A Review on Effect of Some important Parameters on the bending Strength and Surface Durability of Gears

Vikash Chauhan

Department of Mechanical Engineering, ISM Dhanbad, Dhanbad – 826004, Jharkhand

Abstract- Calculation of bending and pitting strength of gear drive is one of the important concerns towards the efficiency improvement program in modern industries. Some preliminary experiments uses calculation methods based on uniform load distribution model on the gear teeth.The results obtained by such experiments are not parallel to recent estimations based on the non-uniform load distribution on gear teeth. Due to different mesh stiffness on different load points, non-uniform load distribution is the actual loading condition. American Gear Manufacturers Association(AGMA) and ISO standards assumes uniform model of load distribution which is not accurate. In order to create coherence among the results obtained by AGMA and ISO, a number of modifications factors have been applied in derived mathematical relations for gears. Latest methods such as minimum elastic potential energy method have been introduced to overcome drawbacks of these mathematical relations.This review article highlights a number of such methods in chronological order so that reader may obtain a overview of contribution of various scientists in development of gears having least bending and contact stress.

Index Terms- addendum modification, bending stress, contact ratio, contact stress, Hertzian model, rim thickness

I. Introduction

A gear is a mechanical rotating toothed part which meshes with other similar toothed parts to transmit torque and power. Two or more gears working in mutual arrangement is called transmission and produce mechanical advantage.

Speed, torque and direction of mechanical components are changed by the operation of gear system. This variation is very useful in the working of modern manufacturing system. The modern power transmission industry requires highly efficient gears so as to own high strength, longer life and less prone to failure. To achieve this, it is vital to reduce unnecessary stresses on gears which are responsible for weakness of the gear teeth. The study of stresses and methods to reduce stress in order to improve performance of gears is an important aspect of modern machinery design processes. Because of its influence not only on the operating cost and operating lives but also on the environment associated with failure of teeth [1], it can't be neglected. Much advancement has been achieved in the investigation of various types of stresses and methods to reduce them in the past centuries. The study of gear tooth stresses and associated strain and deformation is particularly useful in: (i) predicting the critical values at which gear teeth are subjected to failure. (ii) calculating various gear parameters associated with critical stress and working stress so as to obtain optimum design with maximum efficiency. (iii) understanding the behavior of various materials used in gear manufacturing such that it can be used to manufacture various mechanical parts. When the load applied on the gear transmission crosses threshold value, as a consequence of repeated stress crossing maximum stress limit due to defected structure of gear transmission, inappropriate tooth contact between two gears, stress concentration around a fillet and other factors, usually a fatigue fracture of gear teeth takes place. Power transmission is discontinued and the link between driven and driving gear is broken when the gear teeth fail. The tooth fillet is the location where the fracture tends to start. Thus, tooth root stress is the dominant factor to evaluate the tooth strength of involute profile gear with asymmetric [2] and symmetric gear [3] with static and dynamic loading conditions [4]. A considerable number of methodologies have been adopted in the past few decades to investigate the stresses in spur gear teeth. These methodologies can be grouped under three broad classes, namely analytical, experimental and finite element methods [5]. Analytical methods involve bi-potential equations, and using Airy stress function [6], finite prism method [7], or Muskhelishvili method applied to circular elastic rings [8]. Experimental methods involve using electric resistance wire strain gauges [9, 10], or utilizing photo-elastic gauges [10, 11], digital photo-elastic system with real time imaging process [12]. Numerical methods [10, 13] involve the use of FEM based simulation [14]. Performing analytical analysis, experimental work and comparing these with numerical work with finite element methods validates the accuracy of analysis result. Theoretical analysis and numerical analysis are less laborious and less expensive, thus presently, often experimental work is avoided. Analytical and numerical work usually provides required accurate result. It is common observation that gear geometrical parameters, various manufacturing methods and dimensional exactness affect the strength of gear directly. Change in these factors may be the cause of gear tooth failure. Therefore, utmost care is taken by the designer for the consideration of these factors while special purpose gear is to be designed. The geometry and dimensional accuracy of gears depends on various gear design parameters, machine tools, processes, etc. [15]. International standards like AGMA and ISO are introduced to evaluate the values of bending and contact pitting stress on the gear teeth. The relations provided by both standards are based on the Lewis equation for the calculation of bending stress and Hertzian contact stress theory for the calculation of tooth surface contact stress respectively. Both approach calculates only approximate values and are based on various approximations [16]. They are not sufficient to calculate stresses accurately due to various assumptions. In order to avoid approximations and to obtain accuracy of results, a number of researchers have presented their models based on the correction of geometrical parameters or application of some traditional mathematical or scientific principle. This was necessary because of increasing the chances of failure of gear teeth due to increasing demand of load capacity and speed of gear with the advancement of Industrial development. Traditional methods of stress calculation in spur and helical gear teeth for the design proposed in literature assume uniform load distribution based on the Navier's equation and Hertz equation [17] which is not suitable, because it depends on the meshing stiffness [18] which is different at different points along the load distribution line, so it can't be considered as uniform. Although several correction factors are applied to the basic formula, yet, they are unable to bring proper accuracy. A non-uniform model of load distribution along the line of contact [19] developed in the last decade is the brilliant step toward accuracy of stresses in spur and helical gear teeth. This model is based on the hypothesis of minimum elastic potential energy [20]. The elastic potential energy of meshing teeth is calculated and expressed as load point function. Variational technique is used to minimize the total elastic potential energy along with restriction of total load equal to the sum of load at each gear tooth. The same approach was adopted both for spur and helical gear with some modification. The approximate analytical equation is developed by Pedrero et al. [21] for inverse unitary potential which exclusively depends on the transverse contact ratio. This approach provides a much accurate result than other traditional models. Similarly, other methodologies like addendum modification of gear teeth [22], change in fillet radius [23], reducing the possibility of vacuum gearing [24], reducing friction to avoid wear on the teeth and pitting, etc. are adopted by various researchers.

