left((-0.6)^2 - 1\right)}{3!}\left(\frac{1.3694 + 1.33039}{2}\right) +$$

$$+\frac{(-0.6)^2\left((-0.6)^2 - 1\right)(-0.03935}{4!} = 1.96563288$$

Bessel's Interpolation Formula gives:

$$y(3.8) = \frac{2.64575 + 2.23607}{2} + \left(-0.6 - \frac{1}{2}\right)(0.40968) + \frac{-0.6(-0.6-1)}{2!}\frac{-0.09434 + 1.23605}{2} + \frac{-0.6\left(-0.6 - \frac{1}{2}\right)(-0.6-1)}{3!}(1.33039) +$$

$$+\frac{(-0.6)\left((-0.6)^2-1\right)(-0.6-2)}{4!}\left(\frac{-0.03935+0}{2}\right)=2.03094224$$

Previously Proposed Formula by Faruq Abdulla et. al. gives:

$$y(3.8)=\frac{1.73205+2.23607}{2}+\left(-0.6+\frac{1}{2}\right)(0.50402)+\frac{-0.6(-0.6+1)}{2!}\frac{(-0.22808-0.9434)}{2}+\frac{(-0.6)\left((-0.6)^2-\frac{1}{2}\right)(-0.6+1)(-0.03935)}{3!}=2.00362644$$

Proposed Interpolation Formula gives:

$$y(3.8)=\frac{1+2.2360}{2}+\left[1-\frac{-0.6}{2}\left(1+\frac{0.50402}{0.73205}\right)\right]0.73205+\frac{(-0.6+1)}{2!}\left[\frac{(-0.6+2)(0)+(-0.6)(1.33039)}{2}\right]+$$

$$+\frac{(-0.6+1)}{3!}\left[\frac{(-0.6+2)(-0.6+3)(0)+(-0.6)(-0.6-1)(1.36974)}{2}\right]+\frac{(-0.6)(-0.6+1)(-0.6+2)}{4!}\left[\frac{(0.6+3)(0)+(-0.6-1)(-0.03935)}{2}\right]=1.94283156$$

**Table 5: Summary/Comparison Table**

| Problem | Gauss's Forward Formula | Gauss's backward Formula | Stirling's Formula | Bessel's formula | Proposed Formula by Abdulla et al. | New(proposed) | True Value |
|---|---|---|---|---|---|---|---|
| Problem 1 | 70.75 | 70.75 | 70.75 | 70.75 | 70.75 | 70.75 | 70.75 |
| Problem 2 | 14.687 | 14.687 | 14.687 | 14.687 | 14.687 | 14.687 | 14.687 |
| Problem 3 | 0.7898912 | 0.787583 | 0.7877036 | 0.7897072 | 0.78866172 | 0.7885492 | 0.7885492 |
| Problem 4 | 5.748277 | 5.748271 | 5.7482761 | 5.7482761 | 5.806047233 | 5.748276 | 5.748276 |
| Problem 5 | 2.0445882 | 2.031731 | 1.9656329 | 2.0309422 | 2.0036244 | 1.9428316 | 1.9493589 |

**Error Analysis:**

We shall now carry out an error analysis using the concept of mathematical *norm*, to determine which numerical method of interpolation from **Table 5** above, is best.

**Table 6:** Comparing the Actual Values of the functions and the values obtained using Gauss's Forward Interpolation Formula:

| Actual(A) | Gauss's Forward($P_1$) | $K_1$ = \|A-$P_1$\| | %Error |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.788549 | 0.7898912 | 0.001342 | 0.170186 |
| 5.748276 | 5.748277 | 9.98E-07 | 1.74E-05 |

| | | | |
|---|---|---|---|
| 1.949359 | 2.0445882 | 0.095229 | 4.88516 |

$$\|K_1\|_1 = \begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7898912|+|5.74827601-5.748277009| \\ +|1.94935887-2.044588174| \end{bmatrix}$$

