Modelling and Forecasting LKR/USD Monthly Exchange Rates using Constant Elasticity of Variance (CEV) Model

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Abstract- This paper model and forecast monthly exchange rates of US Dollar to Sri Lankan Rupee using Constant Elasticity of Variance (CEV) Model. In this study, five models are formulated considering five elasticity factors $0, {}^{1}/_{2}, 1, {}^{3}/_{2}$ and 2. Monthly exchange rates from January, 1995 to December, 2016 were obtained from the official website of Central Bank of Sri Lanka. Among this data, 254 observations were used to estimate the parameters and other 10 observations were used to test the validity of the models. To estimate the parameters, maximum likelihood estimation method is used and the exchange rates are generated using Euler-Maruyama method. The Monte Carlo technique is used for simulation and the accuracy of the forecasts is compared with Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). The model which has the minimum RMSE and MAPE is chosen as the best model to predict the monthly exchange rate of LKR/USD.

Index Terms- Constant Elasticity of Variance Model, Maximum likelihood estimation, Monte Carlo technique, Euler- Maruyama method, Exchange rate

INTRODUCTION

Most countries use their own currencies as a medium of exchange, similar to the Rupee in Sri Lanka and the Dollar in the United States. Whenever a country with its own unique currency has to make payments to other countries which have different currencies, it has to exchange its currency with other currencies at a given rate of exchange. The rate at which one currency may be exchanged against another is called "the exchange rate". The exchange rate is formally defined as the number of units of one currency that can be exchanged for a unit of another [3]. Exchange rate is very important in financial market, because it determines the level of imports and exports. Hence modeling and forecasting exchange rate is most important for the people who are doing international businesses to minimize the risk.

There are two pure approaches to forecast foreign exchange rates, fundamental and technical approach. The fundamental approach is based on a wide range of data regarded as fundamental economic variables that determine exchange rates. The technical approach focuses on a smaller subset of the available data. In general, it is based on price information. The analysis is "technical" in the sense that it does not rely on a fundamental analysis of the underlying economic determinants of exchange rates or asset prices, but only on extrapolations of past price trends. Technical analysis looks for the repetition of specific price patterns. Generally, econometric models, time series models or a combination of both methods are used to forecast the exchange rate.

Stochastic differential equation (SDE) models can be used in many fields of science especially in finance. Black-Scholes model is the famous model among them. The constant elasticity of variance (CEV) model is a diffusion model with the instantaneous volatility specified to be a power function of the underlying spot price.

In this research, LKR/USD exchange rate is modeled using the CEV model and forecast the exchange rate using that model. Five elasticity factors $0, {}^{1}/_{2}, 1, {}^{3}/_{2}$ and 2 were considered and the best model is selected from these five models. Monthly exchange rates from January, 1995 to December, 2016 were obtained from the official website of Central bank of Sri Lanka. Among these data, 254 observations are used to estimate the parameters and other 10 observations are used to test the validity of the model. Maximum likelihood estimation method is used to estimate the parameters and. to approximate the solution of the SDE, Euler- Maruyama method is used. The Monte Carlo technique is used for simulation purpose and the accuracy of the forecasts is compared with Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE).

I. LITERATURE REVIEW

In literature, many studies were carried out to model and forecast the exchange rate. Booth and Glassman (1987), compared the exchange rate forecasting models considering forecast accuracy and profitability. Tenti (1996), proposed the use of recurrent neural networks to forecast foreign exchange rates. Nabni et al. (1996), described how modern machine learning techniques can be used in conjunction with statistical methods to forecast short term movements in exchange rates. Vinod and Samantha (1997), applied eight SDE models to exchange rate dynamics and evaluated the best model. They used three estimation methods, generalized method of moments (GMM), small sigma asymptotic estimating function (SSA-EF) and numerical conditional variance (NCV) method. Soofi and Cao (1999), performed out-of-sample predictions on daily Peseta–Dollar spot exchange rates using a simple nonlinear deterministic technique of local linear predictor. Gencay (1999), investigated the predictability of spot foreign exchange rate returns from past buy-sell signals of the simple technical trading rules by using the nearest neighbors and the feed forward network regressions.

Majhi, Panda and Sahoo (2009), developed two novel ANN models, functional link artificial neural network (FLANN) and cascaded functional link artificial neural network (CFLANN) involving nonlinear inputs and simple ANN structure with one or two neurons. Rime, Sarno and Sojli (2010) examined the linkages between exchange rate movements, order flow and expectations of macroeconomic variables. Jalil and Fleriden (2010) explained the exchange rate movements in the Pakistan foreign exchange market using the market micro structure approach. Simpson and Grossman (2010) used a relative purchasing power parity (PPP) model to construct a time-varying equilibrium exchange rates.

Abdorrahman et al. (2011), developed three decision making models to maximize profit of trades during a specific period and forecasted the direction of exchange rate over a specific period on the basis of values of indicators in previous time period. Pacelli (2012) analyzed and compared different mathematical models such as artificial neural networks, ARCH and GARCH models. Irena and Andrius (2013) discussed the advantages and drawbacks of the main fundamental exchange rate forecasting models. Yuan (2013), presented the polynomial smooth support vector machine (PSSVM) learning model. Minakhi et al. (2014), proposed a hybrid prediction model by combining an adaptive autoregressive moving average (ARMA) architecture and differential evolution (DE) based training of its feed-forward and feed-back parameters. Mehreen et al. (2014), explored Neuro- evolution and evaluated for its application in devising prediction models for foreign currency exchange rates. Tlegenova (2015), modeled yearly exchange rates between USD/KZT, EUR/KZT and SGD/KZT, and compared the actual data with developed forecasts using time series analysis over the period from 2006 to 2014. Urrutia et al. (2015), formulated a mathematical model to forecast exchange rate of the Philippines from the 1st Quarter of 2015 up to the 4th Quarter of 2020 using Autoregressive integrated Moving Average (ARIMA).

II. THE METHODOLOGY

The monthly exchange rate of LKR/USD for the period from January, 1995 to December, 2016 is represented in Figure 1. It can be observed that there is an upward trend of monthly exchange rate of LKR/USD in this period.