The basic aim of this review paper is to introduce the research work associated with investigation of critical bending stress and contact stress of gear teeth with different approach adopted in the last decades and to review the methods to mitigate the effect of these stresses in order to improve performance of gear.

II. EVALUATION OF TOOTH BENDING AND SURFACE CONTACT STRESSES IN GEAR

So many methods are adopted by various researchers in order to estimate tooth root stress and surface contact stress in gear drive. These methods are associated with analytical, experimental or numerical analysis. The analytical method which uses some mathematical relation related to geometry of gear or some other physical quantity associated with the gear, basically requires some innovative and rigorous mathematical practices besides good knowledge of scientific principles related to gear geometry and various parameters related to its working principles. Experimental method needs a model of gear setup with proper geometrical configuration and specific scientific instruments which are capable to measure stress, strain, deflection, vibration up to desired accuracy level. Numerical analysis requires modeling of considered gear sample on any CAD software like AUTOCAD, SOLIDWORKS, PRO-E, etc. Then it is transferred to FEM software like ANSYS, COMSOL, etc. Various load, stress and vibration analysis are performed using FEM.

Timoshenko et al. [25] put considerable effort to investigate the strength in gear tooth. It was discovered by Tuplin [26] that number of stress cycles resulting in failure of gear under particular stress depends on the rate of stress repetition with respect to time. Pitch error and profile error cause actual error much greater than the nominal error ,due to which failure of high speed gear takes place at much lower stress than the fatigue limit. In order to analyze this situation, spring-mass model of a gear pair in contact was developed. Individual stiffness at each contact point was calculated and then equivalent stiffness was obtained. By considering various types of error, dynamic load on each gear tooth was calculated. The most significant work regarding the analysis of gear in mesh was performed by Harris [27]. The elementary task of Harris was called Harris map. But the Harris map was valid only for normal contact ratio (from 1.2 to 1.6). It was not enough for dealing with high contact ratio gears. Wellauer et al. [28] correlated cantilever plate theory to long gear teeth and performed some experimental work for analysis of stress and deflection of gear teeth of practical configuration and dimension. A semi-empirical relation was developed for finite cantilever plate under transverse load on the surface of the gear tooth. The solution was based on the principle of superposition. The proposed moment-image method was highly favorable with the result obtained from strain-gage analysis of a cantilever plate simulating the gear tooth. The influence of line load and load intensity variation with the load distribution in the tooth root was also investigated. Dynamic factor for spur and helical gear was investigated by Seireg and Houser [29]. Variation of gear mesh stiffness in critical stress and effect of error in the case of spur and helical gear was investigated. Dynamic load on the tooth and torque at the shaft attached with the gears was calculated. Four sets of gears were taken for this analysis. Semi-empirical formula was developed for dynamic tooth load. This formula considers dynamic characteristic of the system, manufacturing parameter and gear geometry. Wilcox and Coleman [30] explained the analytical method of finite element analysis to analyze the gear tooth stress. The details necessary for simulation of two dimensional gear tooth shape with finite element analysis were outlined and a technique for determining stress value at the tooth surface in the root fillet was given. Stress data obtained were used to develop a new simplified stress formula that provided tensile fillet stress as a function of the geometric tooth shape and general loading condition. Wallace and Seireg [31] presented a technical paper which main focus was on simulation of dynamic stress, deformation and fracture of gear teeth with the help of computer programming. Chabert et al. [32] Introduced a journal in which stresses induced by static load applied to gear tooth was evaluated. For gears of different gear ratios with 20 degree pressure angle and standard addendum proportion, the stresses and deflection were computed by finite element method. The formula was derived allowing simple calculation of maximum stress and the result was compared with that of AGMA and ISO standards. Cornell and Westervelt [33] provided a solution for dynamic model of spur gear system which was valid for all contact ratios. The dynamic response of the gear transmission system and the tooth load with stress was obtained in this analysis. This dynamic model assumed two gears as a rigid inertia and the teeth were assumed as variable springs of a dynamic system which were excited by the meshing action of the gear teeth. The influence of various parameters like nonlinearity of the tooth pair stiffness [34] during mashing, the tooth errors and the tooth profile modifications were included in this study. It was concluded that damping and system inertia, tooth profile modification and critical speeds of the system affect the dynamic gear tooth load and stresses substantially. Two researchers Ramamurti and Gupta [35] used FEM to study tooth stresses in spur gear. Plane stress condition along with fixed boundary was adopted for this analysis. The demerit of this model was the exclusion of consideration of adjacent teeth which is not appropriate. So this model required further correction.