= 0.096572

$$\|K_1\|_2 = \sqrt{\begin{aligned}&(|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.7898912|)^2+ \\ &+(|5.74827601-5.748277009|)^2+(|1.94935887-2.044588174|)^2\end{aligned}}$$

= 0.095239

$$\|K_1\|_\infty = \max\begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7898912|+|5.74827601-5.748277009|+ \\ +|1.94935887-2.044588174| \end{bmatrix}$$

= 0.095229

**Table 7:** Comparing the Actual Values of the functions and the values obtained using Gauss's Backward Interpolation Formula

| Actual(A) | Gauss's Backward($P_2$) | $K_2 = \|A- P_2\|$ | %Error |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.788549 | 0.787583 | 0.000966 | 0.122529 |
| 5.748276 | 5.748271 | 4.96E-06 | 8.64E-05 |
| 1.949359 | 2.031731 | 0.082372 | 4.225599 |

$$\|K_2\|_1 = \begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.787583|+|5.74827601-5.748271047|+ \\ +|1.94935887-2.03173096| \end{bmatrix}$$

=0.083343

$$\|K_2\|_2 = \sqrt{\begin{aligned}&(|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.787583|)^2 \\ &+(|5.74827601-5.748271047|)^2\end{aligned}}$$

=0.082378

$$\|K_2\|_\infty = \max\begin{bmatrix}|70.5-70.5|+|14.687-14.687|+|0.7885492-0.787583|+\\+|5.74827601-5.748271047|+|1.94935887-2.03173096|\end{bmatrix}$$

= 0.082372

**Table 8:** Comparing the Actual Values of the functions and the values obtained using Stirling's Formula:

| **Actual(A)** | **Stirling's Formula($P_3$)** | **$K_3$ = \|A- $P_3$\|** | **%Error** |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.788549 | 0.7877036 | 0.000846 | 0.107235 |
| 5.748276 | 5.7482761 | 9.50E-08 | 1.65E-06 |
| 1.949359 | 1.9656329 | 0.016274 | 0.834839 |

$$\|K_3\|_1 = \begin{bmatrix}|70.5-70.5|+|14.687-14.687|+|0.7885492-0.7877036|+|5.74827601-5.748276106|+\\+|1.94935887-1.96563288|\end{bmatrix}$$

= 0.01712

$$\|K_3\|_2 = \sqrt{\begin{matrix}(|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.7877036|)^2\\+(|5.74827601-5.748276106|)^2+(|1.94935887-1.96563288|)^2\end{matrix}}$$

= 0.016296

$$\|K_3\|_\infty = \max\begin{bmatrix}|70.5-70.5|+|14.687-14.687|+|0.7885492-0.787583|+\\+|5.74827601-5.748271047|+|1.94935887-1.96563288|\end{bmatrix}$$

= 0.016274

**Table 9:** Comparing the Actual Values of the functions and the values obtained using Bessel's Interpolation Formula:

| **Actual(A)** | **Bessel's Formula($P_4$)** | **K = \|A-$P_4$\|** | **%Error** |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.788549 | 0.7897072 | 0.001158 | 0.146852 |
| 5.748276 | 5.7482761 | 8.20E-08 | 1.43E-06 |
| 1.949359 | 2.0309422 | 0.081583 | 4.185139 |

$$\|K_4\|_1 = \begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7897072|+|5.74827601-5.748276093|+ \\ +|1.94935887-2.03094224| \end{bmatrix}$$

= 0.082741

$$\|K_4\|_2 = \sqrt{\begin{matrix} (|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.7897072|)^2 \\ +(|5.74827601-5.748276093|)^2+(|1.94935887-2.03094224|)^2 \end{matrix}}$$

= 0.081592

$$\|K_4\|_\infty = \max\begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7897072|+ \\ +|5.74827601-5.748276093|+|1.94935887-2.03094224| \end{bmatrix}$$

= 0.081583

**Table 10:** Comparing the Actual Values of the functions and the values obtained using the proposed interpolation formula by Faruq Abdulla et al.