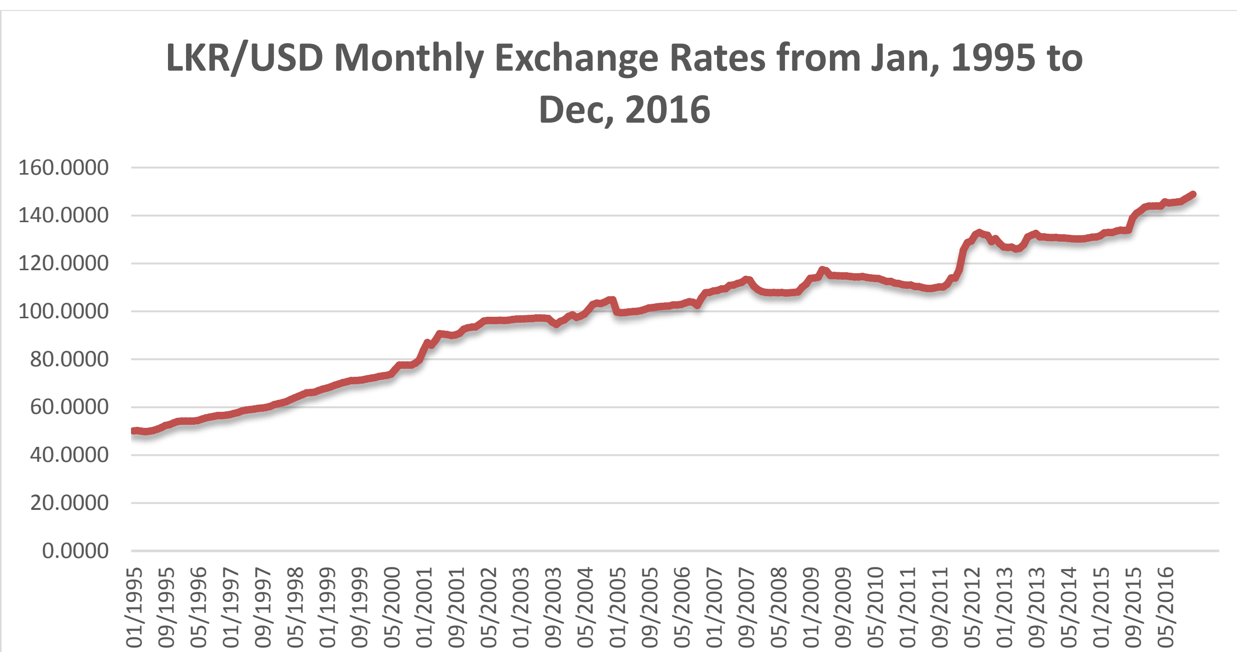

Figure 1: Monthly Exchange Rates of LKR/USD from January, 1995 to December, 2016

The diffusion process of $X(t)$ at time t in a CEV model can be expressed as,

$$dX(t) = \mu X(t)dt + \sigma\big(X(t)\big)^{\beta/2} dW(t) \qquad 0 \le \beta < 2 \qquad \textbf{(1)}$$

where $X(t)$ is the exchange rate at month t, $\mu X(t)$ is the drift coefficient, $\sigma\big(X(t)\big)^{\beta/2}$ is the diffusion coefficient, $W(t)$ is a Weiner process and β is the elasticity factor. If $\beta = 2$, the CEV model returns to the conventional Black–Scholes model in which the variance rate is independent of the stock price. If $\beta = 0$, it is the Ornstein–Uhlenbeck model.
The solution of the equation (3.1) can be approximated using Euler-Maruyama method and it is given by,

$$X(t) = X(t-1) + \mu X(t-1)\Delta t + \sigma X(t-1)^{\beta/2}\big(W(t) - W(t-1)\big) \qquad \textbf{(2)}$$

In this research, five cases of β were considered. They are $0, \frac{1}{2}, 1, \frac{3}{2}$ and 2. First of all, the parameters of the equation (1) must be calculated. For this purpose, maximum likelihood estimation method [1] is used.

Let $x_0, x_1, x_2, \dots, x_N$ are observed values of $X(t)$ at the respective uniformly distributed times $t_i = i\Delta t$ for $i = 0, 1, \dots, N$ where, $\Delta t = T/N$. Let $p(t_k, x_k | t_{k-1}, x_{k-1};\, \theta)$ be the transition probability density of (t_k, x_k) starting from (t_{k-1}, x_{k-1}) given the vector θ. Suppose that the density of the initial state is $p_0(x_0|\theta)$.

In maximum likelihood estimation of θ, the joint density

$$D(\theta) = p_0(x_0|\theta)\prod_{k=1}^{N} p(t_k, x_k | t_{k-1}, x_{k-1};\, \theta)$$

is maximized over $\theta \in \mathbb{R}^m$. It is more convenient to minimize the function

$$L(\theta) = -ln\,(D(\theta))$$

which has the form

$$L(\theta) = -\ln\big(p_0(x_0|\theta)\big) - \sum_{k=1}^{N} \ln\big(p(t_k, x_k | t_{k-1}, x_{k-1};\, \theta)\big)$$

One difficulty in finding the optimal value θ^* is that the transition densities are not generally known. However, by considering the Euler approximation and letting $X(t_{k-1}) = x_{k-1}$ at $t = t_{k-1}$

$$X(t_k) \approx x_{k-1} + \mu x_{k-1}\Delta t + \sigma x_{k-1}{}^{\beta/2}\sqrt{\Delta t}\,\eta_k$$

where $\eta_k \sim N(0, \Delta t)$.
This implies that

$$p(t_k, x_k | t_{k-1}, x_{k-1};\, \theta) = \frac{1}{\sqrt{2\pi\sigma_k{}^2}} exp\left(\frac{-(x_k - \mu_k)^2}{2\sigma_k{}^2}\right)$$

where $\mu_k = x_{k-1} + \mu x_{k-1}\Delta t$ and $\sigma_k = \sigma x_{k-1}{}^{\beta/2}\sqrt{\Delta t}$.

This transition density can be substituted into the expression for $L(\theta)$ which can subsequently be minimized over $\mathbb{R}^m$.
Then, the parameters $\hat{\mu}$ and $\hat{\sigma}$ can be calculated using the equations:

$$\hat{\mu} = \frac{\sum_{k=0}^{N} \frac{(x_{k+1} - x_k)}{x_k{}^{\beta-1}}}{\Delta t \sum_{k=0}^{N} x_k{}^{2-\beta}}$$

and

$$\hat{\sigma} = \sqrt{\frac{\sum_{k=0}^{N} x_{k+1}\left(x_k{}^{-\beta/2} - (1 + \mu\Delta t)x_k{}^{1-\beta/2}\right)^2}{\Delta t}}$$

for $0 \le \beta < 2$.

After estimating parameters, sample paths for the exchange rates can be generated for each value of β. To simulate these sample paths, Monte Carlo method is used. Monte Carlo means using random numbers as a tool to compute something that is not random. It can be used to solve any problem having a probabilistic interpretation. By the law of large numbers, integrals described by the expected value of some random variable can be approximated by taking the empirical mean of independent samples of the variable.