It was found that if gear is manufactured by joining the gear teeth with the disk through welding, the stress produced in this condition and that of the gear manufactured by other processes like gear hobbing shows quite different stress distributional behavior. This is due to the difference in the amount and location of stress concentration. Sayama et al. [36] analyzed the root stress and bending fatigue strength of welded gear. The welding parameters dramatically influenced the gear parameters and stress components.

To determine dynamic load in spur gear, an extended model was developed by Kasuba and Evans [37]. In this analysis, a large scale digitized approach was used for static and dynamic analysis of the spur gear. Variable mesh stiffness was calculated directly as a function of gear profile error, gear hub torsional deformation, transmitted load, gear tooth deflection, and position of contacting profiles points. This method could be used for both normal as well as high contact ratio gears. It was concluded that simulated sinusoidal profile errors and pitting interrupts the normal gear mesh stiffness function and so, increases the dynamic loading. In order to determine the deflection curve of a 14.5 degree, 1 diametral pitch standard spur gear tooth corresponding to number of gear teeth 14, 21, 26 and 34 for the midpoint, pitch point and tip loading was analyzed by Muthukumar and Raghavan [38]. It was found that for every situation, deflection of the tooth at the loading point obtained by finite element method was similar to that obtained by experimental method. In the finite element method, 8 node, 4 sided isoparametric elements were used. In the case of pitch point and mid-point loading, the point of contraflexure in the deflection curve at the point of loading was investigated correctly by the finite element method. Previous mathematical models could not predict the point of contraflexure correctly. The deflection produced at the point of loading calculated in the case of Timoshenko and Baud tapered cantilever beam model was in good agreement with the finite element model in the case of tip and mid- point loading, whereas for pitch point loading, error in deflection was found 17-19%.

The paper presented by Fukunaga [39] explained the strength analysis of spur gear considering contact ratio. This analysis was based on the root stress measurement and finite element method. Spur gear used in this analysis was having module of 6.0, number of teeth 18, face width 10 mm and a boss with a total length of 45 mm. Finite element analysis was carried out in Two-Dimension and Three-Dimension. The deflection was analyzed for single tooth, considering contact ratio. The most important conclusions of this analysis were : (i) if gear possesses a boss, then the stiffness of gear becomes larger but tooth deflection tends to be smaller. (ii) the top clearance does not have any significant effect on the load distribution factor and (iii) even if the deflection of the teeth is clearly known, the load distribution factor cannot be known precisely.

A research paper authored by Simon [40] was published in which the evaluation of stress and load distribution along the line of contact for spur and helical gear teeth was presented. The basic theme of calculation in this paper was tooth profile modification, crowning, manufacturing and alignment error of gear, local contact deformation of teeth, tooth deflection, gear hub bending and deformation and deflection of attached shaft. With the help of regression analysis, relations to evaluate the load and stress distribution factor were derived.

A method to find a solution for dynamic analysis of elastic contact problem with rigid body motion with small deformation was presented by Huh and Kwak [41]. The surface of contact was assumed to be frictionless and unbounded. A variational formulation with constraint for geometric compatibility for contact surface was used for dynamics and its equivalence with equation of equilibrium was presented. For numerical approximation, FEM was used. Integral form of dynamic response included function in the time variable. Approximation of displacement was done by admissible function and during contact, discontinuity of velocity was also considered. With this method, a form of the linear complementary problem formulation was obtained on which numerical method could be applied. Another research paper authored by Kwak [42] was published in which he formulated general three dimensional frictional contact problem. This complementary problem in three dimensions was of nonlinear nature and quite different from two dimensional approach in which equation obtained was of linear type. This problem was solved numerically with the implementation of Newton approach or by the polyhedral law of friction [43].

Refaat and Meguid [44] adopted the technique of variational inequalities for the analysis of contact stresses in spur gear pair. They solved the variational inequalities [45] for general contact stress problem taking into account the effect of interfacial friction. An algorithm employing Lagrange's multiplier [46] and quadratic programming was implemented to solve the variational inequalities and hence contact and tooth root stresses in gear was obtained. This method eliminated the need of most of approximation and difficulties existing in the traditional methodologies. The displacement and stress at the contact surface are usually constraint in such a manner that these can be represented in form of inequalities.We can formulate the problem of contact between two or more than two bodies in the form of inequalities with the condition that one body is rigid and fixed with respect to the chosen frame of reference. Such type of problem is called Signorini problem.