| Actual(A) | Formula by Faruq Abdulla et. al ($P_5$) | K = $\|A-P_5\|$ | %Error |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.7885492 | 0.78866172 | 0.000113 | 0.014269 |
| 5.74827601 | 5.806047233 | 0.057771 | 1.005018 |
| 1.94935887 | 2.0036244 | 0.054266 | 2.783763 |

$$\|K_5\|_1 = \begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.78866172|+|5.74827601-5.806047233|+ \\ +|1.94935887-2.0036244| \end{bmatrix}$$

= 0.112149

$$\|K_5\|_2 = \sqrt{\begin{matrix} (|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.78866172|)^2 \\ +(|5.74827601-5.806047233|)^2+(|1.94935887-2.0036244|)^2 \end{matrix}}$$

=0.079261

$$\|K_5\|_\infty = \max\begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7885492|+ \\ +|5.74827601-5.748276011|+|1.94935887-1.94283156| \end{bmatrix}$$

= 0.057771

**Table 11:** Comparing the Actual Values of the functions and the values obtained using the New (proposed) Interpolation Formula:

| Actual(A) | Proposed Formula ($P_6$) | K = $\|A-P_6\|$ | %Error |
|---|---|---|---|

| | | | |
|---|---|---|---|
| 70.75 | 70.75 | 0 | 0 |
| 14.687 | 14.687 | 0 | 0 |
| 0.788549 | 0.7885492 | 0 | 0 |
| 5.748276 | 5.748276 | 0 | 0 |
| 1.949359 | 1.9428316 | 0.006527 | 0.334844 |

$$\|K_6\|_1 = \begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7885492|+|5.74827601-5.748276011|+ \\ +|1.94935887-1.94283156| \end{bmatrix}$$

$= \ 0.006527$

$$\|K_6\|_2 = \sqrt{\begin{matrix} (|70.5-70.5|)^2+(|14.687-14.687|)^2+(|0.7885492-0.7885492|)^2 \\ +(|5.74827601-5.748276011|)^2+(|1.94935887-1.94283156|)^2 \end{matrix}}$$

$= 0.006527$

$$\|K_6\|_\infty = \max\begin{bmatrix} |70.5-70.5|+|14.687-14.687|+|0.7885492-0.7885492|+ \\ +|5.74827601-5.748276011|+|1.94935887-1.94283156| \end{bmatrix}$$

$= 0.006527$

**Table 11: Error Comparison**

| Norm | Gauss's Forward | Gauss's Backward | Stirling's Formula | Bessel's Formula | Formula by Abdulla et al. | Proposed Formula |
|---|---|---|---|---|---|---|
| $\|1-norm\|$ | 0.096572 | 0.083343 | 0.01712 | 0.082741 | 0.112149 | 0.006527 |
| $\|2-norm\|$ | 0.095239 | 0.082378 | 0.016296 | 0.081592 | 0.079261 | 0.006527 |
| $\|\infty-norm\|$ | 0.095229 | 0.082372 | 0.016274 | 0.081583 | 0.05771 | 0.006527 |

## 4. CONCLUSION

This paper is on a New (proposed) Formula for Interpolation and Comparison with existing models of interpolation, using the concept of mathematical norm. The new model given in equation (8), is center based i.e. when the value to be interpolated is from the centre region in a given data set. The New formula was obtained by retreating the subscript in Gauss's Forward Interpolation Formula by two units and replacing $u$ by $u+2$, then the resulting equation was added to the Gauss's Backward Interpolation Formula and the mean taken to obtain the New (proposed) Model. The New (proposed) Formula for Interpolation was then tested against the existing Formulae which includes: Gauss's Forward Interpolation Formula, Gauss's Backward Interpolation Formula, Stirling's Interpolation

Formula, and Bessel's Interpolation Formula and a Formula previously proposed by Faruq Abdulla et al. The results obtained, was analyzed using the concept of *Mathematical Norm*, and it was discovered that the New model has the minimum errors with respect to *1-norm*, *2-norm* and *infinity-norm*. Therefore, the New (proposed) model, is best for central difference based Interpolation

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