As an example, let X be a random variable and write its expected value as $A = E(X)$. If we can generate $X_1, X_2, \dots, X_n$, n independent random variables with the same distribution, then we can make the approximation,

$$\widehat{A_n} = \frac{1}{n}\sum_{i=1}^{n} X_i.$$

By law of large numbers, $\widehat{A_n} \to A$ as $n \to \infty$. X_i and $\widehat{A_n}$ are random and could be different each time we run the program. Still, the target number A is not random.

In this research work, forecasting is an important task. According to the five CEV models which are considered, future exchange rates can be forecasted using Monte Carlo simulation. Finally, the accuracy of forecasting is calculated using two methods, root mean square error (RMSE) and mean absolute percentage error (MAPE). The equations are given by,

$$RMSE = \sqrt{\sum_{i=1}^{m} \frac{(x_i - \widehat{x_i})^2}{m}}$$

and

$$MAPE = \left(\left(\sum_{i=1}^{m} \left| \frac{x_i - \widehat{x_i}}{x_i} \right| \right) \Big/ m \right) \times 100$$

where x_i is the observed value, $\widehat{x_i}$ is the forecasted value at time $t = i$ and m is the number of observations. Minimum value of these values indicate the best model.

III. RESULTS AND FINDINGS

In this research, five CEV models are considered. According to the value of elasticity factor(β), parameter estimations of each CEV model is given in the Table I. To estimate the parameters, 254 observations are used.

Table I: Estimated Parameter Values for Five CEV Models

Elasticity factor (β)	$\hat{\mu}$	$\hat{\sigma}$
0	0.0417	61.7786
$^1/_2$	0.0435	19.1334
1	0.0456	5.9481
$^3/_2$	0.0481	1.8568
2	0.0508	0.5822

By substituting the values of the Table I to the equation (2), Euler-Maruyama approximations of each model can be obtained. Then, by taking the initial value as the exchange rate of January, 1995 more sample paths can be generated. From Figure 2 to Figure 6 represent five sample paths for each CEV model.

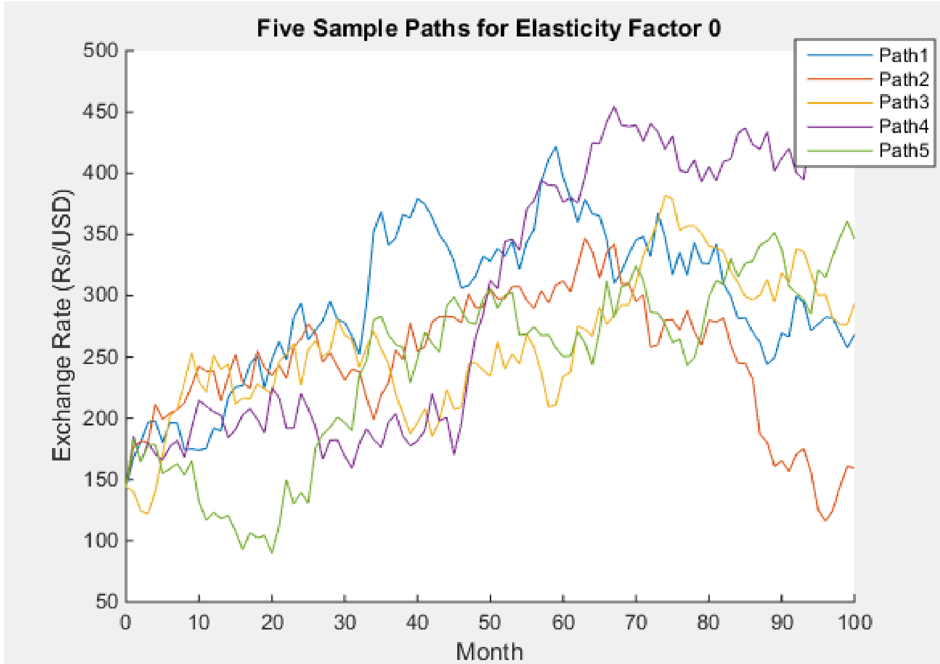

Figure 2: Five Sample Paths for the CEV Model $dX(t) = 0.0417X(t)dt + 61.7786dW(t)$

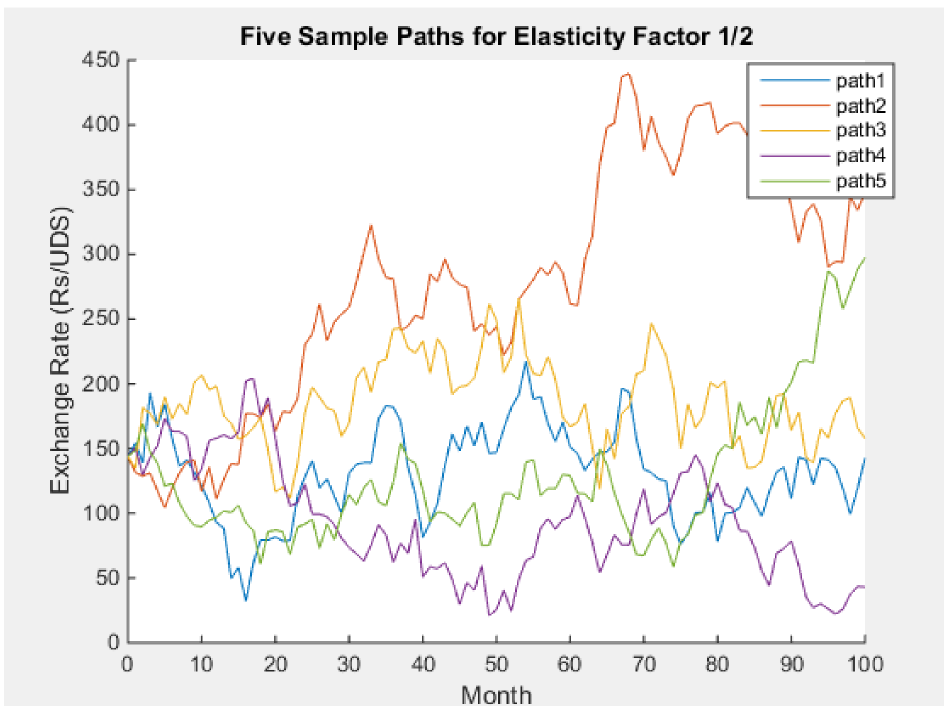

Figure 3: Five Sample Paths for the CEV Model $dX(t) = 0.0435X(t)dt + 19.1334\left(X(t)\right)^{1/4}dW(t)$