Representing the Signorini problem in term of displacement vector u everywhere applying the variational inequality, following inequation is obtained:

$$B(u, w-u) + j(u, w) - j(u, u) \geq f(w-u) \quad \textbf{(1)}$$

for all w

Subjected to following boundary and contact conditions:

$$u = w = 0 \quad on \quad \Gamma_d$$

and $N.w - g \le 0$ on Γ_c

With the gap g *given by* : $g = N_3(\psi - \phi)$ **(2)**

where

$$B(u,v) = \oint \sigma(u)_{ij}\,\varepsilon_{ij}(v)\,dxdydz$$
$$J(u,v) = \oint -\mu\sigma_n(u)|v_1|ds$$
$$f(v) = \int f_i\,v_i\,dxdydz + \int t_i\,v_i\,ds \quad \textbf{(3)}$$

Here w is a virtual displacement vector, N is the normal vector represented as $[N_1\ N_2\ N_3]^T$ with the expression for N_1, N_2 *and* N_3 *as*:

$$N_1 = \frac{\frac{\partial\psi}{\partial x}}{\sqrt{1+\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2}}, \quad N_2 = \frac{\frac{\partial\psi}{\partial y}}{\sqrt{1+\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2}}$$

$$N_3 = \frac{-1}{\sqrt{1+\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2}},$$

ψ *and* ϕ is the function representing contact stress. σ_{ij} *and* ε_{ij} are the stress tensor and strain tensor respectively. Γ_c *and* Γ_d are the actual contact surface and boundary with prescribed displacement field respectively. μ is the coefficient of friction. v is a displacement vector.

The result for contact stress and tooth root stress obtained by this method was sufficiently accurate and could be used for the design of much reliable gears for the gear transmission system.

Static contact stress analysis of spur gear with the help of finite element method considering the effect of friction was studied by Vijayarangan and Ganesan [47]. Two dimensional FEM along with the Lagrangian multiplier technique was used to determine the contact stress. To investigate the friction effect on the mating gears, the static friction coefficient ranged between 0 to 3.0 were used. The actual length of contact was compared with calculated length of contact. The contact stress variation along the contact surface in a direction perpendicular to mating surface gave an idea about the depth of hardening required. In order to analyze the shear effect of involute gear teeth, Rayleign-Ritz method can be applied. This method was used by Yau et al. [48]. Tapered plate model subjected to concentrated load was implemented to simulate the gear tooth. Theoretical values associated with plate deflection along with the shear deformation were compared with the experimental values. It was found that deflection for the shear plate model was more than those for the thin plate model for which shear effect was neglected. The experimental result showed the highest values in comparison to shear plate model due to base flexibilities of experimental model. The results obtained for deflection in shear model were closely resembled with those obtained by finite element method [49]. Analysis of tooth surface contact and stress produced for the double circular arc helical gear was performed by Lu et al. [50]. Meshing and contact with gear drive were simulated by computer programming. Elastic deformation of teeth and surface mismatch causes position error and this position error was the basis of load sharing between the neighboring pair of teeth. The contact ratio for aligned and non-aligned gear drive and condition of load sharing was also investigated. The contact stress and elastic deformation of teeth was analyzed by finite element method. The contact stress, for which the basic theory was a Hertzian theory [51], was assumed to be distributed in elliptic section. The result obtained for the double-circular helical gear was compared with the result as obtained in the other technical paper. These results were used to plot various graphs to represent the effect of variation in parameters on the stress and deformation.

Due to static force exerted on the gear tooth, a flat section of it is affected by static stress. Filiz and Eyercioglu [52] studied the application of FEM to evaluate the static stress induced in the fillet of the gear tooth. This method was implemented by a computer program with which effects of fillet radius, contact ratio, pressure angle, module and number of driving and driven gear teeth were analyzed. The result so achieved was compared with that of previous results. The result gaving better accuracy was adopted.

A static transmission error produced in the gear teeth can be minimized if we try some modification in the gear tooth profile. The minimization of transmission error in the gear tooth lowers the dynamic tooth load in the meshing cycle which ultimately reduces the vibration and noise in the power transmission system in the gear. In order to verify this phenomena, a dynamic analysis of gear with involute profile and modified tooth profile using cubic splines was performed by Yoon and Rao [53]. Firstly, the deformation in the tooth was obtained and then dynamic load on the tooth was obtained at all allowable speed. In this parametric study, it was found that modified tooth profile reduces the transmission error effectively. Some other tooth profile based on the linear and parabolic tip relief was also used to investigate the effect on dynamic load and results were compared.

Finite element analysis is very effective tool for studying various physical phenomena occurring in the mechanical components such as power transmitting elements like gear. In fifth international congress on sound and vibration, a paper was presented by Sirichai et al. [54], in which application of finite element analysis of gear in mesh was performed. In this paper, torsional stiffness of two meshing gears was estimated in which one gear was restrained to rotate while other gears was given the torque input lead. It was found that variation of mess stiffness of gear takes place while the gear rotates. The resulting torsional stiffness fluctuates significantly as the meshing of teeth changes from single to double pair meshing and vice versa. This was represented as a function of mesh points in mesh cycle. The result so obtained showed the change in torsional stiffness when a double pair of tooth contact takes place.

While analyzing the transmission error in the gear, using meshing stiffness of gear teeth, whole body of the gear should be considered under investigation because during power transmission, whole along with the deflection in the gear teeth, gear hub also suffers slight deflection.Thus while performing finite element analysis, whole gear should be considered for investigation. For the analysis, model of gear is produced and discretized in four node plane strain quadratic Two-Dimensional elements. The contact point should be on the intersection point between the involute curve and pressure line. Equivalent gear tooth meshing stiffness is given by:

$$K_{eq} = f\,\frac{K_1 K_2}{K_1 + K_2} \quad \textbf{(4)}$$

where K_1 and K_2 are the meshing stiffness of individual gear teeth. f is the correction factor whose value lies between 1 and 33. This correction factor takes into account effect of friction, the number of actual nodes in contact, meshing size, etc.

