Satellites: The differential equations of motion of the system

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Abstract- This paper deals with the analyses of the effect of earth's oblateness and magnetic force on the motion of a system of two particles .The relative motion of the particles ,for case of circular motion of the center of mass of the system; The satellites may oscillate about some equilibrium position of the system in elliptical orbit. In three dimensional spaces, the particle is bound to attain the sphere .At a certain moment and after that the motion will take place with tight string and the particle will start moving on the surface of the sphere.

Index Terms- Motion ,sphere, circular orbit,Equilibrium,tight string

I. Introduction

The analysis of the combined effects of eartws oblateness and magnetic force on the motion of two satellites. The equations of motion have been integrated in terms of elementary functions.

The free motion is bound to be converted into constrained motion with tight string. We also analyse the condition of the constraint and certain kinematical relations that of a dumb-bell satellite. We also interested in particular equilibrium solutions of the equation of motion regarding the equations of motion as linear one and the nature of the motion of the system .We also emphasis on the Liapunov stable as

$$(i)\emptyset = 0 \qquad ; \psi = sin^{-1}\left[\frac{A}{3-5A_0}\right]$$

$$(ii)\emptyset = 0 \qquad ; \psi = 0$$

Mathematical Approach

Suppose the center of mass of the system moves in a circular orbit so that $e = 0\,; \rho = 1; \rho' = 0$
The equation of the system becomes

$$\left.\begin{aligned} x'' - 2y' - 3x &= \lambda\alpha^{x} - 4A_0^{x} - Acosi \\ y'' + 2x' &= \lambda\alpha^{y} + A_0^{y} \end{aligned}\right\} - - - - - (1)$$

$$z'' + z = \lambda\alpha^{z} + A_0^{z} - \frac{A}{\mu_E}(3p^3 - \mu_E)\sin(v + w)\,sini$$

We have

$$A = \frac{m_1}{m_1 + m_2}\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu E}{\sqrt{\mu\rho}}$$

$$\lambda\alpha = \frac{\rho^3\rho^4}{\mu}\left(\frac{m_1 + m_2}{m_1 . m_2}\right).\lambda$$

$$A_0 = \frac{-3k_2}{\rho^2}$$

The condition of the constraint is

$$x^2 + y^2 + z^2 \leq 1 - - - -(2)$$

There are three cares arise

(i) Loose string
(ii) Tight string
(iii) Alternately loose and tight string

Now for (i)

$$x^2 + y^2 + z^2 \leq 1 \;\; and\; \lambda_\alpha = 0$$

$$\therefore m_2 moves\ inside\ the\ sphere\ x^2 + y^2 + z^2 \leq 1 - - - -(3)$$

The equation (i)becomes

$$\left.\begin{aligned} x'' - 2y' - 3x &= -4A_0 - Acosi \\ y'' + 2x' &= A_0{}^{.y} \end{aligned}\right\} - - - - - - (4)$$

$$z'' + z = A_0{}^{.z} - \frac{A}{\mu_E}(3\rho^3 - \mu_E)\sin(v + w)\, sini$$

The first two equations be reduced as

$$\left.\begin{aligned} x'' - 2y' - (3 - 4A_0)x &= -Acosi \\ y'' + 2x' &= A_0{}^{.y} \end{aligned}\right\} - - - - - - (5)$$

The homogeneous equation

$$z'' + z(1 - A_0) = 0$$

$$z'' + z(1 - A_0) = 0$$

$$z' = -((1 - A_0)z\ in\ simple\ H.metion\ with\ period\ \frac{2\pi}{\sqrt{(1 - A_0)}}$$

$$\therefore z = c\cos\sqrt{(1 - A_0)}.\, v + D.\sin\sqrt{(1 - A_0)}.\, v$$

We take trivial sol^n

$$\left.\begin{aligned} x &= Me^{mv} + \frac{Acosi}{(3-4A_0)} \\ y &= Ne^{mv} \end{aligned}\right\} - - - - - (6)$$

$$\left.\begin{aligned} x' &= M.me^{mv}; y' = N.me^{mv} \\ x'' &= M.m^2e^{mv}; y'' = N.m^2e^{mv} \\ x''' &= M.m^3e^{mv}; y''' = N.m^3e^{mv} \end{aligned}\right\} - - - - - - - - -(7)$$

From (4); we replace the values (7)

$$Mm^3 - 2Nm^2(3 - 4A_0)Mm = 0$$

$$Nm^2 + 2Mm - A_0.N = 0$$

$$i.e\ M(m^2+4A_0-3)+(-2m)N=0$$
$$and\ M(2m)+(m^2-A_0)N=0$$

Condition for solvability

$$\begin{vmatrix} m^2+4A_0-3 & -2m \\ 2m & m^2-A_0 \end{vmatrix}=0$$
$$m^4+m^2(3A_0+1)+(3-4A_0)A_0=0$$

$$(m^2)^2+m^2(3A_0+1)+(3-4A_0)A_0=0$$

$$m^2=\frac{-(1+3A_0)\pm\sqrt{(25{A_0}^2-6A_0+1)}}{2}$$
$$=-1/2[(1+3A_0)\mp(25{A_0}^2-6A_0+1)]$$

Taking positive sign

$$m^2=-\frac{1}{2}\left[(1+3A_0)+\sqrt{\left(25{A_0}^2-6A_0+1\right)}\right]$$

$$m_1,m_2=\pm\frac{i}{\sqrt{2}}\sqrt{(1+3A_0)+\sqrt{\left(25{A_0}^2 6A_0+1\right)}}$$

Taking Negative sign

$$m^2=\frac{1}{2}\left[\sqrt{\left(25{A_0}^2-6A_0+1\right)}-(1+3A_0)\right]$$
$$m_3,m_4=\pm\frac{1}{\sqrt{2}}\sqrt{\left(25{A_0}^2-6A_0+1\right)}-(1+3A_0)$$

$$The\ general\ solution$$
$$x=Mm_1.e^{m_1v}+Mm_2e^{m_2v}+Nm_3.e^{m_3v}+Nm_4.e^{m_4v}$$
$$Now$$
$$N(-2m)+(m^2-4A_0-3)M=0$$

$$\frac{M}{2m}=\frac{N}{m^2-4A_0-3}$$

If it possible

$$m=m_1$$
$$M=Mm_1$$

$$N = Nm_1$$

$$\frac{Mm_1}{2m_1} = \frac{Nm_1}{m^2{}_1 - 4A_0 - 3} = \alpha_1(say)$$
$$Nm_2 = (m^2{}_2 - 4A_0 - 3)\alpha_2$$
$$Nm_3 = (m^2{}_3 - 4A_0 - 3)\alpha_3$$
$$Nm_4 = (m^2{}_4 - 4A_0 - 3)\alpha_4$$

By observing the general solution of them we say that appearing term $e^{m_{3v}}\ in\ x\ and\ y, m_3$ are real and positive. It then in definite increase of time V, the expression for x and y will increase indefinitely .We say that m_2 is bound to touch the circle $x^2 + y^2 = 1$

But in three dimensional spaces (R^3) the particle m_2 bound to attain the sphere $x^2 + y^2 + z^2 = 1$

II. Conclusion

A certain moment the motion will take place with tight string and the particle will start moving on the surface of the sphere.

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