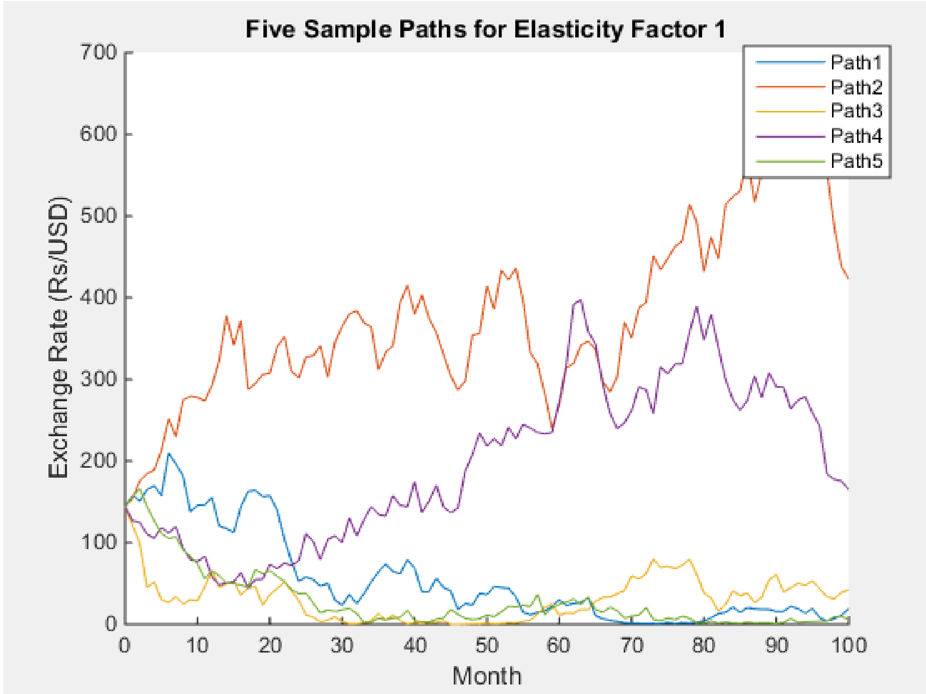

Figure 4: Five Sample Paths for the CEV Model $dX(t) = 0.0456X(t)dt + 5.9481(X(t))^{1/2}dW(t)$

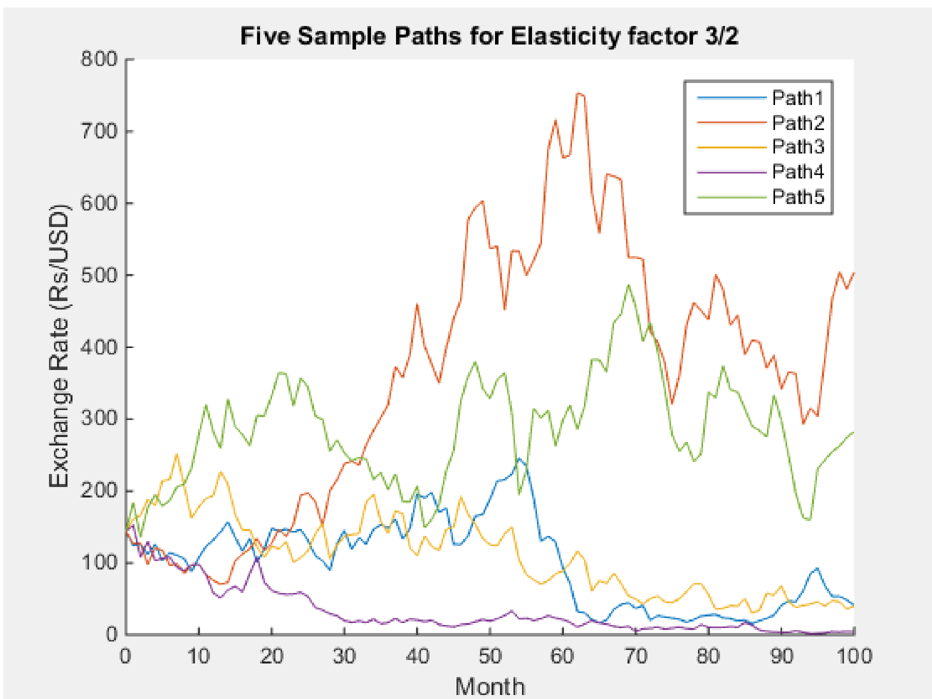

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Figure 5: Five Sample Paths for the CEV Model $dX(t) = 0.0481X(t)dt + 1.8568(X(t))^{3/4}dW(t)$

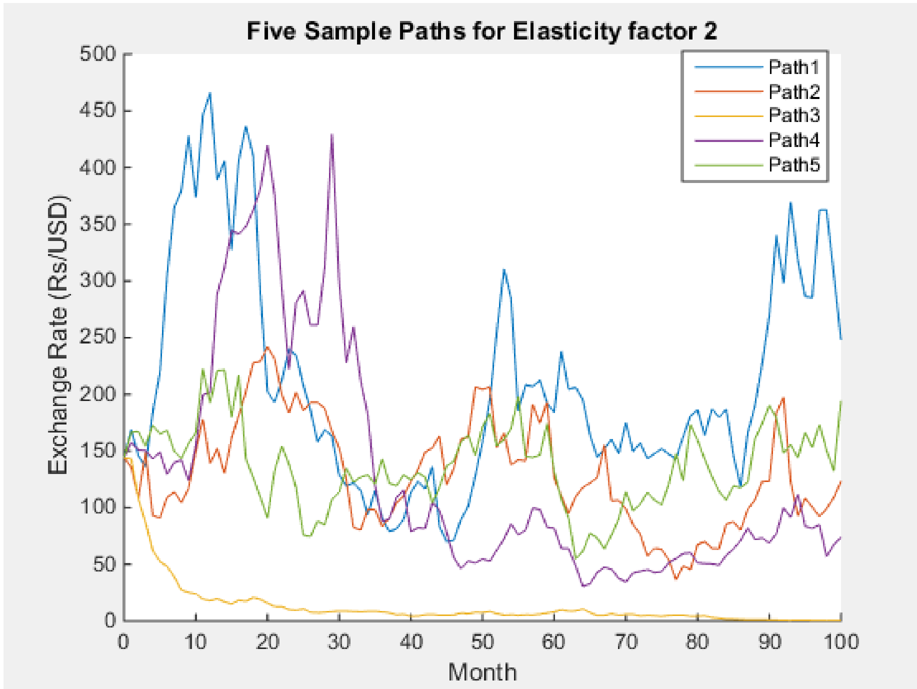

Figure 6: Five Sample Paths for the CEV Model $dX(t) = 0.0508X(t)dt + 0.5822X(t)dW(t)$

In this study, LKR/USD monthly exchange rates for nine months are forecasted by considering the initial value as 143.9594 Rs/$ which is the exchange rate of March, 2016.