While calculating the meshing stiffness, stiffness only at the point of contact is taken into consideration. Only two contact nodes at the point of contact are considered. By restraining the surrounding nodes and Appling vertical load on the contact point in both upward and downward direction, and calculating corresponding deflection, meshing stiffness can be easily calculated.

Three dimensional analyses of bevel and spur gear was performed by Ramamurti et al. [55] by FEM using the concept of cyclic symmetry. The calculation was performed for a gear tooth for each Fourier harmonic component of the contact line load. Then summation of all the components was done to obtain total displacement. Static stress was calculated with the help of this displacement. By using submatrices elimination scheme, natural frequency and mode shape was also obtained. The use of the cyclic symmetry of the Finite element method was demonstrated by this presentation. This method helped in reducing the computational effort and managing computer memory. Utilizing geometric periodicity methods and submatrices elimination scheme, dynamic analysis of the gear tooth was successfully performed.

Using the finite element method and elastic coupling between the gear teeth, a model was presented by Vedmar and Henriksson [56] to determine dynamic forces in spur gears taking into account off line of action and nonlinear wheel stiffness. A determination of contact point was done by knowing the common normal using the undeformed tooth shape. In order to prevent the contact singularity in offline of action points, the tooth was having a tip rounding. The effect of offline of action and elastic coupling was studied through the comparison of various models.

It is found that if the asymmetric gear drive is used in place of symmetric gear drive, the noise, contact and bending stresses reduce significantly. This was confirmed by Litvin et al. [57]. If the different pressure angle corresponds to the driving and driven side, such type of system is called an asymmetric gear system. In order to perform this action, driven and driving gear drive were chosen so that they had different tooth profile. The stress analyses were performed for three cases: (i) asymmetric spur gear having a larger pressure angle of 35^o. (ii) ssymmetric spur gear having a smaller pressure angle of 20^o, and (iii) symmetric gear teeth having pressure angle on pinion and gear both are 25^o. The driven member was having an involute tooth profile while driving member retained double crowned geometry. Due to the modification, better localization and stabilization in bearing contact and the reduced transmission was achieved. The model was created and simulated with the help of computer programming. A method of generation of double crowned gear was developed by the author. Simultaneously, the stress analysis of symmetric as well as un-symmetric gear was analyzed. Confirm reduction of tooth contact and bending stress was noticed.

Contact stress and tooth root bending stress calculation with machining error, assembly error and tooth modification by three dimensional finite element method was done by Li [58]. First of all, position of parallel shaft spur gear pair was defined along with machining error, assembly error and tooth modification in three-dimensional coordinate system. Then, tooth contact was assumed on a reference face around the geometrical contact line. In this face, contact model, first of all reference points on the reference face of first gear was assumed, then the effect of errors and tooth modification was applied to find a responsive contact point in the face of other gear. With the help of three-dimensional FEM, the deformation influence coefficient was calculated and by the mathematical programming, loaded tooth contact analysis was performed. The tooth contact length of a gear pair with lead crowning was evaluated and compared with the measured value. Root strain and tooth contact pattern along with addendum modification was obtained and then compare with the results obtained by experiments. Calculated values and measures values were in equivalence to each other. It was also seen that error and addendum modification affect tooth root stress and contact stress significantly.

To analyze the stress generated in the meshing gear teeth, there are two ways in finite element methods. First one is directly applying the concentrated load at the loading point and calculating bending stress generated in the spur gear. The demerit of this method is that the contact stress can't be calculated from it. The second method is applying torque on the modeled meshing gear. By using this method for two-dimensional finite element model, contact stress on the mating gear can be evaluated. Such type of contact stress analysis for meshing gear during rotation was performed by Hwang et al. [59]. Variation of contact stress at different contact position was investigated. Comparison was made between variation of contact stress during rotation and contact stress at the lowest point of single tooth contact (LPSTC) and the AGMA equation for contact stress. The conclusion of this analysis was that the design that considers contact stress is more severe than the AGMA standard. The values obtained by finite element method were lower than contact fatigue strength of material. Thus, they ensured appropriate safety and strength.

III. EFFECT OF GEAR PARAMETERS ON THE STRENGTH AND PERFORMANCE OF GEARS

(1) ***Effect of contact ratio****:* Load sharing in high contact ratio gears is between two or three gear teeth during meshing. That's why load per tooth for such gear pair is less than a low contact ratio gears. Thus, the less dynamic load is exerted on them and the noise produced is less. High contact ratio can be obtained by the following methods: (i) by increasing addendum. (ii) by decreasing pressure angle and (iii) by larger pitch. It is observed that very high contact ratio gears have tendency to tooth sliding and have less bending strength. This can be improved by tooth profile modification [60]. The significant advantage of high contact ratio spur gear is less bending and tooth root stress, thus increased load carrying capacity [61]. High contact ratio gears can decrease bending stress by twenty percent and contact stress by thirty percent [62]. Higher power to weight ratio, longer service life and more reliability are some advantages of high contact ratio spur gear.