The generated monthly exchange rates of a time point is different to one sample path to another. Hence by generating one sample path, a fixed value for the exchange rate cannot be obtained. Because of that, Monte- Carlo simulation is used to obtain the convergent monthly exchange rate. Figure 7 to Figure 11 illustrate the convergence of exchange rate when the large number of sample paths are generated.

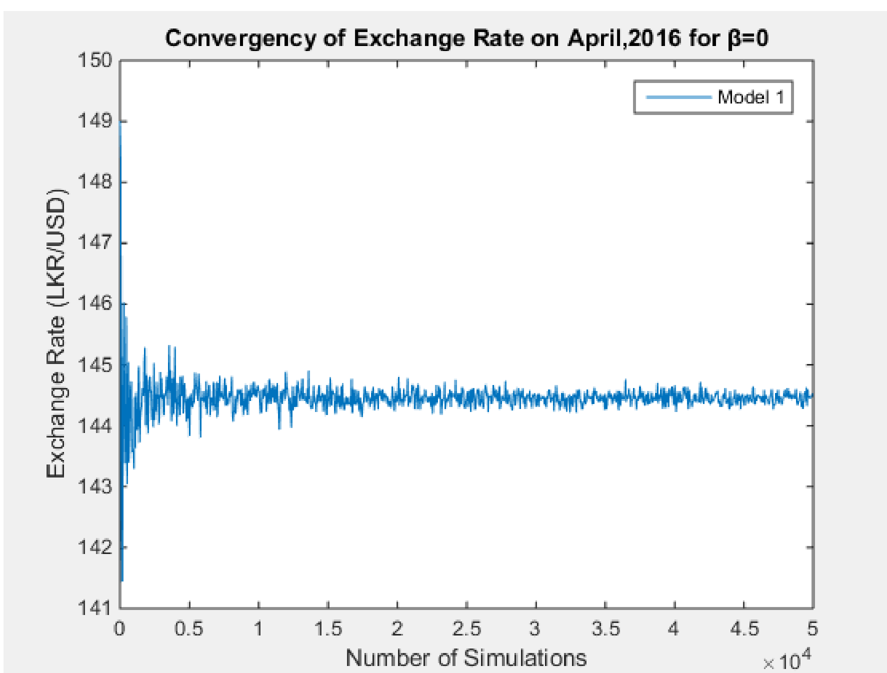

Figure 7: Convergence of Exchange Rate on April, 2016 for the model $dX(t) = 0.0456X(t)dt + 5.9481\left(X(t)\right)^{1/2}dW(t)$

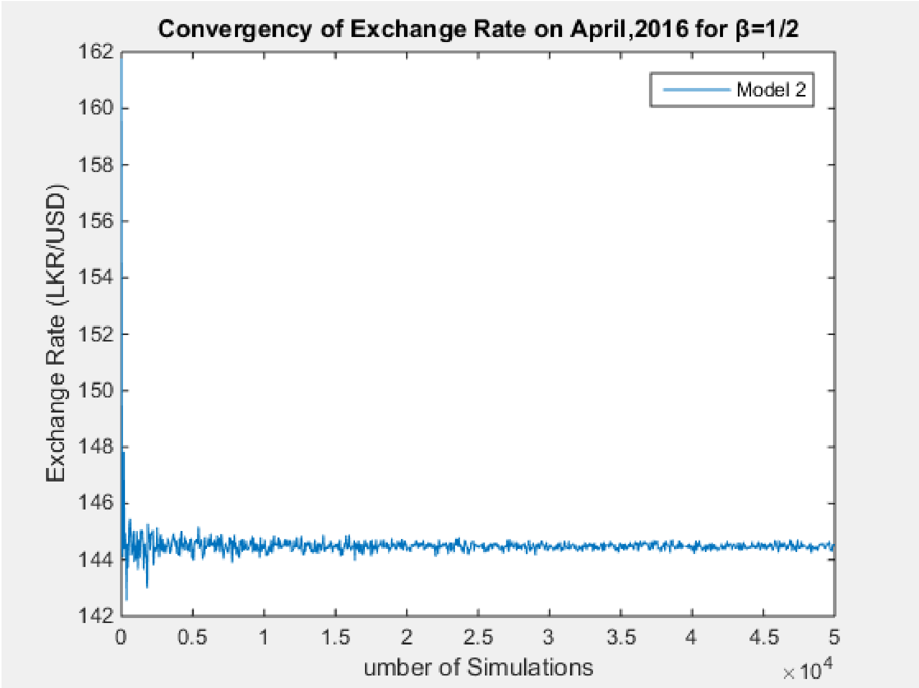

Figure 8: Convergence of Exchange Rate on April, 2016 for the model $dX(t) = 0.0435X(t)dt + 19.1334\big(X(t)\big)^{1/4}dW(t)$

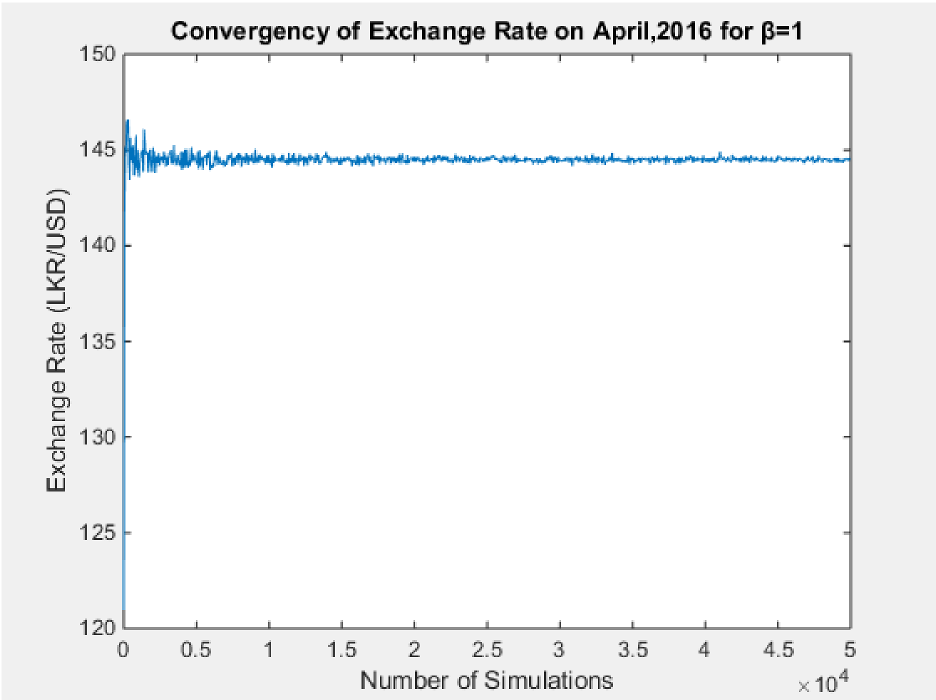

Figure 9: Convergence of Exchange Rate on April, 2016 for the model $dX(t) = 0.0456X(t)dt + 5.9481\big(X(t)\big)^{1/2}dW(t)$

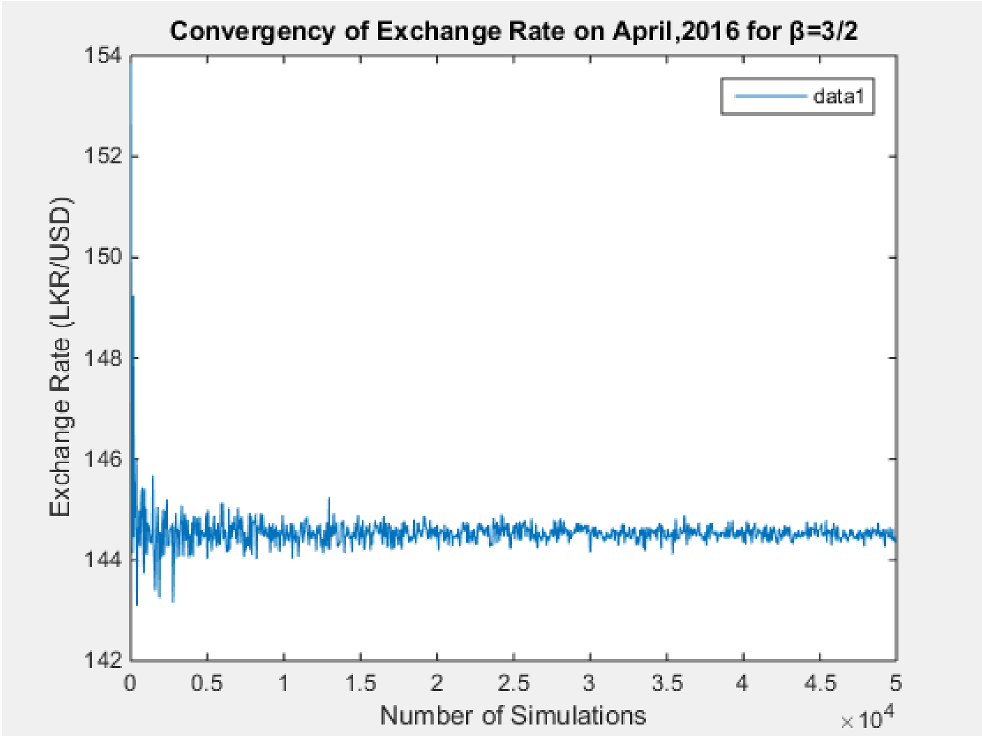

Figure 10: Convergence of Exchange Rate on April, 2016 for the model $dX(t) = 0.0481X(t)dt + 1.8568\big(X(t)\big)^{3/4}dW(t)$

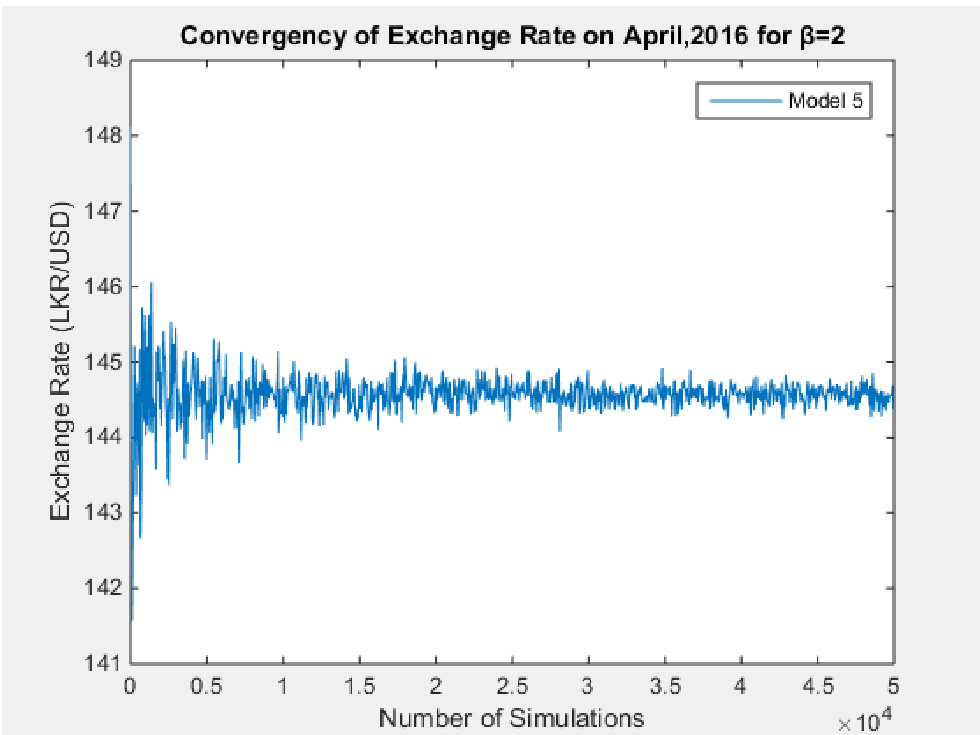

Figure 11: Convergence of Exchange Rate on April, 2016 for the model $dX(t) = 0.0508X(t)dt + 0.5822X(t)dW(t$

By generating 200000 sample paths and taking the average of each point forecasted exchange rates are obtained. Table II represents the actual exchange rates and forecasted values for each model from April, 2016 to December, 2016.