A research paper was published by Staph [63], in which a method for special computer program development was elaborated to design external spur gear with normal contact ration (< 2.0) and high contact ratio (≥ 2.0). Various effect of change in gear parameters on the performance of high contact ratio gear was analyzed by the computer programming. It was concluded from this research that high contact ratio gear obtained by changing the addendum of normal contact ratio gear suffers much less compressive and bending stress which is favorable outcome. On the contrary, doing so increases flash temperature and friction heat generation which is an unfavorable outcome. Elkholy [64] provided solution for the calculations of load sharing between the gear teeth in mesh having high contact ratio. In this analysis, the tooth deflection, profile modification and spacing error were assumed to be equal for all the tooth pairs in contact. Besides, the maximum normal load was assumed to be equal to the sum of normal loads taken by gear tooth pairs. Gear tooth fillet stress and contact stress were determined using tooth geometrical attributes after individual load was calculated. The results of experimental analysis were compared with analytical analysis. A research paper was published by Anderson and Loewenthal [65] in which power loss prediction method was extended for nonstandard gears. The effect of tooth thickness, modified addendum, gear center distance, etc. can be analyzed by this method. Gears with high contact ratio were also analyzed by this method. Using high contact ratio, highly efficient gears can be manufactured.

Computer simulation for dynamic response of high contact ratio spur gear transmission was performed by Lee et al. [66]. High contact ratio gears are sensitive to gear tooth profile error. Based on this fact, a number of profile modifications, under actual loading condition were examined. The dynamic load and dynamic stress under the effect of these profile modification were analyzed. It was found that combination of profile modification along with toque carried by gears affects the system dynamics significantly. One important noticeable thing was that high contact ratio gears require less profile modification than the normal contact ratio gears.

The effect of contact ratio on dynamic loading on spur gear without profile modification was analyzed by Liou et al. [67]. The way in which gear contact ratio influences the dynamic loading on spur gear transmission was simulated by computer. Various gear parameters, e.g. pressure angle, tooth addendum, diametral pitch, central distance, addendum, etc. affects the contact ratio a lot. By altering the length of addendum, the contact ratio was varied. The significant influence of the contact ratio on the gear dynamics was also noticed. It was investigated that a contact ratio of 2.0 reduces the dynamic load. The dynamic load minimized by high contact ratio gear is better than standard low contact ratio gear. Excessive profile modification affects high contact gears a lot as compared to that under modification.

Tirumurugan and Muthuveerappan [68] published a research paper in which they calculated maximum contact and fillet stress for normal and high contact ratio gear. Their research was based on load contact ratio implementing finite element method and was performed for single point load model, multipoint load model and multipoint contact model. The effect of various gear parameters such as pressure angle, teeth number, gear ratio, tooth size and addendum on the load sharing ratio and corresponding stress was investigated. Calculation of maximum fillet and contact stress in the case of normal contact ratio gear and high contact ratio gear using the load sharing ratio was performed.

(2) ***Effect of Addendum Modification****:* Addendum modification is the amount by which addendum of a gear is increased or decreased. In order to avoid interference in gear teeth, addendum modification is done. Undercutting at the root and keeping minimum number of teeth at some pressure angle are the methods to avoid interference. But these methods contribute toward weakening of the teeth. So addendum modification is done to avoid all these less advantageous methods. The addendum modified gear is also called profile shifted gear. By altering the dimension of gear tooth geometry, the load carrying capacity of gear can be greatly improved. Reduced vibration and reduced noise in gear operation can also be achieved by profile shifting. Modified amount of the addendum is equal to product of module (m) and a non-dimensional coefficient called as addendum modification coefficient(x). Analysis of effect of addendum modification on bending fatigue strength of cast iron and cast steel was performed by Oda et al. [69]. Effect of profile shift on the true stress at the fillet section along with most critical point and bending fatigue strength was analyzed in this study. Addendum modification factor was derived by analytical and experimental method. Bending fatigue strength of the addendum modified gear was calculated with high accuracy. The influence of profile shift on the contact ratio was also studied and contact ratio factor was obtained. Following conclusions were drawn from this study:

(i) As the addendum modification coefficient is increased, the root stress factor decreases for both highest points of single tooth loading and tooth tip loading.

(ii) If the proper amount of addendum modification is selected, the bending fatigue limit load of can be raised more than forty percent over standard gear.

(iii) If the addendum modification factor $(B = 1 + 0.5x)$ is applied to cast iron and cast steel, the bending fatigue strength of profile shifted gear can be estimated with very high accuracy.

(iv) If the addendum modification of gears is increased, the contact ratio of profile shifted gear decreases.