Table II: Actual and Forecasted LKR/USD Exchange Rates from April, 2016 to December, 2016

Month	Actual	$\beta = 0$	$\beta = 1/2$	$\beta = 1$	$\beta = 3/2$	$\beta = 2$
03/2016	143.9594	143.9594	143.9594	143.9594	143.9594	143.9594
04/2016	143.9001	144.4051	144.5337	144.5532	144.4990	144.4974
05/2016	145.6502	144.8608	145.0134	145.0565	145.0581	145.0748
06/2016	145.2836	145.3562	145.5455	145.5918	145.6683	145.7518
07/2016	145.4070	145.8443	146.0579	146.1818	146.2689	146.3140
08/2016	145.6010	146.3502	146.5258	146.7334	146.8723	146.8614
09/2016	145.7849	146.8095	147.0654	147.2083	147.4450	147.3881
10/2016	146.8723	147.3390	147.6002	147.7664	148.0006	148.0006
11/2016	147.7710	147.9532	148.1652	148.2576	148.6318	148.6318
12/2016	148.8820	148.5570	148.7373	148.8140	149.1268	149.2442

Graphically, The Actual and forecasted exchange rates for each model can be compared as follows:

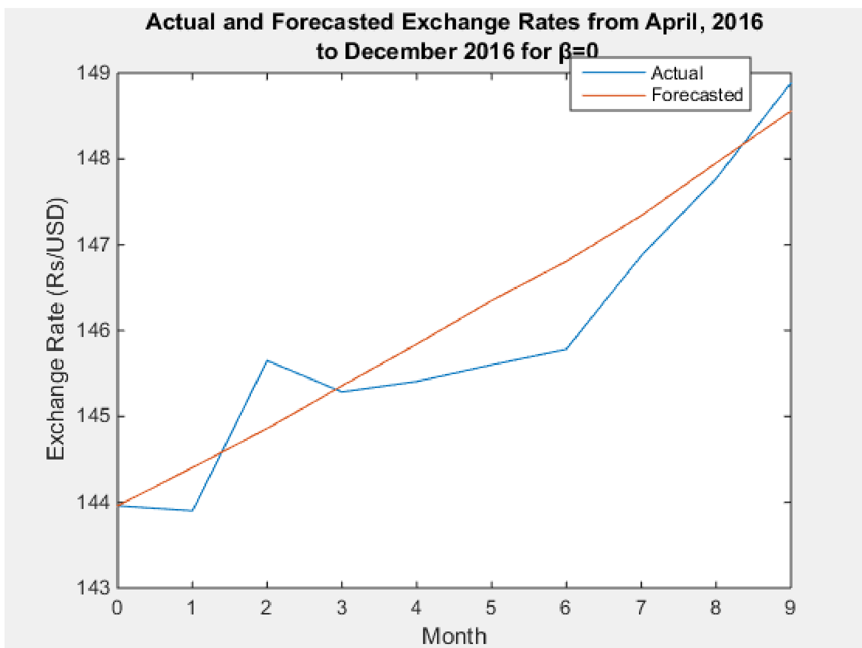

Figure 12: Actual Rates and Forecasted Rates for the CEV Model $dX(t) = 0.0417X(t)dt + 61.7786dW(t)$

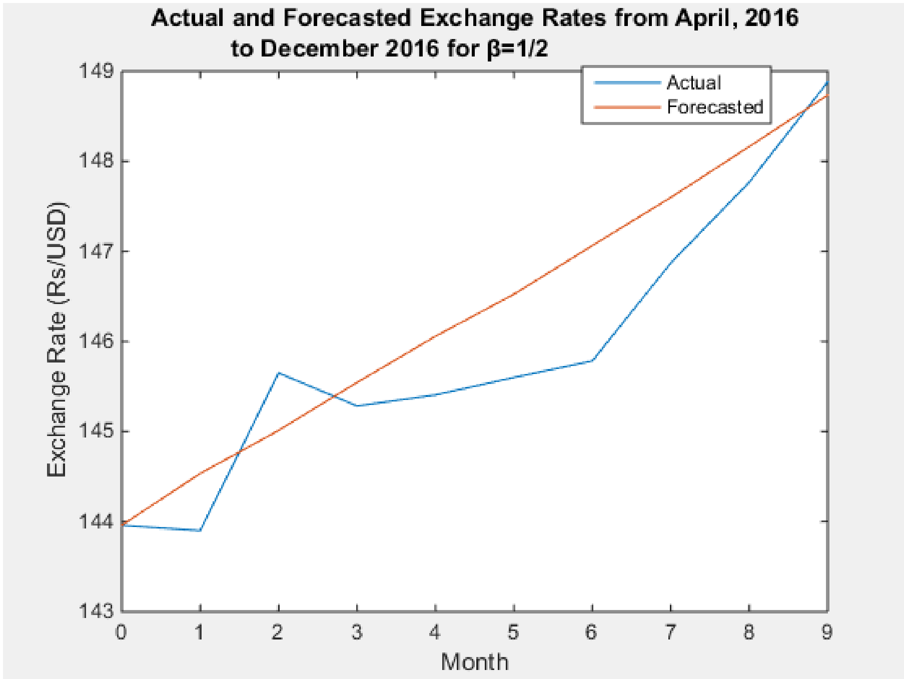

Figure 13: Actual Rates and Forecasted Rates for the CEV Model

$$dX(t) = 0.0435X(t)dt + 19.1334\left(X(t)\right)^{1/4}dW(t)$$

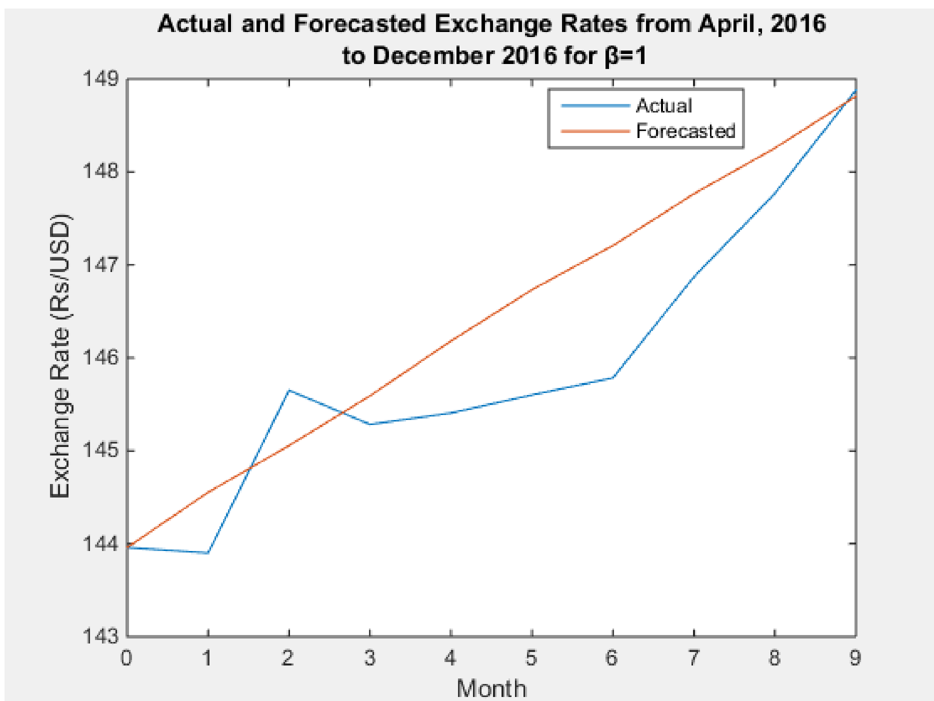

Figure 14: Actual Rates and Forecasted Rates for the CEV Model $dX(t) = 0.0456X(t)dt + 5.9481\left(X(t)\right)^{1/2}dW(t)$

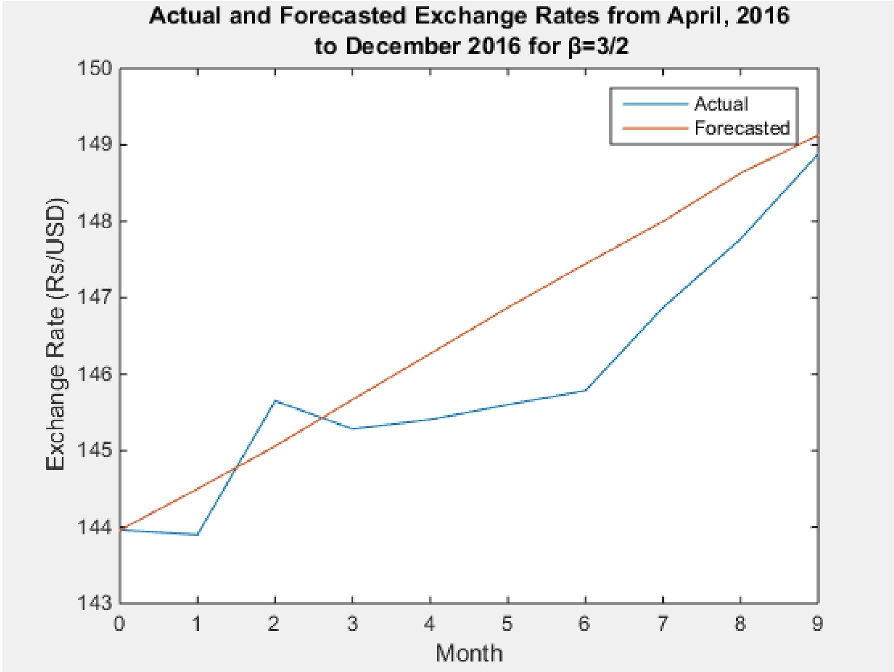

Figure 15: Actual Rates and Forecasted Rates for the CEV Model $dX(t) = 0.0481X(t)dt + 1.8568\left(X(t)\right)^{3/4}dW(t)$

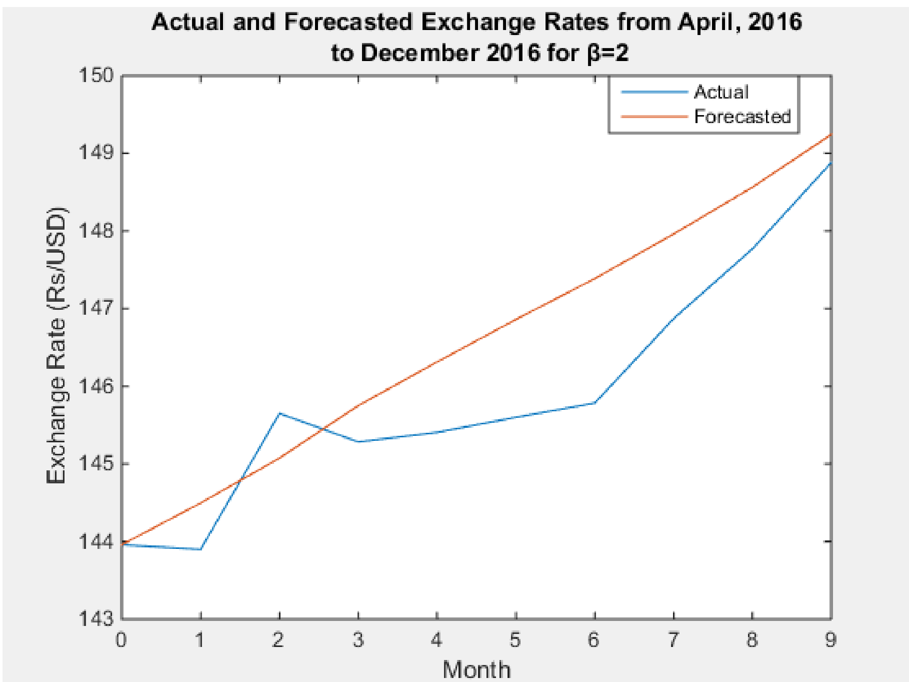

Figure 16: Actual Rates and Forecasted Rates for the CEV Model $dX(t) = 0.0508X(t)dt + 0.5822X(t)dW(t)$

Among these five CEV models, the best model can be selected by checking forecasting accuracy measures RMSE and MAPE. Table III represented RMSE and MAPE values for each model.

Table III: RMSE and MAPE Values for Five CEV Models

β	RMSE	MAPE
0	0.5516	0.3113
1/2	0.6709	0.3862
1	0.7631	0.4321
3/2	0.8970	0.5173
2	0.8853	0.5210

According to the Table 3, the minimum RMSE and MAPE values can be observed when $\beta = 0$. Therefore it can be concluded that the best CEV model to forecast LKR/USD exchange rate among the five models is the model which has elasticity factor 0.
Hence, the best CEV model is,

$$dX(t) = 0.0417X(t)dt + 61.7786dW(t)$$

It can be observed that the model is similar to the Ornstein- Uhlenbeck model.

IV. CONCLUSION

According to the results and findings, it can be concluded that the best model to predict LKR/USD monthly exchange rates from April, 2016 to December, 2016 is Ornstein-Uhlenbeck model. The best model may be changed to another model if one consider another data set. Because of that, by updating the exchange rates one can predict any future exchange rate using these models and the best model may be different to Ornstein-Uhlenbeck model.

In this research only five elasticity factors were considered. As a future research work, one can develop this study by considering more than five elasticity factors. Since β is continuous, if one can simulate results for more elasticity factors exchange rates can be predicted more accurately.

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