In a technical paper, authored by Oda and Koida [70], the surface durability and pitting failure of spur gear with a smaller number of teeth with different amount of addendum modification were investigated with the help of running tests. Ferritic nitrocarburized gear with a small number of teeth was used by Oda et al. Various tests were performed for different addendum modification and compared with the standard one. Calculated values of hertz stress and specific sliding of spur gear with different number of teeth and addendum modification co-efficient were compared with the experiment results. Effect of addendum on bending strength,contact strength and performance parameters of spur gears was studied by Li [71]. Mathematical programming model, Face contact teeth model and three-dimensional finite element method was used to perform loaded tooth contact analysis. Stress and deformation in the spur gear with various values of contact ratio and addendum height were calculated by applying these methods. It was noticed that contact stress at the tooth surface along the tooth profile was not symmetric around the contacts point at distant locations from the pitch point like tooth tip, tooth root, etc. Location of maximum contact stress was not the geometric contact point. Li observed that increasing addendum definitely reduces contact stress in gear tooth. In the case of bending stress, this is not surely true. This is due to the fact that the increase in addendum increases the tooth length also. This results in an increment of bending stress instead of decrement. So, in order to implement longer addendum with high contact ratio, calculation of the scoring strength of the root and tip of the gear is also necessary.

In high gear ratio and addendum modified gears, maximum stress on the gear and the pinion at the fillet section are unequal. Removing this maximum fillet stress, load carrying capacity of the gear can be increased. This was studied by Sekar and Muthuveerappan [72]. They concluded that designing the gear with uniform fillet stress with the replacement of unbalanced fillet stress, performance of the gear can be improved. Changing the tooth thickness of basic racks from standard tooth thickness for non-standard one, uniform fillet strength can be achieved. The effect of cutter tip radius, backup ratio, addendum modification factor in the maximum fillet stress was studied with the help of finite element method taking the various values of tooth thickness coefficient. The optimum value of the tooth thickness coefficient suitable for proper fillet strength was obtained. The effect of the module, gear teeth and addendum height on the load sharing and corresponding stress was analyzed by Marimuthu and Muthuveerappan [73]. In order to calculate stress due to applied load at highest pressure angle, they developed a multi-point contact model for finite element analysis. For the parametric study, they developed ANSYS parametric design language code. It was seen that in the application of load at the critical loading point, an increase in addendum height increases the bending stress. On the contrary, Increase in module and number of teeth favorably decreases the bending stress.

(3). *Effect of Gear Rim Thickness*: In the past decades, thin rim gears became popular due to their great load carrying capacity, light weight and high speed. Along with study stresses in solid gears, thin rim gears have also been paid attention a lot. Oda et al. [74] studied the effect of fillet stress on the strength of thin rimmed gear. The bending fatigue strength of thin rimmed gear was studied by investigating the fillet stress in gear tooth along with the stress on the adjacent teeth. It was found that the fillet stress on the thin rimed gear was quite different from that of the solid gear. This was most probably due to larger elastic deformation in the thin rim of the gear. The deformation in the thin rim was analyzed by finite element method in two-dimension taking triangular element. Result for a number of thin rimmed gears was obtained by finite element method. The obtained result was in good agreement with the measured one. The stress on the tooth root and the location of critical section was also obtained with the help of these investigations.

In the orthogonal coordinate system, the nodes of the element are assigned by the symbols i, j and k having the coordinates of the nodes as (x_i, y_i), (x_j, y_j), (x_k, y_k). The component of nodal force at node i in the x and y direction are X_i and Y_i. Correspondingly the components of nodal displacement at the node i in the x and y direction is U_i and V_i.

The displacement function can be represented in equation form as follows:

$$u = \alpha + \beta x + \gamma y \quad \textbf{(5)}$$

$$v = \delta + \varepsilon x + \epsilon y \quad \textbf{(6)}$$

where $\alpha, \beta, \gamma, \delta, \varepsilon, \epsilon$ are constants whose values are supposed to be evaluated.

The mesh pattern in the tooth of the gear and the nearest tooth is in the form of the triangular elements. The variation of tooth root compressive stresses with tangential angle θ for different values of rim thickness(R) is represented by graphical method. Rim thickness is represented in the form of module $m.$ It can be observed that the maximum stress on the compressive side takes place at the position of $\theta = 55\ degree.$

Usually, the tooth root stress is critical in the fillet zone of the gear, but when the rim thickness decreases, the position of critical root stress shifts from $\theta = 30\ degree$ to $\theta = 55\ degree.$ In this case the magnitude of critical stress also increases. As the value of rim thickness decreases, the magnitude of tooth root stress in the compressive side increases.

Chong et al. [75] presented a paper in which results for various loading conditions, rim thickness and support condition were obtained. SAP -IV finite element code was used to obtain results. It was found that the maximum stress exists on the tooth root surface and it increases as the rim thickness is decreased for the looser fitting hub. For tight fitting hub, tooth root stress decreases with a decrease in the rim thickness. It was also found that as the fillet radius decreases, correspondingly the tooth root stress also increases. This variation was found valid only for tooth surface stress. The internal tooth root stress was unaffected by the change in the parameters.

The effect of chordal tooth thickness at the critical location of tooth, tooth rim thickness, fillet radius and various conditions on the strength of thin rimmed gear was investigated by Chong and Kubo [76]. Mathematical formula was developed with the help of finite element method in order to calculate fillet stress and tooth root stress in thin rimmed spur gear. The result obtained by the empirical formulae and that obtained by finite element method showed good agreement with each other. In the same year, by using approximation method, tooth fillet and tooth root stress for an internal spur gear, fixed by bolt or supported by pinning coupling was calculated by Chong and Kubo [77]. This method was quite successful in tooth fillet and root stress calculation in all the teeth of internal spur gear.

A technical memorandum was prepared by Bibel et al. [78] about the effect of rim thickness on bending stress in spur gear. Assuming a rim thickness as significant design parameter, finite element method was applied to a particular segment of thin rim gear. Location and magnitude of maximum bending stress were calculated with the variation of rim thickness. The effect of rim thickness of spur gear on the bending stress was studied by Bibel et al. [79]. It was found that the bending stress generated in the tooth root and fillet section is different from that generated in the normal solid gears. This variation is due to the deformation produced in the rim. In order to study the effect of gear rim on these stresses, finite element analysis was performed on a segment of thin rim gear. Taking a different thickness of the rim, the numerical value and exact location of the generated stress was noted. The result obtained was compared with previous results and presented in tabulated form.

Gear tooth contact analysis of three-dimensional thin rimmed gear, combining the finite element method with the mathematical programming was performed by Li [80]. A face contact and the whole gear deformation model were used to analyze thin rimmed gear. Three dimensional finite element method was developed to study the thin rimed gear. Using FEM, three-dimensional load distribution, tooth root stress and strain was obtained. Results obtained was compared with the experimental one and found to be in good agreement.

Due to the rotational motion of gear, centrifugal force is exerted on it outwards. At the high speed, this force becomes dominant and affects the performance of gear. The effect of this centrifugal force on the contact strength and tooth bending strength for thin rim, gear spur gear rotating at high speed was analyzed by Li [81]. The three-dimensional finite element technique was implemented to study stresses in thin walled gear model. The centrifugal stress and the centrifugal deformation for the model with offset web were studied by finite element technique in the speed range of 5000-40000 RPM. It was concluded that the effect of centrifugal force can be neglected below the speed of 10000 RPM. However, when this speed becomes greater than critical speed, the centrifugal force becomes dominant. The variation in surface contact stress, tooth bending stress, load sharing ratio, and tooth contact pattern due to the effect of centrifugal load produced in thin-rim spur gear was also analyzed by Shuting Li. It was observed that at the high speed, the centrifugal load have dominating effect and consequently, it affects the various gear parameters significantly.

A research paper was published by Marunic [82] in which deformation in the middle web of thin rimmed involute spur gear in mesh with solid spur gear was analyzed. The deformation of the web was expressed in the form of displacement as non-dimensional form. For the selected face width, the effect of rim thickness and web thickness of deformation of gear was investigated. Besides this, the contribution of web deformation and rim deformation was separately analyzed and mutually compared. It was concluded that maximum web displacement variation with an increase in displacement is more for thicker rim when web thickness decreases and for thinner web when rim thickness decreases. Other phenomenan which was observed that, deformation in the rim takes place significantly as compared to the web. Rim thickness for the thinnest web and rim took approximately forty four percent of maximum gear displacement. Web displacement took about seventeen percent of whole gear displacement. The whole analysis was performed by three dimensional finite element method.

IV. CONCLUSION

Based on the finding and implementation of methods adopted so far by the various scientists for analyzing the two dominant stresses (surface contact stress and bending stress) generated in spur gear, it can be concluded that three methods namely, analytical methods, experimental methods and numerical methods using FEM are equally important. In a particular situation, one method may be dominant on the other method for accuracy of result. Thus, it is essential to apply as many methods as possible for analysis and validation of result by comparing to each other. AGMA and ISO gear standards assumes uniform model of tooth load distribution which is not appropriate. Non-uniform load distribution model gives much accurate results than uniform load distribution model. For analyzing bending stress in gear tooth, it is assumed as cantilever whereas for contact or pitting stress on gear teeth, Hertzian contact theory is used. Minimum elastic potential energy model with implementing non-uniform load distribution gives accurate results closely resembled with experimental result. Profile shifting, change in fillet radius, vacuum gearing reduction and avoiding teeth wear also reduce unnecessary stress on gear teeth. Various parameters such as tooth contact ratio, addendum modification, gear rim thickness, etc. affect nature of stress generated in gear body and gear teeth. While analyzing stress on one gear teeth, adjacent tooth also must be considered because its presence affects the nature and quantity of stress. For the dynamic analysis of gear, meshing teeth are assumed to have meshing stiffness. Dynamic force and stresses are evaluated with the help of measured deflection and meshing stiffness. Stress on gear tooth can be calculated by two methods available in FEM. First is directly applying concentrated load on single gear tooth. Another one is meshing of two gears in such a manner that their teeth are in mutual contact. Only bending stress can be calculated by the first method. From second method, both bending stress and contact stress are calculated. Increase in contact ratio decreases the bending stress on gear teeth. Thus, load carrying capacity increases. Addendum modification, decrease in pressure angle and adopting larger pitch are the methods for increase in contact ratio. Proper amount of profile shifting helps to decrease tooth stress and undercutting. The deflection of thin-rimmed gear is more than solid bulky gear. Thus stress pattern are quite different in both type of gears. Adopting proper thickness, stress in gear can be controlled. These parameters affect directly the performance and efficiency of gears. Thus, while designing gears it is strictly advisable to consider most of the parameters, so that strength and performance of the gear would be up to the mark.

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AUTHORS

First Author – Vikash Chauhan,Master of Technology,ISM Dhanbad,vikashchauhan2009@gmail.com